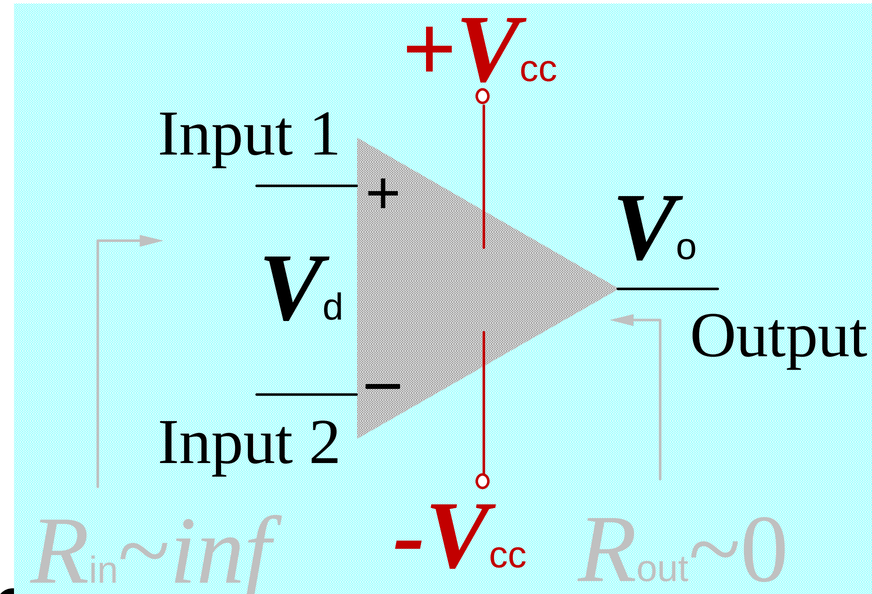
Lecture 1 Op-Amp

- Introduction of Operation Amplifier (Op-Amp)
- Analysis of ideal Op-Amp applications
- Comparison of ideal and non-ideal Op-Amp
- Non-ideal Op-Amp consideration

Operational Amplifier (Op-Amp)

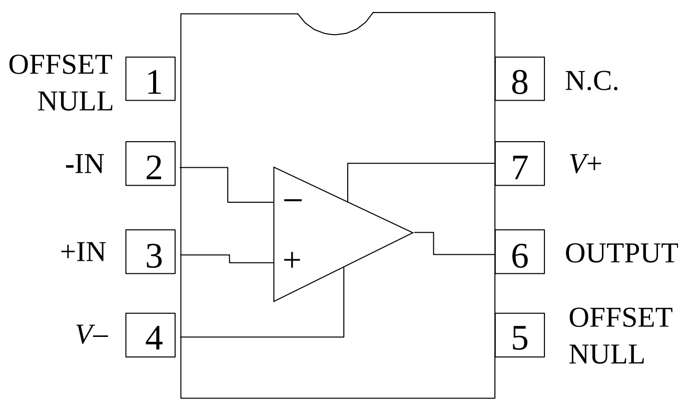
- Very high differential gain
- High input impedance
- Low output impedance
- Provide voltage changes (amplitude and polarity)
- Used in oscillator, filter and instrumentation
- Accumulate a very high gain by multiple stages



