UNIT – I

GOVERNORS

Syllabus

Characteristics of centrifugal governors. Gravity controlled governors such as Porter and Proell governors. Spring controlled centrifugal governors such as Hartung and Hartnell governors. Performance parameters, sensitivity, stability, isochronism and hunting. Governor effort and power.

1.1 Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load. When the load on the engine, its speed decreases, therefore, it becomes necessary to increase the supply of working fluid. When the load on the engine decreases, its speed increases and less working fluid is required.

The governor automatically controls the supply of working fluid to the engine with varying load conditions and keeps the mean speed within certain limits. When the load increases or decreases, the configuration of the governor changes and a valve is moved to increase or decrease the supply of working fluid respectively

1.2 Difference between a flywheel and a governor

The function of a flywheel in an engine is to control the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation.

It does not control the speed variations caused by a varying load.

The governor meets the varying demand for power by regulating the supply of working fluid.

1.3 Types of Governors

The governors may be classified as

1. Centrifugal governors

2. Inertia governors

The centrifugal governors may further be classified as follows:

1. Pendulum type – watt governor

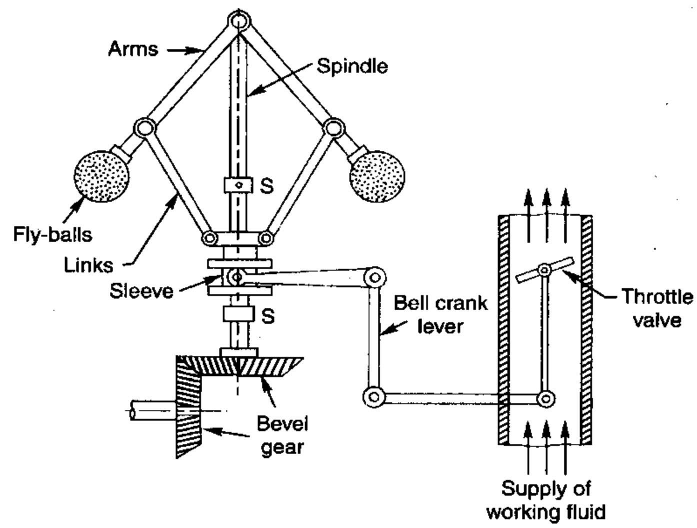
2. Loaded type

(a) Dead weight governors – Porter governor, Proell governor

(b) Spring controlled governors – Hartnell governors, Hartung governor, Wilson Hartnell governor, Pickering governor

1.4 Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force. It consists of two balls of equal mass, which are attached to the arms. The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis.



The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve.

The supply of the working fluid decreases when the sleeve rises and decreases when it falls. When the load on the engine increases, the engine and governor speed decreases. This results in the decrease of centrifugal force in the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply o working fluid and thus engine speed is increased. The extra power output is provided to balance the increased load.

When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upwards movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case the power output is reduced.

1.5 Terms Used in Governors

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis.

2. Equilibrium speed. It is the speed at which the governor balls, arms etc. are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. Mean equilibrium speed. It is the speed at the mean position of the balls or the sleeve.

4. Maximum and minimum equilibrium speeds. The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

5. Sleeve lift. It is the vertical distance which the sleeve travels due to change in equilibrium speed.

1.6 Watt Governor

It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways.

1. The pivot P may be on the spindle axis

2. The pivot may be offset from the spindle axis and the arms when produced intersect at O.

3. The pivot P may be offset, but the arms cross the axis at O.

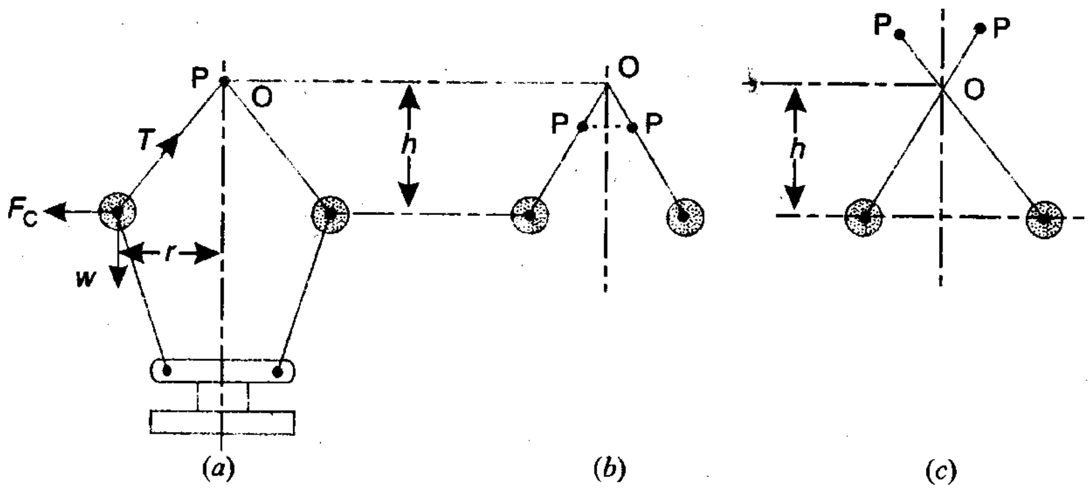


Fig. 18.2. Watt governor.

Let m = Mass of the ball in kg.

w = Weight of the ball in newtons = m.g

T = Tension in the arm in newtons

ω = Angular velocity of the arm and ball about the spindle axis in rad/s.

r = Radius of the path of rotation of the ball

F_c = Centrifugal force acting on the balls in newtons = $m.\omega^2.r$

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. The ball is in equilibrium under the action of

1. the centrifugal force (F_c) acting on the ball, **2.** the tension (T) in the arm, and **3.** the weight (w) of the ball.

Taking moments about point O

$Fc \times h = w \times r = m.g.r$

or $m.\omega^2.r.h = m.g.r$ or $h = g/\omega^2$

or $$h = \frac{895}{N^2}$$

The height of a governor is inversely proportional to N^2. Therefore, at high speeds the value of h is small.

At such speeds the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds i.e. from 60 to 80 rpm

1.7 Porter Governor

The Porter governor is a modification of a Watt's governor, with the central load attached to the central sleeve. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Consider the forces acting on one-half of the governor

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = m.g,

M = Mass of the central load in kg,

r = radius of rotation in metres,

h = Height of governor in metres,

N = Speed of the ball in rpm

ω = Angular speed of the balls in rad/s

F_c = Centrifugal force acting on the ball in newtons

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons

α = Angle of inclination of the arm to the vertical

β = Angle of inclination of the link to the vertical

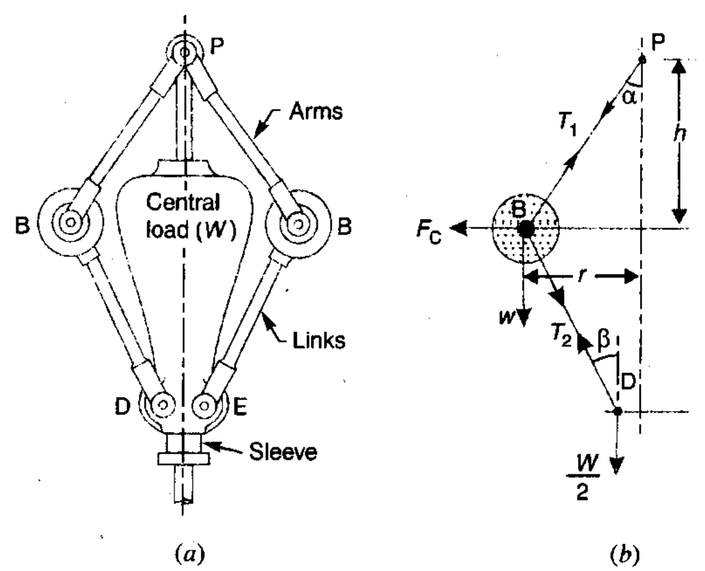


Fig. 18.3. Porter governor.

Method of resolution of forces

Considering the resolution of forces acting at D

$$T_2 \cos\beta = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta} \qquad \textbf{(i)}$$

Again, considering the equilibrium of forces acting at B. The point B is in equilibrium under the action of following forces

(i) The weight of the ball (w = m.g)

(ii) The centrifugal force (F_c)

(iii) The tension in the arm (T_1)

(iv) The tension in the link (T_2)

Resolving the forces vertically

$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g$ (ii)

Resolving the forces horizontally

$T_1 \sin\alpha + T_2 \sin\beta = F_c$

$T_1 \sin\alpha = F_c - \frac{M.g}{2} \times \tan\beta$ …(iii)

Dividing equation (iii) by equation (ii)

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_c - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or $\frac{M.g}{2} + m.g = \frac{F_c}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$

Substituting $\frac{\tan\beta}{\tan\beta} = q, and \; \tan\alpha = \frac{r}{h}$ we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q$$

$$or \qquad m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$h = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

$$\therefore \qquad N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$or \quad \omega^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h} \qquad \text{(v)}$$

Notes:**1.** When the length of arms is equal to the length of links and the points P and D lie on the same vertical line, then

tan α = tan β or q = tan α / tan β = 1, equation (v) becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

2. When the loaded sleeve moves up and down the spindle, the friction force acts on it in a direction opposite to that of the motion of sleeve.

If F = Frictional force acting on the sleeve in newtons, then the equations (v) and (vi) may be written as

$$N^2 = \frac{m.g + \left[\frac{M.g \pm F}{2}\right](1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \qquad (When\, q = 1)$$

The + sign is used when the sleeve moves upwards or the governor speed increases.

On comparing the equation (vi) with equation (ii) of Watt's governor, we find that the mass of the central load (M) increase the height of governor in the ratio (m + M) / m

1.8 Proell Governor

The Proell has the balls fixed at B and C to the extension of the links DF and EG. The arms FP and GQ are pivoted at P and Q respectively.

Consider the equilibrium of the forces on one ball of the governor. The instantaneous centre (I) lies on the intersection of the line FP produced and the line from D drawn perpendicular to the spindle axis. The perpendicular BM is drawn on ID

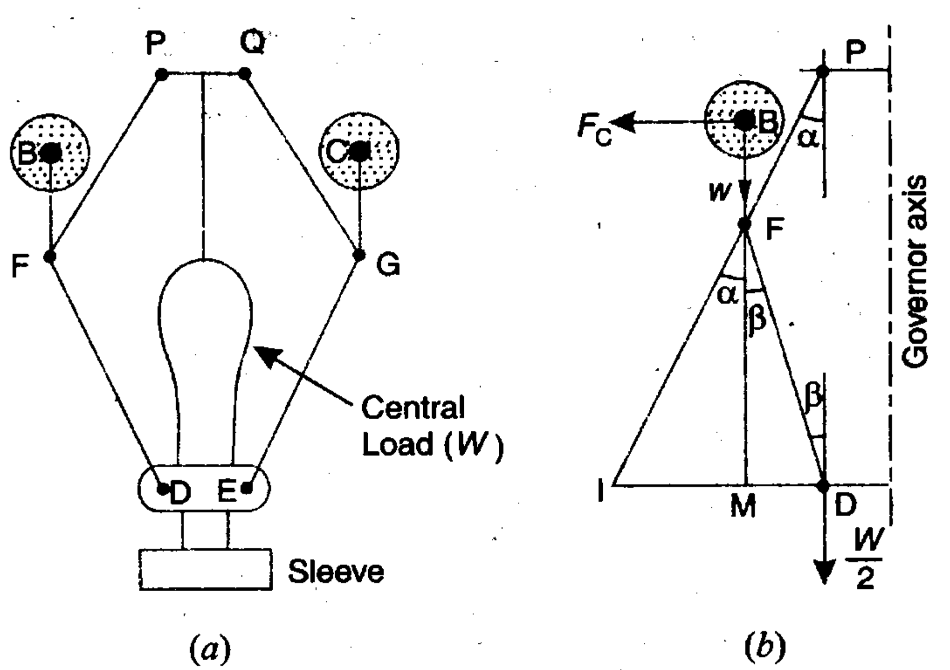


Fig. 18.12. Proell governor.

Taking moments about I

$$F_c \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID \qquad (i)$$

$$\therefore \quad F_c = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad (\because ID = IM + MD)$$

$$\therefore \quad F_c = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

Multiplying and dividing by FM

$$F_c = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}\left(\tan\alpha + \tan\beta\right)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}\left(1+q\right)\right] \quad \textbf{or} \quad \omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}\left(1+q\right)}{m}\right]\frac{g}{h}$$

1.9 Hartnell Governor:

A Hartnell governor is a spring loaded governor. It consists of two bell crank levers pivoted at points O,O to the frame. The frame is attached to the governor spindle. A helical spring in compression provided equal downward forces on the two rollers through a collar on the sleeve.

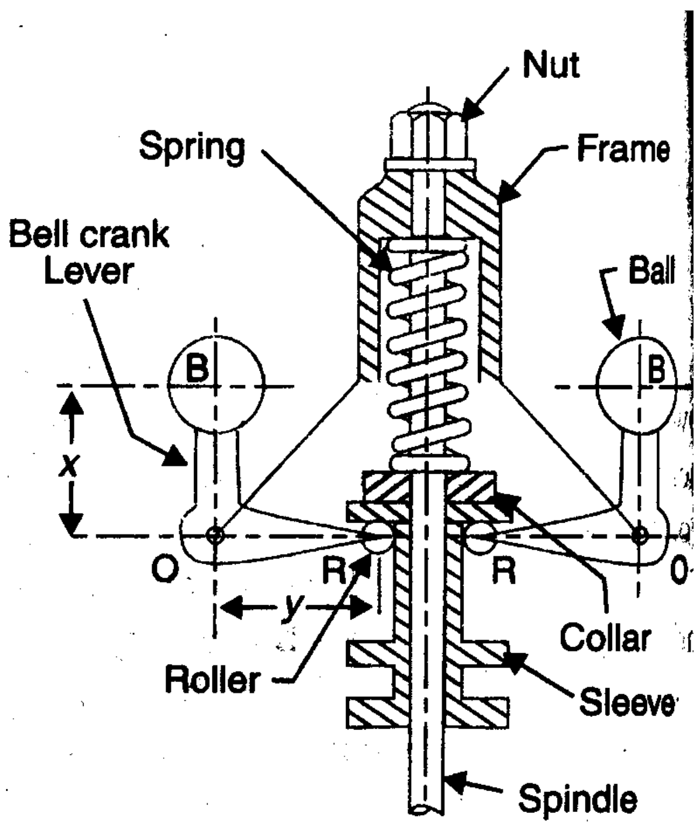


Fig. 18.18. Hartnell governor.

For minimum position

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x}$$

For maximum position

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x}$$

Adding

$$h = \left(r_2 - r_1\right)\frac{y}{x}$$

For minimum position, taking moments about point O

$$\frac{M.g + S_1}{2} \times y_1 = F_{c1} \times x_1 - m.g \times a_1$$

$$M.g + S_1 = \frac{2}{y_1}\left(F_{c1} \times x_1 - m.g \times a_1\right)$$

For maximum position, taking moments about point O

$$\frac{M.g+S_2}{2}\times y_2 = F_{c2}\times x_2 + m.g\times a_2$$

$$M.g+S_2 = \frac{2}{y_2}\left(F_{c2}\times x_2 + m.g\times a_2\right)$$

Subtracting

$$S_2 - S_1 = \frac{2}{y_2}\left(F_{c2}\times x_2 + m.g\times a_2\right) - \frac{2}{y_1}\left(F_{c1}\times x_1 - m.g\times a_1\right)$$

$S_2 - S_1 = h.s$, and $h = (r_2 - r_1)\, y/x$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms and the moment due to weight of the balls,

for minimum position,

$$\frac{M.g+S_1}{2}\times y = F_{c1}\times x \;\; or \;\; M.g+S_1 = 2F_{c1}\times\frac{x}{y}$$

Similarly for maximum position

$$\frac{M.g+S_2}{2}\times y = F_{c2}\times x \;\; or \;\; M.g+S_2 = 2F_{c2}\times\frac{x}{y}$$

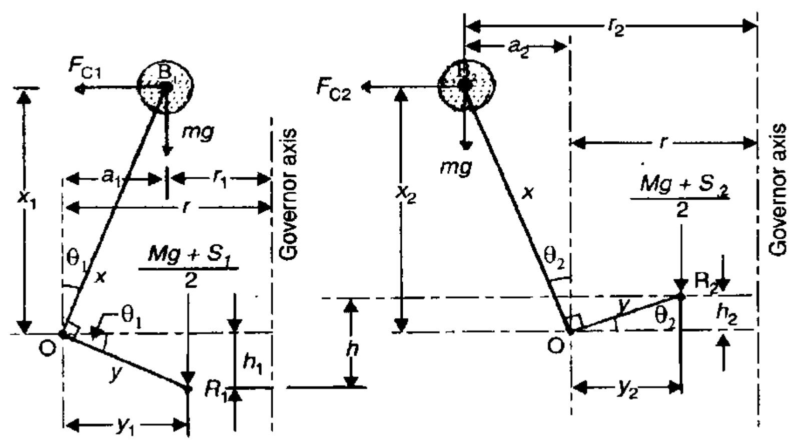
Subtracting

$$S_2 - S_1 = 2\left(F_{c2} - F_{c1}\right)\frac{x}{y}$$

We know that

$$S_2 - S_1 = h.s \quad and \quad h = \left(r_2 - r_1\right)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{c2} - F_{c1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2$$



(*a*) Minimum position. (*b*) Maximum position.

Since the stiffness of a given spring is constant for all positions, therefore for minimum and intermediate positions,

$$s = 2\left(\frac{F_c - F_{c1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2$$

and for intermediate and maximum position,

$$s = 2\left(\frac{F_{c2} - F_c}{r_2 - r}\right)\left(\frac{x}{y}\right)^2$$

From the above equations

$$\frac{F_{c2} - F_{c1}}{r_2 - r_1} = \frac{F_c - F_{c1}}{r - r_1} = \frac{F_{c2} - F_c}{r_2 - r}$$

or $$F_c = F_{c1} + \left(F_{c2} - F_{c1}\right)\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{c2} - \left(F_{c2} - F_{c1}\right)\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

1.10 Hartung Governor:

In this types of governor, the vertical arms of bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

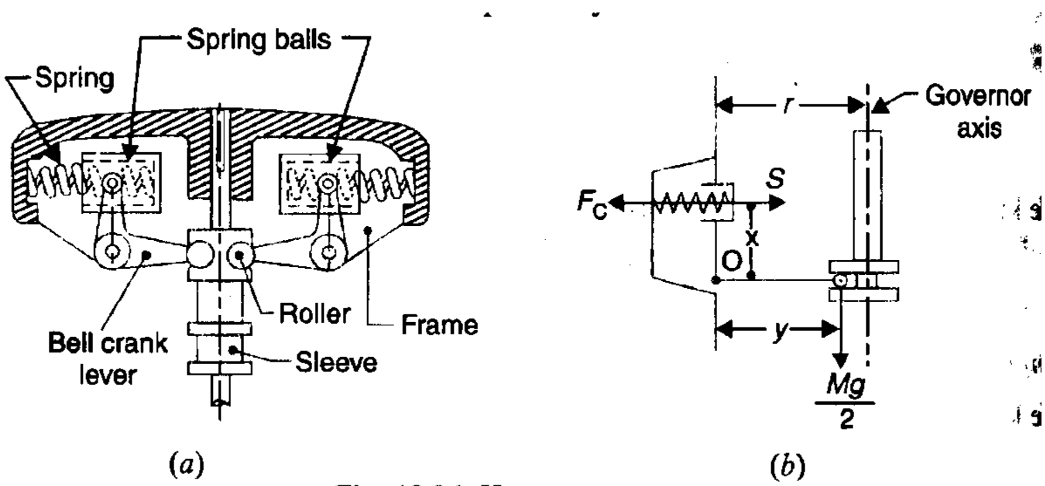


Fig. 18.26. Hartung governor.

