UNIT – 1. Lecture-1

BY

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Introduction to Circuit Theory

- Circuit theory is based on the concept of modeling. To analyze any complex physical system, we must be able to describe the system in terms of an idealized model that is an interconnection of idealized elements.
- By analyzing the circuit model, we can predict the behavior of the physical circuit and design better circuits.

Electric circuit & Network

- A circuit is a conducting path through which an electric current either flow or is intended to flow.
- A network is an interconnection between active & passive elements.

Linear circuit & Non-linear circuit

- A linear circuit is one whose parameters are constant i.e. they do not change with voltage or current.
- It is that circuit whose parameters change with voltage or current.

Lumped Circuits

- Lumped circuits are obtained by connecting lumped elements.
- Typical lumped elements are resistors, capacitors, inductors, and transformers.
- The size of lumped circuit is small compared to the wavelength of their normal frequency of operation.

Lumped Circuit definition

- A lumped circuit is by definition an interconnecting lumped element.
- The two terminal elements are called branches, the terminals of the elements are called nodes.
- The branch voltage and branch current are the basic variables of interest in circuit theory.

Lumped & Distributed elements

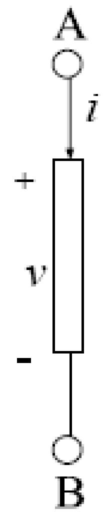
- Physically separate elements such as resistors, capacitors & inductors are known as lumped elements.
- Those elements which are not separable for analytical purpose.

Active & passive devices

- These are those elements in the network which gives out energy which then consumed by passive devices. ex-voltage source, current source.
- The elements which consumed the energy which is given by active devices. ex-R,L,C.

Reference Directions

- A two terminal lumped elements (branch) with nodes A and B.
- The reference directions for the branch voltage v and branch current i are shown in the graph.
- The reference direction is chosen arbitrarily.



Notational conventions

- Total quantities will be represented by lowercase letters with capital subscripts, such as v_T anf i_T.
- The dc components are represented by capital letters with capital subscripts as V_{DC} and I_{DC}; changes or variations from the dc value are represented by v_{ac} and i_{ac}.
- $v_T = V_{DC} + v_{ac}$
- $i_T = I_{DC} + i_{ac}$