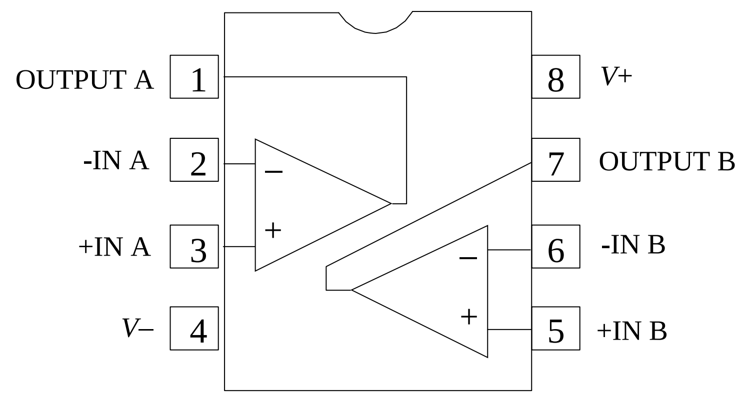
$$V_o = G_d V_d$$

G_d : differential gain normally very large, say 10^5

IC Product

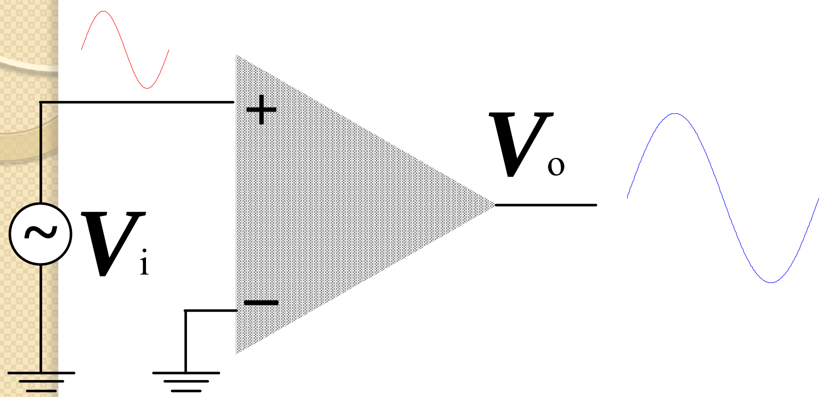


DIP-741

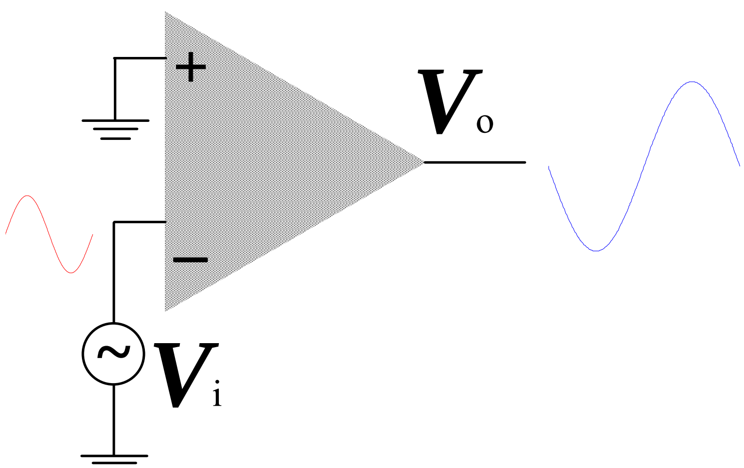


Dual op-amp 1458 device

Single-Ended Input

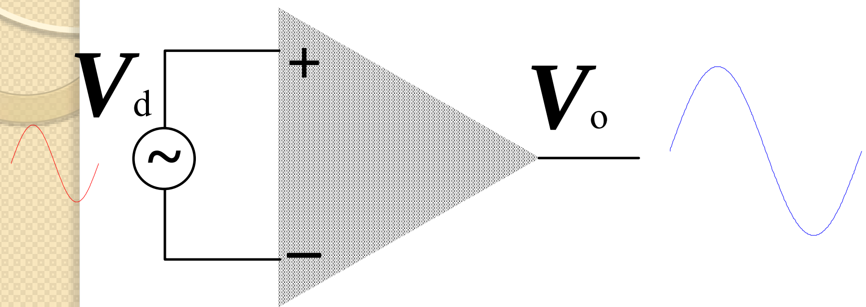


- + terminal : Source
- – terminal : Ground
- 0° phase change



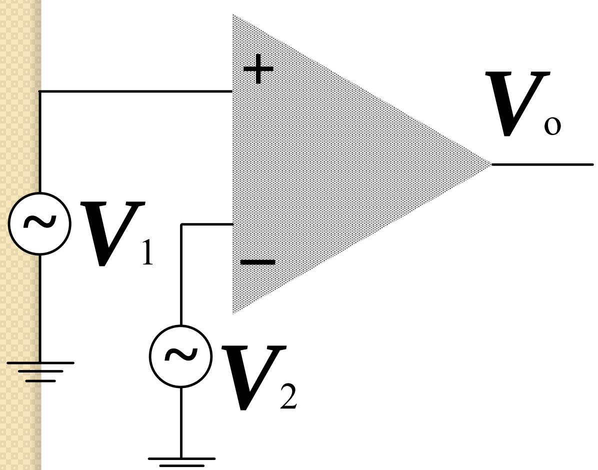
- + terminal : Ground
- – terminal : Source
- 180° phase change

Double-Ended Input

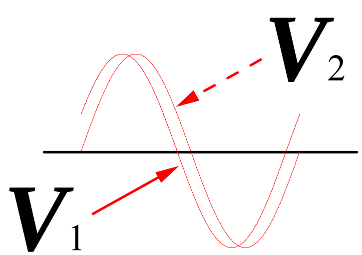


- Differential input
- $V_d = V_+ - V_-$
- 0° phase shift change

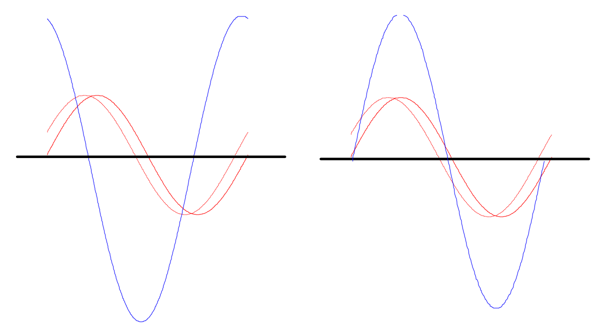
  between V_o and V_d



Qu: What V_o should be if,

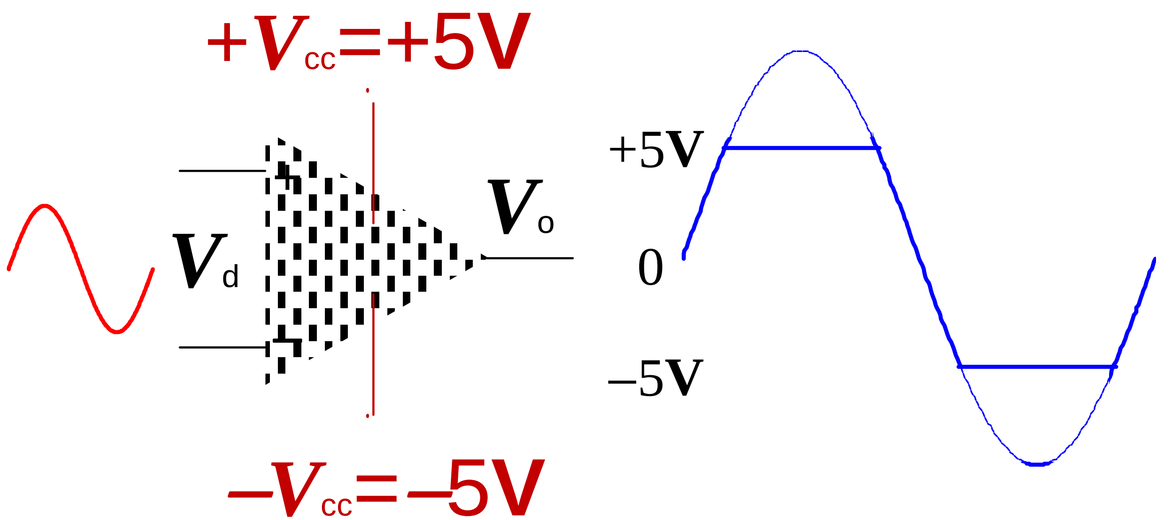


Ans: (A or B) ?



(A) (B)

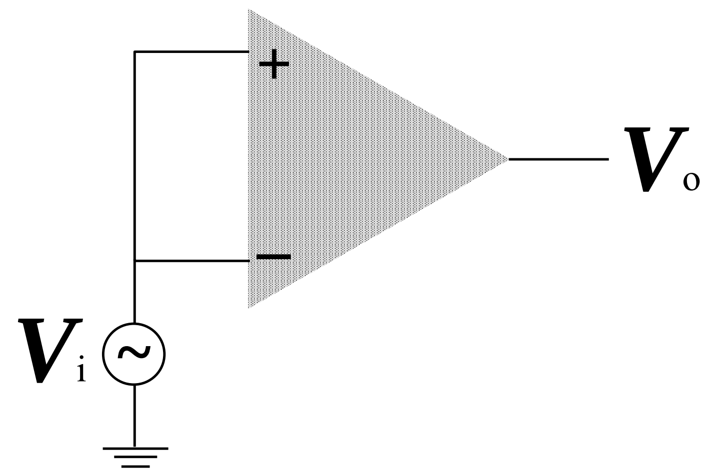
Distortion



The output voltage never excess the DC voltage supply of the Op-Amp

Common-Mode Operation

- Same voltage source is applied at both terminals
- Ideally, two input are equally amplified
- Output voltage is ideally zero due to differential voltage is zero
- Practically, a small output signal can still be measured



Note for differential circuits:
Opposite inputs : highly amplified
Common inputs : slightly amplified

⇒ Common-Mode Rejection

Common-Mode Rejection Ratio (CMRR)

Differential voltage input :

$$V_d = V_+ - V_-$$

Common voltage input :

