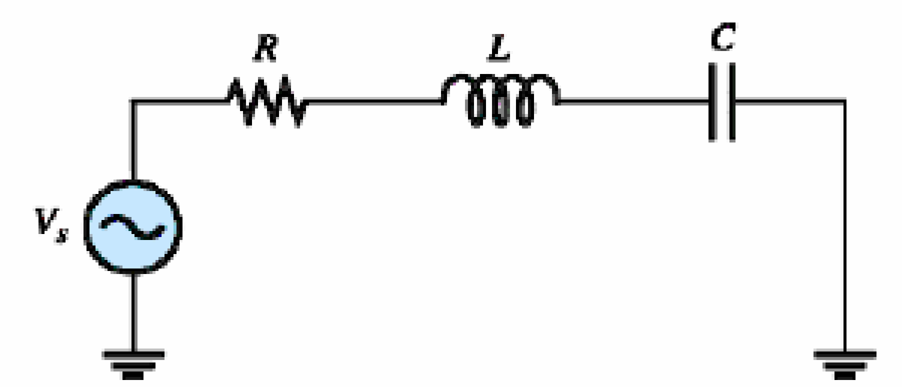
Resonance

Objectives

- Determine the impedance of a series RLC circuit
- Analyze series RLC circuits
- Analyze a circuit for series resonance
- Analyze series resonant filters
- Analyze parallel RLC circuits
- Analyze a circuit for parallel resonance
- Analyze the operation of parallel resonant filters

Impedance of Series RLC Circuits

- A series RLC circuit contains both inductance reactance and capacitance reactance
- Since X_L and X_C are antiphase, the total reactance (X_X) is smaller than the smallest reactance
- $X_X = +jX_L - jX_C$



$$Z = R + jX_L - jX_C$$
$$Z = R + jX_X$$

Analysis of Series RLC Circuits

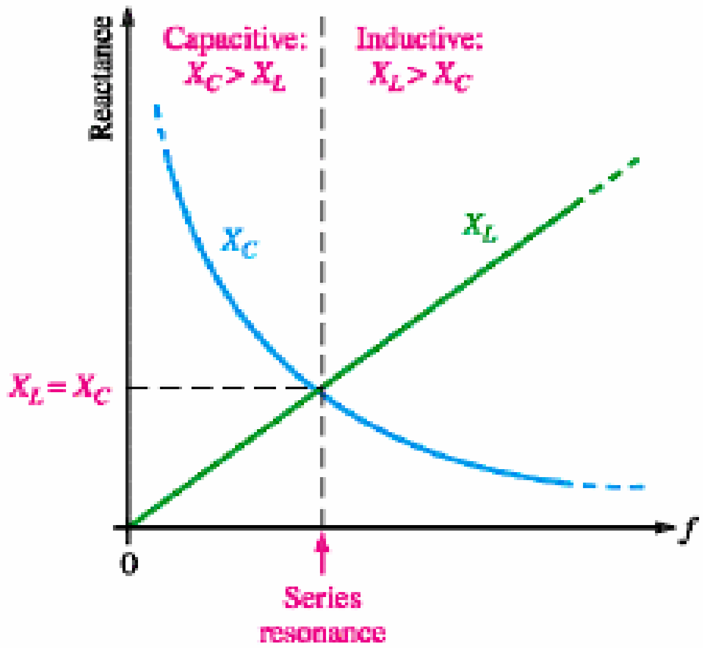
- A series RLC circuit is:
- Capacitive if $|X_C| > |X_L|$
- Inductive if $|X_L| > |X_C|$
- Resonant if $|X_C| = |X_L|$
- At resonance $Z_T = R$
- X_L is a straight line

$$y = mx + b$$

- X_C is a hyperbola

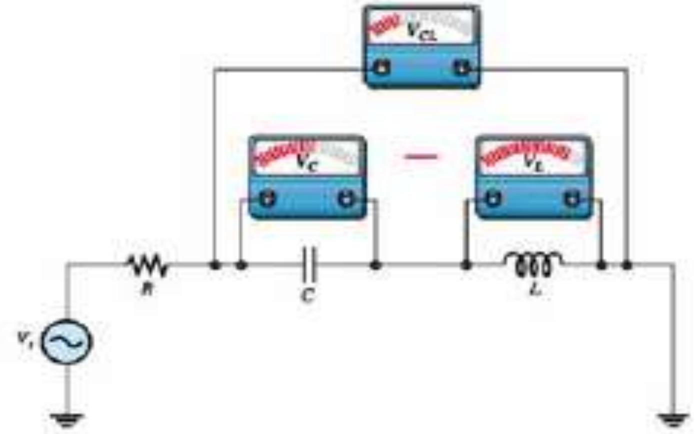
$$xy = k$$

- Resonance occurs where the curve of X_C and X_L intersect



V Across the Series Combination of L and C

- In a series RLC circuit, the capacitor voltage and the inductor voltage are always 180° out of phase (antiphase)
- Because they are antiphase, V_C and V_L have a tendency to cancel each other (and completely cancel at resonance)
- The voltage across the combined L and C is always smaller that the larger individual voltage across either element