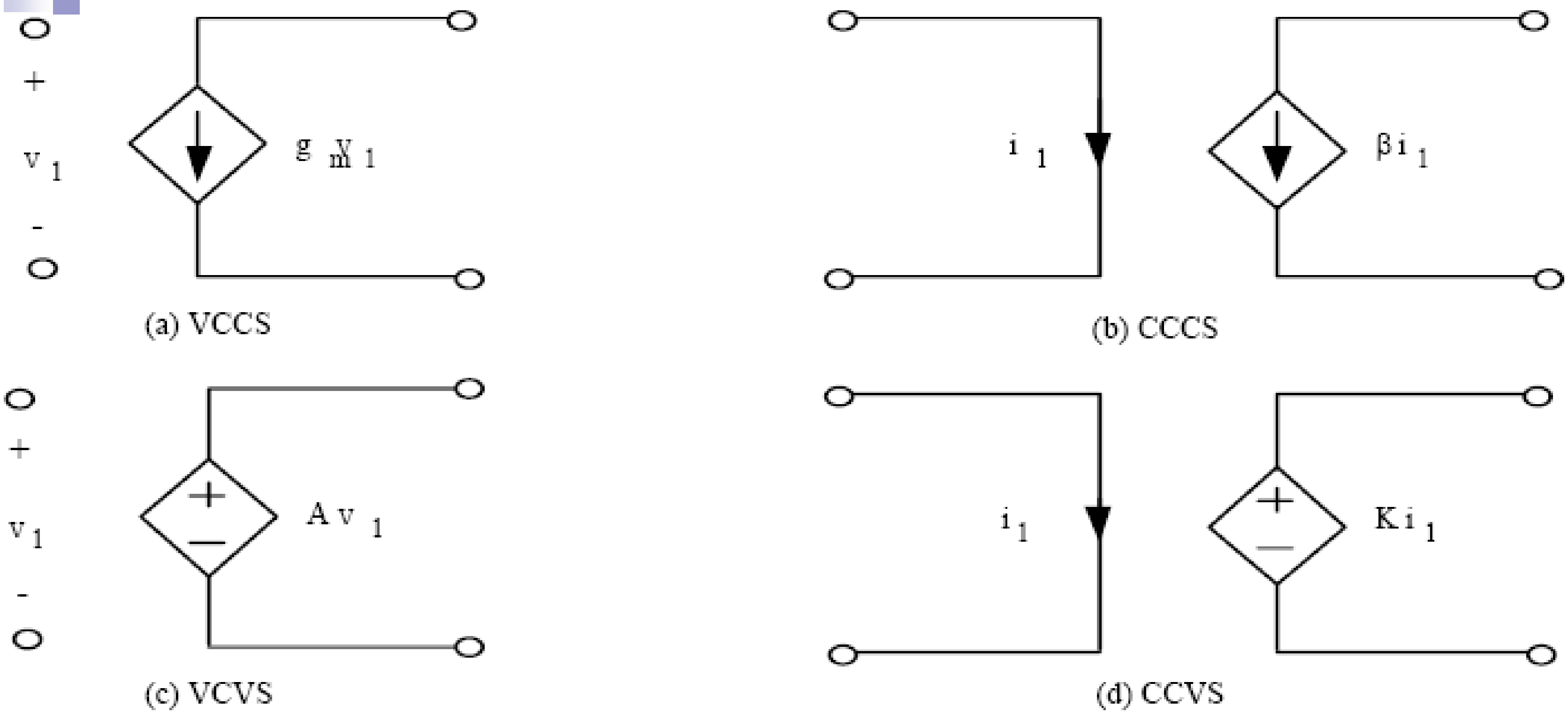


Figure 1.10 - Controlled Sources
(a) Voltage-controlled current source - (VCCS)
(b) Current-controlled current source - (CCCS)
(c) Voltage-controlled voltage source - (VCVS)
(d) Current-controlled voltage source - (CCVS).

Kirchhoff's Current Law (KCL)

- For any lumped electric circuit, for any of its nodes, and at any time, the algebraic sum of all branch currents leaving the node is zero.

KCL Example

- When applying KCL to circuit, first assign reference direction for each branch.
- For node 2, $i_4 - i_3 - i_6 = 0$
- For node 1, $-i_1 + i_2 + i_3 = 0$

Kirchhoff's Voltage Law (KVL)

- For any lumped electric circuit, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loop is zero.

II – Linear electric circuit

A – Definition

We call an electrical network a set of dipoles linked between them by perfect conductor (R=0). The network is called linear if all the dipoles are linear.