1.11 Sensitiveness of Governors

The sensitiveness is defined as the ration of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Let N_1 = Minimum equilibrium speed,
N_2 = Maximum equilibrium speed,

N = Mean equilibrium speed $= \frac{N_1 + N_2}{2}$

- Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2} = \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

1.12 Stability of Governors

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of governor balls at which the governor is in equilibrium. For a stable governor if the equilibrium speed increases, the radius of governor balls must also increase.

Note: A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

1.13 Isochronous Governors

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

1.14 Hunting

A governor is said to hunt if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in speed of rotation takes place.

1.15 Effort of a Governor

The effort of a governor is the mean force exerted at the sleeve for a given percentage change of speed. When the governor is running steadily , there is no force at the sleeve. When the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a maximum value to zero when the governor moves to its new position of equilibrium.

1.16 Power of a Governor

The power of a governor is the work done at the sleeve for a given percentage change of speed. It is the product of mean value of the and the distance through which the sleeve moves. Mathematically,

Power = Mean effort × lift of sleeve

Example 1: A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. the radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor. **K/661**

Solution:

Minimum speed when r_1 = 0.15 mm

$$h_1 = \sqrt{(PB)^2 - (BG)^2} = 0.2\,m$$

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = 17900$$

$$N_1 = 133.8\,r.p.m.$$

Maximum speed when r_2 = 0.2 m
h_2 = 0.15 m
N_2 = 154.5 rpm
Range of speed = $N_2 - N_1$ = 38 rpm

Example 2: In an engine governor of the Porter type, the upper and lower arms are 200 mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and the friction of the sleeve together with the resistance of the operating gear is equal to a load of 24 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30˚ and 40˚, find taking friction into account, range of speed. **K/663**

Solution:

r_1 = BP sin 30˚ = 0.1 m
h_1 = BP cos 30˚ = 0.1732 m

$$DG = \sqrt{(BD)^2 - (BG)^2} = 0.23m$$

tan β_1 = BG/DG = 0.4348
tan α_1 = tan 30˚ = 0.5774

$$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = 0.753$$

$$N_1^2 = \frac{m.g + \left[\frac{M.g - F}{2}\right](1+q_1)}{m.g} \times \frac{895}{h_1} = 33596$$

N_1 = 183.3 rpm
r_2 = BP sin 40˚ = 0.1532 m

$$DG = \sqrt{(BD)^2 - (BG)^2} = 0.2154m$$

tan β_2 = BG/DG = 0.59
tan α_2 = tan 40˚= 0.839

$$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = 0.703$$

Maximum speed is given by

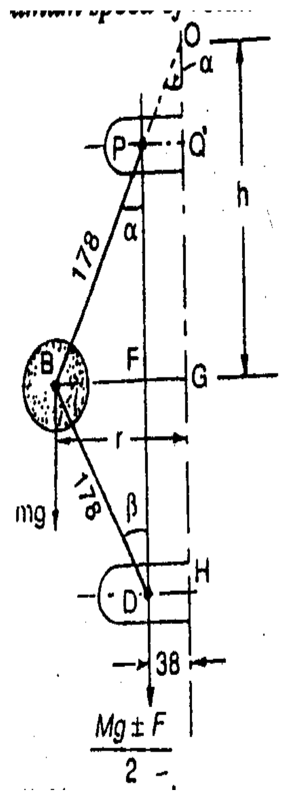
$$(N_2)^2 = \frac{m.g + \left[\frac{M.g + F}{2}\right](1+q_2)}{m.g} \times \frac{895}{h_2} = 49236$$

N_2 = 222 rpm

Range of speed = $N_2 - N_1$ = 38.7 rpm

Example 3: All the arms of a Porter governor are 178 mm long and are hinged at a distance of 38 mm from the axis of rotation. The mass of each ball is 1.15 kg and the mass of the sleeve is 20 kg. The governor begins to rise at 280 rpm when the links are at an angle of 30˚ to the vertical. Assuming the friction force to be constant, determine the minimum and maximum speed of rotation when the inclination of the arms to the vertical is 45˚
K/669/(S08)

Radius of rotation, r = BG = BF + FG
= BP × sin α + FG = 127 mm
Height of governor, h = BG/tan α = 0.22 m



$$N^2 = \frac{m.g + (Mg \pm F)}{m.g} \times \frac{895}{h}$$

± F = 10 N
When α = β = 45˚

r = BG = BF + FG = BP × sinα + FG = 164 mm
Height of the governor, h = BG/tan α = 0.164 mm

$$(N_1)^2 = \frac{m.g + (Mg - F)}{m.g} \times \frac{895}{h}$$

N_1 = 309 rpm

$$(N_2)^2 = \frac{m.g + (Mg + F)}{m.g} \times \frac{895}{h}$$

N_2 = 324 rpm

Example 4: In a Porter governor the upper and lower arms are each 250 mm long and are pivoted on the axis of rotation. The mass of each rotating ball is 3 kg and mass of the sleeve is 20 kg. the sleeve is in its lowest position when the arms are inclined at 30˚ to the governor axis. The lift of the sleeve is 36 mm. Find the force of friction at the sleeve, if the speed at the moment it rises from the lowest position is equal to the speed at the moment it falls from the highest position. Also, find the range of speed of the governor.

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Solution: Given: BP = BD = 250 mm;
m = 3 kg; M = 20 kg; α_1 = 30˚; Lift of sleeve = 36 mm
h_1 = BP cos 30˚ = 250 × cos 30˚ = 216.5 mm = 0.2165 m
h_2 = 0.2165 – 0.036/2 = 0.1985 m
Speed when the sleeve rises = Speed when the sleeve falls

$$\frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_1} = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_2}$$

or $$\frac{m.g + (M.g + F)}{h_1} = \frac{m.g + (M.g - F)}{h_2}$$

$$F + \frac{h_1}{h_2} F = g(m + M)\left(\frac{h_1}{h_2} - 1\right) or$$

$$1.091F = 9.81 \times 23 \times 0.091$$

or F = 9.8 N

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1} = 30317$$

$$N_1 = 174\, r.p.m.$$

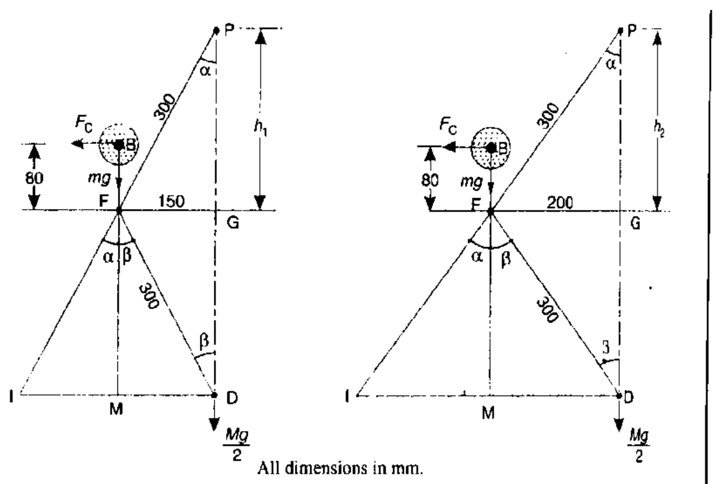
$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2} = 36069$$

$$N_2 = 190\, r.p.m.$$

$N_1 - N_2 = 190 - 174 = 16$ rpm

Example 5: A Proell governor has equal arms of length 300 mm. The upper and lower ends of arms are pivoted on the axis of the governor. The extension arms of lower links are each 80 mm long and parallel to the axis when the radii of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of central load is 100 kg. Determine the range of speed of the governor. **K/671**

Solution:



(*a*) Minimum position. (*a*) Maximum position.

$$h_1 = \sqrt{(PF)^2 - (FG)^2} = 0.26\, m$$

FM = 0.26 m, BM = 0.34 m

$$(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1} = 28956$$

$$N_1 = 170\, r.p.m.$$

h_2 = PG = $\sqrt{(PF)^2 - (FG)^2}$ = 0.224

FM = GD = PG = 0.224 mm
BM = BF + FM = 0.0.304 mm

$$(N_2)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_2} = 28956$$

$$N_2 = 180\, r.p.m.$$

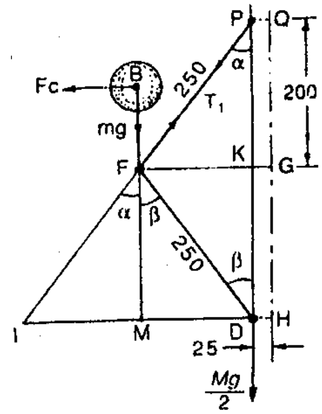
Range of speed = $N_2 - N_1$ = 10 r.p.m.

Example 6: A governor of the Proell type has each arm 250 mm long. The pivots of the upper and lower arms are 25 mm from the axis. The central load acting on the sleeve has a mass of 25 kg. When the governor sleeve is in mid-position, the extension link of the lower arm is vertical and the radius of the path of rotation of the masses is 175 mm. the vertical height of the governor is 200 mm. If the governor speed is 160 r.p.m. when in mid-position, find: 1. length of extension link; and 2. tension in the upper arm.

K/673

Solution:

1. Length of the extension link



$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_2}$$

BM = 0.308 m
BF = BM – FM = 0.108 m = 108 mm

2. Tension in the upper arm

$$PK = \sqrt{(PF)^2 - (FK)^2} = \sqrt{(PF)^2 - (FG - KG)^2} = 200\,mm$$

cos α = PK/PF = 0.8

$$T_1 \cos\alpha = mg + \frac{Mg}{2} = 154\,N$$

T_1 =192.5 N

Example 7: The mass of each ball of a Proell governor is 3 kg and the weight on the sleeve is 20 kg. Each arm is 220 mm long and the pivots of the upper and the lower arms are 20 mm away from the axis. For the mid position of the sleeve, the extension links of the lower arms are vertical, the height of the governor is 180 mm and the speed is 150 rpm. Determine the lengths of the extension links and the tension in the upper arms.

(W 07) R/595/Exercise

Example 8: A Hartnell governor having a central sleeve spring and two right-angled bell crank levers between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : 1. loads on the spring at the lowest and the highest equilibrium speeds and 2. stiffness of the spring.

K/680

Solution:

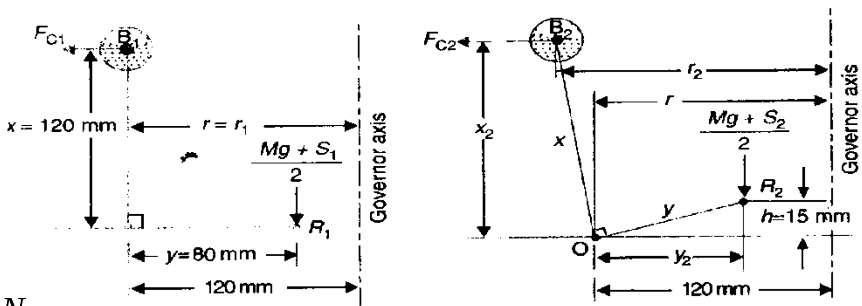
1. Loads on the spring at the lowest and highest equilibrium speeds

Centrifugal force at the minimum speed
$Fc_1 = m\,(\omega 1)^2\, r_1 = 277$ N

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h\frac{x}{y} = 0.1425$$

Centrifugal force at the maximum speed
$F_{c2} = m\,(\omega_2)^2\, r^2 = 376$ N
Neglecting the obliquity effect of arms and the moment due to the weight of the balls, for the lowest position



(a) Lowest position. (b) Highest position.

$$M.g + S_1 = 2F_{c1} \times \frac{x}{y} = 831\,N$$
S_1 = 831 N
and for highest position
$$M.g + S_2 = 2F_{c2} \times \frac{x}{y} = 1128\,N$$
S_2 =1128 N
Stiffness of the spring
$$s = \frac{S_1 - S_2}{h} = 19.8\,N/mm$$

Example 9: In a spring loaded Hartnell type governor , the extreme radii of rotation of the balls are 80 mm and 120 mm. The ball arm and sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 r.p.m., find: 1. the initial compression of central spring, and 2. the spring constant.
K/681/(S 08)

Solution:
Initial compression of the central spring
$F_{c1} = m\,(\omega_1)2\, r_1 = 281$ N
$F_{c2} = m\,(\omega_1)^2\, r^2 = 465$ N
For minimum position
$$M.g + S_1 = 2F_{c1} \times \frac{x}{y}$$
S_1 = 562 N

For maximum position
$$M.g + S_2 = 2F_{c2} \times \frac{x}{y}$$
S_2 = 562 N

Lift of the sleeve
$$h = (r_2 - r_1)\frac{x}{y} = 40\,mm$$

Stiffness of the spring
$$s = \frac{S_2 - S_1}{h} = 9.2\,N/mm$$

Initial compression of the central spring

$$\frac{S_1}{s} = 61mm$$

2. Spring constant

s = 9.2 N/mm

Example 10: A spring loaded governor of the Hartnell type has arms of equal length. The masses rotate in a circle of 130 diameter when the sleeve is in the mid position and the ball arms are vertical. The equilibrium speed for this position is 450 r.p.m., neglecting friction. The maximum speed movement is to be 25 mm and the maximum variation of speed taking into account the friction to be 5 % of the mid position speed.
The mass of the sleeve is 4 kg and the friction may be considered equivalent to 30 N at the sleeve. The power of the governor must be sufficient to overcome the friction by 1per cent change of speed either way at mid-position. Determine, neglecting obliquity effect of arms; The value of each rotating mass; 2. the spring stiffness in N/mm; and 3. The initial compression of spring. **K/682**

Solution:
Minimum speed at mid position $\omega_1 = 0.99\omega = 46.66$ rad/s
Maximum speed at mid position $\omega_2 = 1.01\omega = 47.6$ rad/s
Centrifugal force at the minimum speed $F_{c1} = m\,(\omega_1)^2\,r = 141.5\,m$ N
Centrifugal force at the maximum speed $F_{c2} = m\,(\omega_2)^2\,r = 147.3\,m$ N
For minimum speed at mid-position $S + (M.g - F) = 2\,F_{c1} \times x/y$ or $S + 9.24 = 283\,m$ (i)
For maximum speed at mid-position $S + (M.g + F) = 2\,F_{c2} \times x/y$ or $S + 69.24 = 294\,m$ (ii)
From (i) and (ii)
M = 5.2 kg
2. Spring stiffness in N/mm
Minimum speed considering friction $\omega_1' = \omega - 0.05\omega = 44.8$ rad/s
Maximum speed considering friction $\omega_1' = \omega + 0.05\omega = 49.5$ rad/s
Minimum radius of rotation considering friction $r_1 = r - h_1 \times x/y = 0.0525$ m
Maximum radius of rotation considering friction $r_2 = r - h_2 \times x/y = 0.0775$ m
Centrifugal force at minimum speed considering friction $F_{c1}' = m\,(\omega_1')^2\,r_1 = 548$ N
Centrifugal force at maximum speed considering friction $F_{c2}' = m\,(\omega_2')^2\,r_2 = 987$ N
For minimum speed considering friction $S_1 + (M.g - F) = 2\,F_{c1}' \times x/y$ or $S_1 = 1086.76$ N
For maximum speed considering friction $S_2 + (M.g - F) = 2\,F_{c2}' \times x/y$ or $S_2 = 1904.76$ N
Stiffness of spring

$$s = \frac{S_2 - S_1}{h} = 32.72\,N/mm$$

Initial compression of the spring $= \frac{S_1}{s} = 33.2\,mm$

Example 11: The arms of a Hartnell governor are of equal length. When the sleeve is in the mid-position, the masses rotate in a circle with a diameter of 150 mm (the arms are vertical in mid-position). Neglecting friction the equilibrium speed for this position is 360 rpm. Maximum variation of speed, taking friction into account , is to be 6% of the mid-position speed for a

maximum sleeve movement of 30 mm. The sleeve mass id 5 kg and the friction at the sleeve is 35 N.
Assuming that power of a governor is sufficient to overcome the friction by 1% change of speed on each side of the mid-position, find (neglecting obliquity effect of arms), the

(i) mass of each rotating ball
(ii) spring stiffness
(iii) initial compression of the spring **(W 07) R/572**

Solution: Given: x = y; d = 150 mm or r = 75 mm = 0.075 mm; N = 360 rpm; $\omega = 2\pi N/60 = 37.7$ rad/s; h = 30 mm = 0.03 m; M = 5 kg; F = 35 N.

1. **Value of each rotating mass**

Let m = Value of each rotating mass in kg,
S = spring force on the sleeve at mid position in newtons.
Since the change of speed at mid position to overcome friction is 1 % either way, therefore
Minimum speed at mid position $\omega_1 = \omega - 0.01\omega = 0.99\omega$ rad/s
Maximum speed at mid position $\omega_2 = \omega + 0.01\omega = 1.01\omega$ rad/s
For minimum speed at mid position $S + (M.g - F) = 2F_{c1} \times x/y$ (i)
For maximum speed at mid position $S + (M.g + F) = 2F_{c2} \times x/y$ (ii)
Subtracting (ii) from (i) $2F = 2(F_{c2} - F_{c1})$ or $F = m(\omega_2^2 - \omega_1^2)\,r$
$35 = m \times 37.7^2 (1.01^2 - 0.99^2)\,0.075$ or $m = 8.21$ kg

2. **Spring stiffness in N/mm**

Let s = spring stiffness in N/mm
Since the maximum variation of speed, considering friction is ± 6% of the mid position speed, therefore
Minimum speed considering friction $\omega_1' = \omega - 0.06\omega = 0.94\omega = 35.44$ rad/s
Maximum speed considering friction $\omega_2' = \omega + 0.06\omega = 1.06\omega = 39.96$ rad/s

Minimum radius of rotation considering friction $r_1 = r - h_1 \times \frac{x}{y} = 0.075 - \frac{0.03}{2} = 0.06\,m$

Maximum radius of rotation considering friction $r_2 = r + h_2 \times \frac{x}{y} = 0.075 + \frac{0.03}{2} = 0.09\,m$

Centrifugal force at the minimum speed considering friction
$F'_{c1} = m(\omega'_1)^2 r_1 = 8.21\,(35.44)^2\,0.06 = 618.7$ N
Centrifugal force at the maximum speed considering friction
$F'_{c2} = m(\omega'_2)^2 r_2 = 8.21\,(39.96)^2\,0.09 = 1179.88$ N
Let S_1 = Spring force at minimum speed considering friction
S_2 = Spring force at maximum speed considering friction
For minimum speed considering friction
$S_1 + (M.g - F) = 2F_{c1}' \times x/y$
$S_1 + (5 \times 9.81 - 35) = 2 \times 618.7$ or $S_1 = 1223.35$ N
For maximum speed considering friction
$S_2 + (M.g - F) = 2F_{c2}' \times x/y$
$S_1 + (5 \times 9.81 + 35) = 2 \times 1179.88$
$S_2 = 2275.71$ N
Stiffness of spring

$$s = \frac{S_2 - S_1}{h} = \frac{2275.71 - 1223.35}{30} = 35.078\,N/mm$$

Initial compression

$$or \qquad \frac{F_{s1}}{s} = \frac{1223.35}{35.078} = 34.87\,mm$$

Example 12: Each arm of a Porter governor is 250 mm long and is pivoted on the axis of rotation. The mass of each ball is 5 kg and the sleeve is 25 kg. The sleeve begins to rise when the radius of rotation of the balls is 150 mm and reaches the top when it is 200 mm. Determine the range of speed, lift of the sleeve, governor effort and power. In what way are these values changed if friction at the sleeve is equivalent to 10 N.

