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UNIT – III

Finite Impulse Response (FIR) Filter Design: **Rectangular, Triangular, Hamming, Blackman & Kaiser window.** **Linear Phase and Optimal Filter.**

FIR FILTER

An FIR filter of length M is described by the difference equation

$$y(n) = b_0\, x(n) + b_1 x(n-1) + b_2\, x(n-2) + b_3\, x(n-3) + \dots + b_{M-1}\, x(n-M+1)$$

$$= \sum_{k=0}^{M-1} b_k\, x(n-k)$$

where bk is the set of filter coefficients.

THE the response of the FIR filter depends only on the present and past t samples, whereas for the IIR filter, the present response is a function of the present and past N values of excitation as well as past values of the response.

FIR filters have the following advantages over IIR filters

(i) They can have an exact linear phase.
(ii) They are always stable.
(iii) The design methods are generally linear.
(iv) They can be realised efficiently in hardware.
(v) The filter start-up transients have finite duration.

MAGNITUDE RESPONSE AND PHASE RESPONSE OF DIGITAL FILTERS

The discrete-time Fourier transform of a finite sequence impulse response $h(n)$ is given by

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{M-1} h(n)e^{-j\omega nT} = |H(e^{j\omega})|\,e^{j\,\Phi(\omega)}$$

The magnitude and phase responses are given by

$$M(\omega) = |H(e^{j\omega})| = \{\mathrm{Re}[H(e^{j\omega})]^{2} + \mathrm{Im}\,[H(e^{j\omega}]^{2}\}^{0.5}$$

$$\Phi(\omega) = \tan^{-1}\frac{\mathrm{Im}\,[H(e^{j\omega})]}{\mathrm{Re}\,[H(e^{j\omega})]}$$

Filters can have a linear or non-linear phase depending upon the delay function, namely the phase delay and group delay. The phase and group delays of the filter are given by

$$\tau_p = -\frac{\Phi(\omega)}{\omega} \quad \text{and} \quad \tau_g = -\frac{d\Phi(\omega)}{d\omega},$$ respectively.

LINEAR PHASE FILTER ARE THOSE FILTERS IN WHICH THE PHASE DELAY AND GROUP DELAY ARE CONSTANTS, i.e.INDEPENDENT OF FREQUENCY.(time delay filters)

For the phase response to be linear

$$\frac{\Phi(\omega)}{\omega} = -\tau, \quad -\pi \le \omega \le +\pi$$

$$\Phi(\omega) = \tan^{-1} \frac{\operatorname{Im} H(e^{j\omega})}{\operatorname{Re} H(e^{j\omega})} = -\omega\tau$$

$$h(n) = -h(M - 1 - n)$$

or

$$\tan \omega\tau = \frac{\sum_{n=0}^{M-1} h(n) \sin \omega n}{\sum_{n=0}^{M-1} h(n) \cos \omega n}$$

Simplifying, we get

$$\sum_{n=0}^{M-1} h(n) \sin(\omega\tau - \omega n) = 0$$

and a solution is given by

$$\tau = \frac{(M-1)}{2}$$

and

$$h(n) = h(M-1-n) \text{ for } 0 < n < M-1$$

Example 7.1 The length of an FIR filter is 9. If the filter has a linear phase, show that below is satisfied.

$$\sum_{n=0}^{M-1} h(n)\sin(\omega\tau - \omega n) = 0$$

Example 7.2 The following transfer function characterises an FIR filter (M = 11). *Determine the magnitude response and show that the phase and group delays are constant.*

$$H(z) = \sum_{n=0}^{M-1} h(n)\, z^{-n}$$

Solution The length of the filter, $M = 9$. Therefore, from

$$\tau = \frac{(M-1)}{2} = 4.$$

$h(n) = h(M-1-n)$. Therefore the filter coefficients are $h(0) = h(8)$, $h(1) = h(7)$, $h(2) = h(6)$, $h(3) = h(5)$ and $h(4)$.

Eq. 7.3 can be written as,

$$\sum_{n=0}^{M-1} h(n) \sin(\omega\tau - \omega n) = \sum_{n=0}^{8} h(n) \sin \omega (\tau - n)$$

$$= h(0) \sin 4\omega + h(1) \sin 3\omega + h(2) \sin 2\omega + h(3) \sin \omega + h(4) \sin 0$$

$$+ h(5) \sin(-\omega) + h(6) \sin(-2\omega) + h(7) \sin(-3\omega) + h(8) \sin(-4\omega)$$

$$= 0$$

Thus Eqabove is satisfied.

Solution The transfer function of the filter is given by

$$H(z) = \sum_{n=0}^{M-1} h(n)\, z^{-n}$$

$$= h(0) + h(1)\, z^{-1} + h(2)\, z^{-2} + \ldots + h(10)\, z^{-10}$$

The phase delay, $\tau = \dfrac{M-1}{2} = 5$. Since $\tau = 5$, the transfer function can be expressed as,

$$H(z) = z^{-5}\, [h(0)\, z^{5} + h(1)\, z^{4} + h(2)\, z^{3} + \ldots + h(9)\, z^{-4} + h(10)\, z^{-5}]$$

Since $h(n) = h(M - 1 - n)$,

$$H(z) = z^{-5}\,[h(0)\,(z^5 + z^{-5}) + h(1)\,(z^4 + z^{-4}) + h(2)\,(z^3 + z^{-3}) + h(3)\,(z^2 + z^{-2}) + h(4)\,(z + z^{-1}) + h(5)]$$

The frequency response is obtained by replacing z with $e^{j\omega}$,

$$H(e^{j\omega}) = e^{-j5\omega}\,\{h(0)[e^{5j\omega} + e^{-5j\omega}] + h(1)[e^{4j\omega} + e^{-4j\omega}] + h(2)\,[e^{3j\omega} + e^{-3j\omega}] + h(3)[e^{2j\omega} + e^{-2j\omega}] + h(4)\,[e^{j\omega} + e^{-j\omega}] + h(5)\}$$

$$= e^{-j5\omega}\left[h(5) + 2\sum_{n=0}^{4} h(n)\cos(\tau - n)\,\omega\right] = e^{-j5\omega}\,M(\omega)$$

where $M(\omega)$ is the magnitude response and $\theta(\omega) = -\ 5\omega$ is the phase response. The group delay, τ_g is given by,

$$\tau_p = -\frac{\Phi(\omega)}{\omega} = 5 \quad \text{and} \quad \tau_g = -\frac{d\,[\Phi(\omega)]}{d\,\omega} = -\frac{d\,(-5\,\omega)}{d\,\omega} = 5$$

Thus, the phase delay and the group delay are the same and are constants.

FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)T} \left\{ \sum_{k=0}^{\frac{M-1}{2}} a(k) \cos \omega kT \right\}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} = M(\omega)$$

DESIGN TECHNIQUES FOR FIR FILTERS

.1 Fourier Series Method

.2 Frequency Sampling Method

.3 Window Techniques

Window Techniques

The desired frequency response of any digital filter is

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

where:

$$h(n) = \frac{1}{2\pi} \int_{0}^{2\pi} H\left(e^{j\omega}\right) e^{j\omega n} \, d\omega$$

The Fourier transform of the window function $W(e^{j\omega})$ should have a small width of main lobe containing as much of the total energy as possible.
The Fourier transform of the window function $W(e^{j\omega})$ should have side lobes that decrease in energy rapidly as ω tends to π. Some of the most frequently used window functions are described in the following sections.