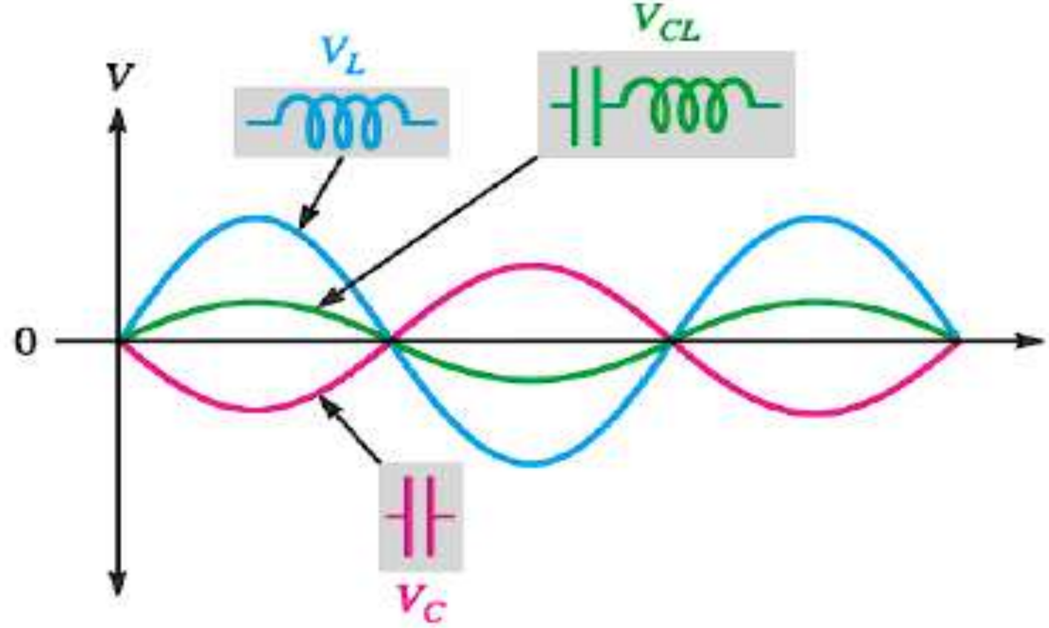


Series Resonance

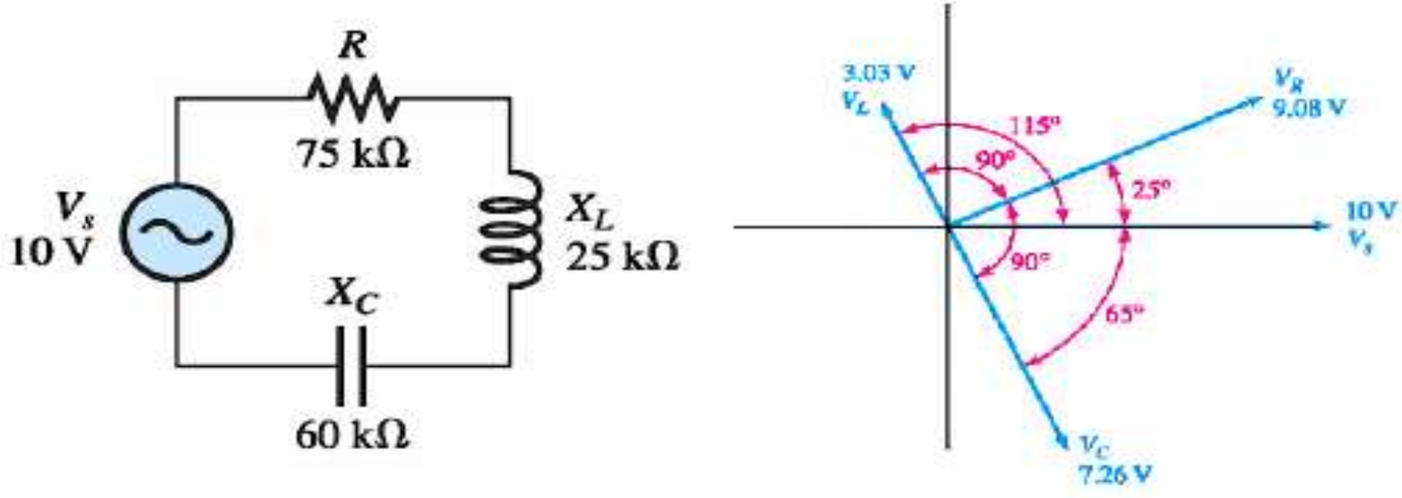
- Resonance is a condition in a series RLC circuit in which the capacitive and inductive reactances are equal in magnitude
- The result is a purely resistive impedance
- The formula for series resonance is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