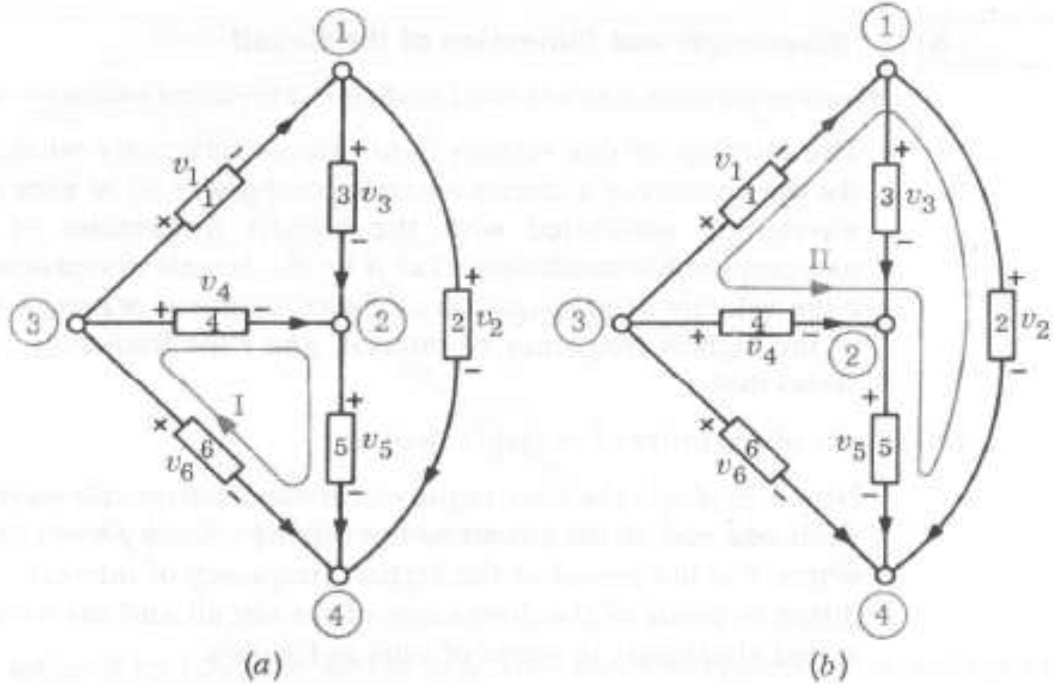
B - Kirchhoff's laws

Gustav Kirchhoff

Gustav Robert Kirchhoff *(March 12, 1824 – October 17, 1887), a German physicist who contributed to the fundamental understanding of electrical circuits, spectroscopy, and the emission of black-body radiation by heated objects. He coined the term "black body" radiation in 1862, and two sets of independent concepts in both circuit theory and thermal emission are named* ***"Kirchhoff's laws"*** *after him.*

Kirchhoff *formulated his circuit laws, which are now ubiquitous in electrical engineering, in 1845, while still a student.*

KVL Example



√Fig. 4.1 Example illustrating KVL; loops I and II are indicated.

Properties of KCL and KVL

- KCL imposes a linear constraint on the branch currents.
- KCL applies to any lumped electric circuit; it is independent of the nature of the elements.
- KCL expresses the conservation of charge at any time.

Properties of KVL and KCL (cont.)

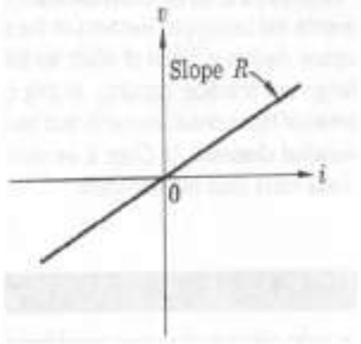
- An example where KCL doesn't apply is the whip antenna. The antenna is about ¼ wavelength so it is not a lumped circuit.
- KVL imposes a linear constraint between branch voltages of a loop.
- KVL is independent of the natural of the elements.

Circuit Elements

- Resistors
- Independent sources
- Capacitors
- Inductors

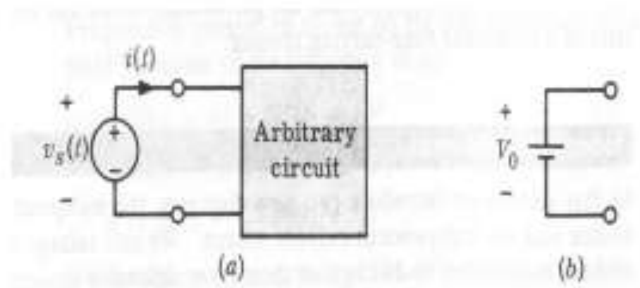
Resistors

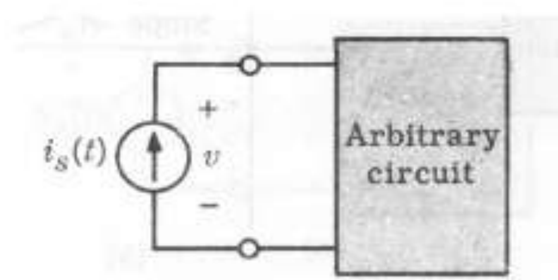
- $v(t) = Ri(t)$ or $i(t) = Gv(t)$
- R is the resistance
- G is called the conductance
- For linear time-invariant resistors, R and G are constants.



Independent Sources

- Independent sources maintains a prescribed voltage or current across the terminals of the arbitrary circuit to which it is connected.





Capacitors

- Capacitors store electrical charges.
- $i(t) = dq/dt$
- $q(t) = Cv(t)$
- $i(t) = Cdv(t)/dt$

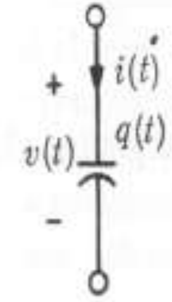


Fig. 3.2 Symbol for a capacitor.