The Fourier cofficients of the series h(n) are identical to the impulse response of a digital filter.There are two difficulties with implementation of equation

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n}$$

for designing a digital filter.

1) Impulse response is of infinite duration.

2)The filter is noncausal and unrealizable. So $H(e^{j\omega})$ is an unrealisable IIR filter.

Solution of above is to convert infinite duration impulse response to finite duration by truncating the infinite series at n=+-N but result in undesirable oscillatons in passband and stopband of the digital filter.

These undesirable oscillatons can be reduced by using a set of time-limited weighting function, w(n), reffered to as WINDOW FUNCTIONS, to modify the fourier cofficients.

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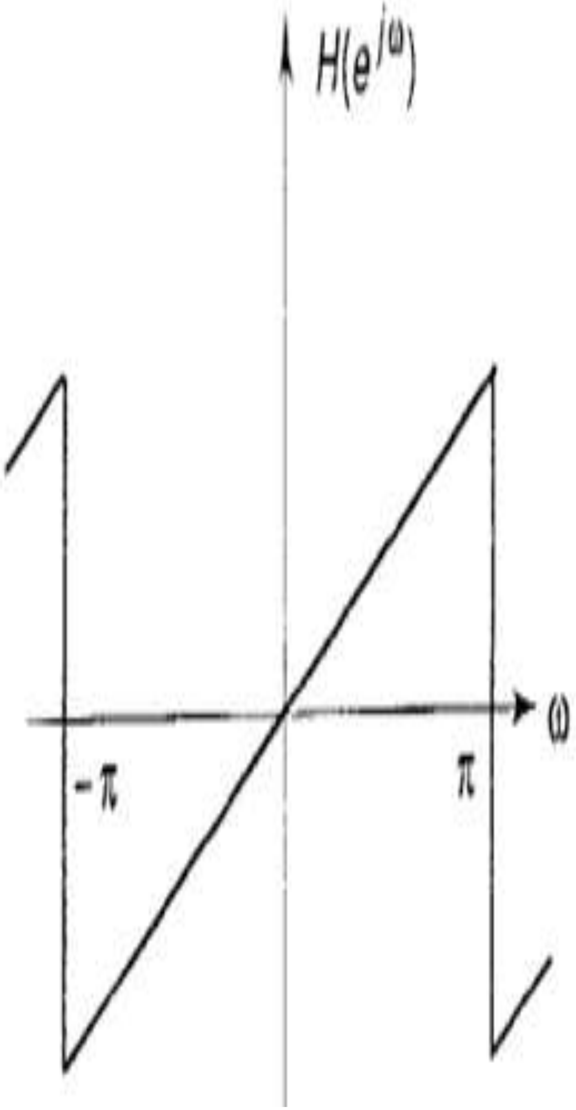

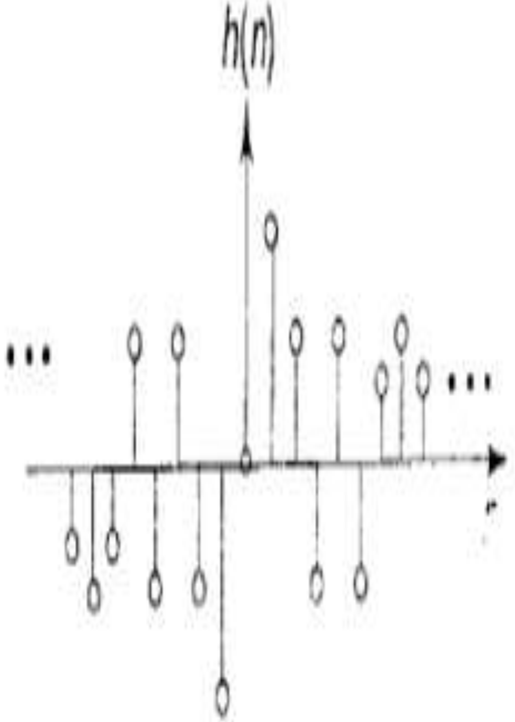

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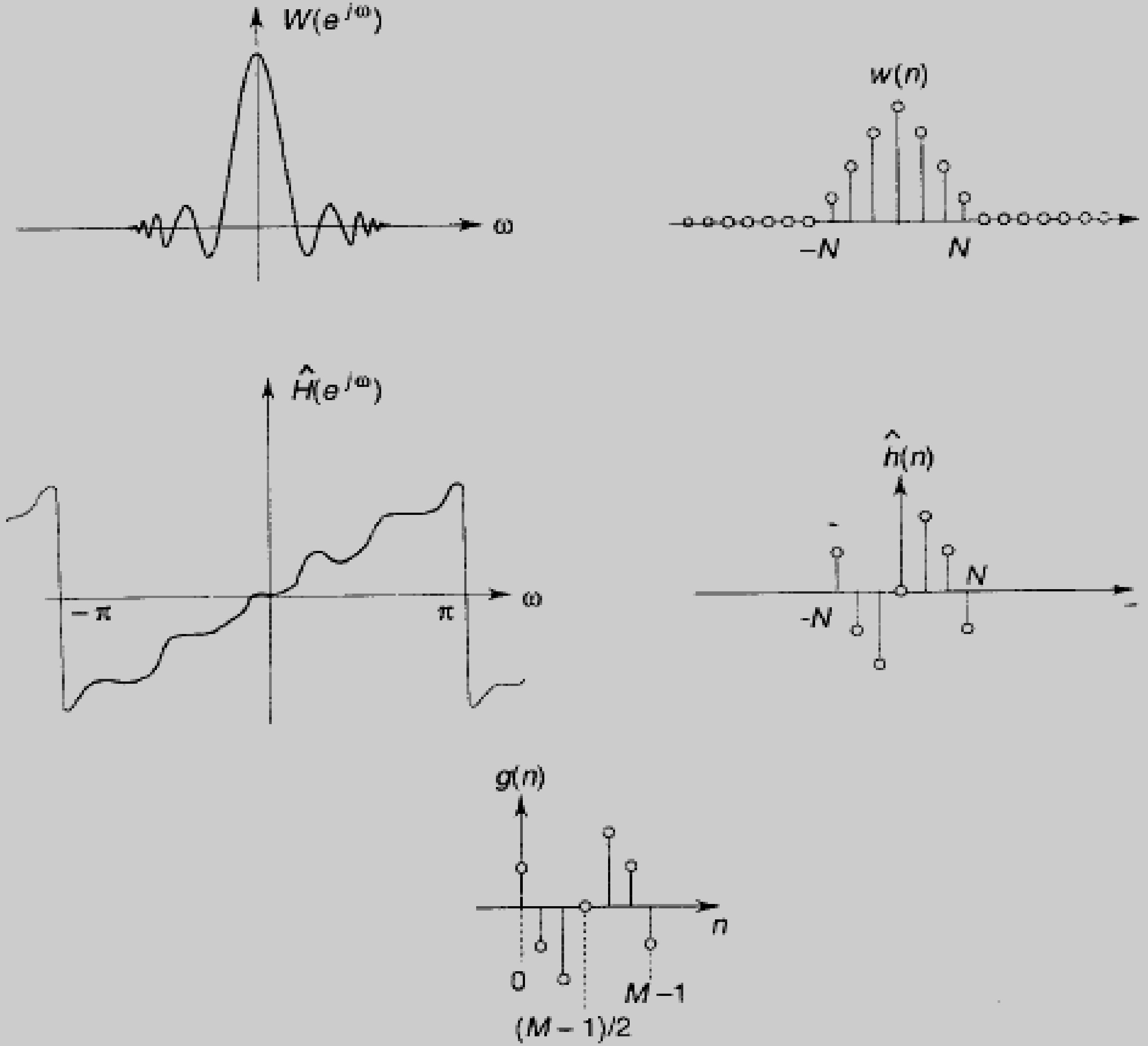