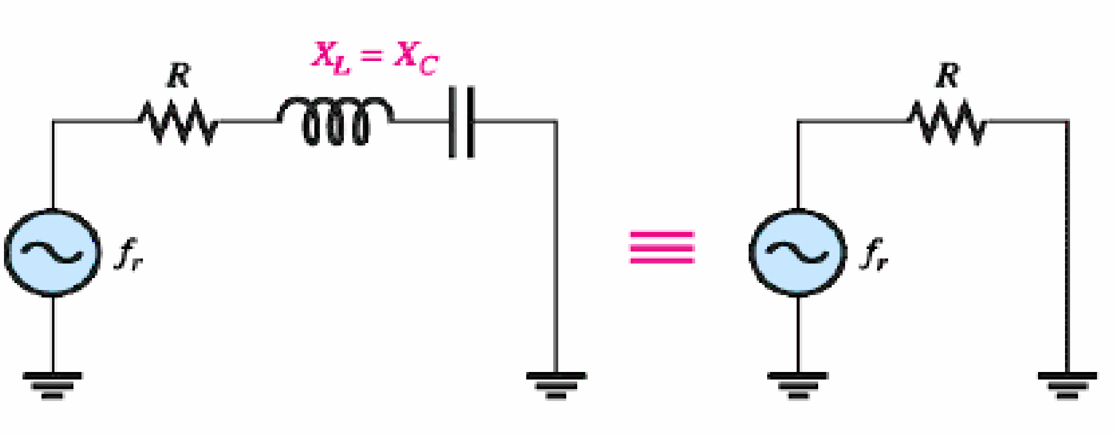
V_L and V_C



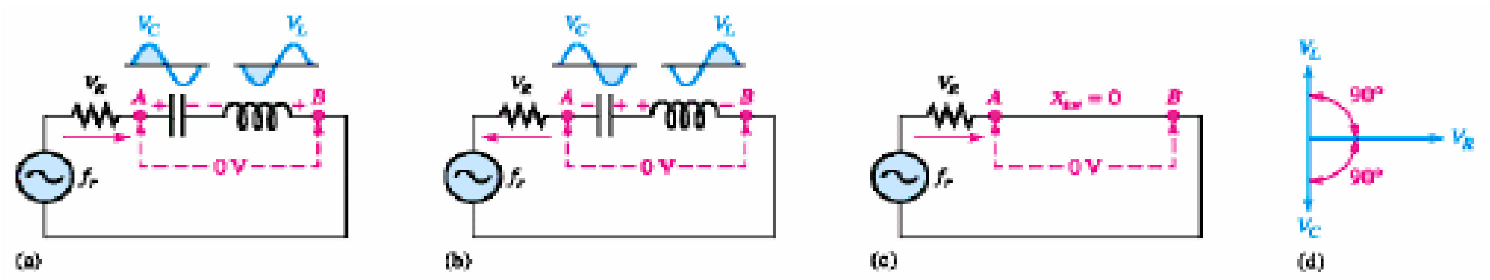
Series RLC Voltage Phasor Diagram



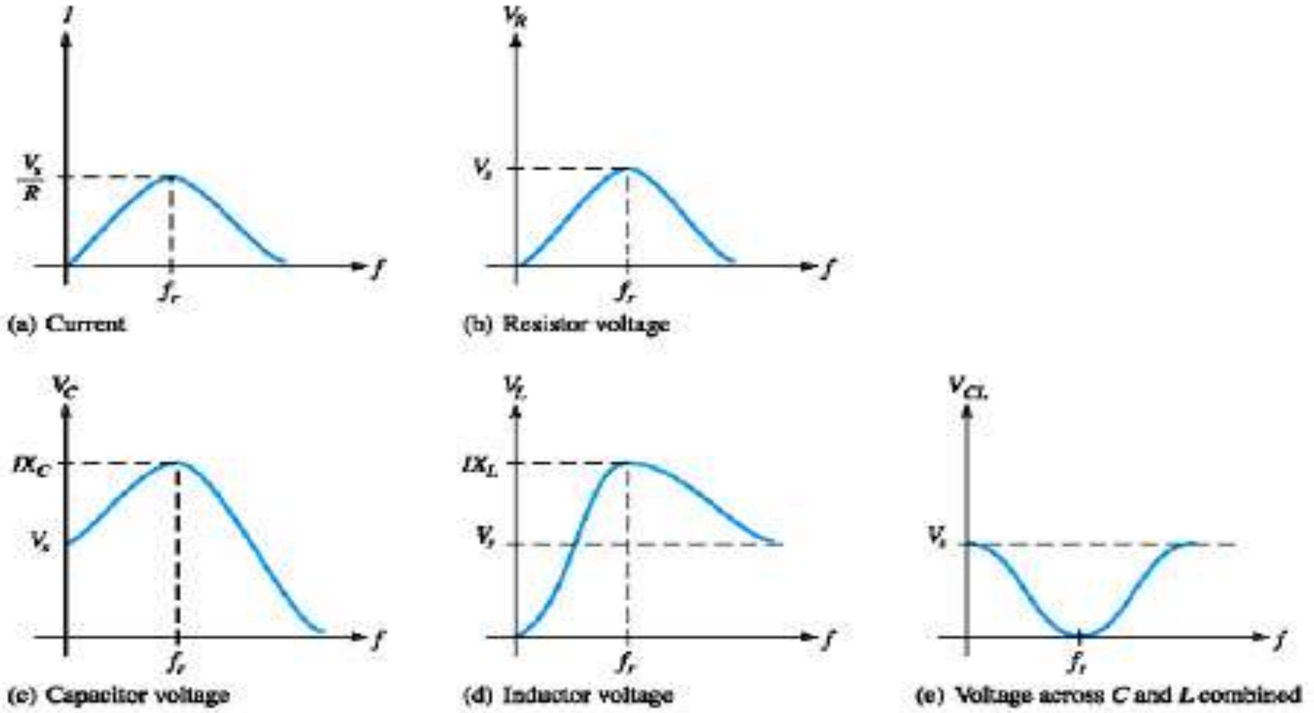
Series Resonant Circuit



Voltage Drops in the Series Resonant Circuit



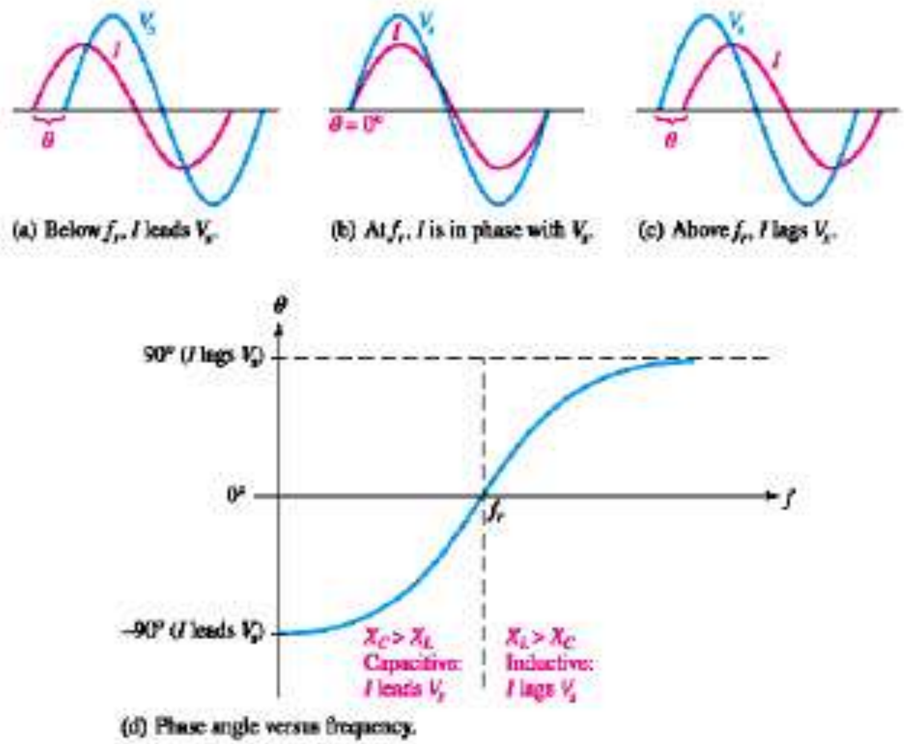
Voltage in the Series RLC Circuit



Current and Voltage in a Series RLC Circuit

- At the series resonant frequency, the current is maximum ($I_{MAX} = V_s/R$)
- Above and below resonance, the current decreases because the impedance increases
- At resonance, impedance is equal to R
