Lecture on

AC Series-Parallel Circuits
Methods of AC Analysis

AC Series-Parallel Circuits

Ac series circuit

Ac parallel circuit

AC Circuits

- The rules and laws which were developed for dc circuits will apply equally well for ac circuits.
- The analysis of ac circuits requires vector algebra.
- Voltages and currents will usually be in phasor form, and will be in rms values.

Ohm's Law

- The voltage and current of a resistor will be in phase.
- The impedance of a resistor is $\mathbf{Z}_R = R\angle 0°$.

Ohm's Law

- The voltage across an inductor leads the current by 90°.

Ohm’s Law

- The current through a capacitor leads the voltage by 90°.

AC Series Circuits

- The current everywhere in a series circuit is the same.
- Impedance is a term used to collectively determine how the resistance, capacitance, and inductance impede the current in a circuit.
- The total impedance in a circuit is found by adding all the individual impedances vectorally.

AC Series Circuits

- All impedance vectors will appear in either the first or the fourth quadrants because the resistance vector is always positive.
- If the impedance vector appears in the first quadrant, the circuit is inductive.
- If the impedance vector appears in the fourth quadrant, the circuit is capacitive.

Voltage Divider Rule

- The voltage divider rule works the same as with dc circuits.
- From Ohm's law:

Kirchhoff's Voltage Law

- KVL is the same as in dc circuits.
- The phasor sum of voltage drops and rises around a closed loop is equal to zero.
- These voltages may be added in phasor form or in rectangular form.
- If using rectangular form, add real parts together, then add imaginary parts together.

AC Parallel Circuits

- The conductance, *G*, is the reciprocal of the resistance.
- The susceptance, *B*, is the reciprocal of the reactance.
- The admittance, **Y**, is the reciprocal of the impedance.
- The unit for all of these is the siemens (S).

AC Parallel Circuits

- Impedances in parallel add together like resistors in parallel.
- These impedances must be added vectorally.
- Whenever a capacitor and an inductor having equal reactance's are placed in parallel, the equivalent circuit of the two components is an open circuit.

Kirchhoff’s Current Law

- KCL is the same as in dc circuits.
- The summation of current phasors entering and leaving a circuit is equal to zero.
- These currents must be added vectorally.

Current Divider Rule

- In a parallel the voltages across all branches are equal.

Series-Parallel Circuits

- Label all impedances with magnitude and the associated angle.
- Analysis is simplified by starting with easily recognized combinations.
- Redraw the circuit if necessary for further simplification.
- The fundamental rules and laws of circuit analysis must apply in all cases.

Frequency Effects of *RC* Circuits

- The impedance of a capacitor decreases as the frequency increases.
- For dc (f = 0 Hz), the impedance of the capacitor is infinite.
- For a series *RC* circuit, the total impedance decreases to *R* as the frequency increases.
- For a parallel *RC* circuit, as the frequency increases, the impedance goes from *R* to a smaller value.

Frequency Effects of *RL* Circuits

- The impedance of an inductor increases as the frequency increases.
- At dc (f = 0 Hz), the inductor looks like a short. At high frequencies, it looks like an open.
- In a series *RL* circuit, the impedance increases from *R* to a larger value.
- In a parallel *RL* circuit, the impedance increases from 0 to *R*.

Corner Frequency

- The corner frequency is a break point on the frequency response graph.
- For a capacitive circuit,

$$\omega_C = 1/RC = 1/\tau$$

- For an inductive circuit,

$$\omega_C = R/L = 1/\tau$$

RLC Circuits

- In a circuit with *R*, *L*, and *C* components combined in series-parallel combinations, the impedance may rise or fall across a range of frequencies.
- In a series branch, at some point the impedance of the inductor may equal that of the capacitor.
- These impedances would cancel, leaving the impedance of the resistor as the only impedance.

Applications

- Any ac circuit may be simplified as a series circuit having resistance and a reactance.
- Also, an ac circuit may be represented as an equivalent parallel circuit with a single resistor and a single reactance.
- Any equivalent circuit will be valid only at the given frequency of operation.