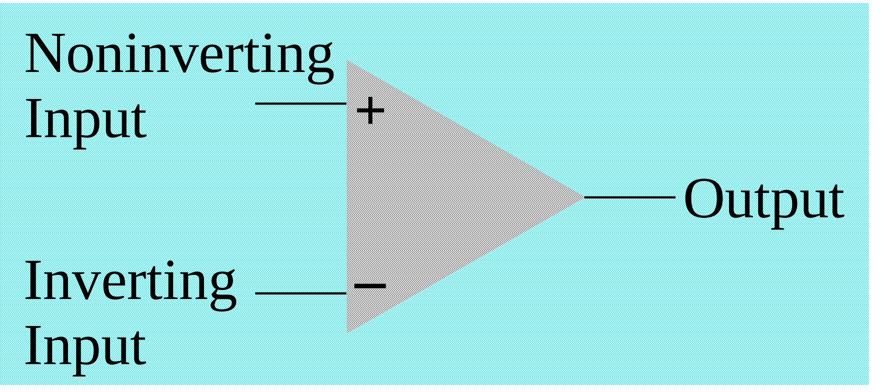
$$V_c = \frac{1}{2}(V_+ + V_-)$$

Output voltage :

$$V_o = G_d V_d + G_c V_c$$

G_d : Differential gain

G_c : Common mode gain



Common-mode rejection ratio:

$$\mathrm{CMRR} = \frac{G_d}{G_c} = 20\log_{10}\frac{G_d}{G_c}\ (\mathrm{dB})$$

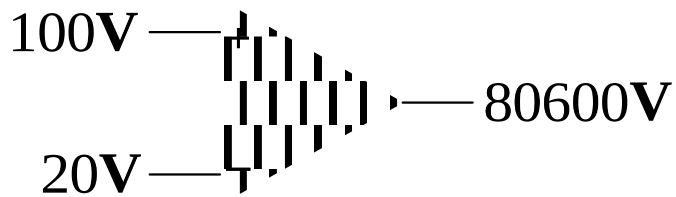
Note:

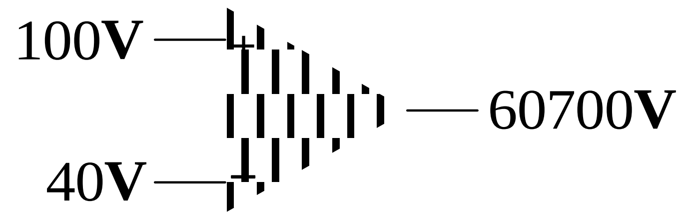
When $G_d >> G_c$ or CMRR $\rightarrow \infty$

$\Rightarrow V_o = G_d V_d$

CMRR Example

What is the CMRR?





Solution :

$$\left.\begin{aligned} V_{d1} &= 100 - 20 = 80\mathbf{V} \\ V_{c1} &= \frac{100+20}{2} = 60\mathbf{V} \end{aligned}\right\} (1) \qquad \left.\begin{aligned} V_{d2} &= 100 - 40 = 60\mathbf{V} \\ V_{c2} &= \frac{100+40}{2} = 70\mathbf{V} \end{aligned}\right\} (2)$$

From (1) $V_o = 80G_d + 60G_c = 80600\mathbf{V}$

From (2) $V_o = 60G_d + 70G_c = 60700\mathbf{V}$

$G_d = 1000$ **and** $G_c = 10$ $\Rightarrow \mathbf{CMRR} = 20\log(1000/10) = 40\mathbf{dB}$

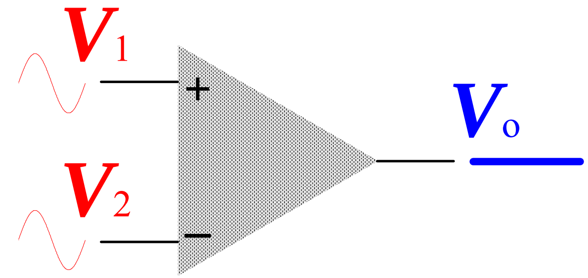
NB: This method is Not work! Why?

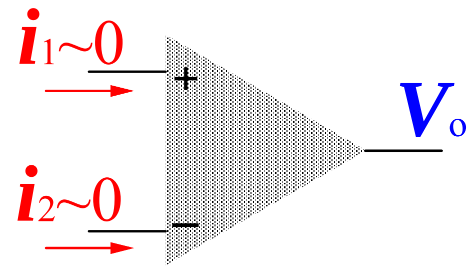
Op-Amp Properties

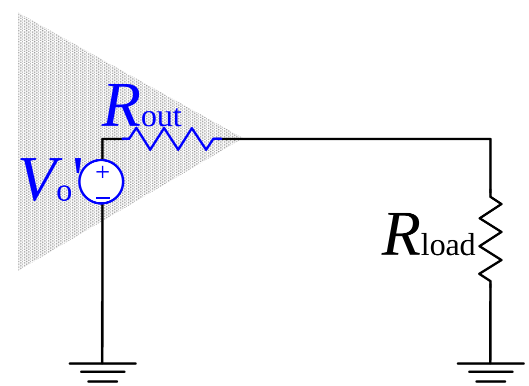
(1) Infinite Open Loop gain
- The gain without feedback
- Equal to differential gain
- Zero common-mode gain
- Pratically, G_d = 20,000 to 200,000

(2) Infinite Input impedance
- Input current i_i ~0A
- T-Ω in high-grade op-amp
- m-A input current in low-grade op-amp

(3) Zero Output Impedance
- act as perfect internal voltage source
- No internal resistance
- Output impedance in series with load
- Reducing output voltage to the load
- Practically, R_{out} ~ 20-100 Ω



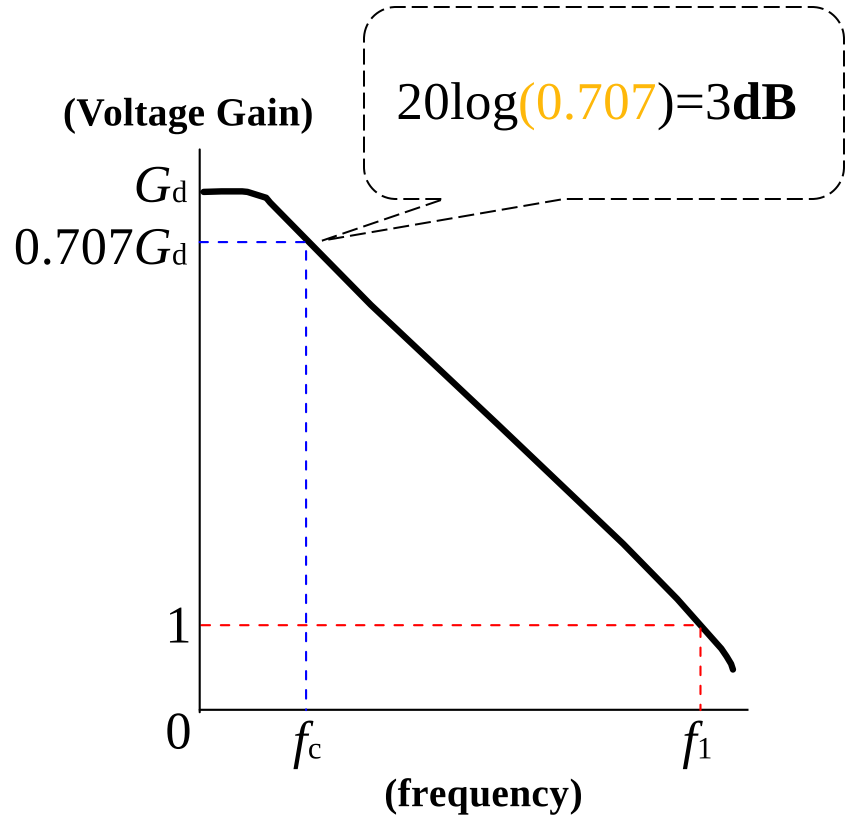




$$V_{load} = V_o' \frac{R_{load}}{R_{load} + R_{out}}$$

Frequency-Gain Relation

- Ideally, signals are amplified from DC to the highest AC frequency
- Practically, bandwidth is limited
- 741 family op-amp have an limit bandwidth of few KHz.
- Unity Gain frequency f_1: the gain at unity
- Cutoff frequency f_c: the gain drop by 3dB from dc gain G_d