- The voltages across L and C are maximum at resonance, but they are also equal in magnitude and 180° out of phase, so they cancel (the total voltage across L and C is zero)

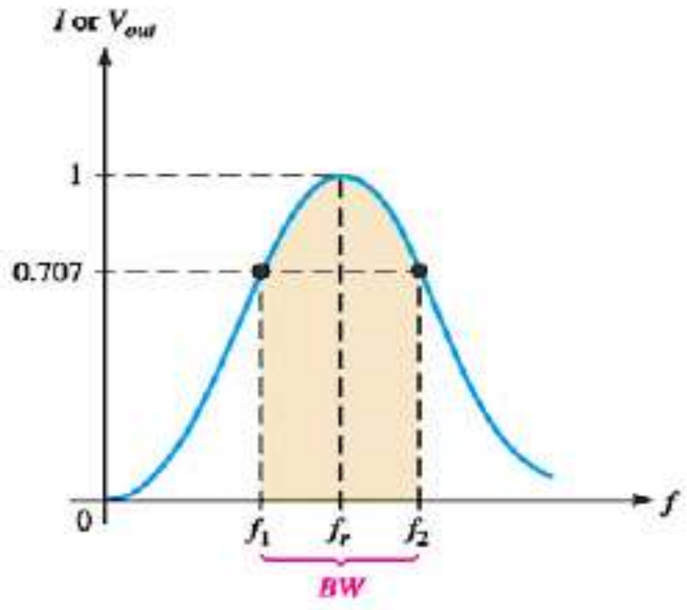
Phase Angle of a Series RLC Circuit



(a) Below f_r, I leads V_s.
(b) At f_r, I is in phase with V_s.
(c) Above f_r, I lags V_s.
(d) Phase angle versus frequency.

Bandwidth of Series Resonant Circuits

- Current is maximum at resonant frequency
- Bandwidth (BW) is the range (f_1 to f_2) of frequencies for which the current is greater than 70.7% of the resonant value



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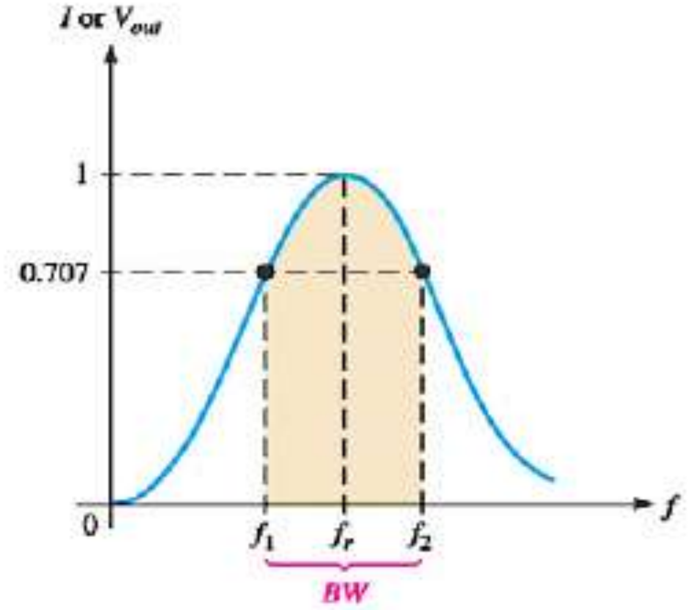
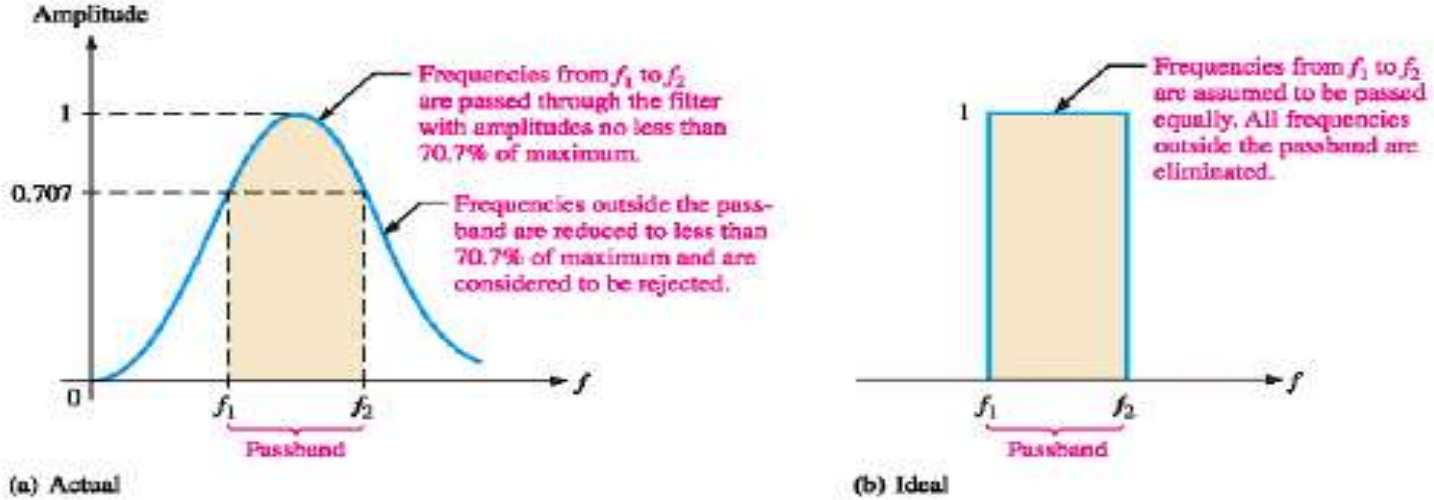