(W 07) R/596/Exercise

Example 13: A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 20 mm when governor is at maximum speed. Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases:

1. When the friction at the sleeve is neglected.
2. When the friction at the sleeve is equivalent to 10 N**.** **(K/704)**

Solution:

When the friction at the sleeve is neglected

$$h_1 = \sqrt{(PB)^2 - (BG)^2} = 0.2\,m$$

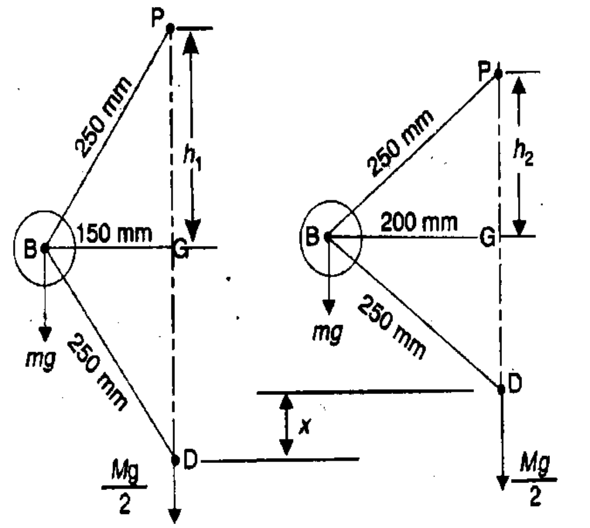
$$h_2 = \sqrt{(PB)^2 - (BG)^2} = 0.15\,m$$

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = 26850$$

$$N_1 = 164\, r.p.m.$$

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = 35800$$

$$N_1 = 189\, r.p.m.$$



(*a*) Minimum position. (*b*) Maximum position.

Range of speed = $N_2 - N_1$ = 25 rpm

Sleeve lift x = 2 ($h_1 - h_2$) = 0.1 m

Governor effort

c.N_1 = $N_1 - N_2$, c = 0.152

Governor effort P = c (m + M) g = 44.7 N

Power of the governor P x = 4.47 N

2. When friction at the sleeve is taken into account

$$(N_2)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1} = 25938$$

N_1 = 192.4 rpm

Range of speed = 31. 4 rpm

Sleeve lift = 0.1 m

Governor effort,

c.N_1 = $N_2 - N_1$ =31.4 rpm

C = 0.195

Governor effort, P = c (m.g + M.g + F) = 57.4 N

Power of the governor = P. x

= 5.75 N-m

Example 14: The radius of rotation of the balls of a Hartnell governor is 80 mm at the minimum speed of 300 rpm. Neglecting gravity effect, determine the speed after the sleeve has lifted by 60 mm. Also determine the initial compression of the spring, the governor effort and the power. The particulars of the governor are given below:

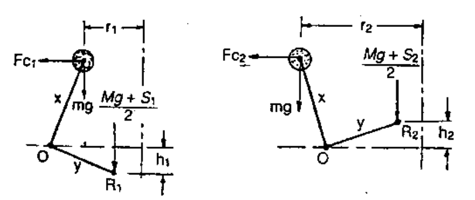
Length of the ball arm = 150mm; length of the sleeve arm = 100 mm; mass of each ball = 4 m kg and stiffness of the spring.

K/707/(W 08)

Solution:

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h \times \frac{x}{y} = 0.17\,m$$



(*a*) Minimum position. (*b*) Maximum position.

F_{c1} = m $(\omega_1)^2$ r_1 = 316 N

Taking moments about fulcrum O

$$F_{c1} \times x = \frac{M.g + S_1}{2} \times y \ or \ S_1 = 2F_{c1} \times \frac{x}{y} = 948\,N$$

$S_2 = S_1$ +h.s = 2448 N

Centrifugal force at the maximum speed,

$$F_{c1} = m(\omega_2)^2 r_2 = m\left(\frac{2\pi N_2}{60}\right)^2 r_2 = 0.00746(N_2)^2$$

Taking moments when in maximum position

$$F_{c2} \times x = \frac{M.g + S_2}{2} \times y$$

N^2 = 331 rpm

Initial compression of the spring

S_1/s = 37.92

Governor effort,

$$P = \frac{S_2 - S_1}{2} = 750\,N$$

Governor power = P× h= 45 N-m

UNIT – II

BALANCING

2.1 Introduction

If all the rotating and reciprocating parts of a high speed engines and other machines are not balanced, the dynamic forces are set up which increase the loads on bearings and stresses in various members. These forces also produce unpleasant and even dangerous vibrations.

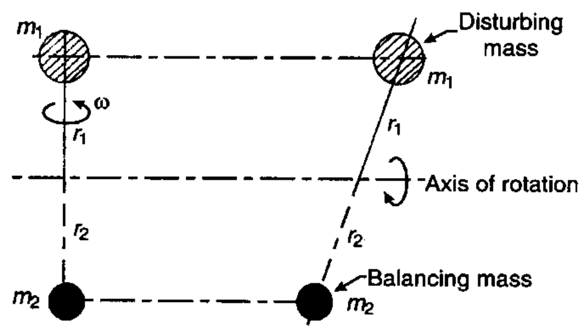
2.2 Balancing of Rotating Masses

Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force. Its effect is to bend the shaft and to produce vibrations in it. In order to prevent this effect another mass is attached to the opposite side of the shaft. Centrifugal of both the masses are made to be equal and opposite.

2.3 Balancing of a Single Rotating Mass by a Single Mass Rotating in the same plane.

Centrifugal force due to disturbing mass is equal to centrifugal force due to balancing mass.

$F_{c1} = F_{c2}$ or $m_1.\omega^2.r_1 = m_2.\omega^2.r_2$



2.4 Balancing of a Single Rotating Mass by Two Masses Rotating in Different Planes.

By introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. But this type of arrangement gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, the two balancing masses are placed in two different planes in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zoro. This requires that the centre of the masses of the system must lie on the axis of rotation. This is the condition for *static balancing*.

2. The net couple due to the dynamic forces acting on the shaft is equal to zero.

The conditions (1) and (2) together give dynamic balancing. The following are two possible methods of attaching the two balancing masses.

1. The plane of the disturbing mass may be in between the planes of two balancing masses.
2. The plane of the disturbing mass may be on the left or right of the two planes containing the balancing masses.

The two conditions are discusses below.

1. When the plane of the disturbing mass lies in between the planes of the two balancing masses.

The net force acing on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$F_c = F_{c1} + F_{c2}$ or $m.\omega^2.r = m_1.\omega^2.r_1 + m_2.\omega^2.r_2$

$m.r = m_1.r_1 + m_2.r_2$ (i)

Taking moments about P

$F_{c1} \times l = F_c \times l_2$ or $m_1.r_1 = m.r \times \frac{l_2}{l}$ (ii)

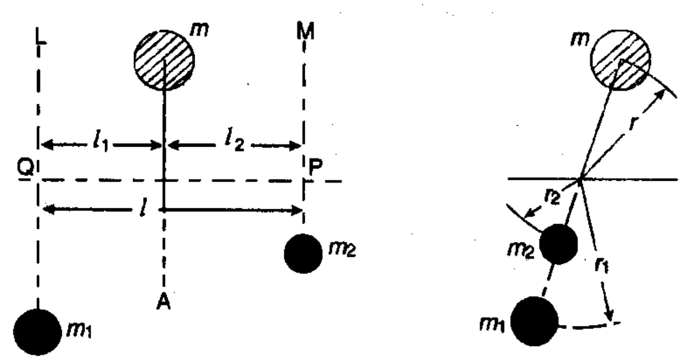
In order to find the balancing force in the plane M, take moments about Q.

Therefore

$F_{c2} \times l = F_c \times l_1$ or $m\ .r_2 = m.r \times \frac{l_1}{l}$ (iii)

Equation (i) represents condition for static balancing .For dynamic balance equations (ii) and (iii) must be satisfied.

2. When the plane of the disturbing mass lies on one end of the planes of the balancing masses.



The following conditions must be satisfied in order to balance the system

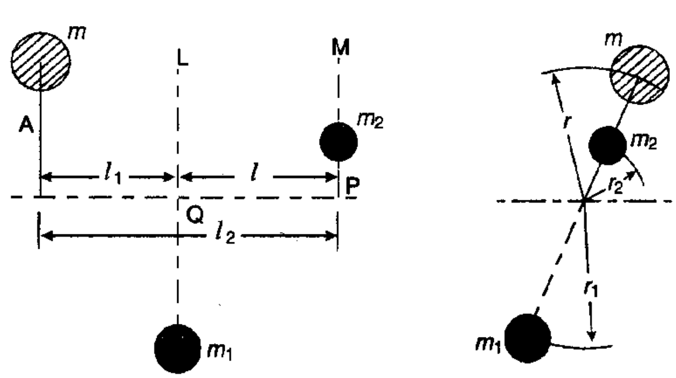
$m.r = m_2.r_2 = m_1.r_1$

Balancing mass in the plane L is given by the equation

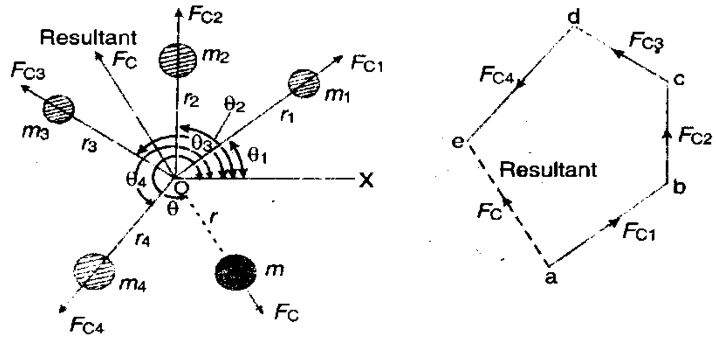
$$m_1.r_1 = m.r \times \frac{l_2}{l}$$

Balancing mass in the plane M is given by the equation.

$$m_2.r_2 = m.r \times \frac{l_1}{l}$$



2.5 Balancing of Several masses rotating in the Same Plane



(a) Space diagram. (b) Vector diagram.

Resolve the centrifugal forces horizontally and vertically and find their sum.

Sum of the horizontal components of the centrifugal forces

$$\Sigma H = m_1.r_1 \cos\theta_1 + m_2.r_2 \cos\theta_2 + \ldots$$

Sum of the vertical components of the centrifugal forces

$$\Sigma V = m_1.r_1 \sin\theta_2 + m_2.r_2 \sin\theta_2 + \ldots$$

Magnitude of the resultant centrifugal force

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

If θ is the angle which the resultant force makes with the horizontal, then

$$\tan\theta = (\Sigma H) \;/\; (\Sigma V)$$

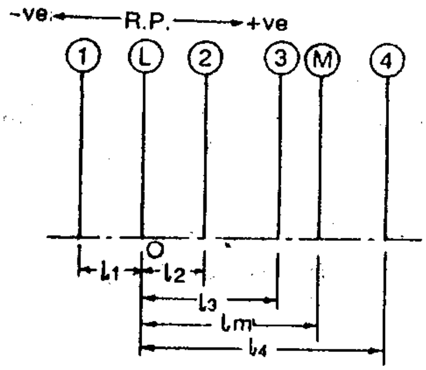
The balancing force is then equal to the resultant force, but in opposite direction

2.6 Balancing of Several Masses Rotating in Different Planes

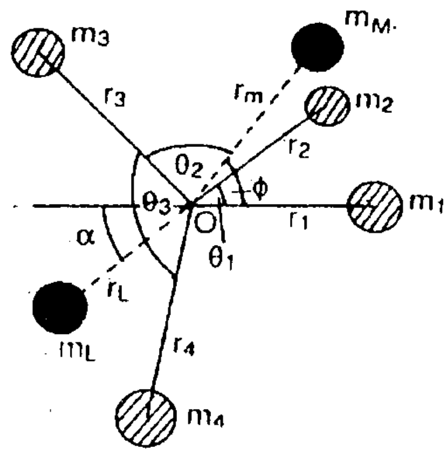
When several masses revolve in different Planes, they may be transferred to a reference plane. The following two conditions must be fulfilled

1. The forces in the reference plane must balance.
2. The couples about the reference plane must balance.

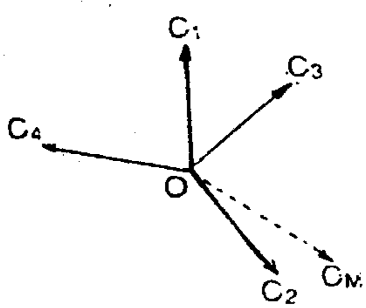
Plane (1)	*Mass (m)* (2)	*Radius (r)* (3)	*Cent. force ÷ ω^2 (m.r)* (4)	*Distance from plane L (l)* (5)	*Couple ÷ ω^2 (m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L (R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



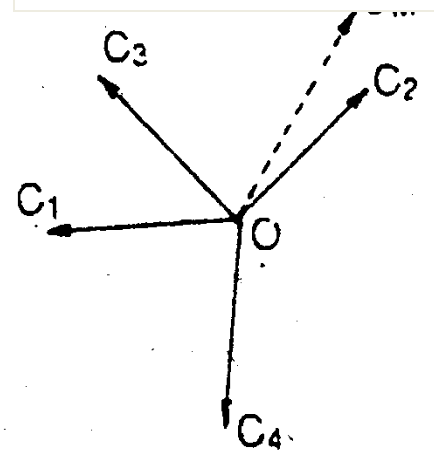
(*a*) **Position of planes of the masses.**



Angular position of the masses



Couple vector diagram



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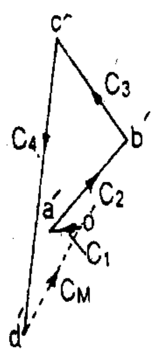
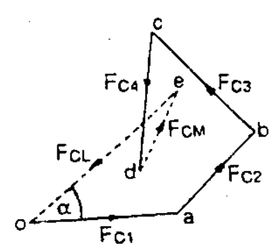
Couple vector turned counter clockwise through a right angle



Couple polygon



Force polygon

2.7 Balancing of Reciprocating Masses.

Consider a horizontal reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts

F_I = Inertia force due to reciprocating parts.

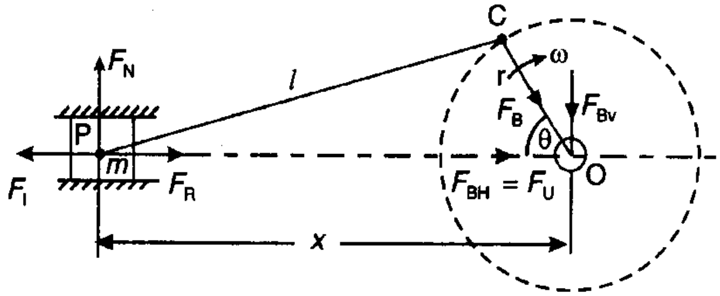


Fig. 22.1. Reciprocating engine mechanism.

F_N = Force on the side of the cylinder walls or normal force acting on the cross-head guides.

F_B = Force acting on the crankshaft bearing or main bearing.

F_R and F_I balance each other.

The force $F_{BH} = F_U$ is an unbalanced force or shaking force and is required to be properly balanced.

The force on the side of the cylinder walls (F_N) and the vertical component of F_B (F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$. Shaking force and shaking couple cause very objectionable vibrations.

2.8 Primary and Secondary Unbalanced Forces of Reciprocating Masses.

Consider a reciprocating engine mechanism.

Let m = mass of the reciprocating parts

l = length of the connecting rod PC

r = radius of the crank PC

ɵ = Angle of inclination of crank with the line of stroke PO,

ω = Angular speed of the crank,

n = ratio of length of connecting rod to the crank radius = l/r

Acceleration of the reciprocating parts is given by the relation

$$a_R = \omega^2 . r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Inertia force due to reciprocating parts $$a_R = m.\omega^2 . r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$F_I = F_R$ = mass × acceleration $= m.\omega^2.r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

Horizontal component of the force exerted on the crankshaft bearing (F_{BH}) is equal and opposite to the inertia force (F_I) and is denoted by F_U

$$F_U = m.\omega^2.r\left[\cos\theta + \frac{\cos 2\theta}{n}\right] = m.\omega^2.r\cos\theta + m.\omega^2.r \times \frac{\cos\theta}{n}$$

The expression $m.\omega^2.r\cos\theta$ is known as primary unbalanced force and

$m.\omega^2.r \times \frac{\cos\theta}{n}$ Is called secondary unbalanced force

The primary unbalanced force is maximum when ɵ = 0˚ or 180˚.

The secondary unbalanced force is maximum when ɵ = 0˚, 90˚. 180˚ or 270˚.

2.9 Partial Balancing of Unbalanced force in a Reciprocating Engine

The primary unbalanced force ($m.\omega^2.r\cos ɵ$) may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius r. Balancing of primary force is considered as equivalent to balancing of a mass m rotating at the crank radius r. This is balanced by having a mass B at radius b, placed diametrically opposite to the crank pin C.

The primary force is balanced if $B.\omega^2.b\cos\Theta = m.\omega^2.r\cos\Theta$ or B.b = m.r.

The centrifugal force produced due to revolving mass B has also a vertical component of magnitude $B.\omega^2.b.\sin\Theta$. This force remains unbalanced. As a compromise let a fraction ‘c’ of the reciprocating masses is balanced, such that

c.m.r = B.b

Unbalanced force along the line of stroke

$= m.\omega^2.r\cos\Theta - B.\omega^2.b.\cos\Theta = m.\omega^2.r\cos\Theta - c.m.\omega^2.r\cos\Theta$

$= (1 - c)\, m.\omega^2.r\cos\Theta$

And unbalanced force along the perpendicular to the line of stroke

$= B.\omega^2.b.\sin\Theta = c.m.\omega^2.b.\sin\Theta$

Resultant unbalanced force at any moment

$$= \sqrt{\left[(1-c)m.\omega^2.r\cos\theta\right]^2 + \left[c.m.\omega^2.r\sin\theta\right]^2} = m.\omega^2.r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$

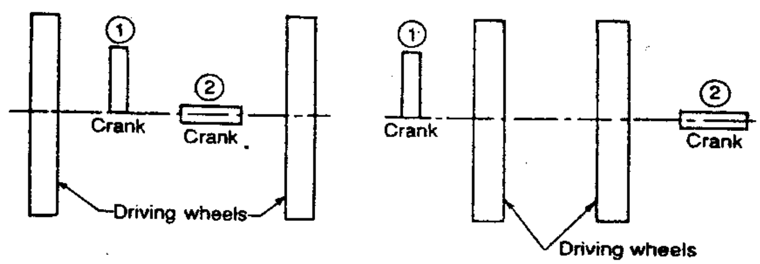
2.10 Partial Balancing of Locomotives

The two cylinder locomotive may be classified as:

1. Inside cylinder locomotives
2. Outside cylinder locomotives.

The locomotives may be

(a) Single or uncoupled locomotives
(b) Coupled locomotives



(a) Inside cylinder locomotives. (b) Outside cylinder locomotives.