GB Product : $f_1 = G_d\, f_c$

GB Product

Example: Determine the cutoff frequency of an op-amp having a unit gain frequency f_1 = 10 MHz and voltage differential gain G_d = 20**V**/m**V**

Sol:

Since f_1 = 10 MHz

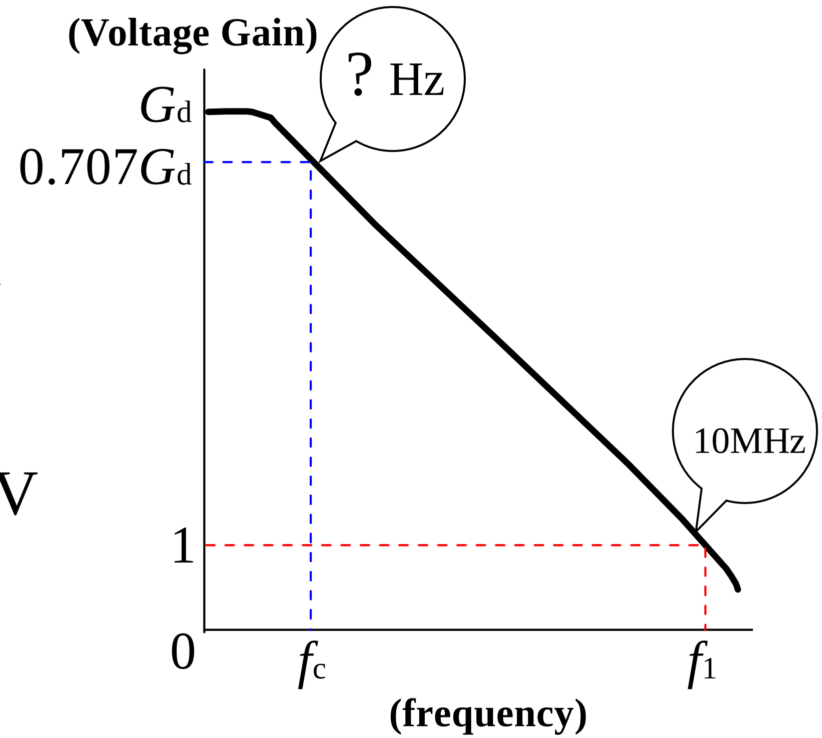
By using GB production equation

$$f_1 = G_d\, f_c$$

$$f_c = f_1 / G_d = 10\text{ MHz} / 20\text{ V/mV}$$

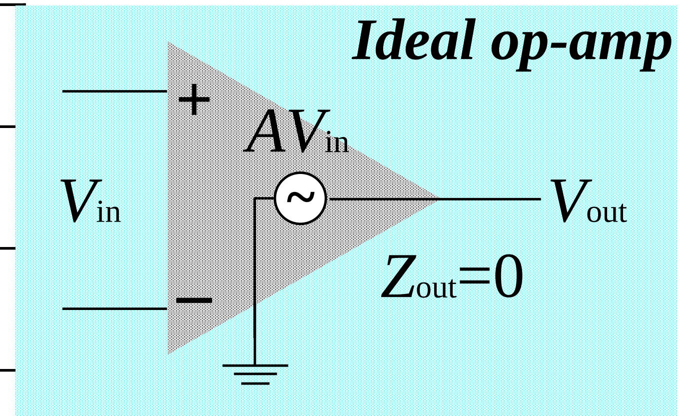
$$= 10 \times 10^6 / 20 \times 10^3$$

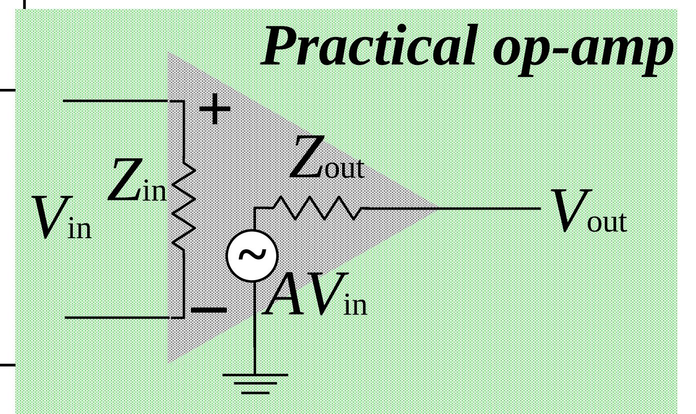
$$= 500\text{ Hz}$$



Ideal Vs Practical Op-Amp

	Ideal	Practical
Open Loop gain *A*	$\propto$	10^5
Bandwidth *BW*	$\propto$	10-100Hz
Input Impedance Z_{in}	$\propto$	>1MΩ
Output Impedance Z_{out}	0 Ω	10-100 Ω
Output Voltage V_{out}	Depends only on $V_d = (V_+ - V_-)$ Differential mode signal	Depends slightly on average input $V_c = (V_+ + V_-)/2$ Common-Mode signal
CMRR	$\propto$	10-100dB





Ideal Op-Amp Applications

Analysis Method :

Two ideal Op-Amp Properties:

(1) The voltage between V_+ and V_- is zero $V_+ = V_-$

(2) The current into both V_+ and V_- termainals is zero

For ideal Op-Amp circuit:

(1) Write the kirchhoff node equation at the noninverting terminal V_+

(2) Write the kirchhoff node eqaution at the inverting terminal V_-

(3) Set $V_+ = V_-$ and solve for the desired closed-loop gain

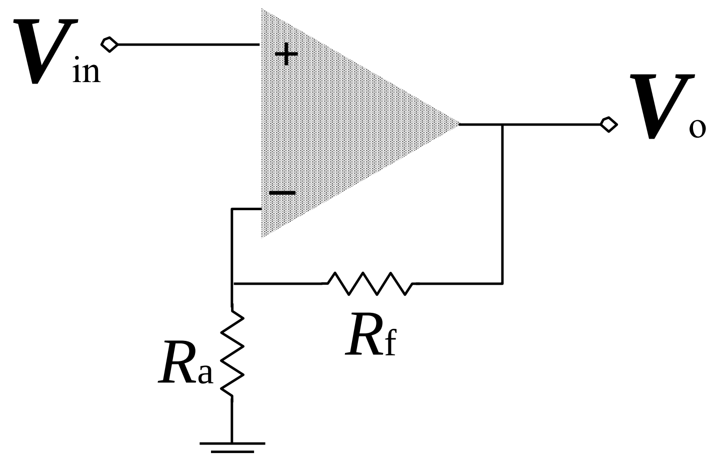
Noninverting Amplifier

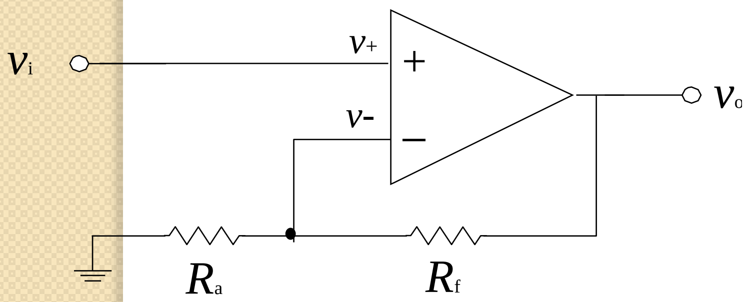
(1) Kirchhoff node equation at V_+ yields, $V_+ = V_i$