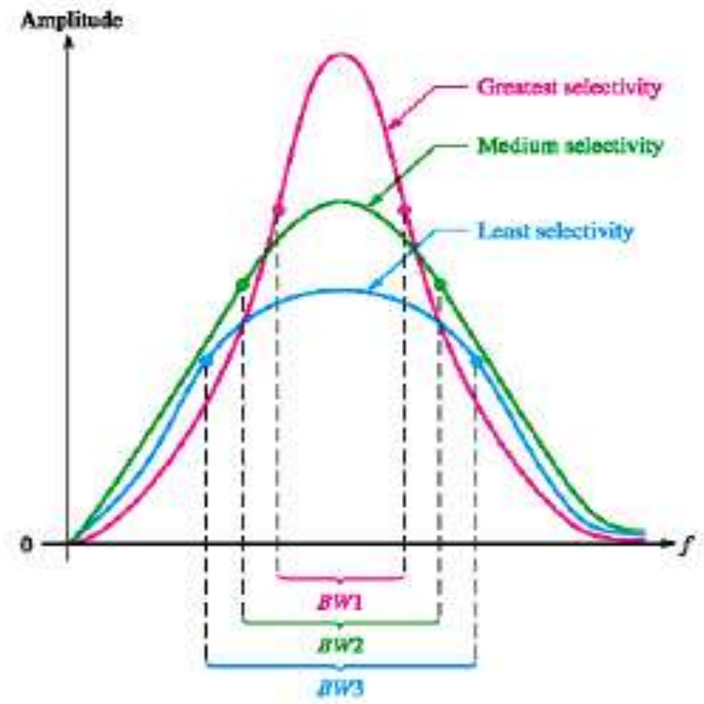
Bandwidth



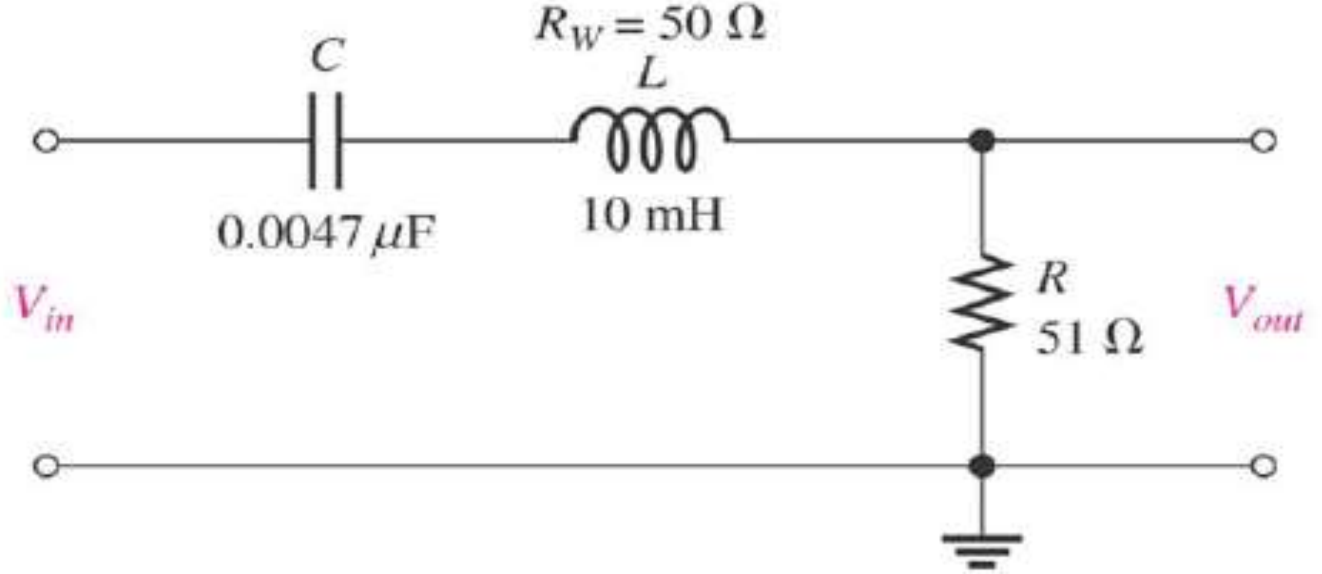
Amplitude
1
0.707
0
f_1
f_2
f
Passband
Frequencies from f_1 to f_2 are passed through the filter with amplitudes no less than 70.7% of maximum.
Frequencies outside the pass-band are reduced to less than 70.7% of maximum and are considered to be rejected.
(a) Actual
1
f_1
f_2
f
Passband
Frequencies from f_1 to f_2 are assumed to be passed equally. All frequencies outside the passband are eliminated.
(b) Ideal