Illustration of the Window Technique

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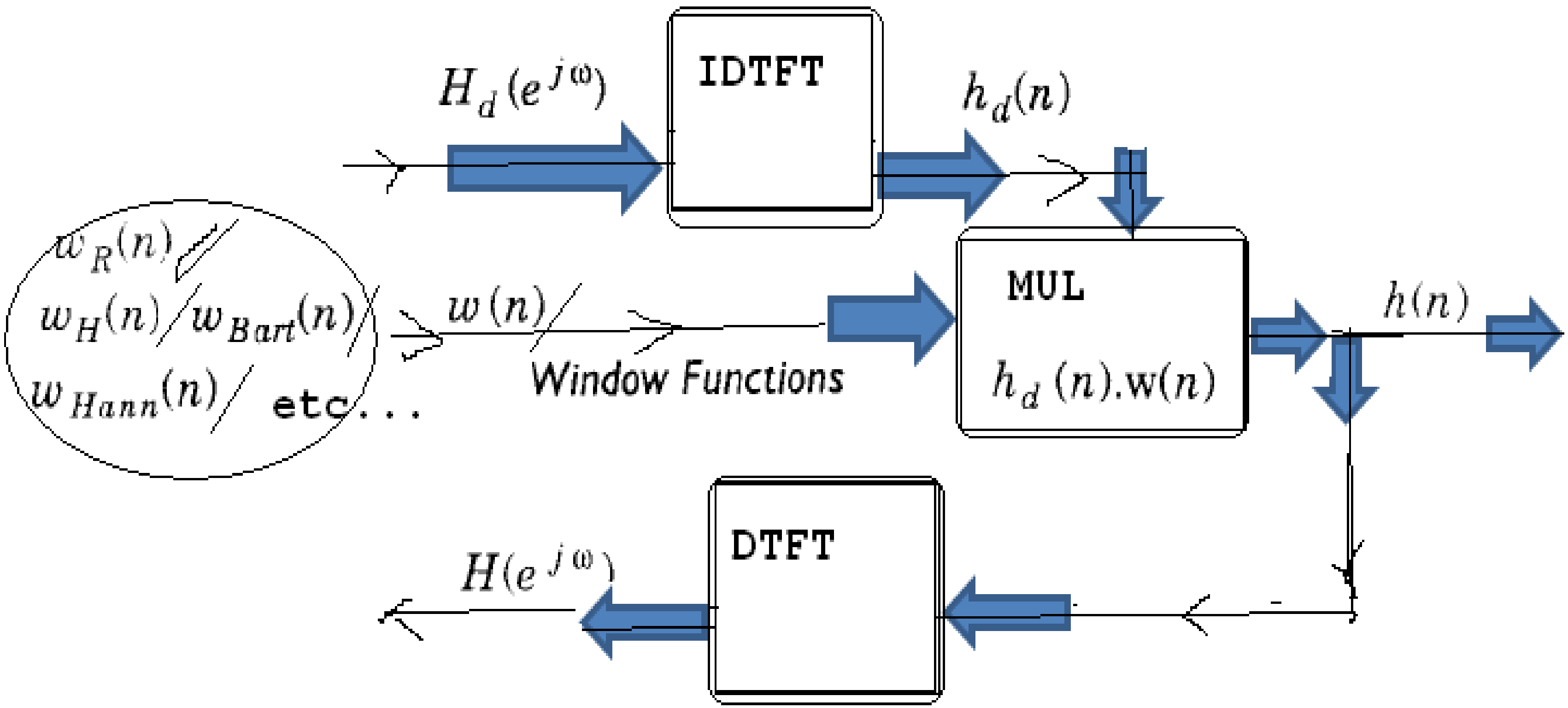
FLOW DIAGRAM FOR DESIGN FIR FILTER USING WINDOW METHOD



frequency response of the designed low-pass filter is $H(e^{j\omega})$

filter coefficients of the filter $h(n) = h_d(n).w(n)$

window function $w(n)$

desired frequency response $H_d(e^{j\omega})$

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TYPES OF WINDOW FUNCTION:

1. **RECTANGULAR WINDOW FUNCTION.**

2. **HAMMING WINDOW FUNCTION.**

3. *HANNING WINDOW FUNTION.*

4. **BLACKMAN WINDOW FUNCTION.**

5. *BARTLET WINDOW FUNCTION.*

6. **TRINGULAR WINDOW FUNTION.**

7. **KAISER WINDOW**

8. _etc…

Kaiser window.

001 **Rectangular Window Function**

The weighting function for the rectangular window is given by

$$w_R(n) = \begin{cases} 1, & \text{for } |n| \le \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$W_R(e^{j\omega T}) = \frac{\sin\left(\dfrac{\omega M T}{2}\right)}{\sin\left(\dfrac{\omega T}{2}\right)}$$