Inductors

- Inductors store enerby in their magnetic fields.
- $V(t) = d\Phi/dt$
- $\Phi(t)=Li(t)$
- $V(t) = L\, di/dt$

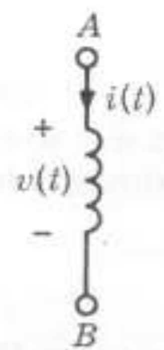


Fig. 4.1 Symbol for an inductor.

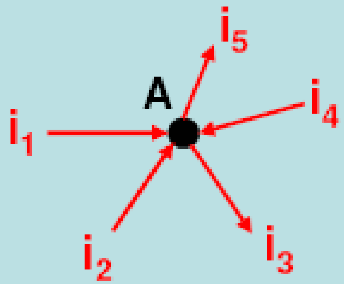
II – Linear electric circuit

These laws are general and valid for DC and AC in the QSSA in any circuit composed by linear dipoles or not. For each network branch, we choose a direction for the current and the voltage.

1 - Nodal analysis

This law is related to the electrical charge conservation. If there is no charge accumulation or deficit in any node of the circuit, then the sum of the current arriving at the node is equal to the sum of the current leaving the node.

⇒ *1st Kirchhoff's law :* $\sum_k \varepsilon_k i_k = 0$



ε_k=+1 for every current arriving to the node
ε_k=-1 for every current leaving the node

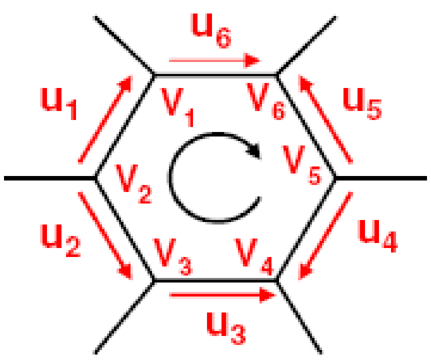
$$i_1 + i_2 + i_4 = i_3 + i_5 \quad \text{or} \quad i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

II – Linear electric circuit

2 - Mesh analysis

We can define at each circuit point an electric potential compared to a reference potential (0V).

The Mesh law is expressed by: the algebraic sum of the p.d along the mesh is null.



$$(V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4) + (V_4 - V_5) + (V_5 - V_6) + (V_6 - V_1) = 0$$

$$u_1 - u_2 - u_3 + u_4 - u_5 + u_6 = 0$$

The mesh law is an algebraic law, we need to choose an arbitrary direction to walk on the mesh.

⇒ *2nd Kirchhoff's law :* $\sum_k \varepsilon_k u_k = 0$

ε_k=+1 if u_k is in the same direction
ε_k=-1 if u_k is in the same direction

II – Linear electric circuit

C – Linear circuit analysis

1 - Calculation using Kirchhoff's laws

a - Principle

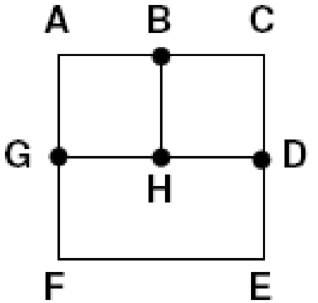
To solve the problem we need to determine the current circulating in each branch of the network. Each branch contains only resistances and generators. The voltage can be written by:

$$u_k = E_k - R_k i_k$$

Using 1st and 2nd Kirchhoff's laws ⇒ set of equations

ⓘ Equations need to be independent ⓘ

If n is the number of nodes in the circuit then the nodal law allows us to write (n-1) linear and independent equations. The other equations will come from the mesh law.



⇒ 4 nodes (B,D,H,G) and 6 branches (GAB,BCD,BH,GH,HD,GFED)

⇒ 3 independent nodal laws so we need 3 independent mesh laws

⇒ (ABHGA,BCDHB,DEFGHD) or (ABHGA,BCDHB,ACEFA)