Methods of AC Analysis

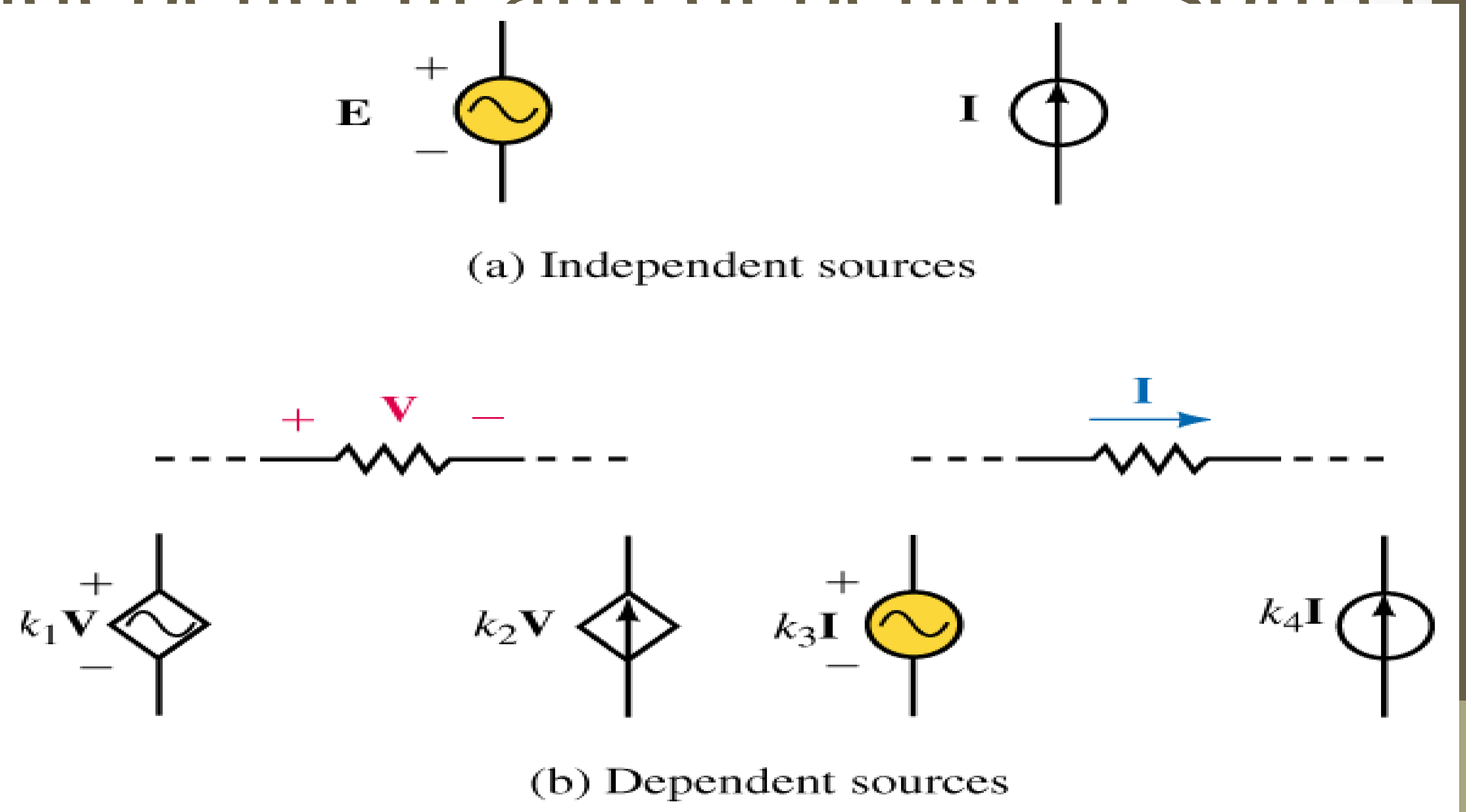
Dependent Sources

- The voltages and currents of independent sources are not any way dependent upon any voltage or current elsewhere in the circuit.
- In some circuits, the operations of certain devices is best explained by replacing the device with an equivalent model.
- These models are dependent upon some internal voltage or current.

Dependent Sources

- The dependent source has a magnitude and phase angle determined by voltage or current at some internal element multiplied by a constant *k*.
- The magnitude of *k* is determined by parameters within the particular model.
- The units of the constant correspond to the required quantities in the equation.

Independent and dependent source



Source Conversion

- A voltage source **E** in series with an impedance **Z** is equivalent to a current source **I** having the same impedance **Z** in parallel.
- **I** = **E/Z**
- **E** = **IZ**
- The voltages and currents at the terminals will be the same; internal voltages and currents will differ.

Source Conversion

- A dependent source may be converted by the same method.
- The controlling element must be external to the circuit.
- If the controlling element is in the same circuit as the dependent source, this procedure cannot be used.

Mesh Analysis

- Convert all sinusoidal expressions into phasor notation.
- Convert current sources to voltage sources.
- Redraw the circuit, simplifying the given impedances.
- Assign clockwise loop currents to each interior closed loop.
- Show the polarities of all impedances.

Mesh Analysis

- Apply KVL to each loop and write the resulting equations.
- Voltages which are voltage rises in the direction of the assumed current are positive; voltages which are drops are negative.
- Solve the resulting simultaneous linear equations.

Systematic approach

- Mutual impedances represent impedances which are shared between two loops.
- Self-impedances are sums of all resistances in a mesh.
- Z_{12} represents the resistor in loop 1 that is shared by loop 1 and loop 2.
- The coefficients along the principal diagonal will be positive.
- All other coefficients will be negative.
- The terms will be symmetrical about the principal diagonal.

Substitute impedances for resistances, use all complex values

$$R_{11}I_1 - R_{12}I_2 - R_{13}I_3 = \sum_1 E$$
$$-R_{21}I_1 + R_{22}I_2 - R_{23}I_3 = \sum_2 E$$
$$-R_{31}I_1 - R_{32}I_2 + R_{33}I_3 = \sum_3 E$$

Systematic mesh analysis

- Convert current sources into equivalent voltage sources.
- Assign clockwise currents to each independent closed loop.
- Write the simultaneous linear equations in the format outline.
- Solve the resulting simultaneous equations.

Nodal Analysis

- Nodal analysis will calculate all nodal voltages with respect to ground.
- Convert all sinusoidal expressions into equivalent phasor notation.
- Convert all voltage sources to current sources.
- Redraw the circuit, simplifying the given impedances and relabelling the impedances as admittances.

Nodal Analysis

- Assign subscripted voltages to the nodes; select an appropriate reference node.
- Assign assumed current directions through all the branches.
- Apply KCL to each node.
- Solve the resulting equations for the node voltages.

Systematic approach

- Mutual admittance is the admittance that is common to two nodes.
- Self-admittance Y_{11} is sum of all admittances connected to a node.
- The mutual admittance Y_{23} is the conductance at Node 2, common to Node 3.
- The admittances at particular nodes are positive.
- Mutual admittances are negative.
- If the equations are written correctly, the terms will be symmetrical about the principal diagonal.

Systematic nodal analysis

- Convert voltage sources into equivalent current sources.
- Label the reference node as ground.
- Label the remaining nodes as V_1, V_2, etc.
- Write the linear equation for each node.
- Solve the resulting equations for the voltages.

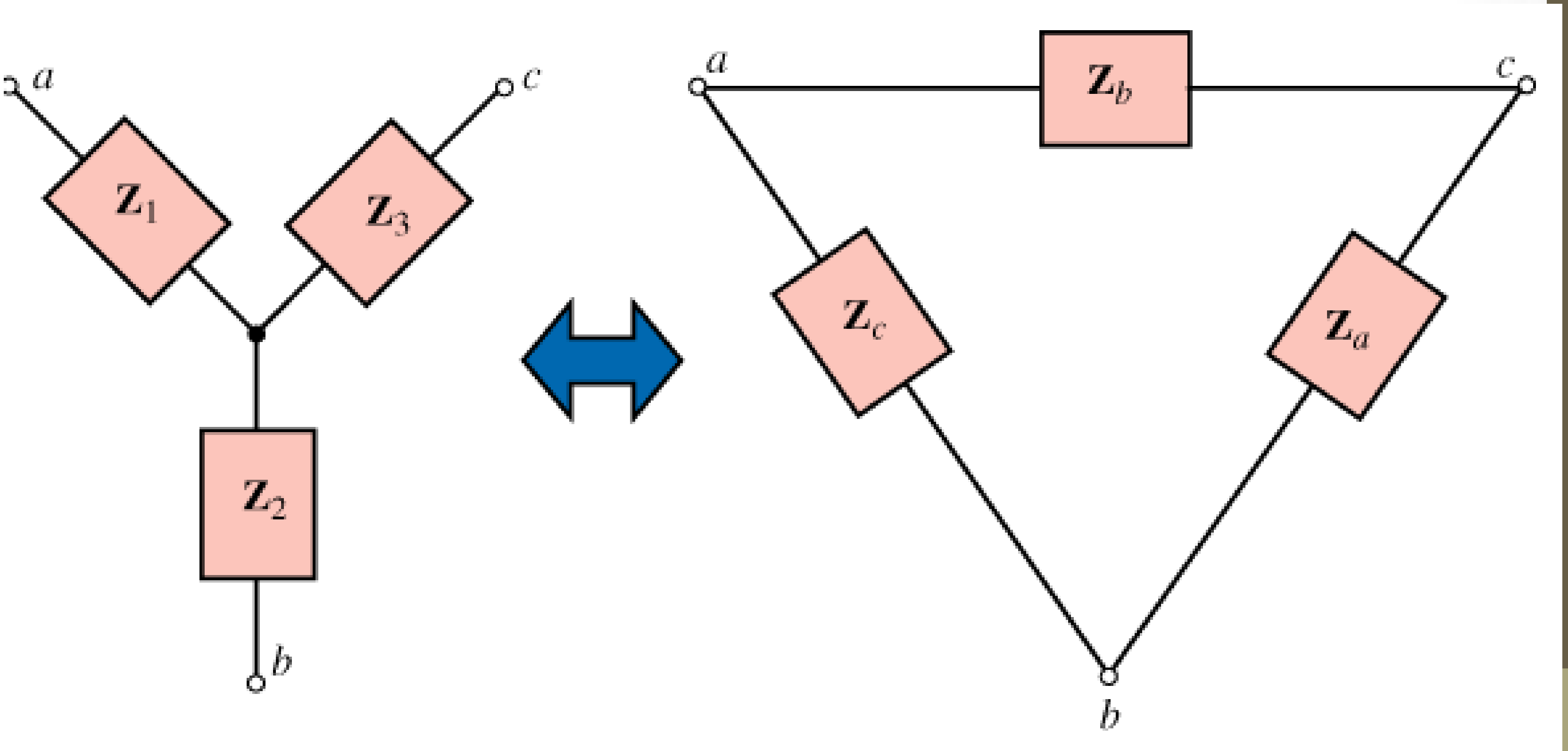
Substitute admittances for conductance's, use all complex values