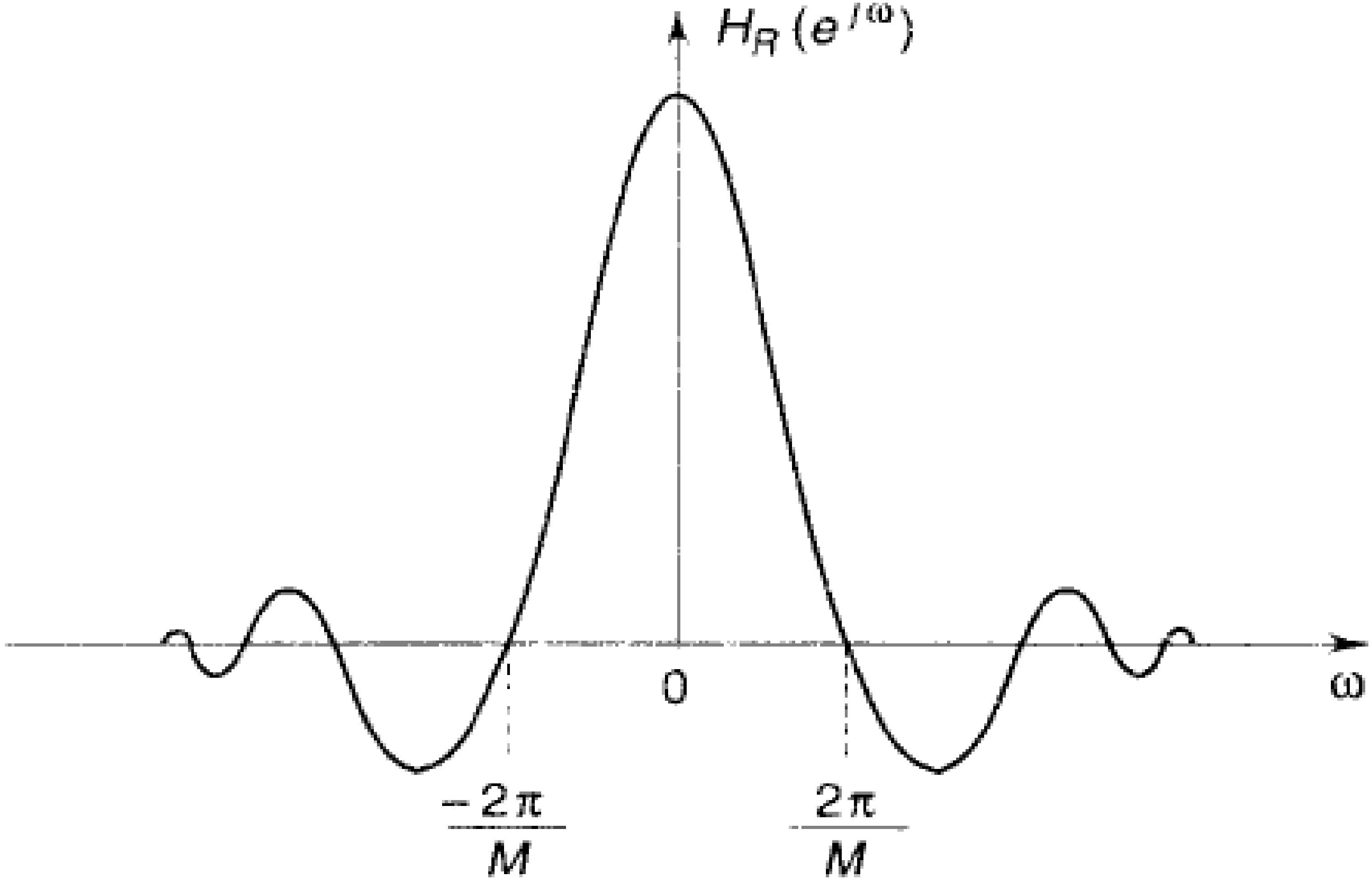
002. **Hamming Window Function**

causal Hamming window function is expressed by

$$w_H(n) = \begin{cases} 0.54 - 0.46 \cos \dfrac{2\pi n}{M-1}, & 0 \le n < M-1 \\ 0, & \text{otherwise} \end{cases}$$

non-causal Hamming window function is given by

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos \dfrac{2\pi n}{M-1}, & \text{for } |n| \le \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$



Frequency Response of a Rectangular Window

003. Blackman Window Function

window function of a causal Blackman window is expressed by

$$\text{Wblack}(n) = \begin{cases} 0.42 - 0.5 \cos \dfrac{2\pi n}{M-1} + 0.08 \cos \dfrac{4\pi n}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

window function of a non-causal Blackman window is given by

$$\text{Wblack}(n) = \begin{cases} 0.42 + 0.5 \cos \dfrac{2\pi n}{M-1} + 0.08 \cos \dfrac{4\pi n}{M-1}, & \text{for } |n| < \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

main lobe is approximately $12\pi/M$ side-lobe is at –58dB.

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004 BARTLETT TRIANGULAR WINDOW

window function of a non-causal Bartlett window is expressed by

$$w_{Bart}(n) = \begin{cases} 1+n, & -\frac{M-1}{2} < n < 1 \\ 1-n, & 1 < n < \frac{M-1}{2} \end{cases}$$

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Frequency-Domain Characteristics of Some Window Functions

TYPES OF WINDOW--	Approximate Transition Width of Main Lobe	Minimum Stopband Attenuation (dB)	Peak of first Sidelobe (dB)
RECTANGULAR-------	$4\pi / M$	–21	– 13
BARTLETT---------	$8\pi / M$	–25	– 27
HANNING-----------	$8\pi / M$	–44	– 32
HAMMING---------	$8\pi / M$	–53	– 43
BLACKMAN----------	$12\pi / M$	–74	–58

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Example 7.5 A low-pass filter is to be designed with the following desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & -\pi/4 \le \omega \le \pi/4 \\ 0, & \pi/4 < |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{j\omega})$ of the designed filter.

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Example 7.5

Solution Given

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & -\pi/4 \le \omega \le \pi/4 \\ 0, & \pi/4 < |\omega| \le \pi \end{cases}$$

Therefore,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)}\, d\omega$$

$$= \frac{1}{\pi(n-2)}\left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{2j}\right]$$

$$= \frac{1}{\pi(n-2)}\sin\frac{\pi}{4}(n-2), \quad n \ne 2$$

For $n = 2$, the filter coefficient can be obtained by applying L'Hospital's rule to the above expression. Thus,

$$h_d(2) = \frac{1}{4}$$

The other filter coefficients are given by

$$h_d(0) = \frac{1}{2\pi} = h_d(4) \quad \text{and} \quad h_d(1) = \frac{1}{\sqrt{2}\,\pi} = h_d(3)$$

The filter coefficients of the filter would be then

$$h(n) = h_d(n).w(n)$$

Therefore,

$$h(0) = \frac{1}{2\pi} = h(4), \quad h(1) = \frac{1}{\sqrt{2}\,\pi} = h(3) \text{ and } h(2) = \frac{1}{4}$$

The frequency response $H(e^{j\omega})$ is given by

$$H(e^{j\omega}) = \sum_{n=0}^{4} h(n)\, e^{-j\omega n}$$

$$= h(0) + h(1)\, e^{-j\omega} + h(2)\, e^{-j2\omega} + h(3)e^{-j3\omega} + h(4)e^{-j4\omega}$$

$$= e^{-j2\omega}\, [h(0)e^{j2\omega} + h(1)e^{j\omega} + h(2) + h(3)e^{-j\omega} + h(4)e^{-j2\omega}]$$

$$= e^{-j2\omega}\, \{h(2) + h(0)\, [e^{j2\omega} + e^{-j2\omega}] + h(1)[e^{j\omega} + e^{-j\omega}]\}$$

$$= e^{-j2\omega} \left\{ \frac{1}{4} + \frac{1}{2\pi}\left[e^{j2\omega} + e^{-j2\omega}\right] + \frac{1}{\sqrt{2}\pi}\left[e^{j\omega} + e^{-j\omega}\right] \right\}$$

The frequency response of the designed low-pass filter is then,

$$H(e^{j\omega}) = e^{-j2\omega} \left\{ \frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos\omega + \frac{1}{\pi}\cos 2\omega \right\}$$

Example 7.6 A filter is to be designed with the following desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 < |\omega| \le \pi \end{cases}$$

MOD

Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{j\omega})$ of the designed filter.