(2) Kirchhoff node equation at V_- yields, $\frac{V_- - 0}{R_a} + \frac{V_- - V_o}{R_f} = 0$

(3) Setting $V_+ = V_-$ yields

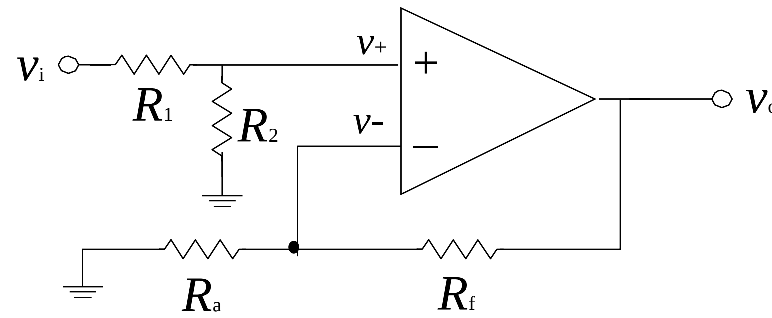
$$\frac{V_i}{R_a} + \frac{V_i - V_o}{R_f} = 0 \quad \text{or} \quad \frac{V_o}{V_i} = 1 + \frac{R_f}{R_a}$$





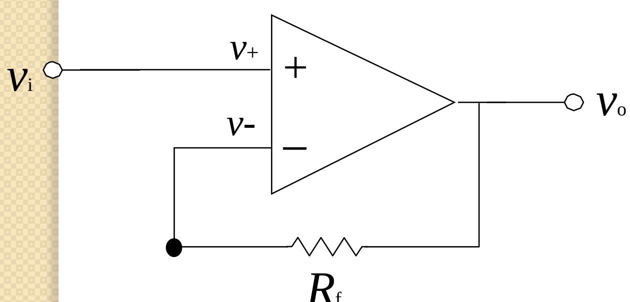
Noninverting amplifier

$$v_o = (1 + \frac{R_f}{R_a})v_i$$



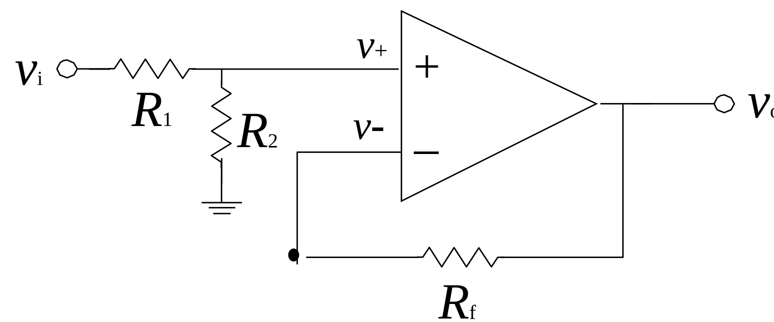
Noninverting input with voltage divider

$$v_o = (1 + \frac{R_f}{R_a})(\frac{R_2}{R_1 + R_2})v_i$$



Voltage follower

$$v_o = v_i$$



Less than unity gain

$$v_o = \frac{R_2}{R_1 + R_2}v_i$$

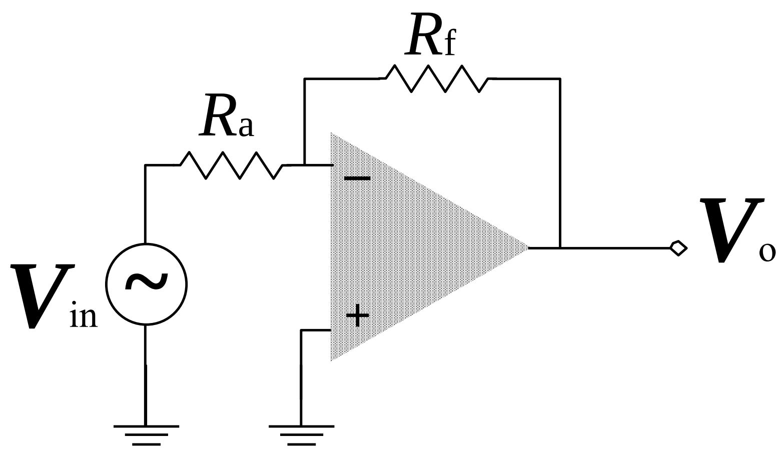
Inverting Amplifier

(1) Kirchhoff node equation at V_+ yields, $V_+ = 0$

(2) Kirchhoff node equation at V_- yields, $\frac{V_{in} - V_-}{R_a} + \frac{V_o - V_-}{R_f} = 0$

(3) Setting $V_+ = V_-$ yields

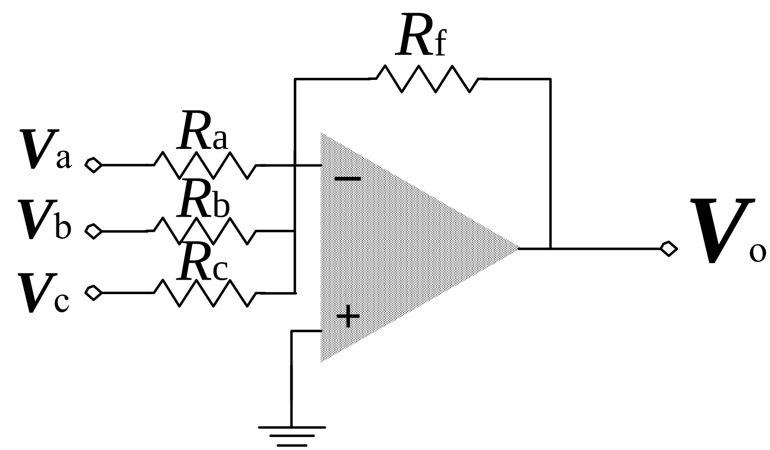
$$\frac{V_o}{V_{in}} = \frac{-R_f}{R_a}$$



Notice: The closed-loop gain V_o/V_{in} is dependent upon the ratio of two resistors, and is independent of the open-loop gain. This is caused by the use of feedback output voltage to subtract from the input voltage.