Selectivity

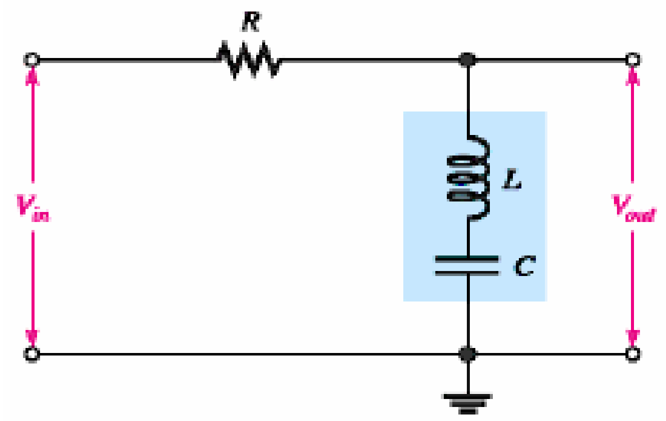
- Selectivity defines how well a resonant circuit responds to a certain frequency and discriminates against all other frequencies
- The narrower the bandwidth, the greater the selectivity
- The steeper the slope of the response curve, the greater the selectivity

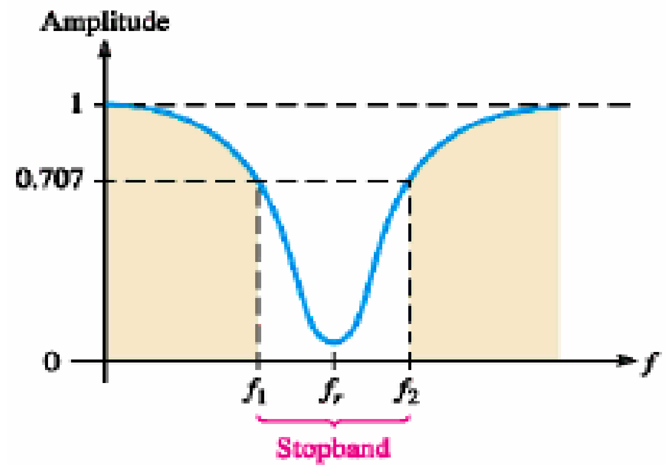


Bandpass Filter



Bandstop Filter





Ideal Parallel Resonance

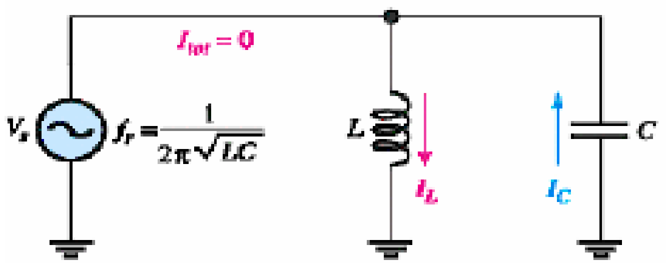
- Parallel resonance occurs when $X_C = X_L$
- At resonance, the two branch currents are equal in magnitude, and 180° out-of-phase with each other
- I_C and I_L cancel
- Since the total current is zero, the impedance of the ideal parallel LC circuit is infinitely large ( )
- Q is the quality factor of the coil, X_L/R_W
- Ideal (no resistance) parallel resonant frequency:

$$f_r = 1/(2\pi\sqrt{LC})$$

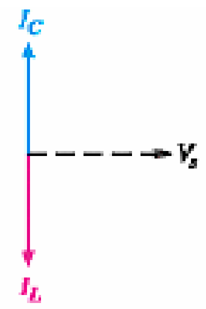
- For nonideal resonant circuits with values of $Q \geq 10$, the parallel resonant frequency is:

$$f_r \cong 1/(2\pi\sqrt{LC})$$

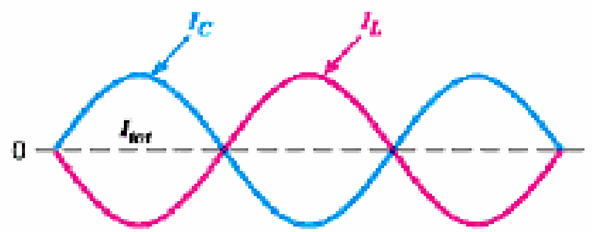
Ideal Parallel Resonant Circuit



(a) Parallel circuit at resonance ($X_C = X_L$, $Z = \infty$)