Example 7.6

Solution Given

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 < |\omega| \le \pi \end{cases}$$

Therefore,

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/4} e^{-j2\omega} e^{j\omega n} \, d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{-j2\omega} e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/4} e^{j\omega(n-2)} \, d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{j\omega(n-2)} \, d\omega$$

$$= \frac{1}{\pi(n-2)} \left\{ \left[\frac{e^{j(n-2)\pi} - e^{-j(n-2)\pi}}{2j} \right] - \left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{2j} \right] \right\}$$

7.6

Solution Given

$$= \frac{1}{\pi(n-2)} [\sin \pi(n-2) - \sin (n-2)\pi/4], \quad n \neq 2$$

The filter coefficients are given by,

$$h_d(2) = \frac{3}{4}, \quad h_d(0) = \frac{1}{2\pi} = h_d(4) \text{ and } h_d(1) = \frac{1}{\sqrt{2}\pi} = h_d(3)$$

and by applying the window function, the new filter coefficients are

$$h(2) = \frac{3}{4}, \quad h(0) = \frac{1}{2\pi} = h(4) \text{ and } h(1) = \frac{1}{\sqrt{2}\pi} = h(3)$$

The frequency response $H(e^{j\omega})$, is obtained as in the previous example,

$$H(e^{j\omega}) = e^{-j2\omega}\left[0.75 - \frac{\sqrt{2}}{\pi}\cos\omega - \frac{1}{\pi}\cos 2\omega\right]$$

Example 7.7 A low-pass filter should have the frequency response given below. Find the filter coefficients $h_d(n)$. Also determine τ so that $h_d(n) = h_d(-n)$.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau}, & -\omega_c \le \omega \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

Example 7.8 The desired response of a low-pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & -3\pi/4 \le \omega \le 3\pi/4 \\ 0, & 3\pi/4 < |\omega| \le \pi \end{cases}$$

Determine $H(e^{j\omega})$ for $M = 7$ using a Hamming window.

Example 7.9 Design an FIR digital filter to approximate an ideal low-pass filter with passband gain of unity, cut-off frequency of 850 Hz and working at a sampling frequency of $f_s = 5000$ Hz. The length of the impulse response should be 5. Use a rectangular window.

Example 7.7

Solution The filter coefficients are given by

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\tau} e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{\sin \omega_c (n-\tau)}{\pi (n-\tau)}, \quad n \neq \tau \text{ and } h_d(\tau) = \frac{\omega_c}{\pi}$$

when $h_d(n) = h_d(-n)$,

$$\frac{\sin \omega_c (n-\tau)}{\pi (n-\tau)} = \frac{\sin \omega_c (-n-\tau)}{\pi (-n-\tau)}$$

That is,

$$\frac{\sin \omega_c (n-\tau)}{\pi (n-\tau)} = \frac{-\sin \omega_c (n+\tau)}{-\pi (n+\tau)} = \frac{\sin \omega_c (n+\tau)}{\pi (n+\tau)}$$

This is possible only when $(n-\tau) = (n+\tau)$ or $\tau = 0$.

Solution The filter coefficients are given by **Example 7.8**

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{-j3\omega}\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{\sin 3\pi(n-3)/4}{\pi(n-3)}, \quad n \neq 3 \text{ and } h_d(3) = \frac{3}{4}$$

The filter coefficients are,

$h_d(0) = 0.0750$, $h_d(1) = -0.1592$, $h_d(2) = 0.2251$, $h_d(3) = 0.75$
$h_d(4) = 0.2251$, $h_d(5) = -0.1592$, $h_d(6) = 0.0750$

The Hamming window function is,

$$w(n) = \begin{cases} 0.54 - 0.46 \cos \dfrac{2\pi n}{M-1}, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, with $M = 7$,

$w(0) = 0.08$, $w(1) = 0.31$, $w(2) = 0.77$, $w(3) = 1$, $w(4) = 0.77$,

$w(5) = 0.31$, $w(6) = 0.08$.

The filter coefficients of the resultant filter are then,

$h(n) = h_d(n).w(n) \quad n = 0, 1, 2, 3, 4, 5, 6.$ agk-RCET-BHILAI

Therefore,

$h(0) = 0.006$, $h(1) = -0.0494$, $h(2) = 0.1733$, $h(3) = 0.75$,

$h(4) = 0.1733$, $h(5) = -0.0494$ and $h(6) = 0.006$.

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0}^{6} h(n)\, e^{-j\omega n}$$

$$= e^{-j3\omega}\,[\, h(3) + 2h(0)\cos 3\omega + 2h(1)\cos 2\omega + 2h(2)\cos \omega]$$

$$= e^{-j3\omega}\,[\, 0.75 + 0.3466 \cos \omega - 0.0988 \cos 2\omega + 0.012 \cos \omega]$$

Example 7.9

Solution The desired response of the ideal low-pass filter is given by

$$H_d(e^{j\omega}) = \begin{cases} 1, & 0 \le f \le 850 \text{ Hz} \\ 0, & f > 850 \text{ Hz} \end{cases}$$

The above response can be equivalently specified in terms of the normalised ω_c. The normalised $\omega_c = 2\pi f_c / f_s = 2\pi\,(850)/(5000) = 1.068$ rad/sec. Hence, the desired response is

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$H_d(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le 1.068 \\ 0, & 1.068 < |\omega| \le \pi \end{cases}$$

The filter coefficients are given by

Therefore,

$$h(0) = 0.34,\ h(1) = 0.2789,\ h(2) = 0.1344,\ h(3) = -0.0066,$$
$$h(4) = -0.072$$

$$h_d(n) = \frac{\sin 1.068\, n}{\pi n}, \quad n \ne 0 \quad \text{and} \quad h_d(0) = \frac{1.068}{\pi} = 0.3400$$

Using the rectangular window function and for $M = 5$,

$$h(n) = h_d(n)\,.\,w(n) \qquad n = 0, 1, 2, 3, 4.$$

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005. Kaiser Window

width of the main lobe praportional to
inversely proportional to the length of the filter.