In coupled locomotives the driving wheels are connected to the trailing and leading wheels by an outside coupling rod.

2.11 Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives.

Due to partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke; and 2. Swaying couple.
2. The effect of an unbalanced force primary force perpendicular to the line of stroke is to produce variation in pressure.on the rails. The maximum magnitude of the unbalanced force perpendicular to the line of stroke is known as hammer blow.

Variation of Tractive Force

Unbalanced force along the line of stroke for cylinder 1

$= (1 – c)\ m.\omega^2.r \cos \Theta$

Unbalanced force along the line of stroke for cylinder 2

$= (1 – c)\ m.\omega^2.r \cos (90^\circ + \Theta)$

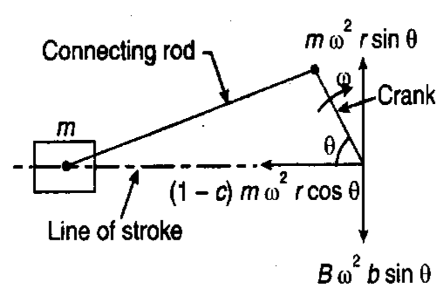


Fig. 22.4. Variation of tractive force.

Tractive force $F_T = (1 – c)\ m.\omega^2.r (\cos \Theta – \sin \Theta)$

The tractive force is maximum or minimum when ($\cos \Theta – \sin \Theta$) is maximum or minimum.

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$

∴ or Θ = 135° or 315°

Maximum or minimum value of tractive force

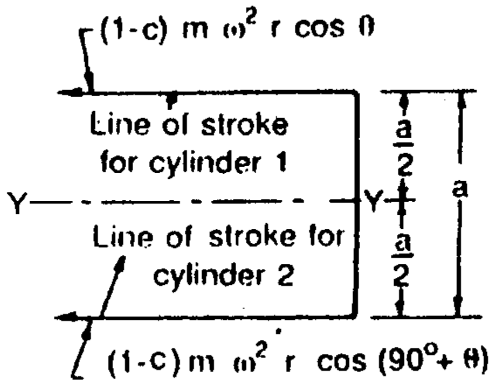
= ± (1- c) m.ω^2.r (cos 135° - sin 135°)

= ± √2 (1– c) m.ω^2.r

2.13 Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about thr centre line YY between the cylinders.

This couple has swaying effect about a vertical axis.



Swaying couple $= (1-c)m.\omega^2.r\cos\theta \times \frac{a}{2} - (1-c)m.\omega^2.r\cos(90^\circ+\theta)\frac{a}{2}$

$= (1-c)m.\omega^2.r\cos\theta \times \frac{a}{2}(\cos\theta + \sin\theta)$

The swaying couple is maximum or minimum when (cos Θ + sin Θ) is maximum or minimum .

he swaying couple is maximum or minimum when Θ = 45° or 225°.

Maximum and minimum value of swaying couple

$$= \pm(1-c)m.\omega^2.r\cos\theta \times \frac{a}{2}(\cos 45^\circ + \sin 45^\circ) \quad = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

2.12 Hammer Blow

Hammer blow = B.ω^2.b

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail.

Let P be the downward pressure on the rails (or static wheel load).

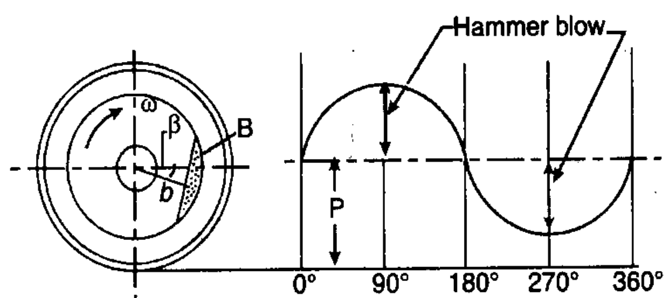


Fig. 22.6. Hammer blow.

Net pressure between the wheel and the rail = P ± $B.\omega^2.b$.

If ($P - B.\omega^2.b$) is negative, then the wheel will be lifted from the rail.. Therefore the limiting condition that the wheel is not lifted from the rail is given by

$$P = B.\omega^2.b$$

$$\omega = \sqrt{\frac{P}{B.b}}$$

2.15 Balancing of Coupled Locomotives

In a coupled locomotive, the driving wheels are connected to the leading and trailing wheels by an outside coupling rod. The coupling rod cranks are placed diametrically opposite to the adjacent main cranks.

2.16 Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same planes and on the same side of the centre line of the crankshafts, are known as In-line engines. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts.

1. The algebraic sum of the primary forces must be equal to zero.
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero.

In order to give the primary balance of reciprocating parts of a multi-cylinder , it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.

Notes: 1. For a two cylinder engine with cranks at 180˚, condition (1) may be satisfied, but an unbalanced couple will remain.

2. For a three cylinder engine with cranks at 120˚ and same reciprocating masses per cylinder , condition (1) will be satisfied but unbalanced couples will remain.

3. For a four cylinder engine, primary balance is possible.

For a four-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided the number of cranks are not less than four.

2.17 Balancing of Secondary Forces of Muti-cylinder In-line Engines

Secondary Force = $F_s = m.\omega^2.r \times \frac{\cos 2\theta}{n}$

The expression may be written as

$$F_s = m.(2\omega)^2.\frac{r}{4n} \times \cos 2\theta$$

The secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed by an imaginary crank of length r/4n and revolving at twice the speed of the actual crank

2.18 Balancing of Radial Engines (Direct and Reverse Cranks Method)

The Primary Forces

Let us suppose that mass (m) of the reciprocating parts is divided into two parts, each equal to m/2. It is assumed that that m/2 is fixed at the primary direct crank pin C and m/2 at the secondary reverse crank pin C'.

The centrifugal force acting on the primary direct and reverse crank =

∴ Component of the centrifugal force acting on the primary direct crank

$$= \frac{m}{2} \times \omega^2 . r \cos\theta$$

and component of the centrifugal force acting on the primary reverse crank

$$= \frac{m}{2} \times \omega^2 . r \cos\theta$$

Total component of the centrifugal force acting along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2 . r \cos\theta = m \times \omega^2 . r \cos\theta = \text{Pr}\,imary\ Force, F_P$$

Secondary Forces

Secondary Force $= m \times (2\omega)^2 . \frac{r}{4n} \times \cos 2\theta = m.\omega^2 r \times \frac{\cos 2\theta}{n}$

2.19 Balancing of V-engines

Inertia force due to reciprocating parts

due to cylinder 1

$$= m.\omega^2 . r \cos(\alpha - \theta) + \frac{\cos 2(\alpha - \theta)}{n}$$

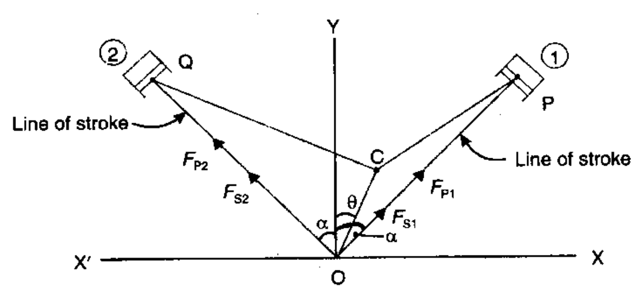


Fig.22.33. Balancing of V-engines.

Inertia force due to reciprocating parts due to cylinder 2

$$= m.\omega^2 . r \cos(\alpha + \theta) + \frac{\cos 2(\alpha + \theta)}{n}$$

$$F_{S1} = m.\omega^2 r \times \frac{\cos 2(\alpha - \theta)}{n}$$

Primary forces
Primary forces acting along the line of stroke of cylinder 1,F_{P1} = m.ω^2 r cos($\alpha - \Theta$)

Component of F_{P1} along the vertical line OY

$$F_{P1}\cos\alpha = m.\omega^2 r\cos(\alpha - \Theta)\cos\alpha$$

Component of F_{P1} along the horizontal line OX

$$F_{P1}\sin\alpha = m.\omega^2 r\cos(\alpha - \Theta)\sin\alpha$$

Primary forces acting along the line of stroke of cylinder 2, F_{P2} = m.ω^2 r cos($\alpha + \Theta$)

$$= F_{S1}\cos\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha-\theta)}{n} \times \cos\alpha$$

Component of F_{P2} along the vertical line OY, $F_{P2}\cos\alpha = m.\omega^2 r\cos(\alpha + \Theta)\cos\alpha$

Component of F_{P1} along the horizontal line OX' $F_{P2}\ \sin\alpha = m.\omega^2 r\cos(\alpha + \Theta)\sin\alpha$

Total component of primary force along the horizontal line OX

$= m.\omega^2 r\cos\alpha\,[\cos(\alpha - \Theta) - \cos(\alpha + \Theta)$

$= m.\omega^2 r\cos\alpha \times 2\cos\alpha\cos\Theta$

$= 2m.\ \omega^2.r\cos^2\alpha.\cos\Theta$

Resultant primary force

$$F_P = \sqrt{\left(F_{PV}\right)^2 + \left(F_{PH}\right)^2}$$
$$= 2m.\omega^2.r\sqrt{(\cos^2\alpha.\cos\theta)^2.+(\sin^2\alpha.\sin\theta)^2}$$

Secondary forces
Secondary force acting along the line of stroke of cylinder 1,

$$F_{S1} = m.\omega^2 r \times \frac{\cos 2(\alpha-\theta)}{n}$$

Component of F_{S1} along the vertical line OY

$$= F_{S1}\cos\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha-\theta)}{n} \times \cos\alpha$$

Component of F_{S1} along the horizontal line OX

$$F_{S1}\sin\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha-\theta)}{n} \times \sin\alpha$$

Secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n}$$

Component of F_{S2} along the vertical line OY

$$F_{S2}\cos\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n} \times \cos\alpha$$

Component of F_{S2} along the Horizontal line OX'

$$F_{S2}\sin\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n} \times \sin\alpha$$

Total component of secondary force along the vertical line OY,

$$F_{SV} = \frac{m}{n} \times \omega^2 .r\cos\alpha\left[\cos 2\left(\alpha-\theta\right)+\cos 2\left(\alpha+\theta\right)\right]$$

$$= \frac{2m}{n} \times \omega^2 .r\cos\alpha \times \cos 2\alpha \cos 2\theta$$

Total component of secondary force along the vertical line OX',

$$F_{SH} = \frac{m}{n} \times \omega^2 .r\sin\alpha\left[\cos 2\left(\alpha-\theta\right)-\cos 2\left(\alpha+\theta\right)\right]$$

$$= \frac{2m}{n} \times \omega^2 .r\sin\alpha \times \cos 2\alpha \sin 2\theta$$

Resultant secondary force

$$F_s = \frac{2m}{n}.\omega^2 .r\sqrt{(\cos\alpha.\cos 2\alpha.\cos 2\theta)^2.+(\sin\alpha\sin 2\alpha.\sin 2\theta)^2}$$

Example 1: Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m. 0.15 m, 0.25 m and 0.3 m respectively. And the angles between the successive masses are 45˚, 75˚ and 135˚. Find the position and magnitude of the balance mass required if the radius of rotation is 0.2 m.

Example 2: A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg,, mass of revolving parts at 150 mm radius 37 kg. If two-third of reciprocating parts and all the revolving parts are to be balanced, find: 1. The balance mass required at a radius of 400 mm, and 2. The residual unbalanced force when the crank has rotated 60˚ from top dead centre.

Solution:

1, Balance mass required

2. Residual unbalanced force

Residual unbalanced force = 712.2 N

B = 26.38 kg Ans.

Example 3: A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm,C to 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45˚, B to C 70˚ and C to D 120˚ The balancing masses are to be placed in planes X and Y. the distance between the planes A and X are 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Solution.

Plane (1)	Mass (m) kg (2)	Radius (r) m 3)	Cent. force÷ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple÷ω^2 (m.r.l) kg-m^2 (6)
A	200	0.08	16	0.1	1.6
X(R.P.)	m_x	0.1	$0.1m_x$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1m_y$	0.4	$0.04m_Y$
D	200	0.08	16	0.6	0.6

The vector d'o' represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y , therefore by measurements

0.04 m_Y = vector d'o' = 7.3 kg.m^2 or m_Y = 182.5 kg.

The angular position of mY is ɵY = 12˚ in the clockwise direction from mass mA.

From force polygon by measurements

0.1 m_X = vector eo = 35.5 kg-m or m_X = 355 kg .

By measurement, the angular position of m_X is Θ_X = 145˚ in the clockwise direction from mass m_A

Example 3: An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

The whole of the rotating masses and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

Solution:

Example: The following particulars relate to a two cylinder locomotive with two coupled wheels on each side:

Stroke	= 650 mm
Mass of reciprocating parts per cylinder	= 240 kg
Mass of revolving parts per cylinder	= 200 kg
Mass of each coupling rod	= 250 kg
Radius of centre of coupling rod pin	= 250 mm
Distance between cylinders	= 0.6 m
Distance between coupling rods	= 1.8 m

The main cranks are at right angles and the coupling rod pins are at 180˚ to their respective main cranks. The balance masses are to be placed at the wheels at a mean radius of 675 mm in order to balance whole of the revolving and 3/4th of the reciprocating masses. The balance mass for the reciprocating masses is to divided equally between the driving wheels and the coupled wheels. Find: 1. The magnitude and angular position of the masses required for the driving and trailing wheels, and 2. The hammer blow at 120 km/h, if the wheels are 1.8 metre diameter.

Solution:

Example 4: A four cylinder vertical engine has cranks 150 mm long. The planes of the rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.

Solution:

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r) kg-m (4)	Distance from plane 3(l) m (5)	Couple÷ω^2 (m.r.l) kg-m^2 (6)
1	50	0.15	7.5	–0.4	-3
2	60	0.15	9	–0.2	-1.8
3(R.P.)	m_1	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

$\Theta_2 = 160^\circ$, $\Theta_4 = 26^\circ$, $m_3 = 60$ kg, $\Theta_3 = 227^\circ$

Example 6: A vee-twin engine has the cylinder axis at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius id 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be

Solution:

$$F_P = m.\omega^2.r$$

$$F_s = \sqrt{2} \times \frac{m}{n}.\times \omega^2.r \sin 2\theta$$

This is maximum when sin 2Θ is maximum i.e. when 2Θ = ±1 or Θ = 45° or 135°.

Maximum resultant secondary force = 8.36 N.

Unit – 3

Gyroscope

3.1 Introduction. 1.When a body moves along a curved path, a force in the direction of centripetal acceleration (centripetal force) has to be applied externally. This external force is known as active force.

2. When a body is moving along a circular path, it is subjected to the centrifugal force radially outwards. This centrifugal force is known as reactive force.

Note: Whenever the effect of any force or couple is to be considered, it should be with respect to reactive force. or couple.

3.2 Precessional Angular Motion

Consider a disc spinning about the axis OX (axis of spin) with an angular velocity ω. After a short interval of time δt, let the disc be spinning about the new axis of spin OX'

Total angular acceleration of the disc

$$= \frac{d\omega}{dt} + \omega.\omega_P$$

Angular velocity of the axis of spin is known as angular velocity of precession. The axis about axis of spin is to turn is known as axis of precession.

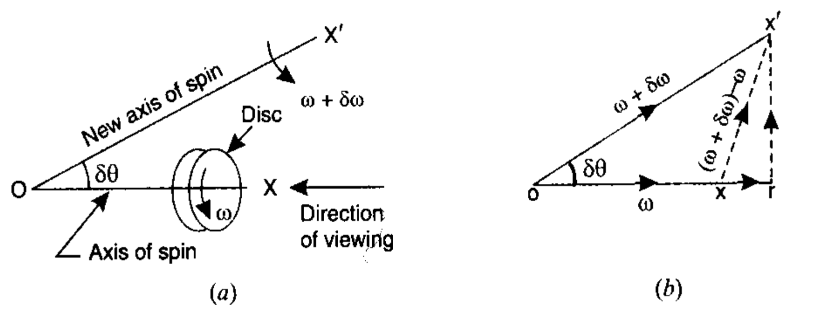


Fig. 14.1. Precessional angular motion.

If the angular velocity of the disc changes direction but remains constant in magnitude, then angular acceleration of the disc is given by

$\alpha_c = \omega.d\Theta/dt = \omega.\omega_P$

The angular acceleration α_c is known as gyroscopic acceleration.

3.3 Gyroscopic Couple

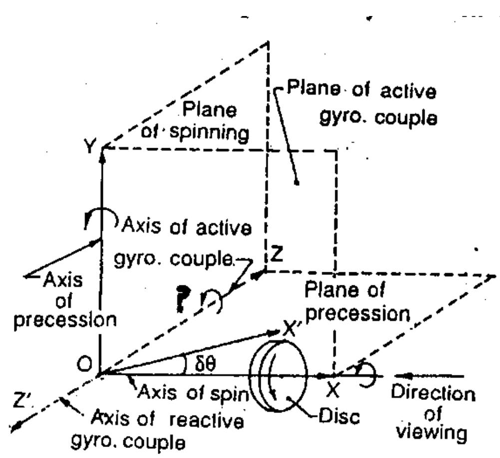
Consider a disc spinning with an angular velocity ω rad/s.

Angular momentum of the disc = I.ω

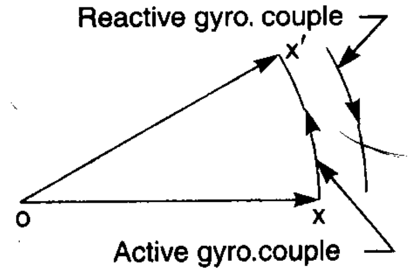
The couple applied to the disc causing precession

C = $I.\omega.\omega_P$

The couple $I.\omega.\omega_P$ in the direction of the



vector xx' (representing the change in angular momentum) is the active gyroscopic couple which has to be applied over the disc.



When the axis of spin moves with an angular velocity ω_P, the disc is subjected to reactive gyroscopic couple which is opposite in direction to that of active couple.