$$G_{11}V_1 - G_{12}V_2 - G_{13}V_3 = \sum_1 I$$

$$-G_{21}V_1 + G_{22}V_2 - G_{23}V_3 = \sum_2 I$$

$$-G_{31}V_1 - G_{23}V_2 + G_{33}V_3 = \sum_3 I$$

Delta-to Wye conversion



Delta- to- Wye Conversion

- The impedance in any arm of a Y circuit is determined by taking the product of the two adjacent Δ impedances at this arm and dividing by the summation of the Δ impedances.

Wye -to- Delta Conversions

- Any impedance in a Δ is determined by summing the possible two-impedance product combinations of the Y and then dividing by the impedance found in the opposite branch of the Y.

Bridge Networks

- When a balanced bridge occurs in a circuit, the equivalent impedance of the bridge is found by removing the central **Z** and replacing it by a short or open circuit.
- The resulting **Z** is then found by solving the series-parallel circuit.
- For an unbalanced bridge, **Z** can be determined by Δ-to-Y conversion or mesh analysis

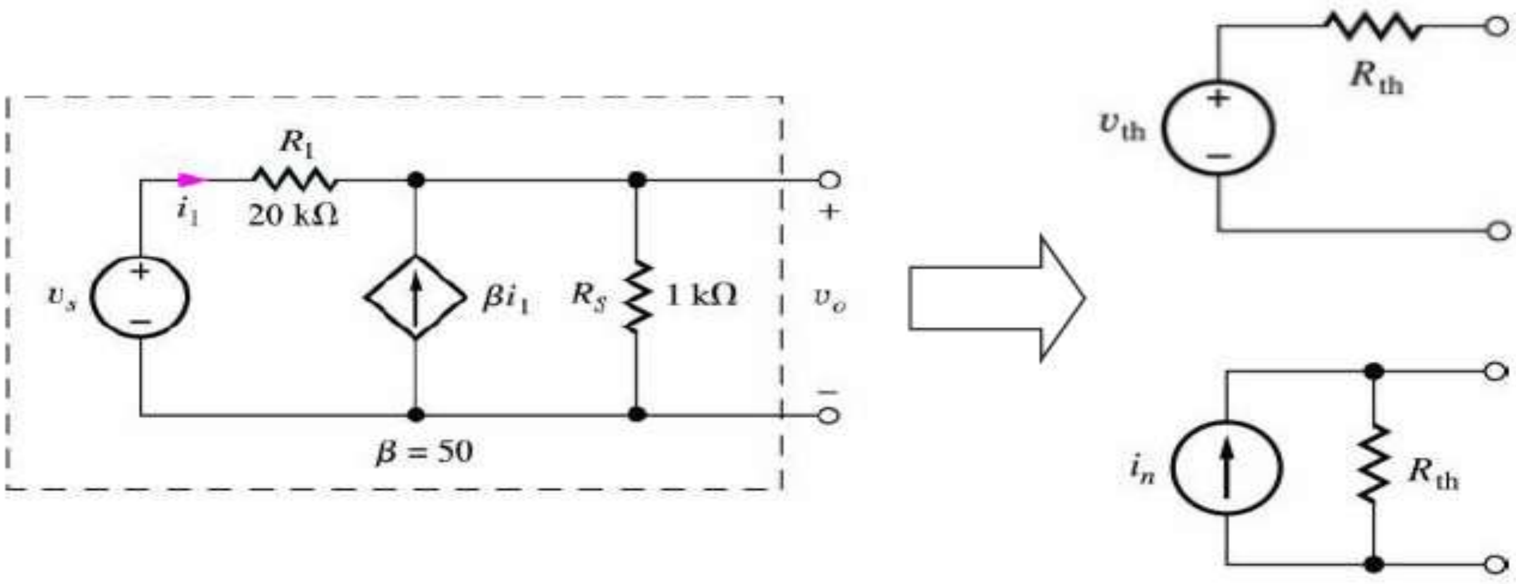
Bridge Networks

- Bridge circuits are used to measure the values of unknown components.
- Any bridge circuit is balanced when the current through the branch between two arms is zero.
- The condition of a balanced bridge occurs when

Equivalent Circuit

- Thevenin and Norton Equivalent circuit represents real-world battery models.
- Complex circuits can be simplified to these representation to help us understand the circuits.