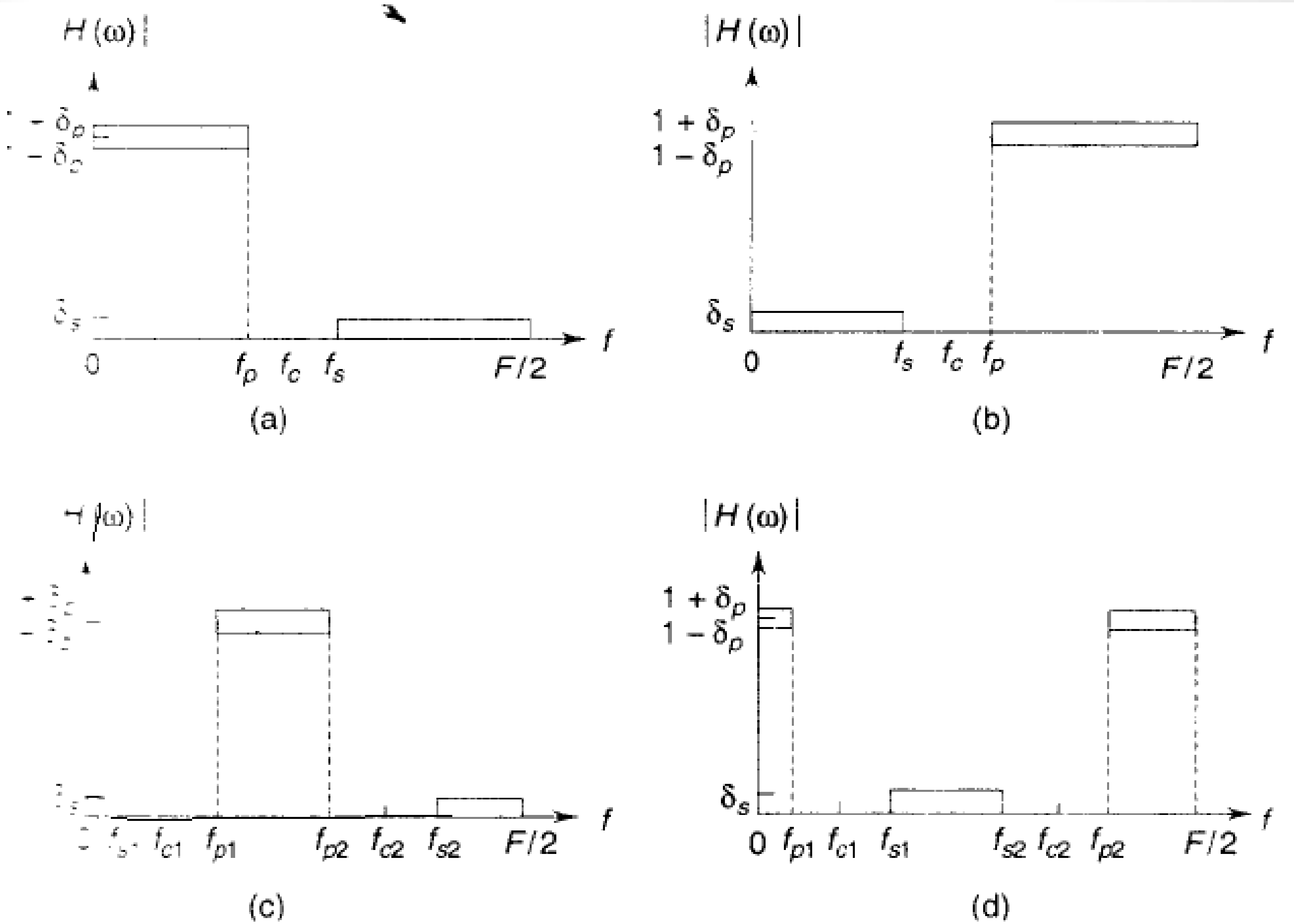
adv over above filter

Kaiser window, the side lobe level can be controlled with respect
mainlobe peak by varying a parameter, α.

The width of the main lobe
can be varied by adjusting the length of the filter.

function is given by

$$w_K(n) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)}, & \text{for } |n| \leq \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$



Idealised Frequency Responses: (a) low-pass filter; (b) high-pass filter; (c) bandpass filter; (d) bandstop filter

Design Using the Kaiser Window Fucntion

Design Specifications

1. Filter type : Low-pass, high-pass, bandpass or bandstop.
2. Passband and stopband frequencies in hertz:
   For low-pass / high-pass : f_P and f_S.
   For band-pass / band-stop : f_{P1}, f_{P2}, f_{S1} and f_{S2} .
3. Passband ripple and minimum stopband attenuation in positive decibels: A_P' and A_S'.
4. Sampling frequency in hertz : F
5. Filter order M - odd.

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OUR AIM TO DETERMINE:

$$h(n) = w_K(n)\,h_d(n),$$

DESIGN PROCEDURE FOR KAISER WINDOW:

Design Procedure

$$w_K(n) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)}, & \text{for } |n| \le \frac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

WHERE:

$$\beta = \alpha\left[1-\left(\frac{2n}{M-1}\right)^2\right]^{0.5}$$

$$A_P = 20\log_{10}\frac{1+\delta_p}{1-\delta_p}\ \text{dB}$$

$$A_S = -20\log_{10}\delta_S\ \text{dB}$$

FIRST STEP:

The length of the filter, M

$$M \ge \frac{FD}{\Delta F} + 1$$

$$D = \begin{cases} 0.9222, & \text{for } A_S \le 21 \\ \frac{A_S - 7.95}{14.36}, & \text{for } A_S > 21 \end{cases}$$

The transition bandwidth is

$$\Delta F = f_S - f_P$$

. Sampling frequency in hertz : F

where A_P and A_S are the actual passband peak-to-peak ripple and minimum stopband attenuation, respectively.

Suppose M=23 so !n!=-13 to -1, then 0 then 1 to 13

The filter coefficients of the non-causal digital filter $\{h(n) = h(-n)\}$

STEP SECOND:

$$w_K(n) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)}, & \text{for } |n| \le \dfrac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

WHERE:

$$\beta = \alpha \left[1 - \left(\frac{2n}{M-1} \right)^2 \right]^{0.5}$$

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k!} \left(\frac{x}{2} \right)^k \right]^2$$

$$= 1 + \frac{0.25x^2}{(1!)^2} + \frac{(0.25x^2)^2}{(2!)^2} + \frac{(0.25x^2)^3}{(3!)^2} + \dots$$

Determine the parameter α from the Kaiser's design equation

$$\alpha = \begin{cases} 0, & \text{for } A_S \le 21 \\ 0.5842(A_S - 21)^{0.4} + 0.07886(A_S - 21), & \text{for } 21 < A_S \le 50 \\ 0.1102(A_S - 8.7), & \text{for } A_S > 50 \end{cases}$$

$$\delta = \min(\delta_P, \delta_S)$$

$$\delta_S = 10^{-0.05 A_S'} \text{ and } \delta_P = \frac{10^{0.05 A_P'} - 1}{10^{0.05 A_P'} + 1}$$

STEP THIRD:

determine $h_d(n)$,

it is depend on type of filter

WHERE:

Low-Pass FIR Filter

$$h_d(n) = \begin{cases} \left(\frac{2f_c}{F}\right) \frac{\sin 2\pi n f_c / F}{2\pi n f_c / F}, & \text{for } n > 0 \\ \frac{2f_c}{F}, & \text{for } n = 0 \end{cases}$$

where

$$f_c = 0.5\,(f_p + f_s) \quad \text{and} \quad \Delta F = f_s - f_p$$

High-Pass FIR Filter

$$h_d(n) = \begin{cases} -\left(\frac{2f_c}{F}\right)\frac{\sin 2\pi n f_c / F}{2\pi n f_c / F}, & \text{for } n > 0 \\ 1 - \frac{2f_c}{F}, & \text{for } n = 0 \end{cases}$$

where

$$f_c = 0.5\,(f_p + f_s) \quad \text{and} \quad \Delta F = f_p - f_s$$

Bandpass FIR Filter

$$h_d(n) = \begin{cases} \frac{1}{n\pi}\left[\sin\left(2\pi n f_{c2} / F\right) - \sin\left(2\pi n f_{c1} / F\right)\right], & \text{for } n > 0 \\ \frac{2}{F}(f_{c2} - f_{c1}), & \text{for } n = 0 \end{cases}$$

where

$$f_{c1} = f_{p1} - \frac{\Delta F}{2} \qquad f_{c2} = f_{p2} + \frac{\Delta F}{2}$$