Multiple Inputs

(1) Kirchhoff node equation at V_+ yields, $V_+ = 0$



(2) Kirchhoff node equation at V_- yields,

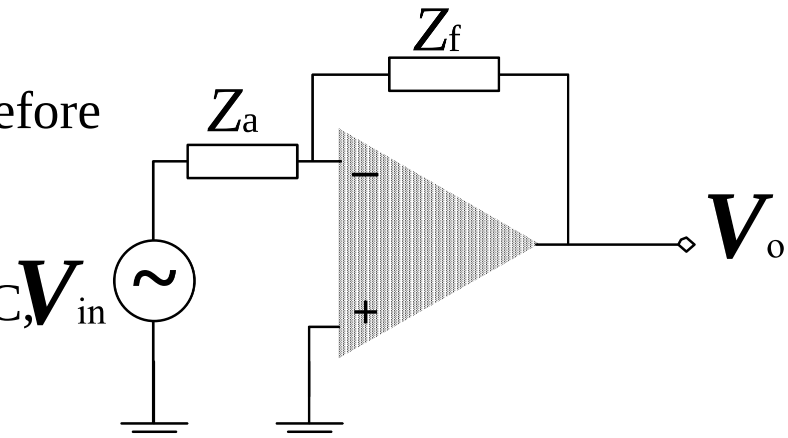
$$\frac{V_- - V_o}{R_f} + \frac{V_- - V_a}{R_a} + \frac{V_- - V_b}{R_b} + \frac{V_- - V_c}{R_c} = 0$$

(3) Setting $V_+ = V_-$ yields

$$V_o = -R_f \left(\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} \right) = -R_f \sum_{j=a}^{c} \frac{V_j}{R_j}$$

Inverting Integrator

Now replace resistors R_a and R_f by complex components Z_a and Z_f, respectively, therefore



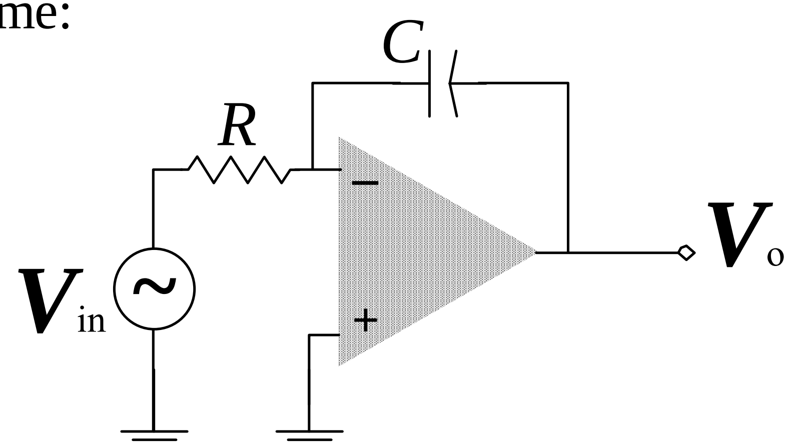
Supposing
$$V_o = \frac{-Z_f}{Z_a} V_{in}$$

(i) The feedback component is a capacitor C, i.e.,

$$Z_f = \frac{1}{j\omega C}$$

(ii) The input component is a resistor R, $Z_a = R$

Therefore, the closed-loop gain (V_o/V_{in}) become:



$$v_o(t) = \frac{-1}{RC} \int v_i(t) dt$$

where

$$v_i(t) = V_i e^{j\omega t}$$

What happens if $Z_a = 1/j\omega C$ whereas, $Z_f = R$?

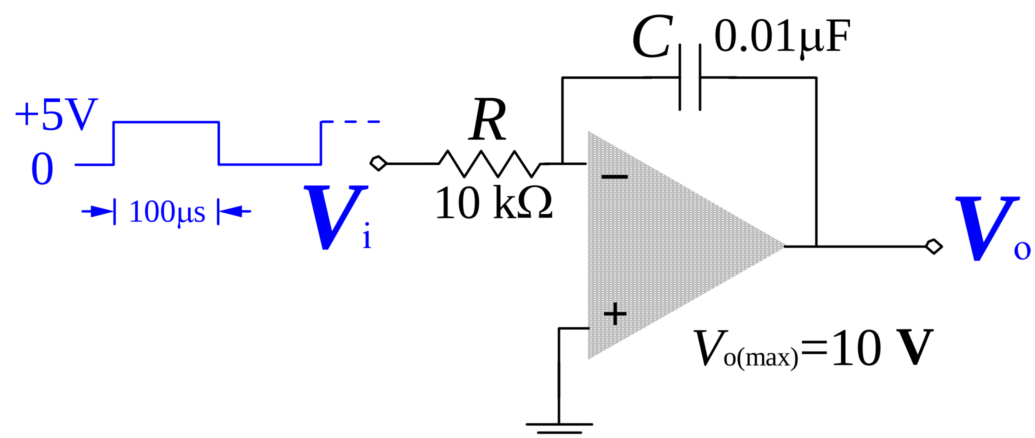
Inverting differentiator

Op-Amp Integrator

Example:

(a) Determine the rate of change of the output voltage.

(b) Draw the output waveform.



Solution:

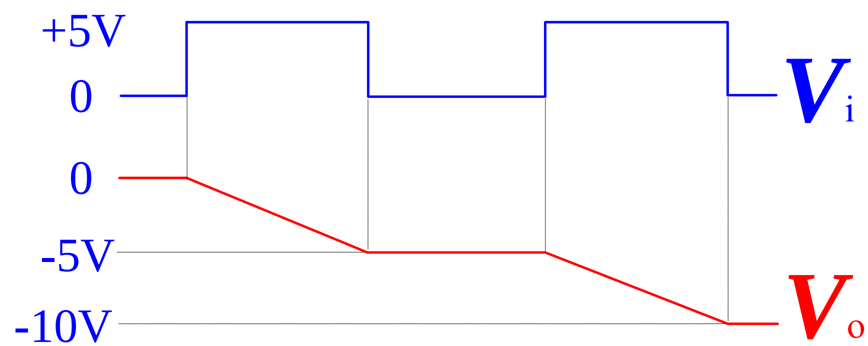
(a) Rate of change of the output voltage

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_i}{RC} = \frac{5\,\mathbf{V}}{(10\,\mathbf{k\Omega})(0.01\,\mu\mathbf{F})}$$