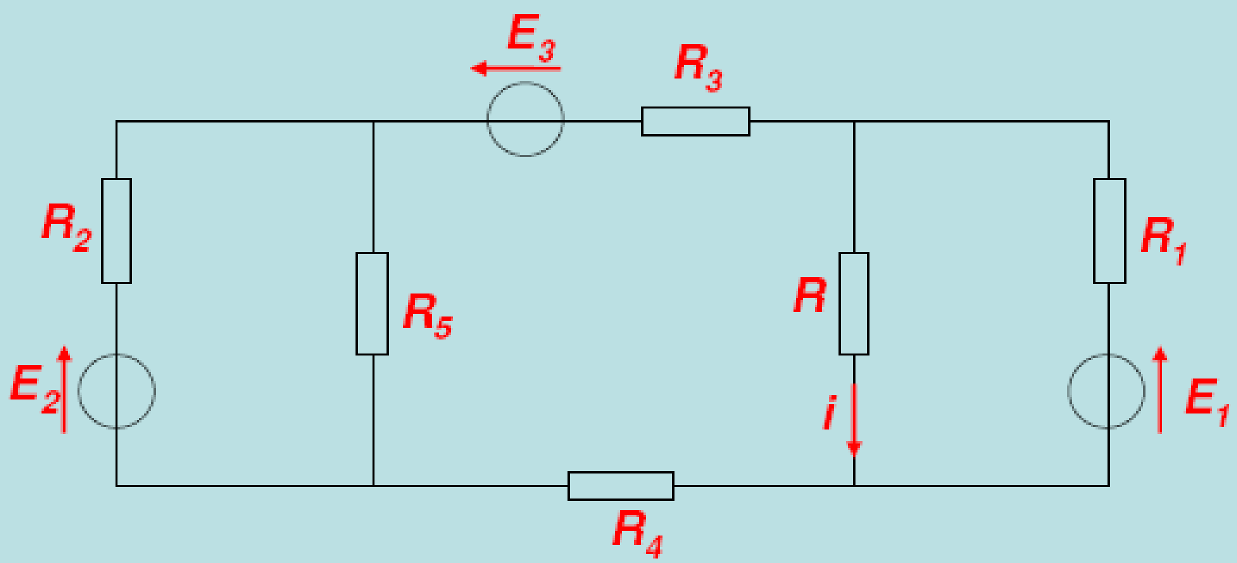
MYcsvtu Notes

ⓘ Do not forget any branch in the circuit ⓘ

II – Linear electric circuit

b - Application

1 - We want to determine the current i in the branch containing the resistance R by applying the Kirchhoff's laws.



Numerical values:

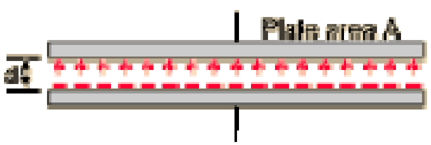
E_1=3V, E_2=30V, E_3=14V

R_1=6kΩ, R_2=3kΩ, R_3=2kΩ, R_4=2kΩ, R_5=6kΩ, R=3kΩ

⇒ *We can simplify by using Thevenin-Norton transformation*

Parallel Plate Capacitance

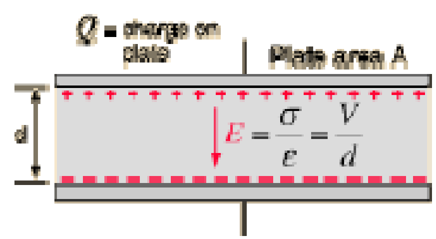
- K = relative permittivity of the dielectric material in between two plates.
- K= 1 for free space, K=3.9 for SiO_2
- High K (K > 3.9)dielectric (e.g. (BaSr)TiO_3, barium strontium titanate for K=160-600 for storage capacitance; zirconium silicate, $ZrSiO_4$ with K=15 for next generation gate oxide)
- Low K (K < 3.9)dielectric for ILD (interlayer dielectrics) to insulate between metal lines (e.g. Porous SiO_2 for K=1.3)



$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

$$E = \frac{\sigma}{\varepsilon} \quad \text{where} \quad \begin{array}{l}\sigma = \text{charge density} \\ \varepsilon = \text{permittivity}\end{array}$$