$$\Delta F_l = f_{p1} - f_{s1} \qquad \Delta F_h = f_{s2} - f_{p2}$$

$$\Delta F = \min\left[\Delta F_l, \Delta F_h\right]$$

Bandstop FIR Filter

$$h_d(n) = \begin{cases} \dfrac{1}{n\pi}\left[\sin\left(2\pi n f_{c1}/F\right) - \sin\left(2\pi n f_{c2}/F\right)\right], & \text{for } n > 0 \\ \dfrac{2}{F}\left(f_{c1} - f_{c2}\right) + 1, & \text{for } n = 0 \end{cases}$$

where

$$f_{c1} = f_{p1} + \frac{\Delta F}{2} \qquad f_{c2} = f_{p2} - \frac{\Delta F}{2}$$

$$\Delta F_l = f_{s1} - f_{p1} \qquad \Delta F_h = f_{p2} - f_{s2}$$

$$\Delta F = \min\left[\Delta F_l, \Delta F_h\right]$$

Example 7.10 Design a low-pass digital FIR filter using Kaiser window satisfying the specifications given below.

Passband cut-off frequency, f_p = 150 Hz, stopband cut-off frequency, f_s = 250 Hz, passband ripple, A_p = 0.1 dB, stopband attenuation, A_s = 40 dB and sampling frequency, F = 1000 Hz.

The length of the filter, $M = 27$

along with the Kaiser window coefficients are listed below.

$h_d(0) = 0.400000006$	$a(0) = 1.000000000$	$h(0) = 0.400000006$
$h_d(1) = 0.302658588$	$a(1) = 0.990451336$	$h(1) = 0.299768597$
$h_d(2) = 0.093381047$	$a(2) = 0.962219954$	$h(2) = 0.089853108$
$h_d(3) = -0.062470987$	$a(3) = 0.916526139$	$h(3) = -0.057256293$
$h_d(4) = -0.075602338$	$a(4) = 0.855326056$	$h(4) = -0.064664647$
$h_d(5) = 0.000160935$	$a(5) = 0.781202257$	$h(5) = 0.000125723$
$h_d(6) = 0.050484315$	$a(6) = 0.697217405$	$h(6) = 0.035198543$
$h_d(7) = 0.026587145$	$a(7) = 0.606746852$	$h(7) = 0.016131667$
$h_d(8) = -0.023507830$	$a(8) = 0.513293743$	$h(8) = -0.012066422$
$h_d(9) = -0.033573132$	$a(9) = 0.420304537$	$h(9) = -0.014110940$
$h_d(10) = 0.000160934$	$a(10) = 0.330991328$	$h(10) = 0.000053268$
$h_d(11) = 0.027559206$	$a(11) = 0.248175934$	$h(11) = 0.006839531$
$h_d(12) = 0.015454730$	$a(12) = 0.174161583$	$h(12) = 0.002691620$
$h_d(13) = -0.014516240$	$a(13) = 0.110641472$	$h(13) = -0.00160609$

Example 7.11 Design a high-pass digital FIR filter using Kaiser window satisfying the specifications given below.

Passband cut-off frequency, f_p = 3200 Hz, stopband cut-off frequency, f_s = 1600 Hz, passband ripple, A_p = 0.1 dB, stopband attenuation, A_s = 40 dB and sampling frequency, F = 10000 Hz.

Solution From Eq. , the value of δ = 0.005756.

The actual stopband attenuation, A_s = 44.796982

The value of α = 3.952357 and D = 2.565946 from Eq.

The length of the filter, M = 18

The desired filter coefficients $\{h_d(n)\}$ are obtained from Eq.

The filter coefficients of the non-causal digital filter $\{h(n) = h(-n)\}$ along with the Kaiser window coefficients are listed below.

The filter coefficients of the non-causal digital filter $\{h(n) = h(-n)\}$ along with the Kaiser window coefficients are listed below.

0) = 0.519999981	$a(0) = 1.000000000$	$h(0) = 0.519999981$
1) = −0.317566037	$a(1) = 0.976742625$	$h(1) = -0.310180277$
2) = −0.019747764	$a(2) = 0.909421921$	$h(2) = -0.017959049$
3) = 0.104217999	$a(3) = 0.805053890$	$h(3) = 0.083901107$
4) = 0.019595038	$a(4) = 0.674255788$	$h(4) = 0.013212068$
5) = −0.060581177	$a(5) = 0.529811919$	$h(5) = -0.032096628$
6) = −0.019342067	$a(6) = 0.384986669$	$h(6) = -0.007446438$