3.4 Effect of the Gyroscopic Couple on an Aeroplane

Let ω = Angular velocity of the engine in rad/s,

m = mass of the engine and propeller in kg,

k = Its radius of gyration in metres,

I – Mass moment of inertia of the engine and propeller in kg-m^2 = $m.k^2$

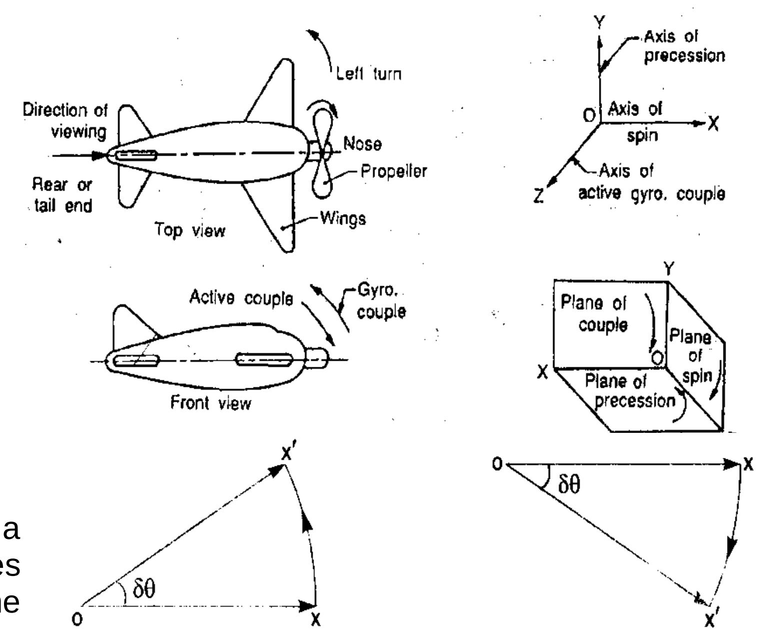
v = Linear velocity of the aeroplane in m/s

R = radius of curvature in metres, and

ω_P = Angular velocity of precession = $\frac{v}{R}$ rad/s

Notes:

1. When the aeroplane takes a left turn, the effect of the reactive gyroscopic couple will be to raise the nose and dip the tail.
2. When the aeroplane takes a right turn, the effect will be to dip the nose and raise the tail.
3. When the engine rotates in anticlockwise direction when viewed from the front and the aeroplane takes a left turn, the effect will be to raise the tail and dip the nose.
4. When the aeroplane takes a right turn and the engine rotates in anticlockwise direction, the effect will be to raise the nose and dip the tail.
5. When the engine rotates in clockwise direction and the aeroplane takes a left turn, the effect will be to raise the tail and dip the nose.
6. When the aeroplane takes a right turn and the engine rotates in clockwise direction, the effect will be to raise the nose and dip the tail.



(a) Aeroplane taking left turn. (b) Aeroplane taking right turn.

Fig. 14.6. Effect of gyroscopic couple on an aeroplane.

3.5 Effect of Gyroscopic Couple on a Naval Ship during Steering

1. When the rotor of the ship rotates in the clockwise direction when viewed from the stern, and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.
2. When the ship steers to the right and the rotor rotates in the clockwise direction, the effect will be to raise the stern and lower the bow.
3. When the rotor rotates in the anticlockwise direction and the ship steers to the left and the effect will be to lower the bow and raise the stern

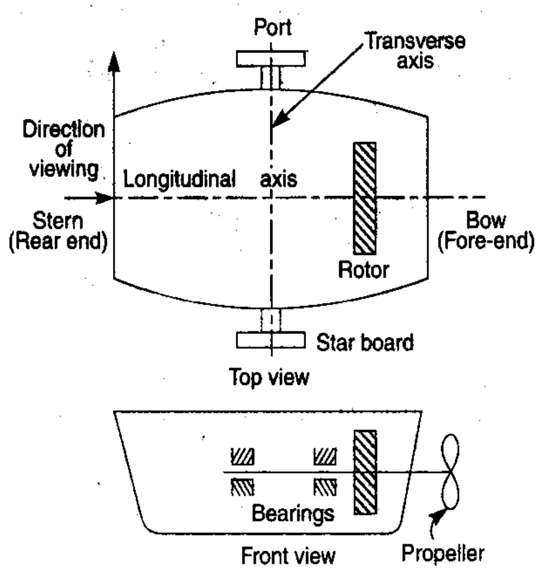


Fig. 14.7. Terms used in a naval ship.

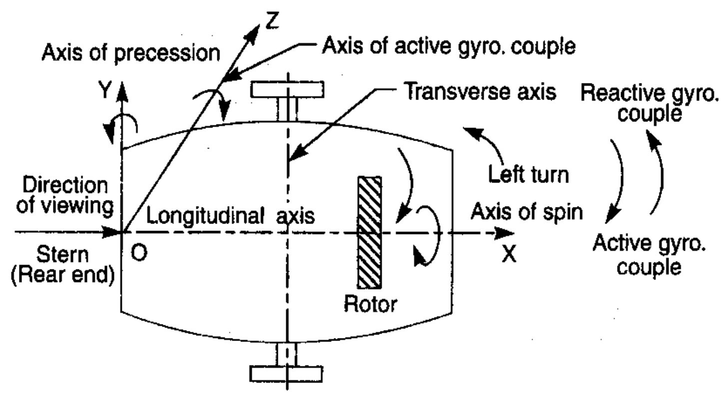


Fig. 14.8. Naval ship taking a left turn.

3.6 Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis. The pitching of the ship is assumed to takes place with simple harmonic motion.

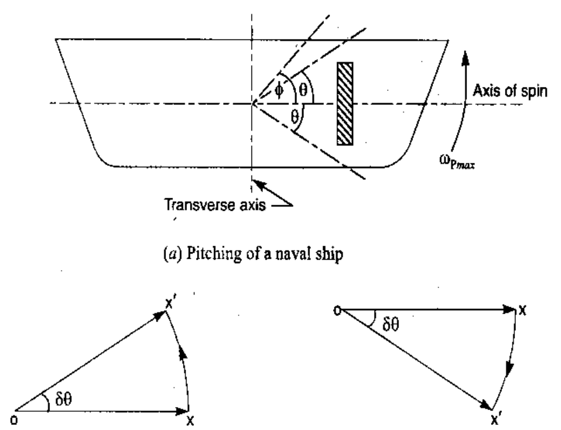
Angular displacement of the axis of spin from mean position after time t seconds

$\theta = \phi \sin \omega_1.t$

where ϕ = Amplitude of swing, t_p

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{Time\ period\ of\ S.H.M.in\ seconds} = \frac{2\pi}{t_p}$$



(a) Pitching of a naval ship

(b) Pitching upward (c) Pitching downward

Fig. 14.10. Effect of gyroscopic couple on a naval ship during pitching.

Maximum angular velocity of precession

$\omega_{Pmax} = \phi.\omega_1 = \phi \times 2\pi/t_P$

Maximum gyroscopic couple $C_{max} = I.\ \omega.\ \omega_{Pmax}$

When the pitching is upward, the reactive gyroscopic couple will try to move the ship toward star-board. If the pitching is downward, the effect is to turn the ship towards port side.

Note: There is no effect of the gyroscopic couple acing on the body of the ship.

3.7 Stability of a Four Wheel Drive Moving in a Curved Path

Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons =m.g,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres,

h = Distance of c.g. vertical above the road surface in metres,

x = Width of track in metres,

I = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W = Angular velocity of the wheels,

I = Mass moment of inertia of the rotating parts of the engine in rad/s,

G = gear ratio = ω_E/ω_W

v = Linear velocity of the vehicle in m/s = $\omega_W.r_W$

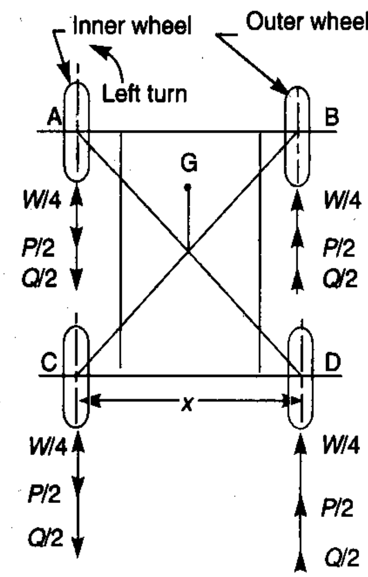


Fig. 14.11. Four wheel drive moving in a curved path.

1. Effect of the gyroscopic couple

Velocity of precession

$\omega_P = v/R$

Gyroscopic couple due to four wheels,

$C_W = 4I_{W.}\omega_P.\omega_P$

Gyroscopic couple due to rotating parts of the engine

$C_E = I_{E.}\omega_E.\omega_P = I_{E.}G\omega_W.\omega_P$

Net gyroscopic couple = $C_W \pm C_E = \omega_W.\omega_P\ (4I_W \pm G.I_E)$

Example 1: A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 rpm. clockwise, looking from the front, with what speed will it precess about the vertical axis. **K/**

Solution:

Example 2: An aeroplane makes a complete half circle of 50 metres radius, towards left, When flying at 200 km/hr. The rotary engine and propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Solution:

Example 3: The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 rpm clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/h and steer to the left in a curve of 75 m radius.

Solution:

UNIT – IV

Mechanical Vibrations

Introduction: When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion.

4.1 Types of Vibratory Motion

1. **Free or natural vibrations.** When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations
2. **Forced vibrations.** When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The external force is a periodic disturbing force created due to unbalance. The vibrations have the same frequency as the applied force. When the frequency of external force is same as that of the natural vibrations, resonance takes place.
3. **Damped vibrations.** When there is reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration. A certain amount of energy is always dissipated in overcoming frictional resistance to the motion.

Types of Free Vibrations

1. Longitudinal vibrations 2. Transverse vibrations 3. Torsional vibrations.

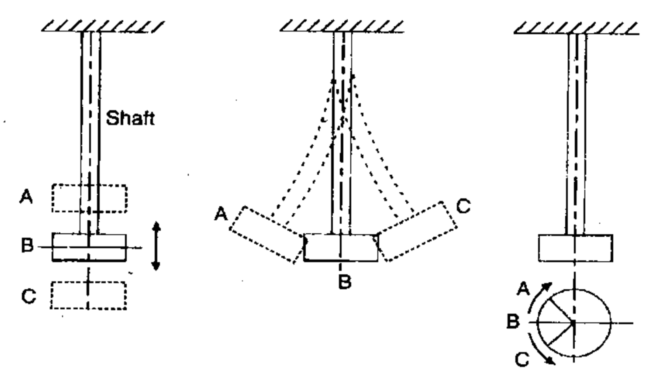
4. (a) Define free vibration and natural frequency. 2

(W08)

iv. (a) The natural frequency (in Hz) of free longitudinal vibrations is equal to

(i) $\frac{1}{2\pi}\sqrt{\frac{s}{m}}$ (ii) $\frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ (iii) $\frac{0.4985}{\sqrt{\delta}}$

(iv) Any one of these 2 marks



B = Mean position ; A and C = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

Fig. 23.1. Types of free vibrations.

4.2 Natural Frequency of Free Longitudinal Vibrations

Equilibrium Method

Consider a constraint (i.e. spring) of negligible mass in an unstrained position.

Let s = stiffness of the constraint.

m = Mass of the body suspended from the constraint in kg.

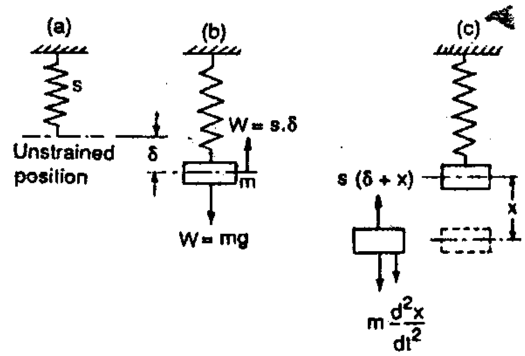
W = Weight of the body in newtons =m.g

δ = Static deflection of the spring in metres due to weight W newtons, and

x = displacement given to the body by the external force

Natural frequency

$$f_n = \frac{1}{t_P} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

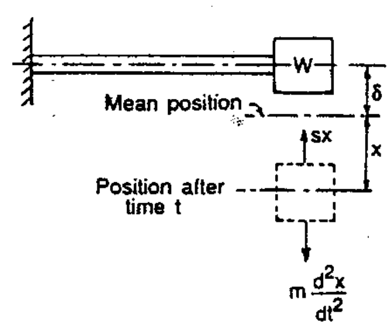


Natural Frequency of Free Transverse Vibrations

Consider a shaft of negligible mass, whose one end is fixed and the other end carries a body of weight W

Natural frequency

$$f_n = \frac{1}{t_P} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$



(b) Derive an expression for the natural frequency of free longitudinal vibrations by equilibrium method.

Natural Frequency of Free Transverse Vibrations

Let s = Stiffness of shaft

δ = Static deflection due to weight of the body

x = Displacement of body from mean position after time t

m = Mass of the body after time t = W/g

7

(W07)

Time period,

$$t_p = 2\pi\sqrt{\frac{m}{s}}$$

Natural frequency $$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

(c) Establish an expression for the natural frequency of free transverse vibrations for a simply supported beam carrying a number of point loads, by

Energy method

Dunkerley's method 7 marks

(W08)

4.3 Natural Frequency of Free Transverse Vibrations for a shaft subjected to a Number of Point Loads

Consider a shaft AB of negligible mass loaded with point loads W_1, W_2, W_3 and W_4 etc. in Newtons. Let m_1, m_2, m_3 and m_4 etc. be the corresponding masses in kg.

1. **Energy (or Rayleigh's) Method**

Let y_1, y_2, y_3, y_4 etc. be total deflection under loads W_1, W_2, W_3, W_4 etc.

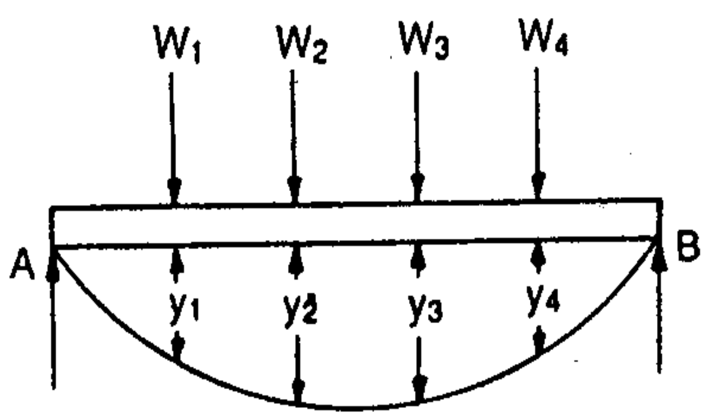


Fig. 23.11. Shaft carrying a number of point.loads.

Maximum potential energy

$$= \frac{1}{2} \times m_1.g.y_1 + m_2.g.y_2 + m_3.g.y_3 + m_4.g.y_4 + ...$$
$$= \frac{1}{2}\sum m.g.y$$

Maximum kinetic energy

$$= \frac{1}{2} \times m_1.(\omega.y_1)^2 + \frac{1}{2} \times m_2.(\omega.y_2)^2 + \frac{1}{2} \times m_3.(\omega.y_3)^2 + \frac{1}{2} \times m_4.(\omega.y_4)^2 + ...$$
$$= \frac{1}{2} \times \omega^2 \sum m.y^2$$

Equating the maximum K.E. to the maximum P.E.

$$\frac{1}{2}\times\omega^2\sum m.y^2 = \frac{1}{2}\sum m.g.y$$

$$\therefore \quad \omega^2 = \frac{\sum m.g.y}{\sum m.y^2} \quad or \quad \omega = \sqrt{\frac{g\sum m.y}{m.y^2}}$$

Natural Frequency of transverse vibrations

$$f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g\sum m.y}{\sum m.y^2}}$$

2. Dunkerley's method

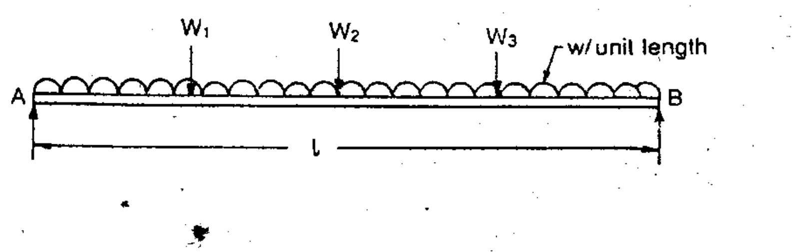
According to Dunkerley's empirical formula

where f_n = Natural frequency of transverse vibrations of the shaft carrying point loads and uniformly distributed load.

f_{n1}, f_{n2}, f_{n3} etc. = Natural frequency of transverse vibration of each point load.

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} +$$

F_{ns} = Natural frequency of transverse vibration of uniformly distributed load (or due to mass of the shaft)



.12. Shaft carrying a number of point loads and a uniformly distributed load.

Consider a shaft loaded as shown in the figure.

Let δ_1, δ_2, δ_3, etc. = Static deflection due to the load W_1, W_2, W_3 etc. when considered separately.

Natural frequency of transverse vibration due to load W_1, $f_{n1} = \frac{0.4985}{\sqrt{\delta_1}}$

Natural frequency of transverse vibration due to load W_2, $f_{n2} = \frac{0.4985}{\sqrt{\delta_2}}$

Natural frequency of transverse vibration due to load W_3, $f_{n3} = \frac{0.4985}{\sqrt{\delta_3}}$

Natural frequency of transverse vibration due to uniformly distributed load or weight of the shaft, $f_{ns} = \frac{0.4985}{\sqrt{\delta_s}}$

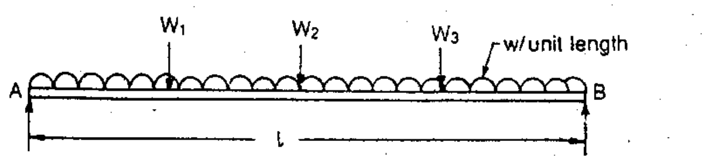
According to Dunkerley's empirical formula, the natural frequency of the whole system,

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} + ... + \frac{1}{(f_{ns})^2}$$

where f_n = Natural frequency of transverse vibrations of the shaft carrying point loads and uniformly distributed load.

f_{n1}, f_{n2}, f_{n3} etc. = Natural frequency of transverse vibration of each point load.

F_{ns} = Natural frequency of transverse vibration of uniformly distributed load (or due to mass of the shaft)



.12. Shaft carrying a number of point loads and a uniformly distributed load.

Consider a shaft loaded as shown in the figure.

Let δ_1, δ_2, δ_3, etc. = Static deflection due to the load W_1, W_2, W_3 etc. when considered separately.