$$\text{and } \sigma = \frac{Q}{A}$$

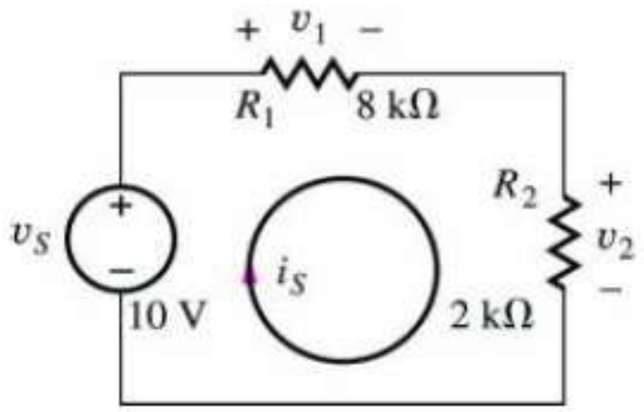


Physical Componenets vs. Circuit Elements

- Range of Operation
- Temperature Effect
- Parasitic effect
- Typical Element Size
  - Resistor : 1ohm to Mohms
  - Capacitor : femto Farad to micro Farad

Circuit Theory Review: Voltage Division

$v_1 = i_s R_1$ and $v_2 = i_s R_2$



Applying KVL to the loop,

$$v_s = v_1 + v_2 = i_s(R_1 + R_2)$$

and $$i_s = \frac{v_s}{R_1 + R_2}$$

Combining these yields the basic voltage division formula:

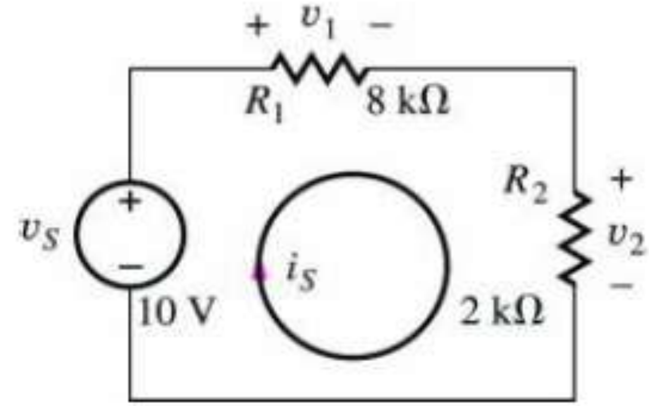
$$v_1 = v_s \frac{R_1}{R_1 + R_2} \qquad v_2 = v_s \frac{R_2}{R_1 + R_2}$$

Circuit Theory Review: Voltage Division (cont.)

Using the derived equations with the indicated values,

$$v_1 = 10\ \text{V} \frac{8\ \text{k}\Omega}{8\ \text{k}\Omega + 2\ \text{k}\Omega} = 8.00\ \text{V}$$

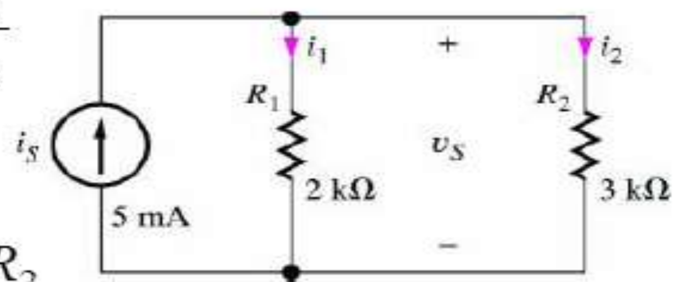
$$v_2 = 10\ \text{V} \frac{2\ \text{k}\Omega}{8\ \text{k}\Omega + 2\ \text{k}\Omega} = 2.00\ \text{V}$$



Design Note: Voltage division only applies when both resistors are carrying the same current.

Circuit Theory Review: Current Division

$i_s = i_1 + i_2$ where $i_1 = \dfrac{v_s}{R_1}$ and $i_2 = \dfrac{v_s}{R_2}$



Combining and solving for v_s,

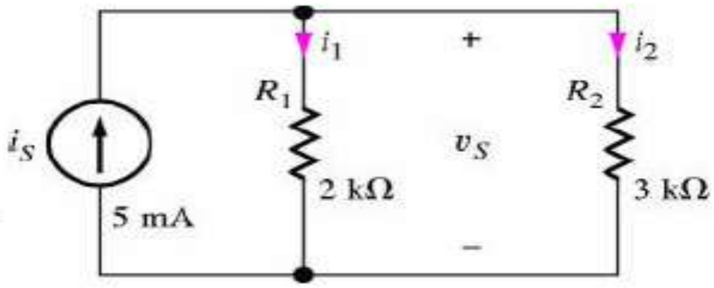
$$v_s = i_s \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = i_s \frac{R_1 R_2}{R_1 + R_2} = i_s R_1 \,\|\, R_2$$

Combining these yields the basic current division formula:

$$i_1 = i_s \frac{R_2}{R_1 + R_2} \qquad \text{and} \qquad i_2 = i_s \frac{R_1}{R_1 + R_2}$$

Circuit Theory Review: Current Division (cont.)