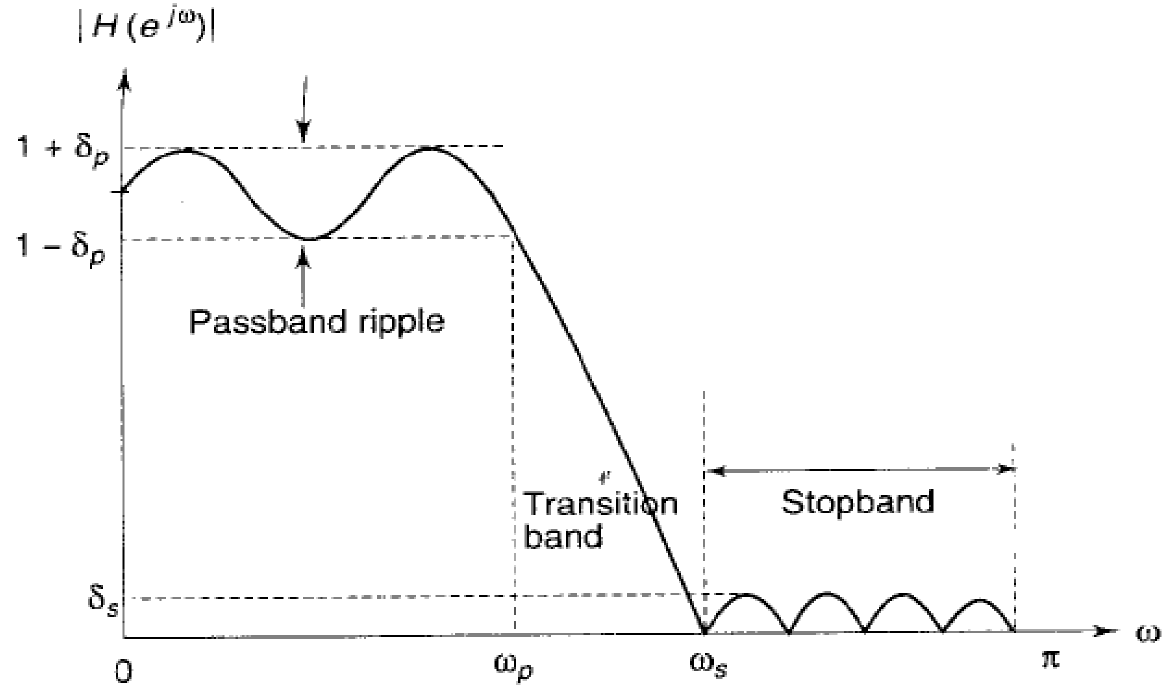
DESIGN OF OPTIMAL LINEAR PHASE FIR FILTERS

$$1-\delta_p \le |H(e^{j\omega})| \le 1+\delta_p, \quad |\omega| \le \omega_p$$

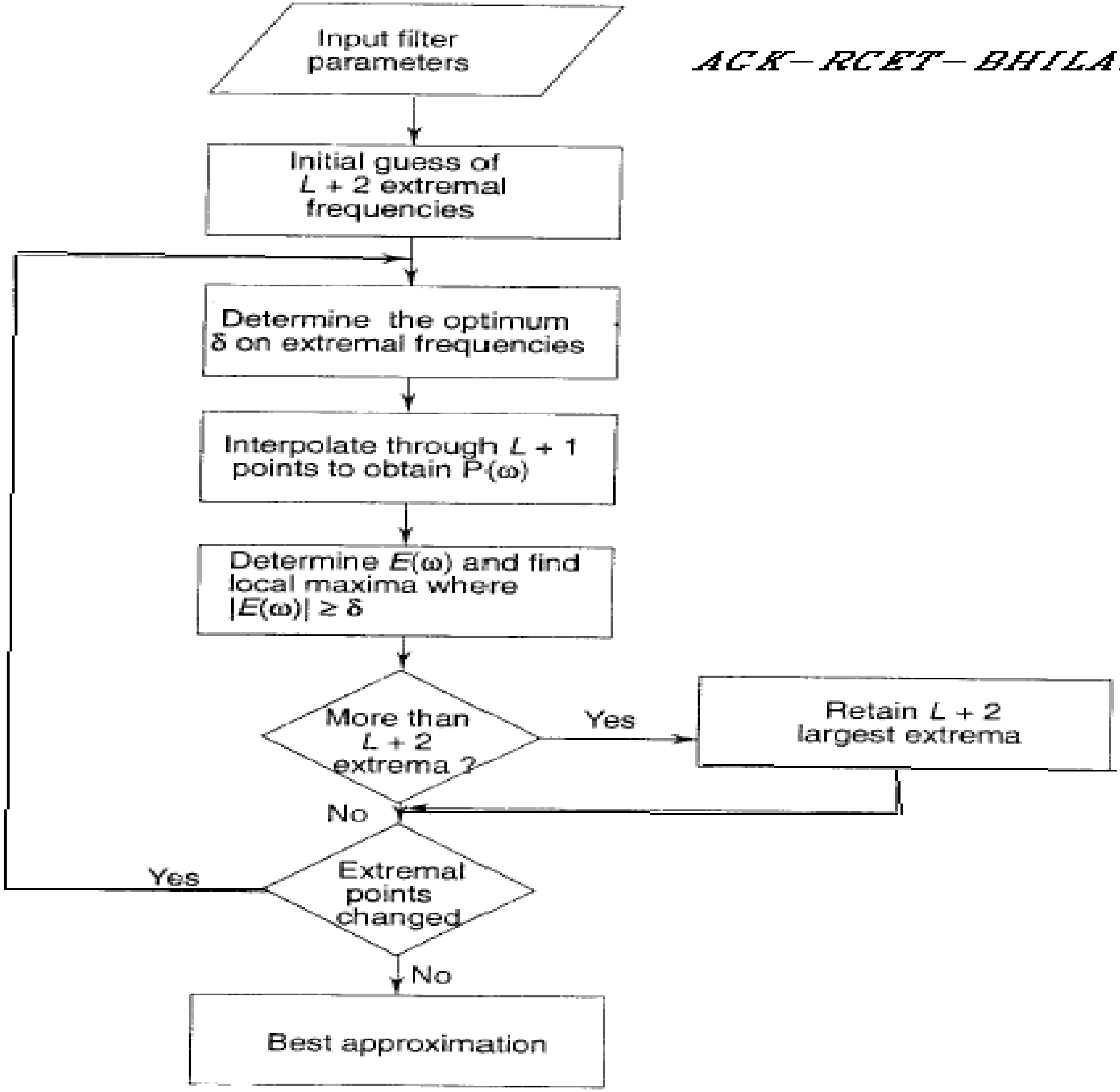
The frequency response in the stopband is

$$-\delta_s \le |H(e^{j\omega})| \le \delta_s, \quad |\omega| \ge \omega_s$$

The term δ_p represents the passband ripple and δ_s is the maximum attenuation in the stopband.



Frequency Response Characteristics of Physically Realisable Filters



ACK–RCET–BHILAI

Flowchart of the Remez Exchange Algorithm

... power.

.13 What are the effects of truncating an infinite Fourier series into a finite series ?

.14 Explain Gibb's phenomenon.

.15 What are the desirable features of the window functions ?

.16 What are the effects of windowing ?

.17 Explain the process of windowing using illustrations.

18. Name the different types of window functions. How they are defined ?

.19 What is a rectangular window function ? Obtain its frequency-domain characteristics.

.20 What is a Hamming window function ? Obtain its frequency-domain characteristics.

.21 What is a Hann window function ? Obtain its frequency-domain characteristics.

.22 Compare the frequency-domain characteristics of the different types of window functions.

.23 The desired frequency response of a low-pass filter is

$$H_d(e^{j\omega}) = \begin{cases} 1, & -\pi/2 \le \omega \le \pi/2 \\ 0, & \pi/2 \le |\omega| < \pi \end{cases}$$

Determine $h_d(n)$. Also determine $h(n)$ using the symmetric rectangular window with window length = 7.

Ans: $h_d(-3) = -0.1061 = h_d(3)$, $h_d(-2) = 0 = h_d(2)$, $h_d(-1) = 0.3183 = h_d(1)$, $h_d(0) = 0.5$; $h(n) = h_d(n).w(n) = h_d(n)$ for $-3 \le n \le 3$

24 The desired frequency response of a low-pass filter is

24 The desired frequency response of a low-pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & -3\pi/4 \le \omega \le 3\pi/4 \\ 0, & 3\pi/4 < |\omega| \le \pi \end{cases}$$

Determine $H(e^{j\omega})$ for $M = 7$ using a rectangular window.

Ans: $H(e^{j\omega}) = e^{-j3\omega}[0.75 + 0.4502 \cos \omega - 0.3184 \cos 2\omega + 0.15 \cos 3\omega]$

25 Design a bandpass filter which approximates the ideal filter with cut-off frequencies at 0.2 rad/sec and 0.3 rad/sec. The filter order is $M = 7$. Use the Hanning window function.

Ans: $h(0) = 0$, $h(1) = 0.0078$, $h(2) = 0.0209$, $h(3) = 0.0232$, $h(4) = 0.0128$, $h(5) = 0.00248$, $h(6) = 0$

26 What is a Kaiser window ? In what way is it superior to other window functions ?

27 Explain the procedure for designing an FIR filter using the Kaiser window.

7.28 *What is an FIR half-band digital filter ? Explain with a suitable illustration.*

7.29 *What is an optimal linear phase FIR filter ? What parameters are optimised in these filters ?*

7.30 *State and explain the alternation theorem.*

7.31 *What are extra ripple filters ?*

7.32 *What are maximal ripple filters ?*

7.33. *Explain the Remez exchange algorithm used in the design of optimal filters.*

THANK YOU

END OF THIRD UNIT

NAMASTE