Circuit Theory Review: Thevenin and Norton Equivalent Circuits

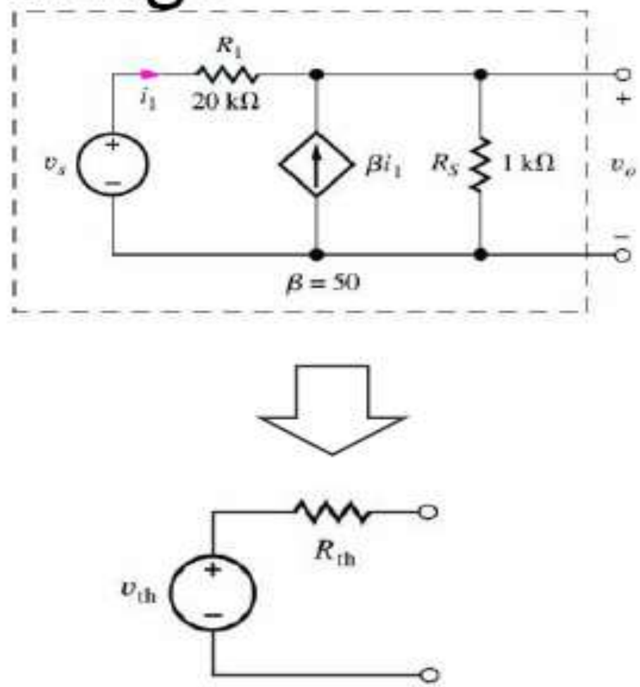


Circuit Theory Review: Find the Thevenin Equivalent Voltage

Problem: Find the Thevenin equivalent voltage at the output.

Solution:

- **Known Information and Given Data:** Circuit topology and values in figure.
- **Unknowns:** Thevenin equivalent voltage v_{TH}.
- **Approach:** Voltage source v_{TH} is defined as the output voltage with no load.
- **Assumptions:** None.
- **Analysis:** Next slide…

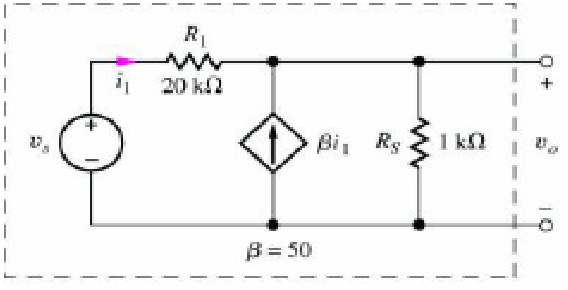


Circuit Theory Review: Find the Thevenin Equivalent Voltage

Applying KCL at the output node,

$$\beta i_1 = \frac{v_o - v_s}{R_1} + \frac{v_o}{R_S} = G_1(v_o - v_s) + G_S v_o$$

Current i_1 can be written as: $i_1 = G_1(v_o - v_s)$



Combining the previous equations

$$G_1(\beta+1)v_s = [G_1(\beta+1) + G_S]v_o$$

$$v_o = \frac{G_1(\beta+1)}{G_1(\beta+1)+G_S} v_s \times \frac{R_1 R_S}{R_1 R_S} = \frac{(\beta+1)R_S}{(\beta+1)R_S + R_1} v_s$$

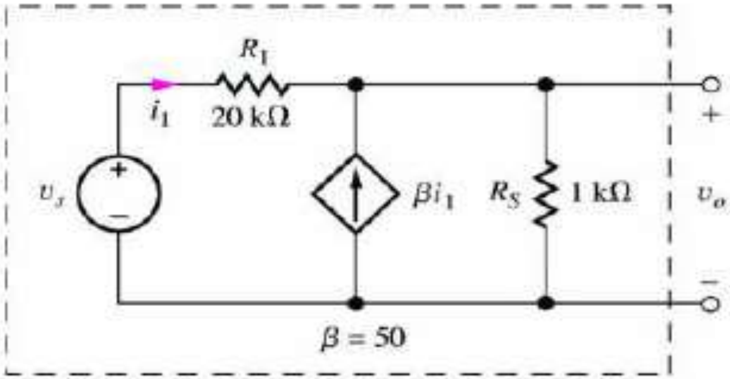
Circuit Theory Review: Find the Thevenin Equivalent Voltage (cont.)

Using the given component values:

$$v_o = \frac{(\beta+1)R_S}{(\beta+1)R_S + R_1} v_s = \frac{(50+1)1\ \text{k}\Omega}{(50+1)1\ \text{k}\Omega + 1\ \text{k}\Omega} v_s = 0.718 v_s$$

and

$$v_{\text{TH}} = 0.718 v_s$$

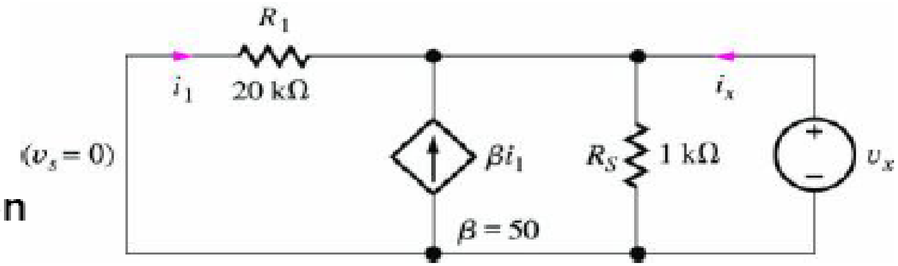


Circuit Theory Review: Find the Thevenin Equivalent Resistance

Problem: Find the Thevenin equivalent resistance.

Solution:

- **Known Information and Given Data**: Circuit topology and values in figure.
- **Unknowns**: Thevenin equivalent resistance R_{TH}.
- **Approach**: R_{TH} is defined as the equivalent resistance at the output terminals with all independent sources in the network set to zero.
- **Assumptions**: None.
- **Analysis**: Next slide…

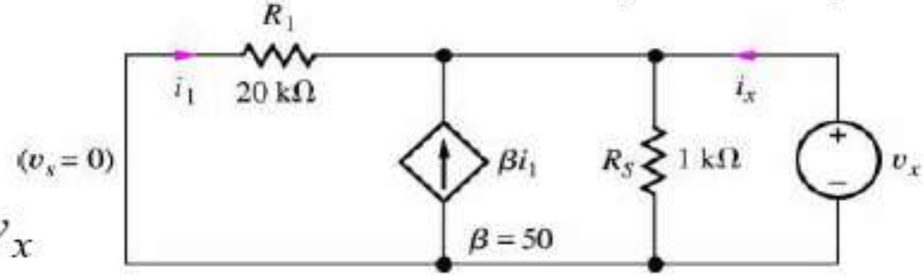


Test voltage v_x has been added to the previous circuit. Applying v_x and solving for i_x allows us to find the Thevenin resistance as v_x/i_x.

Circuit Theory Review: Find the Thevenin Equivalent Resistance (cont.)