Using the derived equations with the indicated values,



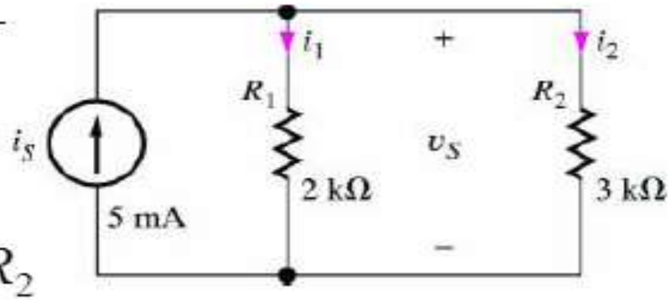
$$i_1 = 5\text{ ma}\frac{3\text{ k}\Omega}{2\text{ k}\Omega + 3\text{ k}\Omega} = 3.00\text{ mA}$$

$$i_2 = 5\text{ ma}\frac{2\text{ k}\Omega}{2\text{ k}\Omega + 3\text{ k}\Omega} = 2.00\text{ mA}$$

Design Note: Current division only applies when the same voltage appears across both resistors.

Circuit Theory Review: Current Division

$$i_s = i_1 + i_2 \text{ where } i_1 = \frac{v_s}{R_1} \text{ and } i_2 = \frac{v_s}{R_2}$$



Combining and solving for v_s,

$$v_s = i_s \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = i_s \frac{R_1 R_2}{R_1 + R_2} = i_s R_1 \parallel R_2$$

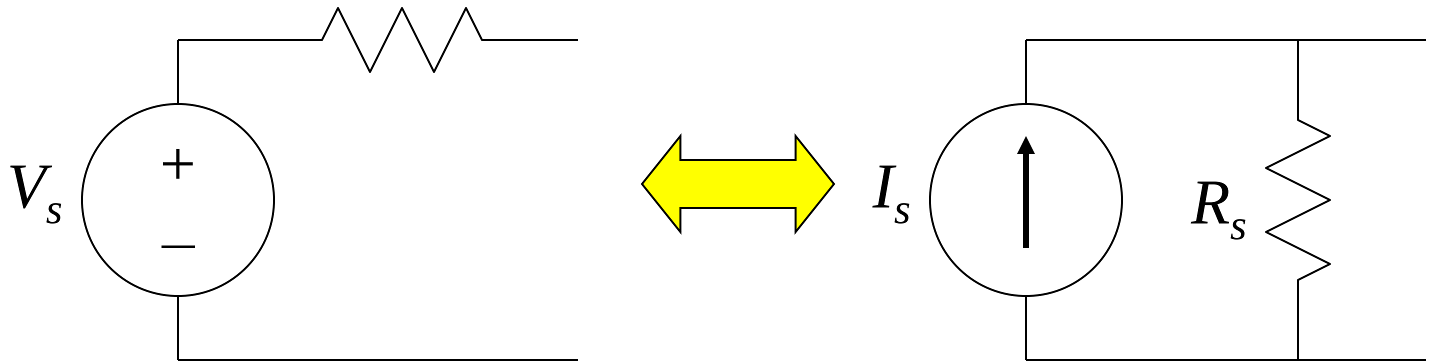
Combining these yields the basic current division formula:

$$i_1 = i_s \frac{R_2}{R_1 + R_2} \qquad i_2 = i_s \frac{R_1}{R_1 + R_2}$$

and

Source Transformation

- Diag.



$$V_s = R_s I_s$$

$$I_s = \frac{V_s}{R_s}$$

Source Transformation

- Equivalent sources can be used to simplify the analysis of some circuits.
- A voltage source in series with a resistor is transformed into a current source in parallel with a resistor.
- A current source in parallel with a resistor is transformed into a voltage source in series with a resistor.