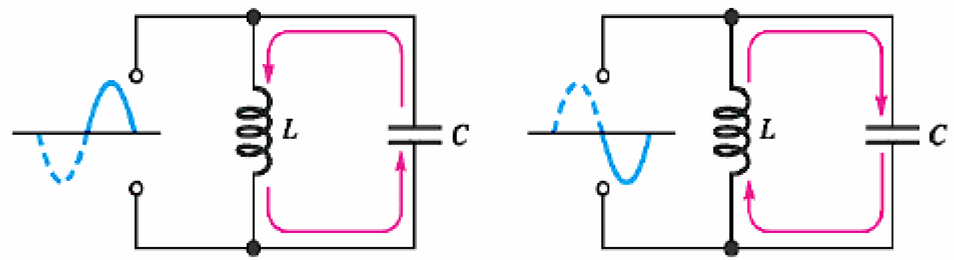
(b) Current phasors



(c) Current waveforms

Tank Circuit

- A parallel resonant circuit stores energy in the magnetic field of the coil and the electric field of the capacitor. The energy is transferred back and forth between the coil and capacitor



(a) The coil deenergizes as the capacitor charges.

(b) The capacitor discharges as the coil energizes.

Non-ideal Parallel Resonant Circuit

- A practical treatment of parallel resonant circuits must include the coil resistance R_W
- At parallel resonance:

$$X_L = X_C$$

- Q is the quality factor of the coil, X_L / R_W
- The total impedance of the non-ideal tank circuit at resonance can be expressed as the equivalent parallel resistance:

$$Z_T = R_W(Q^2 + 1)$$

Current and Phase Angle at Resonance

- Ideally the total current from the source at resonance is zero because the impedance is infinite
- In the non-ideal case when the coil resistance is considered, there is some total current at the resonant frequency:

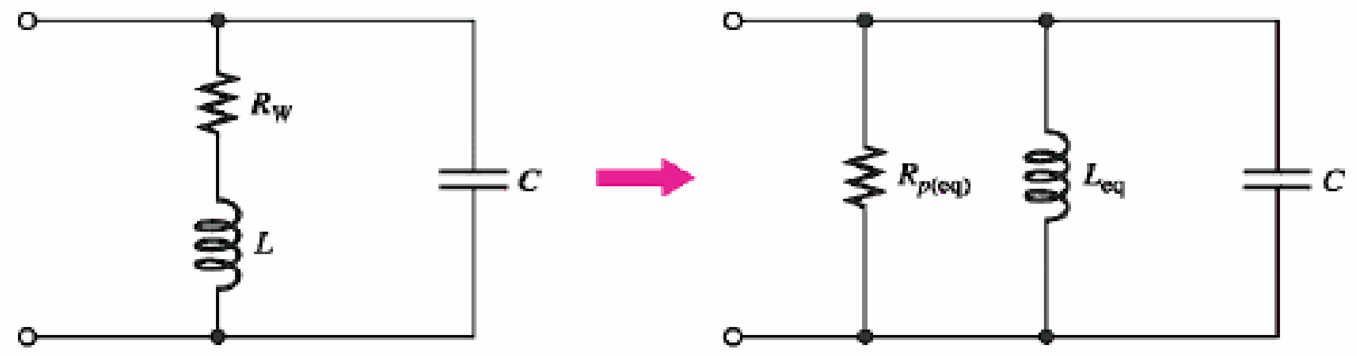
$$I_{tot} = V_s/Z_r$$

- Since the impedance is purely resistive at resonance, the phase angle is 0°

Parallel Resonant Circuits

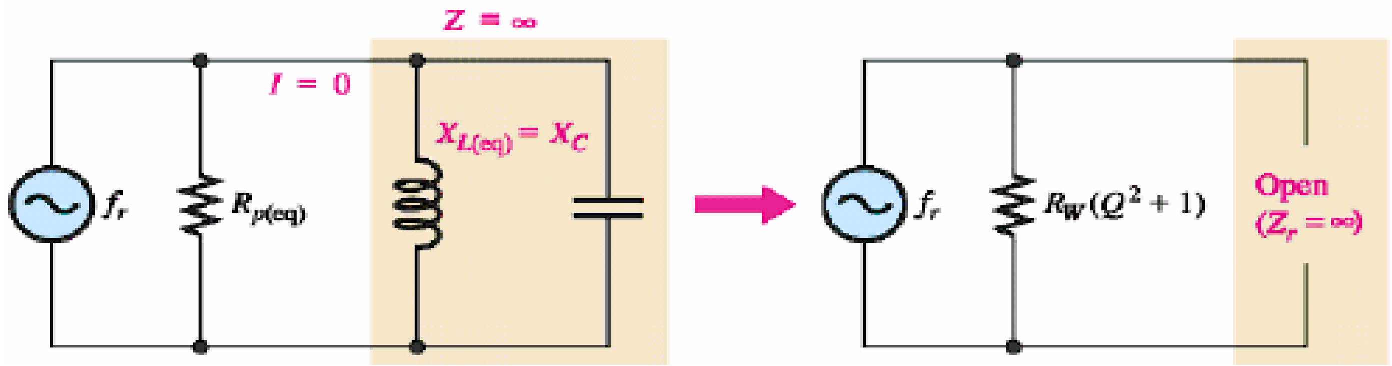
- For parallel resonant circuits,
- **the impedance is maximum** at the resonant frequency
- **Total current is minimum** at the resonant frequency
- **Bandwidth is the same as for the series resonant circuit**; the critical frequency impedances are at $0.707Z_{max}$