Applying KCL,

$$i_x = -i_1 - \beta i_1 + G_S v_x$$
$$= G_1 v_x + \beta G_1 v_x + G_S v_x$$
$$= \left[G_1(\beta+1) + G_S\right] v_x$$



$$R_{th} = \frac{v_x}{i_x} = \frac{1}{G_1(\beta+1) + G_S} = R_S \left\| \frac{R_1}{\beta+1} \right.$$

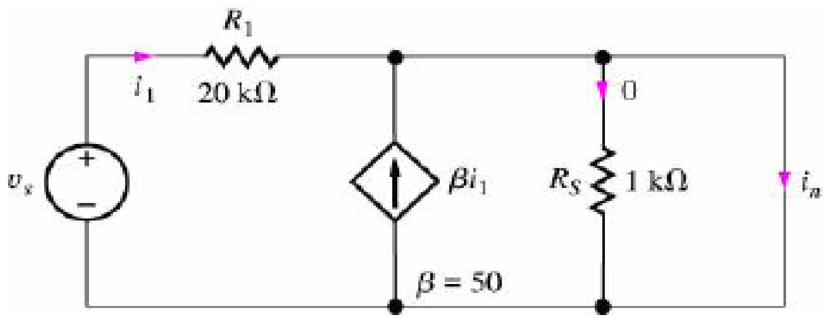
$$R_{th} = R_S \left\| \frac{R_1}{\beta+1} = 1\text{ k}\Omega \right\| \frac{20\text{ k}\Omega}{50+1} = 1\text{ k}\Omega \| 392\ \Omega = 282\ \Omega$$

Circuit Theory Review: Find the Norton Equivalent Circuit

Problem: Find the Norton equivalent circuit.

Solution:

- **Known Information and Given Data**: Circuit topology and values in figure.
- **Unknowns**: Norton equivalent short circuit current i_N.
- **Approach**: Evaluate current through output short circuit.
- **Assumptions**: None.
- **Analysis**: Next slide…

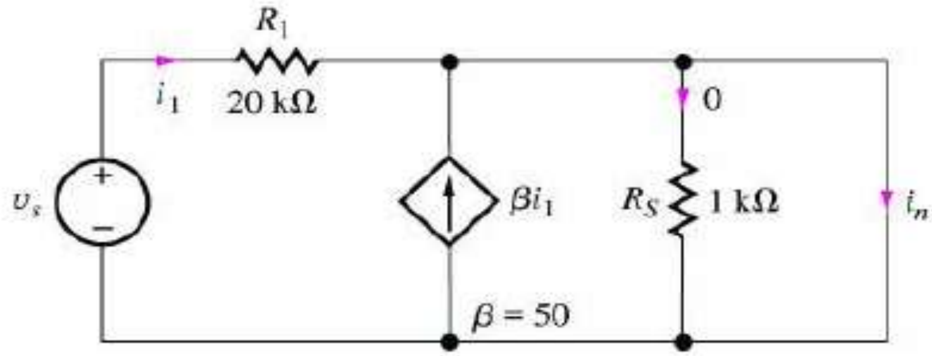


A short circuit has been applied across the output. The Norton current is the current flowing through the short circuit at the output.

Circuit Theory Review: Find the Thevenin Equivalent Resistance (cont.)

Applying KCL,

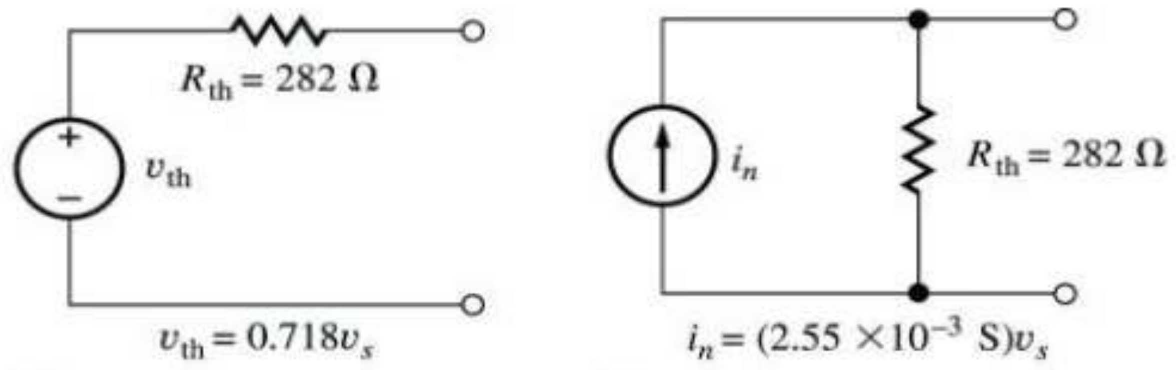
$$i_N = i_1 + \beta i_1$$
$$= G_1 v_s + \beta G_1 v_s$$
$$= G_1(\beta+1)v_s$$
$$= \frac{v_s(\beta+1)}{R_1}$$



Short circuit at the output causes zero current to flow through R_S. R_{th} is equal to R_{th} found earlier.

$$i_N = \frac{50+1}{20\ \text{k}\Omega} v_s = \frac{v_s}{392\ \Omega} = (2.55\ \text{mS})v_s$$

Final Thevenin and Norton Circuits



Check of Results: Note that $v_{TH}=i_N R_{th}$ and this can be used to check the calculations: $i_N R_{th}$=(2.55 mS)v_s(282 Ω) = 0.719v_s, accurate within round-off error.

While the two circuits are identical in terms of voltages and currents at the output terminals, there is one difference between the two circuits. With no load connected, the Norton circuit still dissipates power!

Example : Circuit with a controlled source

- Applying KVL around the loop containing v_s yields
- $v_s = i_s R_1 + i_2 R_2 = i_s R_1 + (i_s + g_m v_1) R_2$ (1.33)
- $v_1 = i_s R_1$ (1.34)
- $v_s = i_s (R_1 + R_2 + g_m R_1 R_2)$ (1.35)
- $R_{eq} = v_s / i_s = R_1 + R_2 (1 + g_m R_1)$ (1.36)
- $R_{eq} = 3K\Omega + 2K\Omega[1+0.1S*3K\Omega] = 605\ k\Omega$. This value is far larger than either R1 or R2.