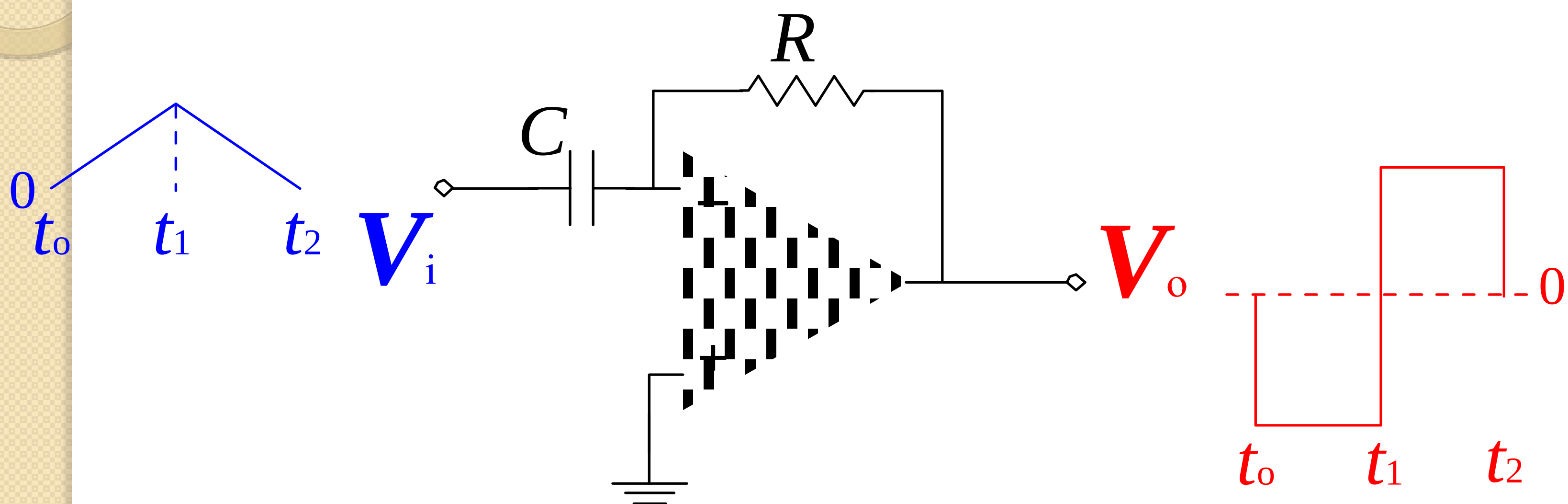
$$= -50\,\mathbf{mV}/\mu\mathbf{s}$$

(b) In 100 μs, the voltage decrease

$$\Delta V_o = (-50\,\mathbf{mV}/\mu\mathbf{s})(100\boldsymbol{\mu}\mathbf{s}) = -5\mathbf{V}$$

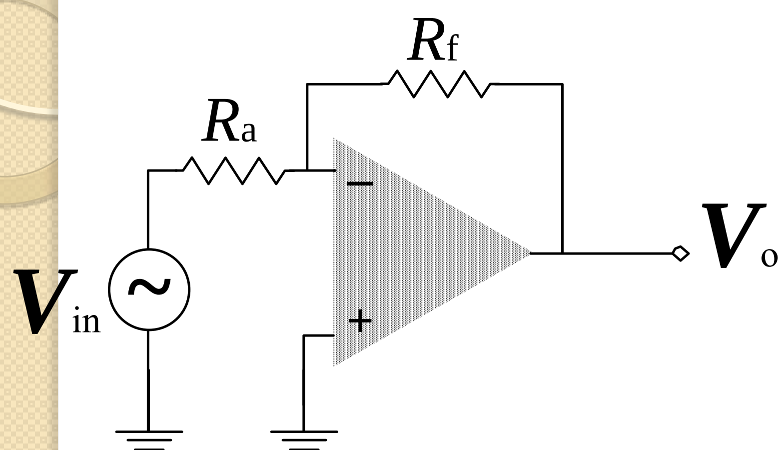


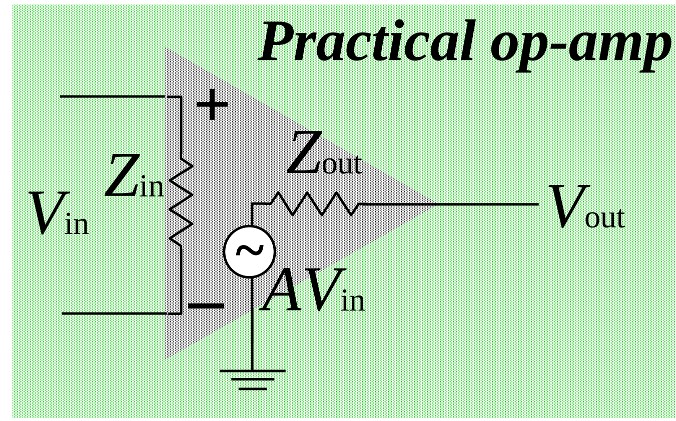
Op-Amp Differentiator



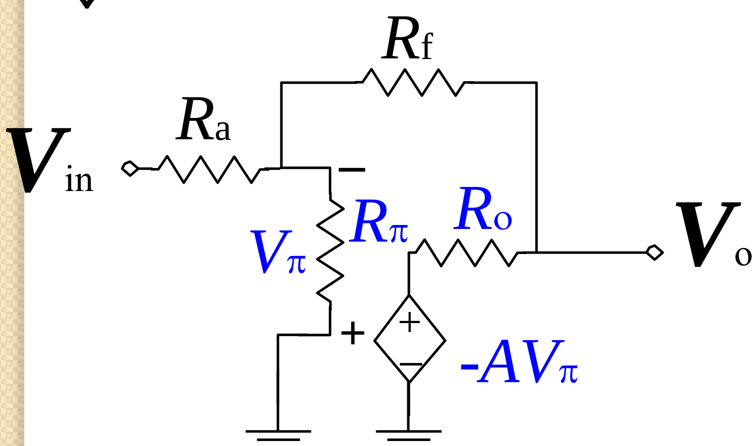
$$v_o = -\left(\frac{dV_i}{dt}\right)RC$$

Non-ideal case (Inverting Amplifier)





⇓ Equivalent Circuit



3 categories are considering

- ❑ Close-Loop Voltage Gain
- ❑ Input impedance
- ❑ Output impedance

Close-Loop Gain

Applied KCL at V– terminal,

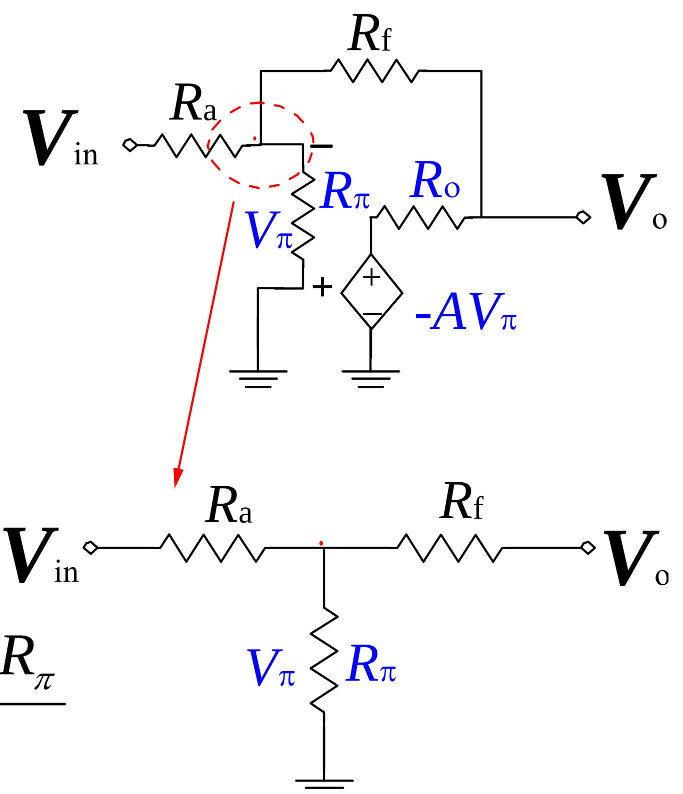
$$\frac{V_{in}-V_{\pi}}{R_a}+\frac{-V_{\pi}}{R_{\pi}}+\frac{V_o-V_{\pi}}{R_f}=0$$