Practical Parallel Resonant Circuit



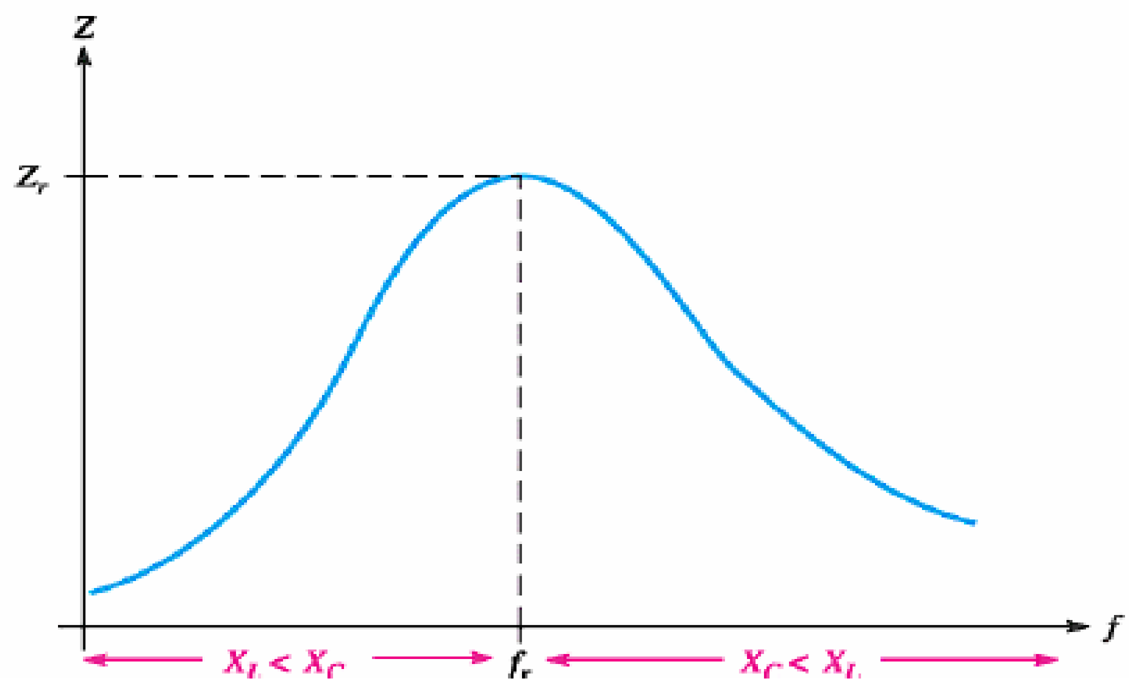
(a) Nonideal tank circuit (b) Parallel *RLC* equivalent

Practical Parallel Resonant Circuit

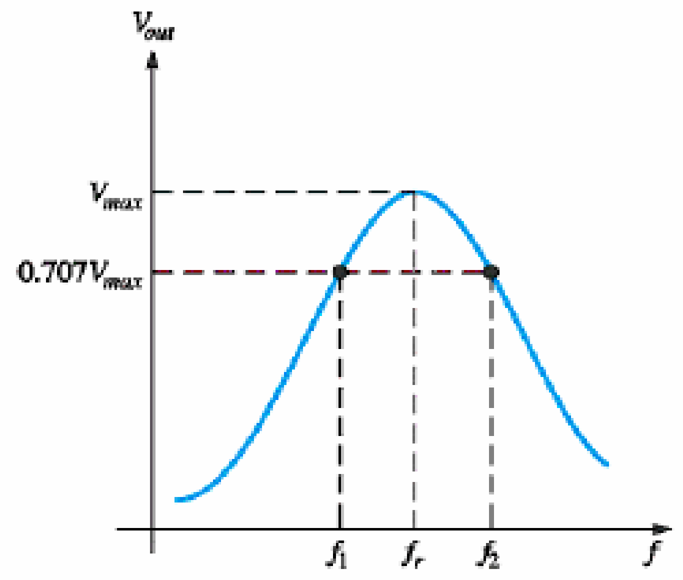


At resonance, the parallel LC portion appears open and the source sees only $R_{p(eq)}$, which equals $R_W (Q^2 + 1)$.

Z vs f in the Parallel Circuit

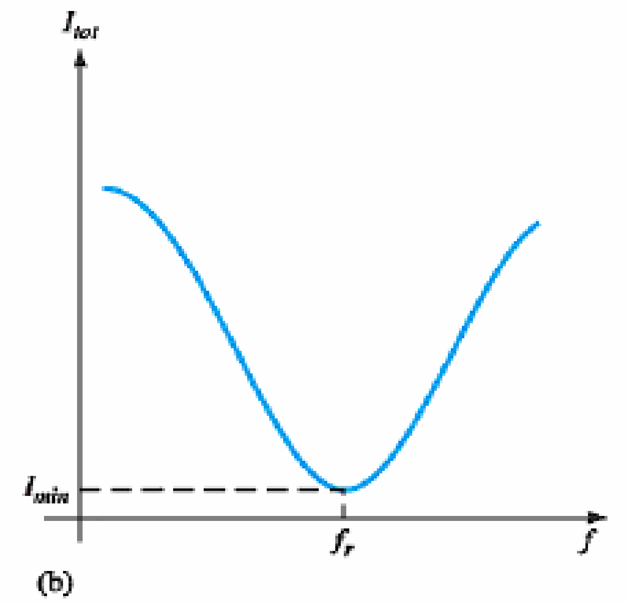


V vs f in the Parallel Circuit



(a)

I vs f in the Parallel Circuit



Parallel Resonance BW, Q, and fr

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$BW = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_o}{BW}$$

$$Q = \omega_o RC = \frac{R}{\omega_o} = R\sqrt{\frac{C}{L}}$$