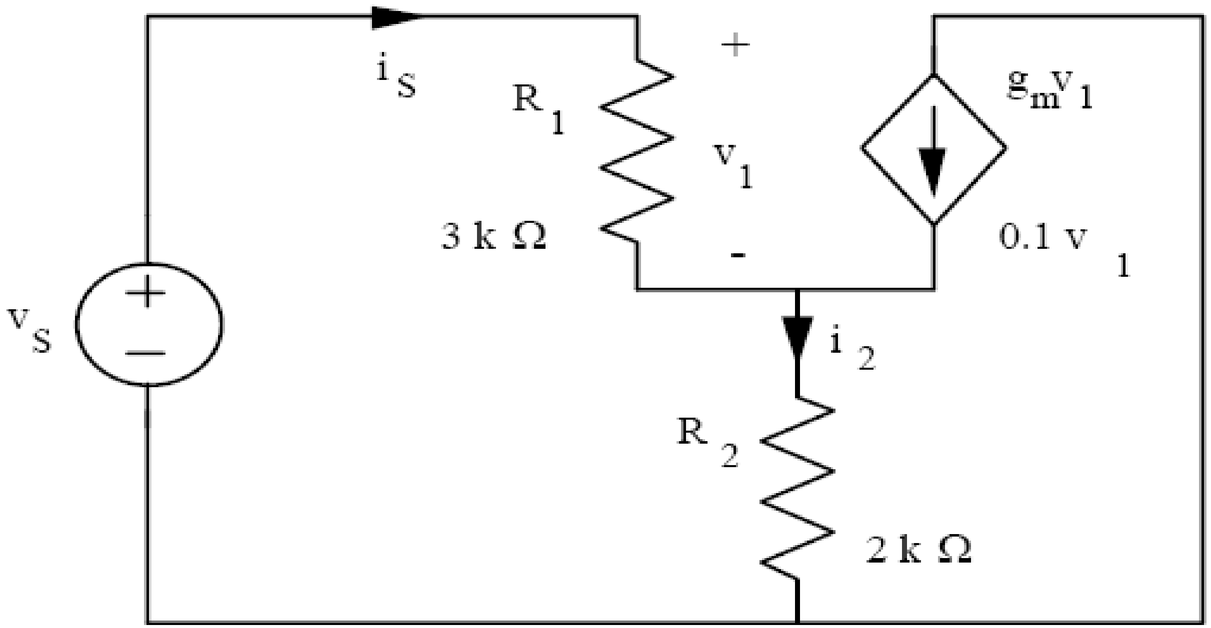


Figure 1.17 - Circuit containing a voltage-controlled current source

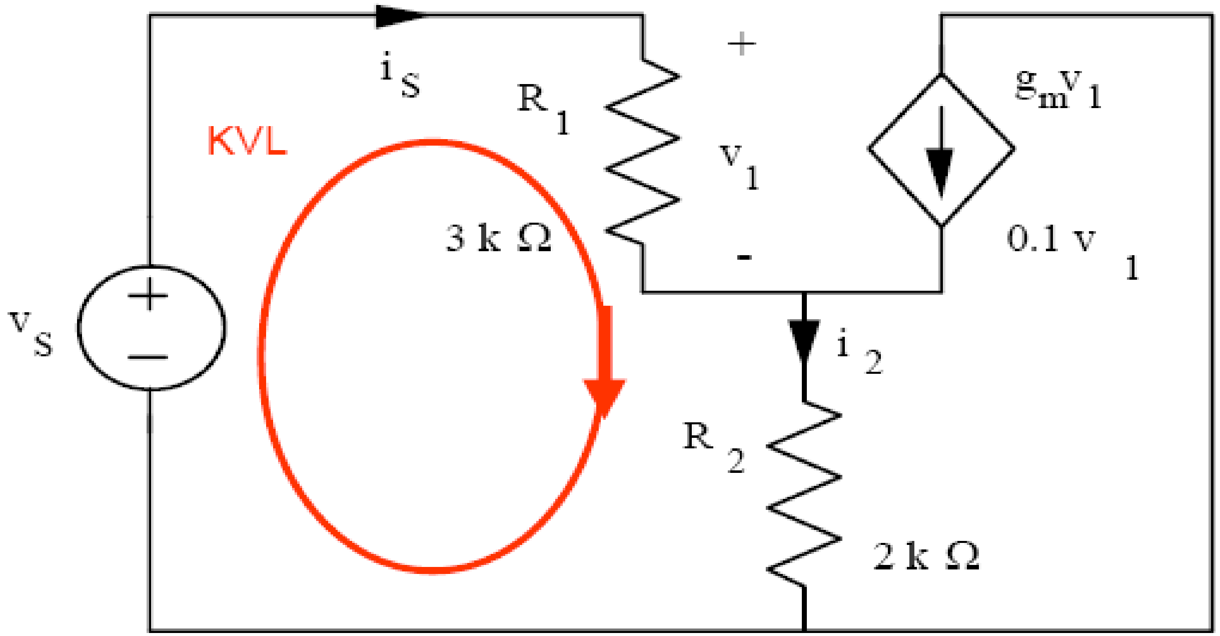


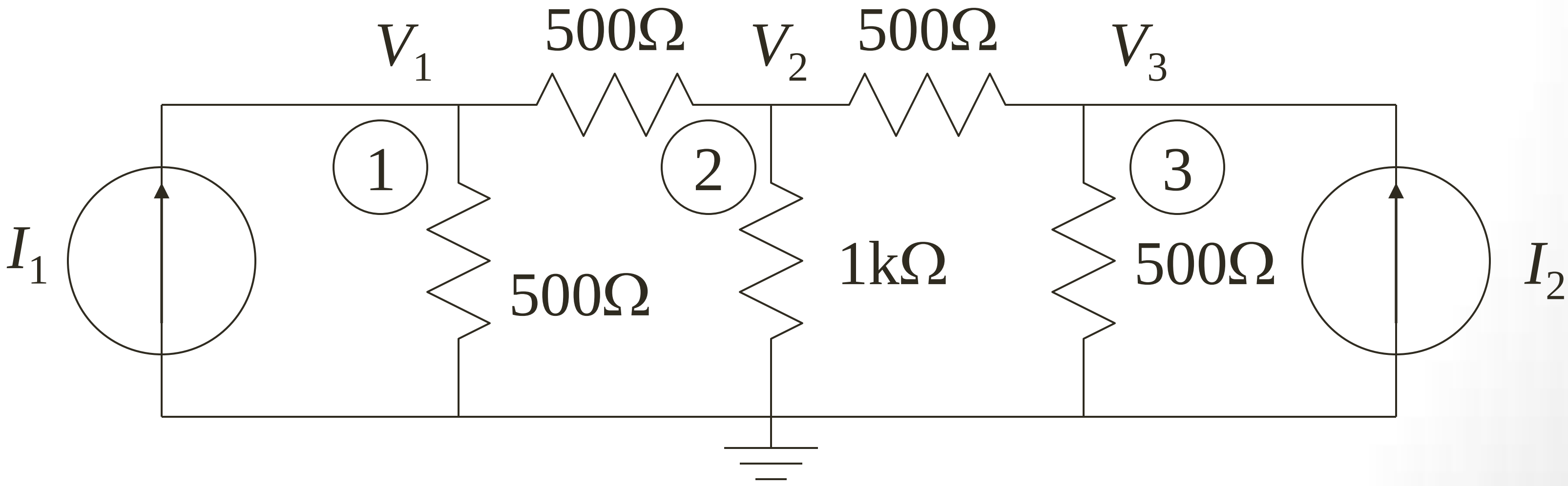
Figure 1.17 - Circuit containing a voltage-controlled current source

Nodal Analysis

1. Choose a reference node.
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference-express currents in terms of node voltages.
4. Solve the resulting system of linear equations.