By using the open loop gain,

$$V_o=-AV_{\pi}$$

$$\Rightarrow \quad \frac{V_{in}}{R_a}+\frac{V_o}{AR_a}+\frac{V_o}{AR_{\pi}}+\frac{V_o}{R_f}+\frac{V_o}{AR_f}=0$$

$$\Rightarrow \quad \frac{V_{in}}{R_a}=-V_o\,\frac{R_{\pi}R_f+R_aR_f+R_aR_{\pi}+AR_aR_{\pi}}{AR_aR_{\pi}R_f}$$



The Close-Loop Gain, A_v

$$A_v=\frac{V_o}{V_{in}}=\frac{-AR_{\pi}R_f}{R_{\pi}R_f+R_aR_f+R_aR_{\pi}+AR_aR_{\pi}}$$

Close-Loop Gain

When the open loop gain is very large, the above equation become,

$$A_v \sim \frac{-R_f}{R_a}$$

Note : The close-loop gain now reduce to the same form as an ideal case

Input Impedance

Input Impedance can be regarded as,

$$R_{in} = R_a + R_\pi // R'$$

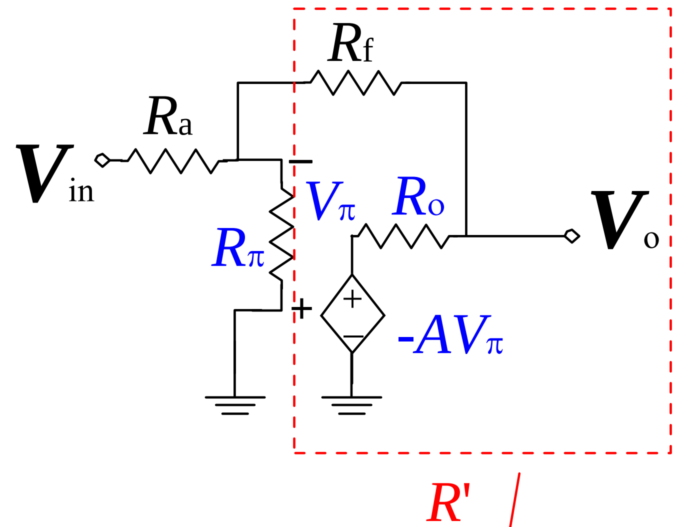
where R′ is the equivalent impedance of the red box circuit, that is

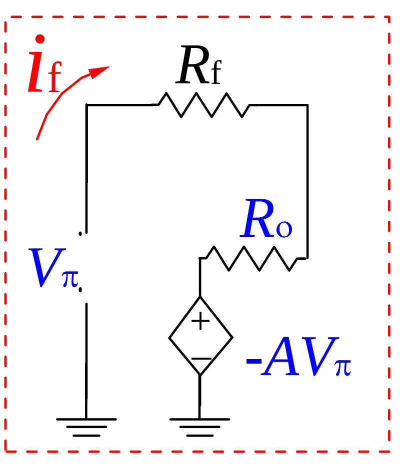
$$R' = \frac{V_\pi}{i_f}$$

However, with the below circuit,

$$V_\pi - (-AV_\pi) = i_f (R_f + R_o)$$

$$\Rightarrow R' = \frac{V_\pi}{i_f} = \frac{R_f + R_o}{1 + A}$$





Input Impedance

Finally, we find the input impedance as,

$$R_{in} = R_a + \left[\frac{1}{R_\pi} + \frac{1+A}{R_f + R_o}\right]^{-1} \quad \Rightarrow \quad R_{in} = R_a + \frac{R_\pi (R_f + R_o)}{R_f + R_o + (1+A)R_\pi}$$

Since, $R_f + R_o << (1+A)R_\pi$, R_{in} become,

$$R_{in} \sim R_a + \frac{(R_f + R_o)}{(1+A)}$$

Again with $R_f + R_o << (1+A)$

$$R_{in} \sim R_a$$

Note: The op-amp can provide an impedance isolated from input to output

Output Impedance

Only source-free output impedance would be considered, i.e. V_i is assumed to be 0

Firstly, with figure (a),

$$V_\pi = \frac{R_a // R_\pi}{R_f + R_a // R_\pi} V_o \Rightarrow V_\pi = \frac{R_a R_\pi}{R_a R_f + R_a R_\pi + R_f R_\pi} V_o$$

By using KCL, $i_o = i_1 + i_2$

$$i_o = \frac{V_o}{R_f + R_a // R_f} + \frac{V_o - (-AV_\pi)}{R_o}$$

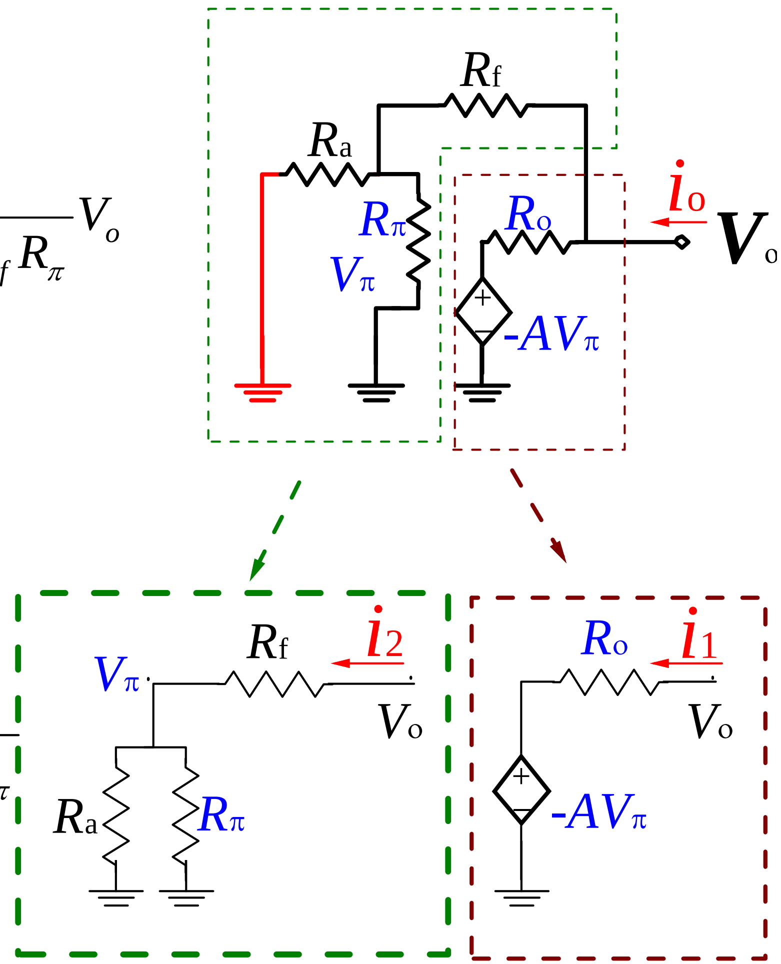
By substitute the equation from Fig. (a),

The output impedance, R_{out} is

$$\frac{V_o}{i_o} = \frac{R_o (R_a R_f + R_a R_\pi + R_f R_\pi)}{(1 + R_o)(R_a R_f + R_a R_\pi + R_f R_\pi) + (1 + A) R_a R_\pi}$$

$\therefore R_\pi$ and A comparably large,

$$R_{out} \sim \frac{R_o (R_a + R_f)}{A R_a}$$



(a) (b)