Natural frequency of transverse vibration due to load W_1, $f_{n1} = \frac{0.4985}{\sqrt{\delta_1}}$

Natural frequency of transverse vibration due to load W_2, $f_{n2} = \frac{0.4985}{\sqrt{\delta_2}}$

Natural frequency of transverse vibration due to load W_3, $f_{n3} = \frac{0.4985}{\sqrt{\delta_3}}$

Natural frequency of transverse vibration due to uniformly distributed load or weight of the shaft,

$$f_{ns} = \frac{0.4985}{\sqrt{\delta_s}}$$

According to Dunkerley's empirical formula, the natural frequency of the whole system,

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} + ... + \frac{1}{(f_{ns})^2}$$

$$= \frac{\delta_1}{(0.4985)^2} + \frac{\delta_2}{(0.4985)^2} \frac{\delta_3}{(0.4985)^2} + ... \frac{\delta_s}{(0.4985)^2}$$

$$= \frac{1}{(0.4985)^2}\left[\delta_1 + \delta_2 + \delta_3 + ... + \frac{\delta_s}{1.27}\right]$$

or $$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + ... + \frac{\delta_s}{1.27}}}$$

The value of simply supported shaft may be obtained from the relation

$$\delta = \frac{Wa^2b^2}{3EIl}$$

Q. What is whirling or critical speed?

4.4 Critical or Whirling Speed of a Shaft

A rotating shaft carries different mountings and accessories in the form of gears and pulleys. The centre of gravity of these mountings does not coincide with the axis of the shaft. As a result, when the shaft rotates, it is subjected to centrifugal force. This force will bend the shaft which will further increase the distance of the c.g.

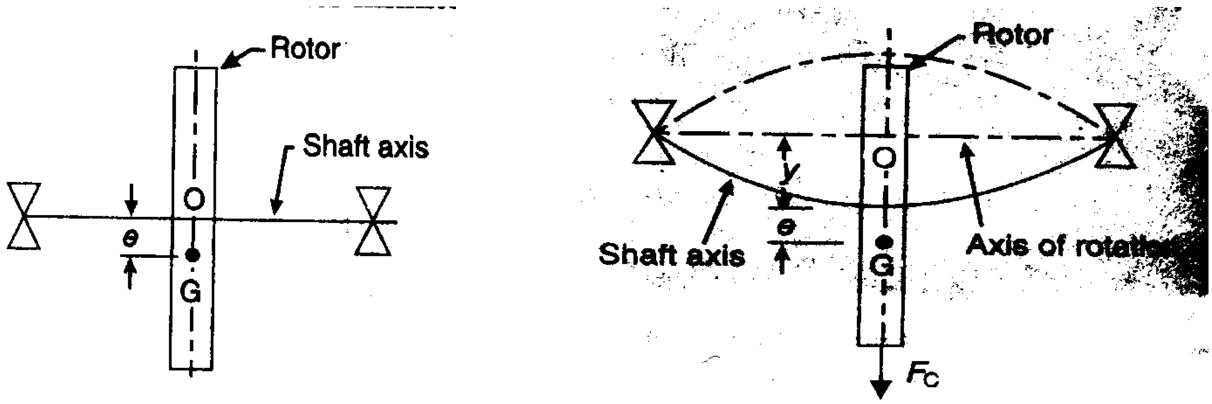
This correspondingly increases the value of centrifugal force which further increases the distance of c.g. This effect is cumulative and ultimately the shaft fails. The bending of shaft not only depends upon the value of eccentricity but also upon the speed of the shaft.

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.

$$N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}} r.p.s.$$

$$\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}}$$

Critical or whirling speed is the same as the natural frequency of transverse vibrations. Its unit is revolutions per second.



(*a*) When shaft is stationary.

(*b*) When shaft is rotating.

4.6 Frequency of Free Damped Vibrations

(Viscous Damping)

The motion of a body is resisted by frictional forces. In vibrating systems, the effect of friction is referred to as damping. The damping provided by fluid resistance is known as viscous damping.

In damped vibrations the amplitude of the resulting vibration gradually diminishes.

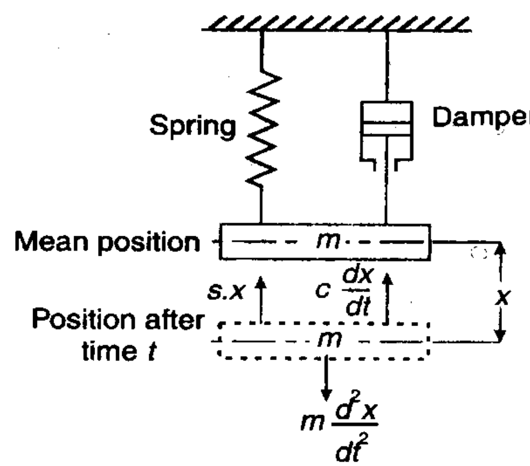


Fig. 23.17. Frequency of free damped vibrations.

Damping force or frictional force on the mass acting in opposite direction to the motion of the mass = $c \times \frac{dx}{dt}$

Accelerating force on the mass = $m \times \frac{d^2x}{dt^2}$

Spring force on the mass, acting in opposite direction to the motion of the mass = s.x

Therefore the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -\left(c \times \frac{dx}{dt} + s.x\right)$$

(Negative sign indicates that force opposes the motion)

or $$\frac{d^2x}{dt^2} + \frac{c}{m} \times \frac{dx}{dt} + \frac{s}{m} \times x = 0$$

This is a differential equation of the second order. Assuming a solution of the form $x = e^{kt}$ where k is a constant to be determined. The differential equation reduces to

$$k^2.e^{kt} + \frac{c}{m} \times k.e^{kt} + \frac{s}{m} \times e^{kt} = 0$$

or $$k^2 + \frac{c}{m} \times k + \frac{s}{m} = 0$$

and $$k = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4 \times \frac{s}{m}}}{2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}$$

The two roots of the equation are

$$k = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}} \quad \text{and} \quad k = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}$$

The most general solution of the differential equation is

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t}$$

where C_1 and C_2 are two arbitrary constants which are to be determined from the initial conditions of the motion of the mass. The roots k_1 and k_2 may be real, complex conjugate or equal.

When the roots are real (overdamping)

If $\left(\frac{c}{2m}\right)^2 > \frac{s}{m}$ then the roots k1 and k2

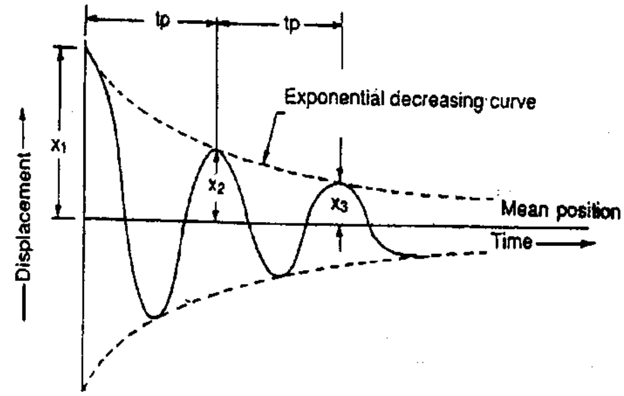
are real but negative. This is a case of overdamping and the mass moves slowly to the equilibrium position. This motion is known as aperiodic.

In actual practice overdamped vibrations are avoided.

When the roots are complex conjugate (underdamping)

If $\frac{s}{m} > \left(\frac{c}{2m}\right)^2$, then the radical becomes negative. The two roots are then known as complex conjugate. This is most practical case of damping and it is known as underdamping .

The two roots are



$$k_1 = -\frac{c}{2m} + \iota\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

and $$k_2 = -\frac{c}{2m} - \iota\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

For the sake of mathematical calculations, let

$$\frac{c}{2m} = a; \frac{s}{m} = (\omega_n)^2 ; and$$
$$\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2} = \omega_d = \sqrt{(\omega_n)^2 - a^2}$$

Therefore the two roots may be written as

k_1 = -a + í ω_d and k_1 = -a + í ω_d

General solution of a differential equation is

If t is measured from the instant at which the mass m is released after an initial displacement A, then

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t} = C_1 e^{(-a+\iota\omega)t} + C_2 e^{(-a-\iota\omega)t}$$
$$= e^{-at}\left(C_1 e^{\iota\omega_d . t} + C_1 e^{-\iota\omega_d . t}\right)$$

$x = Ae^{-at} \cos\omega_d .t$

where $$\omega_d = \sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{(\omega_n)^2 - a^2}; \; a = \frac{c}{2m}$$

The motion is S.H.M. whose circular damped frequency is ω_d and the amplitude is Ae^{-at} which diminishes exponentially with time. The oscillations may take some considerable time to die away.

Periodic time of vibration

$$t_P = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

Frequency of damped vibration

$$f_d = \frac{1}{t_P} = \frac{\omega_d}{2\pi} = \frac{1}{2\pi}\sqrt{(\omega_n)^2 - a^2} = \frac{1}{2\pi}\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

(a) What do you understand by over-damped, under damped and critical damped system?

2 marks

(S08)

(a) Show that the ratio of two successive amplitudes of oscillations is constant in a damped vibratory system. 7 marks

(W08)

3. When the roots are equal (critical damping)

When $\left(\frac{c}{2m}\right)^2 = \frac{s}{m}$ the two roots are equal. This is known as critical damping. In this case frequency of damped vibrations (f_d) is zero. This type of damping is also avoided because the mass moves back rapidly to its equilibrium position in the shortest possible time.

The critical damping coefficient

$$c_c = 2m\sqrt{\frac{s}{m}} = 2m \times \omega_n$$

The critical damping coefficient is the amount of damping required for a system to be critically damped.

Damping Factor or Damping Ratio

The ratio of the actual damping coefficient (c) to the critical damping coefficient (c_c) is known as damping factor or damping ratio.

Logarithmic Decrement

It is defined as the natural logarithm of the amplitude reduction factor. Amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position.

If x_1 and x_2 are successive values of the amplitude on the same side of the mean position, then the amplitude factor,

$$\frac{x_1}{x_2} = \frac{Ae^{-at}}{Ae^{-a\left(t+t_p\right)}} = e^{at_p}$$

Logarithmic decrement

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = a.t_p = a \times \frac{2\pi}{\omega_d} = \frac{a \times 2\pi}{\sqrt{\left(\omega_n\right)^2 - a^2}}$$

$$= \frac{\frac{c}{2m} \times 2\pi}{\sqrt{\left(\omega_n\right)^2 - \left(\frac{c}{2m}\right)^2}} = \frac{\frac{c}{2m} \times 2\pi}{\omega_n\sqrt{1 - \left(\frac{c}{2m\omega_n}\right)^2}}$$

$$= \frac{c \times 2\pi}{c_c\sqrt{1 - \left(\frac{c}{c_c}\right)^2}} = \frac{2\pi \times c}{\sqrt{\left(c_c\right)^2 - c^2}}$$

In general, amplitude reduction factor,

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \ldots = \frac{x_n}{x_{n+1}} = e^{at_p} = \quad \text{constant}$$

Therefore logarithmic decrement. $\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = a.t_p = \frac{2\pi \times c}{\sqrt{\left(c_c\right)^2 - c^2}}$

(b) Define:

Critical speed

Damping factor

Logarithmic decrement

Underdamping 4 marks

(S09)

(a) Define critical speed of a shaft 01 mark

(b) Explain torsional vibration. 01 mark

(W09)

Frequency of Under Damped Forced Vibrations

Consider a system consisting of spring, mass and damper. Let the system is acted upon by an external periodic (i.e. harmonic) disturbing force.

$F_x = F \cos \omega$

F = Static force

ω = Angular velocity of periodic disturbing force

The displacement x at any time t is given by

$$x = \frac{F}{c^2.\omega^2 + \left(s - m.\omega^2\right)^2} \times \cos\left(\omega.t - \phi\right)$$

Where $\phi = \tan^{-1}\left(\frac{c.\omega}{s - m.\omega^2}\right)$

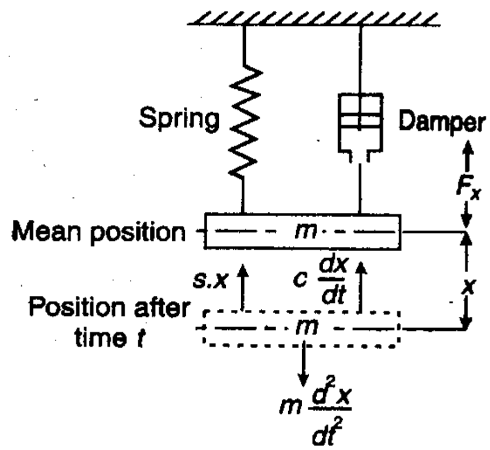


Fig. 23.19. Frequency of under damped forced vibrations.

The equation shows that the motion is S.H.M. whose circular frequency is ω and the amplitude is

$$x = \frac{F}{c^2.\omega^2 + \left(s - m.\omega^2\right)^2}$$

Maximum displacement or amplitude of forced vibration $x_{\max} = \frac{F}{c^2.\omega^2 + \left(s - m.\omega^2\right)^2}$

The equation may be written as $$x_{\max} = \frac{F / s}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{\left(s - m.\omega^2\right)^2}{s^2}}}$$

Substituting $F/s = x_0$ $$x_{\max} = \frac{x_0}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{\left(s - m.\omega^2\right)^2}{s^2}}}$$

where x_0 is the deflection under the static force F

Natural frequency of free vibration is given by

$(\omega_n)^2 = s/m$ $\quad \therefore\ x_{max} = \dfrac{x_0}{\sqrt{\dfrac{c^2.\omega^2}{s^2} + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$

3. When damping is negligible, then c = 0

$$\therefore\ x_{max} = \frac{x_0}{1 - \dfrac{\omega^2}{(\omega_n)^2}} = \frac{x_0(\omega_n)^2}{(\omega_n)^2 - \omega^2} = \frac{x_0 \times s/m}{(\omega_n)^2 - \omega^2} = \frac{F}{m\left[(\omega_n)^2 - \omega^2\right]}$$

$\therefore$

4. At resonance $\quad \omega = \omega_n = \sqrt{\dfrac{s}{m}}\ rad/s$

and $\quad x_{max} = x_0 \times \dfrac{s}{c.\omega_n}$

Magnification Factor or Dynamic Magnifier

It is the ratio of maximum displacement of the forced vibrations (x_{max}) to the deflection due to the static force F(x_0)

Maximum displacement or the amplitude of forced vibration $x_{max} = \dfrac{x_0}{\dfrac{c^2.\omega^2}{s^2} + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}$

$\therefore$ Magnification factor or dynamic magnifier $D = \dfrac{x_{max}}{x_0} = \dfrac{1}{\sqrt{\dfrac{c^2.\omega^2}{s^2} + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}} = \dfrac{1}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$

$$\left[\because\ \frac{c.\omega}{s} = \frac{2c.\omega}{2m \times \dfrac{s}{m}} = \frac{2c.\omega}{2m(\omega_n)^2} = \frac{2c.\omega}{c_c.\omega_n}\right]$$

The magnification factor gives the factor by which the static deflection (i.e. x_0) must be multiplied in order to obtain the maximum amplitude of the forced vibration (i.e. x_{max}) by the harmonic force F cos ω.t

$X_{max} = x_0 \times D$

Notes: 1. If there is no damping, then c = 0 and $\quad D = \dfrac{x_{max}}{x_0} = \dfrac{1}{\sqrt{1 - \dfrac{\omega^2}{(\omega_n)^2}}} = \dfrac{(\omega_n)^2}{(\omega_n)^2 - \omega^2}$

2. At resonance $\omega_n = \omega$, Therefore $D = \frac{x_{max}}{x_0} = \frac{s}{c.\omega_n}$

Vibration Isolation and Transmissibility

When an unbalanced machine is installed on the foundation, it produces vibration in the foundation. In order to prevent these vibrations or to minimise the transmission of forces to the foundation, the machines are mounted on springs or on some vibration isolation material. The ratio of the force transmitted (F_T) to the force applied (F) is known as isolation factor or transmissibility ratio of the spring support.

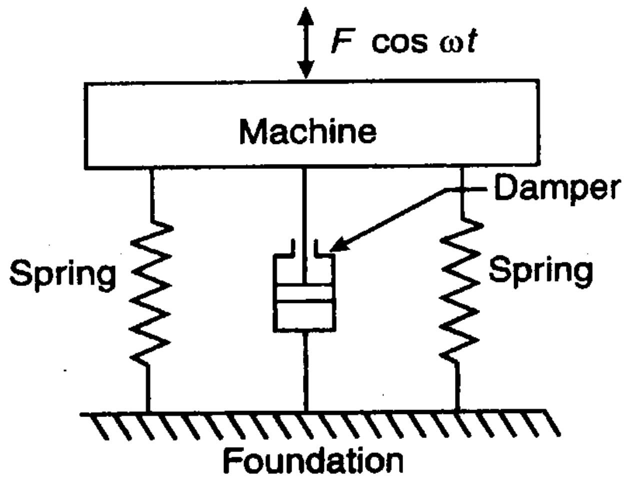


Fig. 23.22. Vibration isolation.

Natural Frequency of Free Torsional Vibrations

Let Θ = Angular displacement of the shaft from mean position after time t in radians,

m = Mass of disc in kg,

I = Mass moment of inertia of disc in kg-m^2

= m.k^2

r = Radius of gyration in metres.

q = Torsional stiffness of the shaft in N-m

Restoring force = q.Θ …(i)

Accelerating force = $I \times \frac{d^2\theta}{dt^2}$ …(ii)

Equating equations (i) and (ii) the equation of motion is

$$I \times \frac{d^2\theta}{dt^2} = -q.\Theta \quad \text{or} \quad I \times \frac{d^2\theta}{dt^2} + q.\Theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{q}{I} \times \theta = 0 \qquad \text{(iii)}$$

The fundamental equation of S.H.M. is

$$\frac{d^2\theta}{dt^2} + \omega^2.x = 0 \qquad \text{(iv)}$$

$$\omega = \sqrt{\frac{q}{I}}$$

Comparing equations (iii) and (iv)

Time period = $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

Natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

$$\frac{d^2\theta}{dt^2} + \omega^2 \times \theta = 0 \qquad \text{(iv)}$$

Comparing equations (iii) and (iv)

$$\omega = \sqrt{\frac{q}{I}}$$

Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

Natural frequency**,** $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

Note: 1. The value of torsional stiffness may be obtained from the torsion equation,

$$\frac{T}{J} = \frac{C.\theta}{l} \quad or \quad \frac{T}{\theta} = \frac{C.J}{l}$$

$\therefore$ $q = \frac{C.J}{l}$

where C = Modulus of rigidity of the shaft

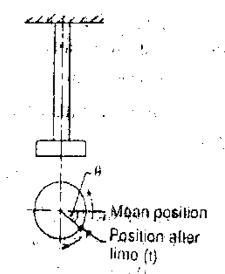
material

J = Polar moment of inertia

Free Torsional Vibrations of a Single Rotor System

For a shaft fixed at one end and carrying a rotor at the free end, the natural frequency of torsional vibration,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{C.J}{l.I}}$$



The section of the shaft whose amplitude of torsional vibration is zero, is known as node.

Effect of Inertia of the Constraint on Torsional Vibrations

Let m_c = total mass of constraint = m.l

I_c = Total mass moment of inertia of constraint

Natural frequency of vibration,

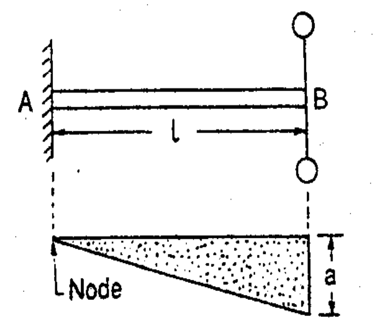


Fig. 24.4. Free torsional vibrations of a single rotor system.