Matrix Formulation & Inspection

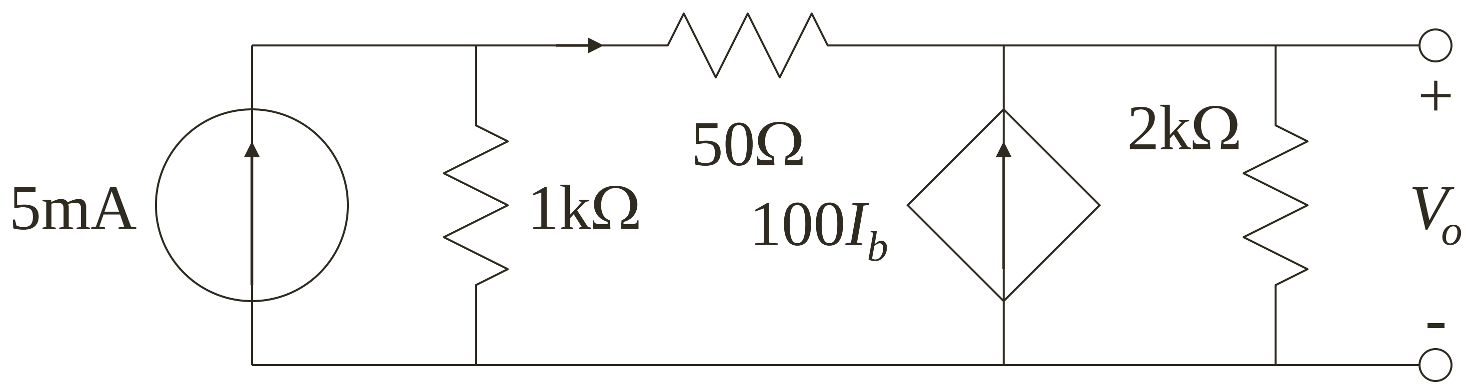
- The three equations can be combined into a single matrix/vector equation.
- Symmetric



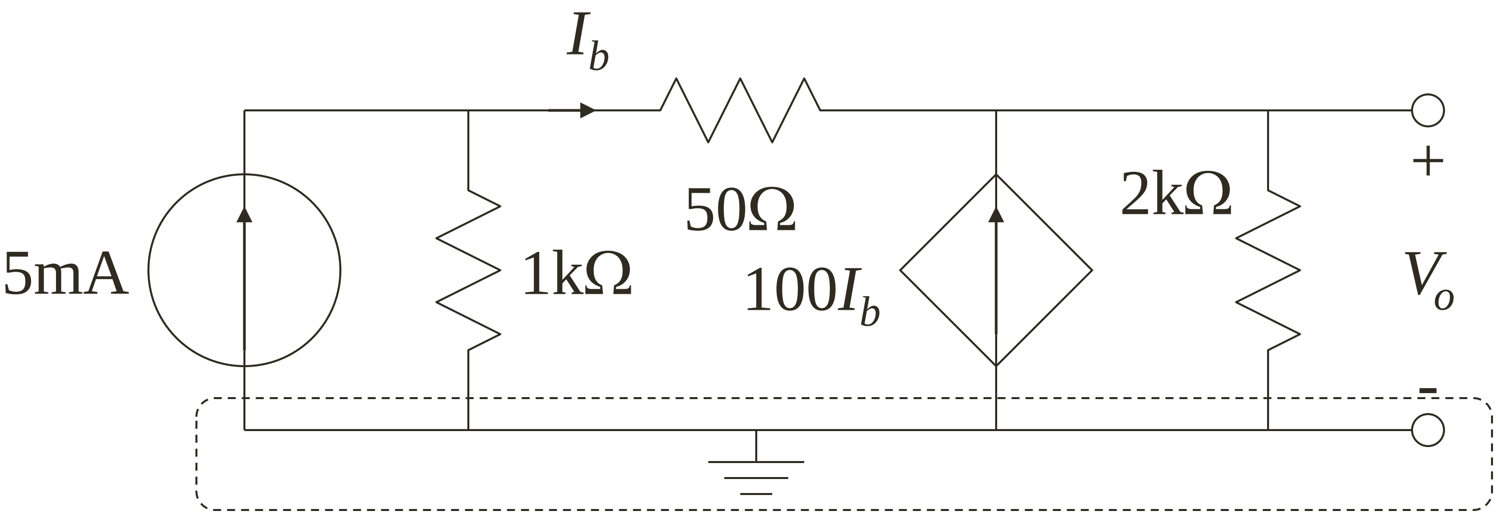
$$\begin{bmatrix} \frac{1}{500\Omega}+\frac{1}{500\Omega} & -\frac{1}{500\Omega} & 0 \\ -\frac{1}{500\Omega} & \frac{1}{500\Omega}+\frac{1}{1k\Omega}+\frac{1}{500\Omega} & -\frac{1}{500\Omega} \\ 0 & -\frac{1}{500\Omega} & \frac{1}{500\Omega}+\frac{1}{500\Omega} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_2 \end{bmatrix}$$

What if there are dependent sources?