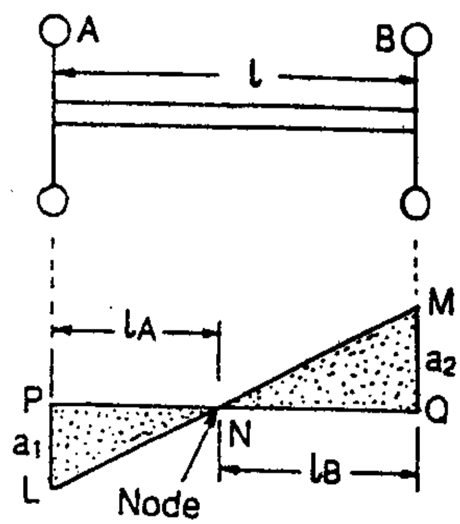
$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I + I_c/3}}$$

Free Torsional Vibrations of a Two Rotor System

Natural frequency of torsional vibration for rotor A,

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_A.I_A}}$$

Natural frequency of torsional vibration for rotor B,



24.5. Free torsional vibrations of a two rotor system.

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_B.I_B}}$$

Since $f_{nA} = f_{nB}$, therefore

$$\frac{1}{2\pi}\sqrt{\frac{C.J}{l_A.I_A}} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_B.I_B}}$$

$$\therefore \quad l_A = \frac{l_B.I_B}{I_B}$$

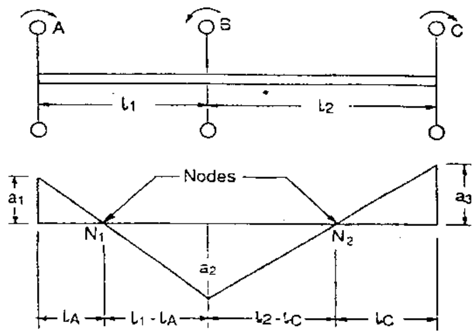
Free Torsional Vibrations of a Three Rotor System

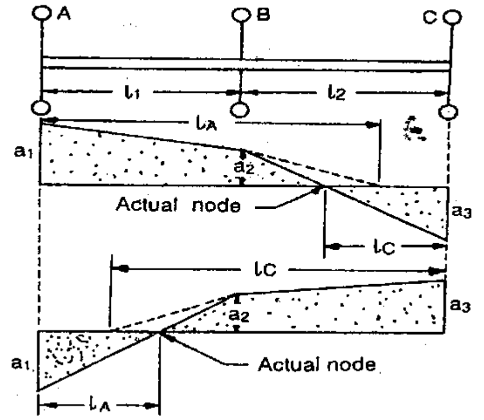
Natural frequency of torsional vibration for rotor A,

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_A.I_A}}$$

Natural frequency of torsional vibration for rotor B

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_B.I_B}}$$





Natural frequency of torsional vibration for rotor C, $f_{nC} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_C.I_C}}$

Torsionally Equivalent Shaft

Since the total angle of twist of the shaft is equal to the sum of the angle of twists of different lengths, therefore

$\Theta = \Theta_1 + \Theta_2 + \Theta_3$

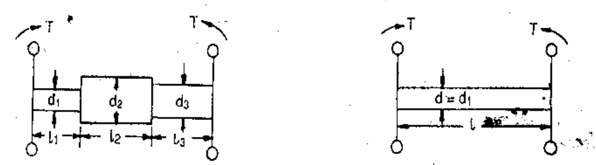
or

$$\frac{T.l}{C.J} = \frac{T.l_1}{C.J_1} + \frac{T.l_2}{C.J_2} + \frac{T.l_3}{C.J_3}$$

$$\frac{l}{J} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\frac{l}{\frac{\pi}{32}\times d^4} = \frac{l}{\frac{\pi}{32}\times(d_1)^4} + \frac{l}{\frac{\pi}{32}\times(d_2)^4} + \frac{l}{\frac{\pi}{32}\times(d_3)^4}$$

$$\frac{l}{d^4} = \frac{l_1}{(d_1)^4} + \frac{l_2}{(d_2)^4} + \frac{l_3}{(d_3)^4}$$



(a) Shaft of varying diameters. (b) Torsionally equivalent shaft.

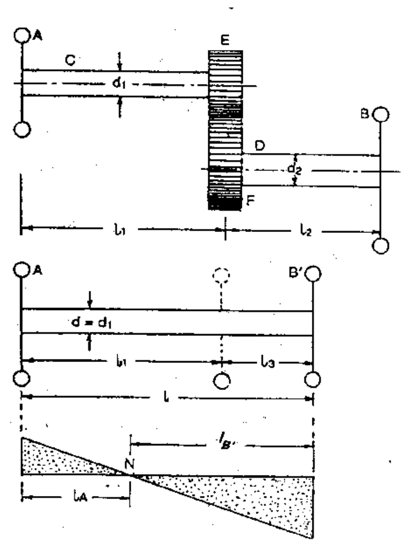
In actual practice it is assumed that the diameter d of the equivalent shaft is equal to one of the diameter of the actual shaft. Let us assume $d = d_1$

$$\therefore \quad \frac{l}{(d_1)^4} = \frac{l_1}{(d_1)^4} + \frac{l_2}{(d_2)^4} + \frac{l_3}{(d_3)^4}$$

$$or \quad l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

This expression gives the length l of the equivalent shaft

Free Torsional Vibration of a Geared System



Let G = Gear ratio = $\dfrac{Speed\ of\ \ pinion\,E}{Speed\ of\ \ pinion\,F} = \dfrac{\omega_A}{\omega_B}$

d = Diameter of the equivalent shaft

l = Length of the equivalent shaft

I_B' = Mass moment of inertia of the equivalent rotor B'

The following two conditions must be satisfied by an equivalent system.

The K.E. of the equivalent system must be equal to the K.E. of the original system.

The strain energy of the equivalent system must be equal to the strain energy of the original system.

If condition (1) is satisfied then

$$I_B^{'} = I_B\left(\frac{\omega_B}{\omega_A}\right)^2 = \frac{I_B}{G^2}$$

If condition (2) is satisfied, then

$$l_3 = G^2.l_2\left(\frac{d_1}{d_2}\right)^4$$

Length of the equivalent shaft

$$l = l_1 + l_3 = l_1 + G^2.l_2\left(\frac{d_1}{d_2}\right)^2$$

Let the node of the equivalent system lies at N, then

$I_A.l_A = I_B.l_B$

Also $l_A + L_B' = l$

When the inertia of gearing is taken into consideration, the an additional rotor (shown dotted) must be introduced. This rotor will have mass moment of inertia

$$I_E^{'} = I_E + \frac{I_F}{G^2}$$

Example 1: A refrigerator unit having a mass of 35 kg is to be supported on three springs, each having a spring stiffness s. The unit operates on 480 rpm. Find the value of stiffness s if only 10% of the shaking force is allowed to be transmitted to the supporting structure. R/645/(W07)

Solution: As no damper is used

$$\varepsilon = \frac{1}{\pm\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$\omega = \frac{2\pi \times 480}{60} = 16\pi$$

$$\therefore \quad 0.1\frac{1}{\pm\left[1-\left(\frac{16\pi}{\omega_n}\right)^2\right]}$$

$$or \ \pm\left[0.1 - 0.1\left(\frac{16\pi}{\omega_n}\right)^2\right] = 1$$

If the positive sign is changed, $\frac{16\pi}{\omega_n} = \sqrt{-9}$

Therefore by taking the negative sign,

$\frac{16\pi}{\omega_n} = \sqrt{11}$ or ω_n = 15.15 rad/s

or $\sqrt{\frac{s}{m}} = \sqrt{\frac{s}{35}} = 15.15$

Equivalent stiffness,

s = 8037 N/m = 8.037 n/mm

Stiffness of each spring = $\frac{8.037}{3} = 2.679\, N/mm$

Example 2: A machine of mass 10 kg is supported on springs and dashpots. The total stiffness of the spring is 5 N/mm and total damping is 0.075 m/mmls. The system is initially at rest and a velocity of 100 m/s is imparted to the mass. Determine (i) the displacement and velocity of mass as function of time (ii) the displacement and velocity after one second. (S08)

Solution:

Example 3: A machine part of mass 2 kg vibrates in a viscous medium. Determine the damping coefficient when a harmonic exciting force of 25 N results in resonant amplitude of 12.5 mm with a period of 0.2 second. If the system is excited by a harmonic force of frequency 2 Hz, what will be percentage increase in the amplitude of vibration when damper is removed as compared with that with damping?

K/959/(W08)

Solution:

ωn = 2π/tp = 31.42 rad/s

Maximum amplitude of vibration at resonance (xmax),

$$0.0125 = \frac{F}{c.\omega_n} = 63.7\, N/m/s$$

Percentage increase in amplitude

Example 4: Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of shaft material is 40 Mg/m^3, and Young's modulus is 200 GN/m^2. Assume the shaft to be freely supported.

K/932/W07

Solution:

$$I = \frac{\pi}{64} \times d^4 = 7.855 \times 10^{-9} m^4$$

Mass of the shaft per metre length,

M_s = Area × length × density = 12.6 kg/m

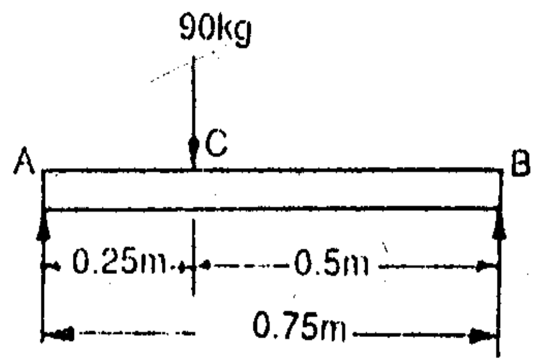
Static deflection due to

Example 5: A shaft of length 1.25 m is 75 mm in diameter for the first 275 mm of its length, 125 mm in diameter for the next 500 mm length, 87.5 mm in diameter for next 375 mm and 175 mm in diameter for remaining 100 mm length. The shaft carries two rotors at two ends. The mass moment of inertia of first and second rotor is 75 kg-m^2 and 50 kg-m^2 respectively. Find the frequency of natural torsional vibration of the system. Take modulus of rigidity of material as 800 N/m^2.

Example 6: A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibrations.

Assume E = 200 GN/m^2 and shaft diameter = 50 mm.

Solution:



Moment of inertia = $\frac{\pi}{64} \times d^4$ = 0.307 × 10-6 m4

Static deflection at the load point

$$\delta = \frac{Wa^2b^2}{3EIl} = 0.1 \times 10^{-3}\, m$$

Natural frequency of transverse vibrations

$$f_n = \frac{0.4985}{\sqrt{\delta}} = 49.85\, Hz$$

UNIT – V

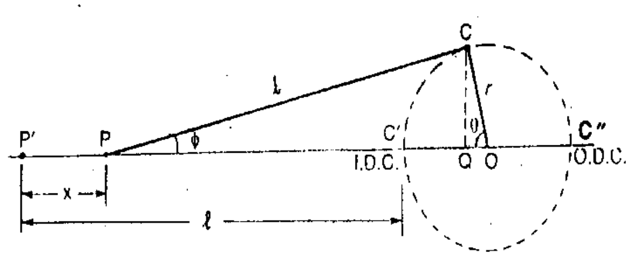
Inertia Force Analysis

5.1 Approximate Analytical Method for Velocity and Acceleration of the Piston

Consider the motion of a crank and connecting rod of a reciprocating steam engine.

Let l = Length of the connecting rod between the centres.

r = Radius of crank



Φ = Inclination of connecting rod to the line of stroke PO

n = Ratio of length of c. rod to the radius of crank = l/r

From the geometry of the figure,

x = P'P = OP' – OP = (P'C' + C'O) – (PQ – QO) = (l +r) – (l cos Φ + r sin Θ)

$$= r\left[(1-\cos\theta)+\frac{l}{r}\left(1-\cos\phi\right)\right]$$

$$= r\left[(1-\cos\theta)+n\left(1-\cos\phi\right)\right] \qquad \text{(i)}$$

From triangles CPQ and CQA,

CQ = l sin Φ = r sin Θ or l/r = sin Θ/sin Φ or sin Φ = sin Θ/n ……….. (ii)

We know that $\cos\phi=\left(1-\sin^2\phi\right)^{\frac{1}{2}}=\left(1-\dfrac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}$

Expanding the above equation by binomial theorem

$$\cos\phi = 1-\frac{1}{2}\times\frac{\sin^2\theta}{n^2}+.....$$ (Neglecting the higher terms)

$$or \quad 1-\cos\phi=\frac{\sin^2\theta}{2n^2} \qquad \text{…… (iii)}$$

Substituting the value of (1- cos Φ) in equation (i)

$$x = r\left[\left(1-\cos\theta\right)+n\times\frac{\sin^2\theta}{2n^2}\right] = r\left[\left(1-\cos\theta\right)+\frac{\sin^2\theta}{2n}\right] \qquad \text{...... (iv)}$$

Differentiating equation (iv) with respect to ϴ,

$$\frac{dx}{dx} = r\left[\sin\theta+\frac{1}{2n}\times 2\sin\theta\cos\theta\right] = r\left[\sin\theta+\times\frac{\sin 2\theta}{2n}\right] \qquad \text{(v)}$$

- Velocity of P with respect to O or velocity of the piston P,

$$v_{PO} = v_P = \frac{dx}{dt} = \frac{dx}{d\theta}\times\frac{d\theta}{dt} = \frac{dx}{d\theta}\times\omega$$

Substituting the value of dx/dϴ from equation (v), we have

$$v_P = \omega.r\left(\sin\theta+\frac{\sin 2\theta}{2n}\right) \qquad \text{..... (vi)}$$

Acceleration of the piston

$$a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\theta}\times\frac{d\theta}{dt} = \frac{dv_P}{d\theta}\times\omega$$

Differentiating equation (vi) with respect to ϴ,

$$\frac{dv_P}{d\theta} = \omega.r\left[\cos\theta+\frac{\cos 2\theta\times 2}{2n}\right] = \omega.r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]$$

Substituting the value of $\frac{dv_P}{d\theta}$ in the above equation

$$a_P = \omega.r\left[\cos\theta+\frac{\cos 2\theta\times 2}{2n}\right]\times\omega = \omega^2.r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]$$

$$a_P = \omega^2.r\left[\cos\theta+\frac{\cos 2\theta}{n}\right]$$

Q. Deduce an expression for the inertia force of reciprocating parts, neglecting the weight of the connecting rod. 7 marks

(S08)

Acceleration of the reciprocating parts

$$a_R = \omega.r\left[\cos\theta + \frac{\cos 2\theta \times 2}{2n}\right]\times\omega = \omega^2.r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

∴ Accelerating force or inertia force of reciprocating parts,

$$F_I = m_R.a_R = m_R.\omega^2.r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

where m_R = Mass of the reciprocating parts

5.2 Angular Velocity and Acceleration of the Connecting Rod

$$\omega_{PC} = \frac{\omega\cos\theta}{\left(n^2 - \sin^2\theta\right)^{1/2}}$$

$$\alpha_{PC} = \frac{-\omega^2\sin\theta\left(n^2 - 1\right)}{\left(n^2 - \sin^2\theta\right)^{3/2}}$$

5.2 Forces on the Reciprocating Parts of an Engine, neglecting the weight of the Connecting rod

1. Piston effort. It is the net force acting on the piston or crosshead pin, along the line of stroke.

Piston Effort,

F_P = Net load on the piston $\mp$ Inertia Force

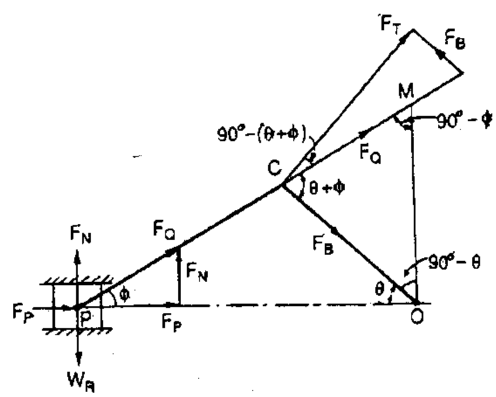
= $F_L \mp F_I$ (Neglecting frictional resistance)

= $F_L \mp F_I - R_F$ (Considering frictional resistance)

2. Force acting along the connecting rod

$$F_Q = \frac{F_P}{\cos\phi}$$

$$\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$



$$\therefore \quad F_Q = \frac{F_P}{\sqrt{1-\frac{\sin^2\theta}{n^2}}}$$

3. Thrust on the sides of the cylinder

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi}\times\sin\phi = F_P\tan\phi$$

4. Crank-pin effort and thrust on crank shaft bearings.

The force acting on the connecting rod F_Q may be resolved into two components, one perpendicular to the crank and other along the crank. The component of F_Q perpendicular to the crank is known as crank-pin effort and is denoted by F_T. The component of F_Q along the crank produces a thrust on the crank shaft bearings and is denoted by F_B

Resolving F_Q perpendicular to the crank,

$$F_T = F_Q\left(\sin\theta+\phi\right) = \frac{F_P}{\cos\phi}\times\sin\left(\theta+\phi\right)$$

Resolving F_Q along the crank,

$$F_B = F_Q\cos\left(\theta+\phi\right) = \frac{F_P}{\cos\phi}\times\cos\left(\theta+\phi\right)$$

5. Crank effort or turning moment or torque on the crank.

The product of the crank-pin effort F_T and the crank pin radius (r) is known as crank effort or turning moment or torque on the crank.

Crank effort

$$T = F_P\times r\left(\sin\theta+\frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

Q. A rigid body, under the action of external forces, can be replaced by two masses placed at a fixed distance apart. The two masses form an equivalent dynamical system, if

i. the sum of the two masses is equal to the total mass of the body,
ii. the centre of gravity of the two masses coincides with the body,
iii. the sum of the moment of inertia of the masses about their centre of gravity is equal to the mass moment of inertia of the body

iv. All of the above. 2 marks

(S09)

5.3 Equivalent Dynamical System

In order to determine the motion of rigid body, under the action of external forces, it is usually convenient to replace the rigid body by two masses placed at fixed distance apart, in such a way that,

1. the sum of their masses is equal to the total mass of the body;
2. the centre of gravity of the two masses coincides with that of the body; and
3. the sum of the mass moment of inertia of the masses about their centre of gravity is equal to the mass moment of inertia of the body.

When these three conditions are satisfied, then it is said to be an equivalent dynamical system.

For the two masses to be dynamically equivalent,

$$m_1 + m_2 = m \qquad \text{(i)}$$

$$m_1 . l_1 = m_2 . l_2 \qquad \text{(ii)}$$

$$\text{and} \quad m_1 (l_1)^2 + m_2 (l_2)^2 = m (k_G)^2$$

From equations (i) and (ii)

$$m_1 = \frac{l_2 . m}{l_1 + l_2} \qquad \text{(iv)}$$

$$\textit{and} \qquad m_2 = \frac{l_1 . m}{l_1 + l_2} \qquad \text{(v)}$$

$$l_1 . l_2 = (k_G)^2$$

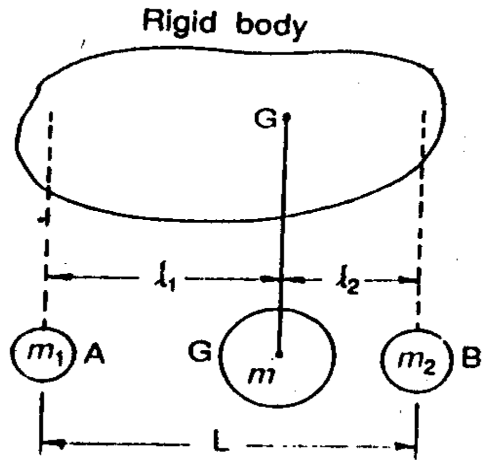


Fig. 15.14. Equivalent dynamical system. . . . (i

5.4 Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

Turning Moment on the crank shaft

$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

where F_p = Piston effort

r = radius of crank
n = Ratio of the connecting rod length and radius of crank

Θ = Angle turned by the crank from inner dead centre

5.5 Turning moment diagram for a Four Stroke Cycle Internal Combustion Engine

In four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions. During the working stroke, work is done by the gases. During exhaust stroke, work is done on the gases.

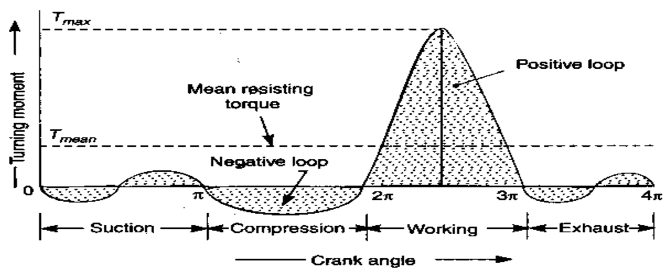


Fig. 16.2. Turning moment diagram for a four stroke cycle internal combustion engine.

5.6 Turning Moment Diagram for a Multi-cylinder Engine

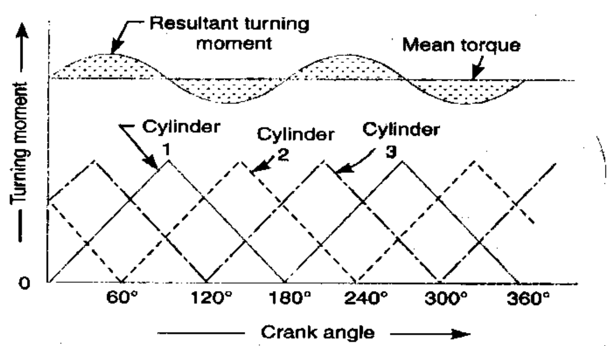


Fig. 16.3. Turning moment diagram for a multi-cylinder engine.

5.7 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed

Let N_1 and N_2 = Maximum and minimum speeds in rpm during the cycle, and

$$N = Mean\,speed\,in\,rpm = \frac{N_1 + N_2}{2}$$

Coefficient of fluctuation of speed

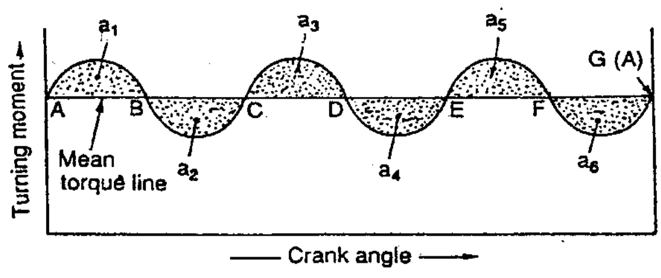
$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{} = \frac{2(\omega_1 - \omega_2)}{}$$

Q. Determine the equation of maximum fluctuation of energy. 2 marks

(W09)

5.8 Determination of Maximum Fluctuation of Energy



The horizontal line AG represents the mean torque line.

Let the energy in the flywheel at A = E,

Energy at B = $E + a_1$

Energy at C = $E + a_1 - a_2$

Energy at D = $E + a_1 - a_2 + a_3$

Energy at E = $E + a_1 - a_2 + a_3 - a_4$

Energy at F = $E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G = $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (cycle repeats after G)

Let us suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in the flywheel = $E + a_2$

Minimum energy in the flywheel

$= E + a_1 - a_2 + a_3 - a_4$

Maximum fluctuation of energy,

$\Delta E = (E + a_1) - (E + a_1 - a_2 + a_3 - a_4)$

$= a_2 - a_3 + a_4$

5.9 Coefficient of Fluctuation of Energy

It may be defined as the ratio of maximum fluctuation of energy to the work done perpendicular cycle

Coefficient of fluctuation of energy

$$C_E = \frac{Maximum\ fluctuation\ of\ energy}{Work\ done\ per\ cycle}$$

1. Work done per cycle = $T_{mean} \times \Theta$

Where T_{mean} = Mean torque

Θ = 2π in case of steam engine and two stroke I.C. engine.

= 4π, in case of four stroke I.C. engines.

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

Where P = Power transmitted in Watts

2. Work done per cycle = $\frac{P \times 60}{n}$

where n = number of working strokes per minute

Q. Describe the function of a flywheel. 2 marks

(W08)

Q. Prove that the maximum fluctuation of energy,

$\Delta E = E \times 2C_S$

where E = mean kinetic energy of the flywheel, and

C_s = coefficient of fluctuation of sped 4 marks

(S09)

5.10 Flywheel

A flywheel used in machines serve as a reservoir, which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply.

A flywheel controls the speed variations caused by the fluctuation of engine turning moment during each cycle of operation.

5.11 Energy Stored in a Flywheel

Mean K.E. of the flywheel, $E = \frac{1}{2} \times m.k^2.\omega^2$

Maximum fluctuation of energy, Δ E = Maximum K.E. – Minimum K.E.

$$= I.\omega^2.\left(\frac{\omega_1 - \omega_2}{\omega}\right) = m.k^2.\omega^2.C_s = \frac{\pi^2}{900} \times m.k^2.N^2.C_s$$

Example 1: The crank and connecting rod of a vertical petrol engine, running at 1800 rpm are 60 mm and 270 mm respectively. The diameter of the piston is 100 mm and the mass of the reciprocating parts is 1.2 kg. During the expansion stroke when the crank has turned through 20˚ from the top dead centre, the gas pressure is 650 kN/m^2. Determine:

(i) the net force on the piston;

(ii) the net load on the gudgeon pin;

(iii) the thrust on the cylinder walls;

(iv) the speed at which the gudgeon pin load is reversed in direction. 10 marks

R/444(W08)

Solution:

Load on the piston $F_L = \frac{\pi}{4} D^2 \times p = 5105\,N$

n = l/r = 270/60 = 4.5

Inertia force on the piston $F_I = m_R.a_R = m_R.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = 2840\,N$

(i). For a vertical engine, net force on the piston

$F_P = F_L - F_I + W_R = 2276.8$ N

(ii) Let Φ = Angle of inclination of the connecting rod to the line of stroke

sin Φ = sin Θ/n Φ = 4.36˚

Resultant load on the gudgeon pin, $F_Q = \frac{F_P}{\cos\phi} = 2283.4\,N$

(iii) Thrust on the cylinder walls

$F_N = F_P \tan \Phi = 173.6$ N

(iv) Speed above which the gudgeon pin load will be reversed in direction

Let N_1 = Required speed in rpm.

The gudgeon pin load i.e. F_Q will be reversed in direction, if F_Q becomes negative. This is only possible if F_P is negative. Therefore, for F_P to be negative F_I must be greater than $(F_L + W_R)$

$$i.e.\quad m_R\left(\omega_1\right)^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) > 5105 + 1.2 \times 9.81$$

ω > 253 rad/s

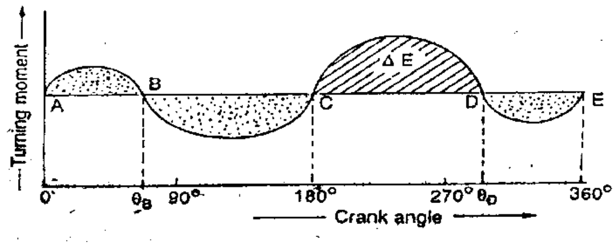
N > 2416 rpm

Example 2: A certain machine requires a torque of (5000 + 500 sin Θ) N-m to drive it, where Θ is the angle of rotation of shaft measured from certain datum. The machine is directly coupled to an engine which produces a torque of (5000 + 600 sin 2Θ) N-m. The flywheel and other rotating parts attached to the engine has a mass of 500 kg at a radius of gyration of 0.4 m. If the mean speed is 150 rpm, find: 1. the fluctuation of energy, 2. the total percentage fluctuation of speed, and 3. the maximum and minimum angular acceleration of the flywheel and the corresponding shaft position. **K/587/(W07)**

Solution:

1. Fluctuation of energy

Change in torque = $T_2 - T_1$ = 600 sin 2Θ – 500 sinΘ



This change is zero when

600 sin 2Θ = 500 sinΘ or 1.2 sin 2Θ = sin Θ

or 2.4 sin Θ cos Θ = sinΘ

Either sin Θ = 0 or cos Θ = 1/2.4 = 0.4167

When sin Θ = 0, Θ = 0˚, 180˚ and 360˚

$\Theta_A = 0^\circ$, $\Theta_C = 180^\circ$, Θ_E 360˚,

When cos Θ = 0.4167, Θ = 65.4˚ and 294.6˚ i.e $\Theta_B = 65.4^\circ$, $\Theta_D = 294.6^\circ$

The maximum fluctuation of energy lies between C and D (i.e. between 180˚ and 294.6˚)

Maximum fluctuation of energy,

$$\Delta E = \int_{180^\circ}^{294.6^\circ} (T_2 - T_1)\, d\theta$$

$$= \int_{180^\circ}^{294.6^\circ} \left[(5000 + 600\sin 2\theta) - (5000 + 500\sin\theta)\right] d\theta$$

$$= \left[-\frac{600\cos 2\theta}{2} + 500\cos\theta\right]_{180^\circ}^{294.6^\circ} = 1204\, N - m$$

2. Total percentage fluctuation of speed

Let C_s = Total percentage fluctuation of speed

Maximum fluctuation of energy (Δ E)

1204 = $m.k^2.\omega^2.C_s$ = 19744 C_s

C_s = 0.061 = 6.1%

3. Maximum and minimum angular acceleration of the flywheel and the corresponding shaft positions

The change in torque must be maximum or minimum when acceleration is maximum or minimum.

Change in torque, $T = T_2 - T_1$

$= (5000 + 600 \sin 2\theta) - (5000 + 500 \sin \theta)$

$= 600 \sin 2\theta - 500 \sin \theta$ … (i)

Differentiating with respect to Θ and equating to zero for maximum or minimum values,

$$\frac{d}{d\theta}(600 \sin 2\theta - 500 \sin \theta) = 0$$

or $1200 \cos 2\theta - 500 \cos \theta = 0$

or $12 \cos 2\theta - 5 \cos \theta = 0$

or $24 \cos^2 \theta - 5 \cos \theta - 12 = 0$

$$\therefore \quad \cos\theta = \frac{5 \pm \sqrt{25 + 4 \times 12 \times 24}}{2 \times 24} = \frac{5 \pm 34.3}{48} = 0.8187 \text{ or } -0.6104$$

$\therefore \quad \theta = 35^\circ$ or 127.6°

Substituting $\theta = 35^\circ$ in equation (i)

$T_{max} = 600 \sin 70^\circ - 500 \sin 35^\circ = 277$ N-m

Substituting $\theta = 35^\circ$ in equation (i)

$T_{min} = 600 \sin 225.2^\circ - 500 \sin 127.6^\circ = -976$ N-m

Maximum acceleration $\quad \alpha_{max} = \frac{T_{max}}{I} = \frac{277}{500(0.4)^2} = 3.46\, rad/s^2\, Ans$

Minimum acceleration $\quad \alpha_{min} = \frac{T_{min}}{I} = \frac{976}{500(0.4)^2} = 12.2\, rad/s^2\, Ans$

Example 3: In a machine, the intermittent operations demand the torque to be applied as follows:

- During the first half-revolution, the torque increases uniformly from 800 N-m to 3000 N-m.
- During the next one revolution, the torque remains constant.

- During the next one revolution, the torque decreases uniformly from 3000 N-m to 800 N-m.
- During the last half-revolution, the torque remains constant.

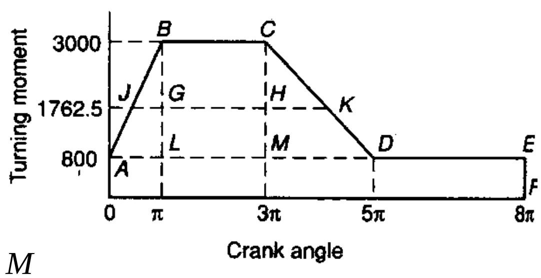
Thus, a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 250 rpm. A flywheel of mass 1800 kg and radius of gyration of 500 mm is fitted to the shaft. Determine the

(i) power of the motor

(ii) total fluctuation of the speed of machine shaft. 10 marks

R/468/(W08)

Solution:



Torque for one complete cycle, T = area OABCDEF

or T = Area OAEF + Area ABL

+ Area LBCM + Area MCD

$$=8\pi\times800+\frac{\pi\times2200}{2}+2\pi\times2200+\frac{2\pi\times2200}{2}=14100\pi\, n-M$$

$$T_{mean}=\frac{14100\pi}{8\pi}=1762.5\,N.m$$

$$P=T_m\omega=176.5\times\frac{2\pi\times250}{60}=46142W=46.142\,kW$$

(ii) $JG=AL\times\frac{BG}{BL}=\pi\times\frac{3000-1762.5}{3000-800}=1.767$

The fluctuation of energy is equal to the area above the mean torque line.

e = Area JBCK = area JBJ + area GBCH + area HCK

$$=\left(3000-1762.5\right)\left[\frac{1.767}{2}+2\pi+\frac{3.534}{2}\right]=11055N.m$$

$$K=\frac{e}{mk^2\omega^2}=\frac{11055}{1800\times\left(0.5\right)^2\times\left(\frac{2\pi\times250}{60}\right)}=0.0358\,or\,3.58\%$$

Example 4: A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes perpendicular minute. It requires a 6 N-m energy per mm^2 of sheared area. If the punching takes 1/10 of a second and the rpm of the flywheel varies from 160 to 140, determine the mass of the flywheel having radius of gyration of 1 metre.

Solution:

Sheared area per hole = π.d.t = 1885 mm^2

Energy required to punch a hole

E_1 = 6 × 1885 = 11 310 N-m

Energy required for punching work per second

= Energy required per hole × No. of holes per second

= 11 310 × 30/60 = 5655 N-m/s

Energy supplied by the motor in 1/10 second,

E_2 = 5655 × 1/10 = 565.5 N-m

Energy to be supplied by the flywheel during punching a hole or maximum fluctuation of energy of the flywheel,

ΔE = $E_1 - E_2$ =10 744.5 N-m

Mean speed of the flywheel, $N = \dfrac{N_1 + N_2}{2} = 150\, rpm$

Maximum fluctuation of energy (ΔE) $10744.5 = \dfrac{\pi^2}{900} \times m.k^2 N\left(N_1 - N_2\right) = 33m$

m = 327 kg

Example 5: The flywheel of a steam engine has a radius of gyration of 1 m and mass 2900 kg. The starting torque of steam engine is 1500 N-m. Determine: 1. the angular acceleration of the flywheel. 2. the kinetic energy of the flywheel after 10 seconds from the start.

K/ /

Solution:

$I = m.k^2 = 2900 \times 1^2 = 2900$ kg-m^2

Starting torque of the engine (T)

$1500 = 2900 \times \alpha$ or $\alpha = 0.52$ rad/s^2

2. Kinetic energy of the flywheel

Let ω_1 = Angular speed at rest = 0

ω_2 = Angular speed after 10 seconds

$\omega_2 = \omega_1 + \alpha\, t = 0.52 \times 10 = 5.2$ rad/s

K.E. of the flywheel = $\frac{1}{2} \times I\left(\omega_2\right)^2 = \frac{1}{2} \times 2900 \times 5.2^2 = 78416\, N-m = 78.4\, kN.m$

Example 6: The turning moment diagram for a petrol engine is drawn to the following scales: Turning moment , 1 mm = 5 N-m; crank angle, 1 mm = 1˚. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2 **.** The rotating parts are equivalent to a mass of 30 kg at a radius of gyration of 150 mm. Deflection the coefficient of fluctuation of speed when the engine runs at 1800 r.p.m

Solution: 1 mm^2 on turning moment diagram = $5 \times \frac{\pi}{180} = \frac{\pi}{36} N-m$

Let the total energy at A = E

Let the total energy at A = E

Energy at B = E + 295 ….(Maximum energy)

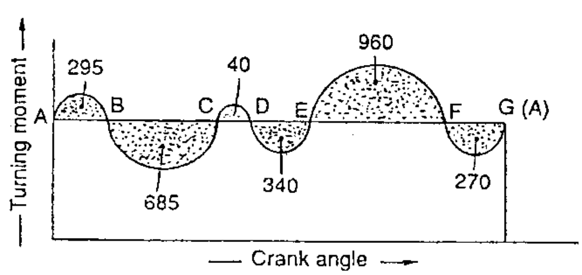
Energy at C = E + 295 - 685 = E – 390

Energy at D = E – 390 + 40 = E – 350

Energy at E = E – 350 – 340 = E – 690 ….(Minimum energy)

Energy at F = E – 690 + 960 = E + 270

Energy at G = E + 270 - 270 = E = Energy at A



Maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy

= (E + 295) – (E – 690) = 985 mm^2

= 985 × = 86 N-m = 86 J

Let C_s = Coefficient of fluctuation of speed

$\Delta E = m.k^2\, \omega^2 .\, C_s$

$86 = 36 \times (0.15)^2 \times (188.52)^2 . C_s$

$C_s = 0.003$ or 0.3%

Example 7: A horizontal cross compound steam engine develops 300 kW at 90 rpm. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is to be kept within ± 0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2 metres. **K/ /(W08)**

Solution:

Mean angular speed, $\omega = 2\pi \times 90/60 = 9.426$ rad/s

Let ω_1 and ω_2 = Maximum and minimum respectively.

Total fluctuation of speed, $\omega_1 - \omega_2 = 0.1\ \omega$

Coefficient of fluctuation of speed, $C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.01$

Work done per cycle = P × 60/N = 200×10^3 N-m

Maximum fluctuation of energy

ΔE = Work done per cycle × $C_E = 20 \times 10^3$ N-m

Maximum fluctuation of energy (ΔE)

$20 \times 10^3 = m.k^2.\omega^2.C_s = 3.554\ m$

m = 5630 kg

Example 8: **T**he crank pin circle radius of a horizontal engine is 300 mm. The mass of the reciprocating parts is 250 kg. When the crank has travelled 60˚ from I.D.C. the difference between the driving and back pressure is 0.35 N/mm^2. The connecting rod length between the centres is 1.2 m and the cylinder bore is 0.5 m. If the engine runs at 250 rpm and if the effect of piston rod diameter is neglected, calculate:

1. pressure on slide bars, 2. thrust in the connecting rod 3. tangential force on the crank-pin and 4. turning moment on the crank shaft. 14

(W09)