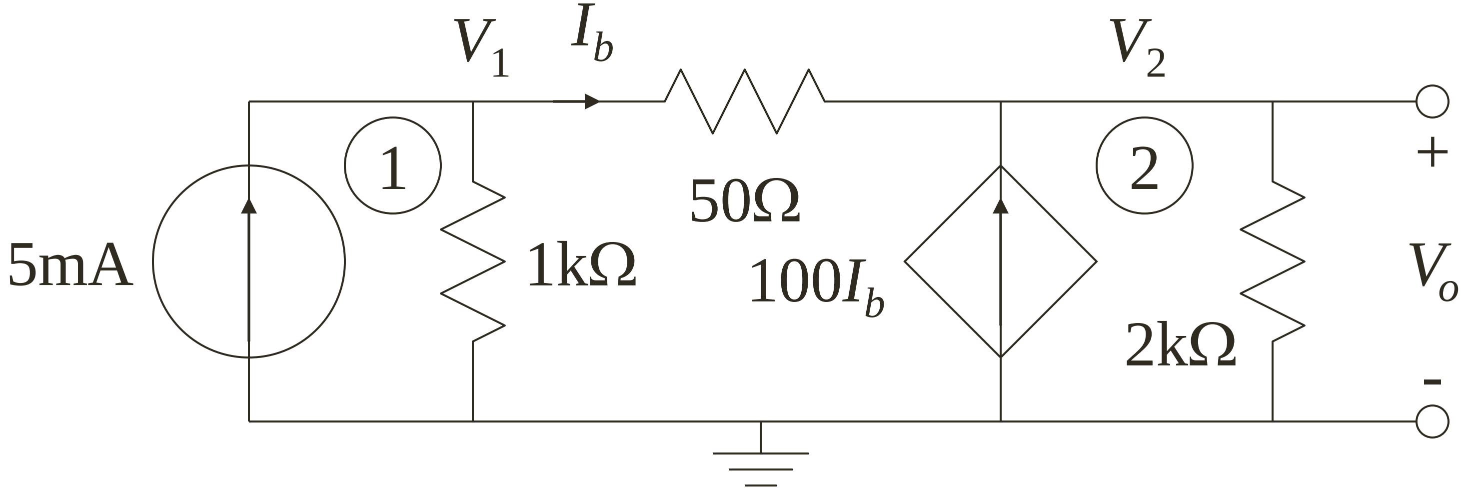
Example:



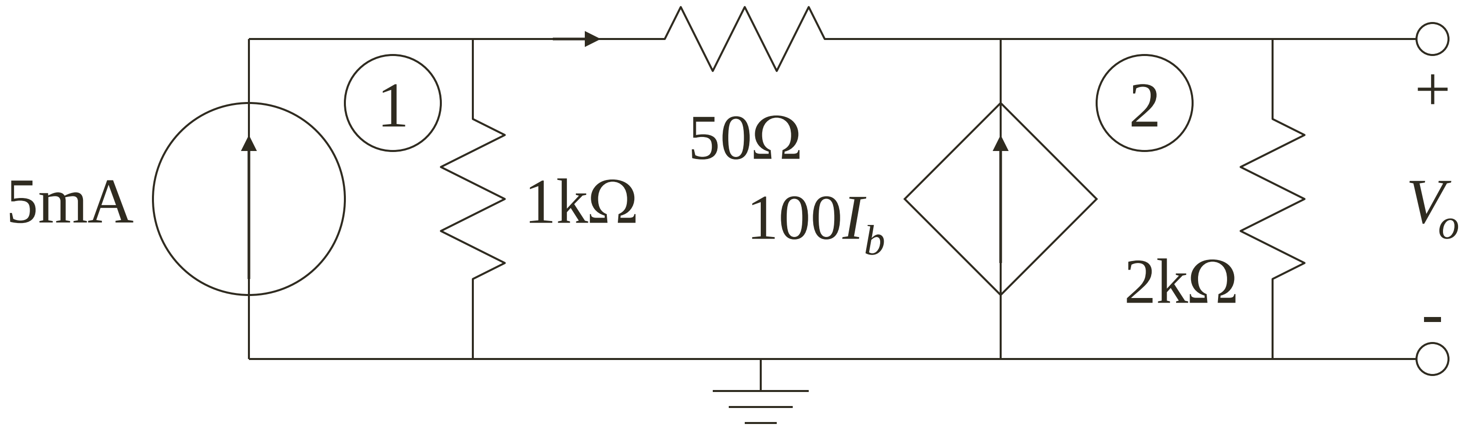
Reference Node



Assign Node Voltages

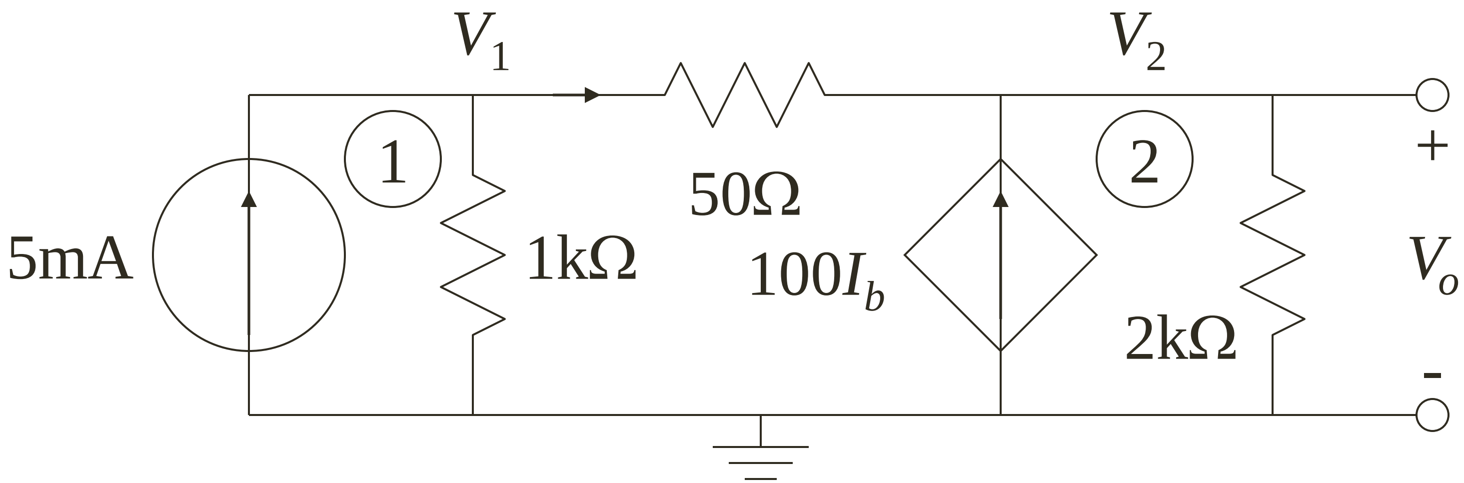


KCL @ Node 1



$$\frac{V_1}{1k\Omega} + \frac{V_1 - V_2}{50\Omega} = 5mA$$

KCL @ Node 2



$$\frac{V_2 - V_1}{50\Omega} - 100I_b + \frac{V_2}{1k\Omega} = 0$$

The Dependent Source

- We must express I_b in terms of the node voltages:
- Equation from Node 2 becomes

$$\frac{V_2 - V_1}{50\Omega} - 100\frac{V_1 - V_2}{50\Omega} + \frac{V_2}{1\text{k}\Omega} = 0$$

System of Equations