Conventions for Describing Networks

2-1. For the controlled (monitored) source shown in the figure, prepare a plot similar to that given in Fig. 2-8(b).

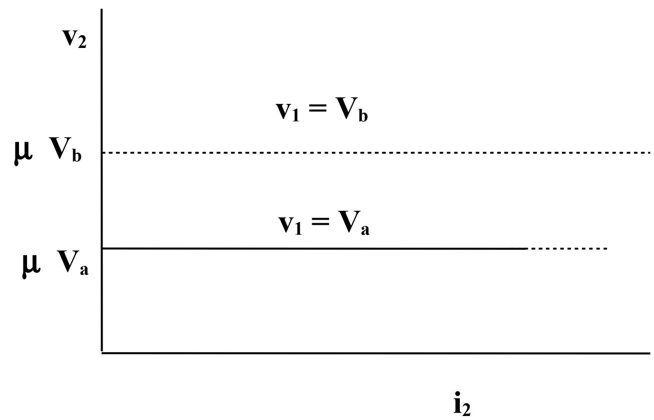
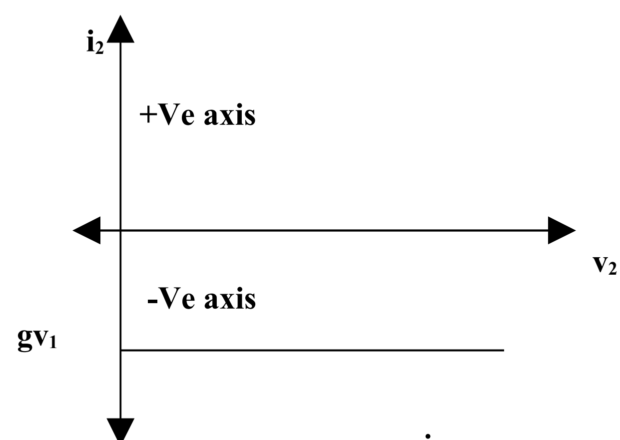


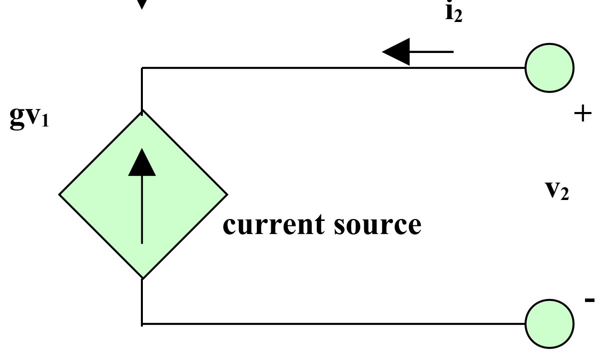
Fig. 2-8 (b)

Solution:

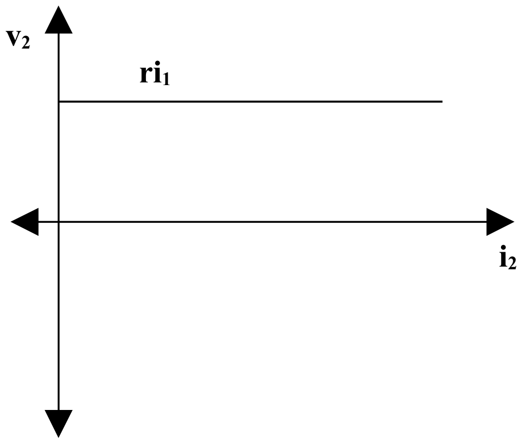
Open your book & see the figure (P/46)

It is voltage controlled current source.





2-2. Repeat Prob. 2-1 for the controlled source given in the accompanying figure.
Solution:
Open your book & see the figure (P/46)
It is current controlled voltage source.



2-3. The network of the accompanying figure is a model for a battery of open-circuit terminal voltage V and internal resistance R_b. For this network, plot i as a function v. Identify features of the plot such as slopes, intercepts, and so on.
Solution:
Open your book & see the figure (P/46)
Terminal voltage

$v = V - iR_b$

$iR_b = V - v$

$i = (V - v)/R_b$

When $v = 0$

$i = (V - v)/R_b$

$i = (V - 0)/R_b$

$i = V/R_b$ **amp**

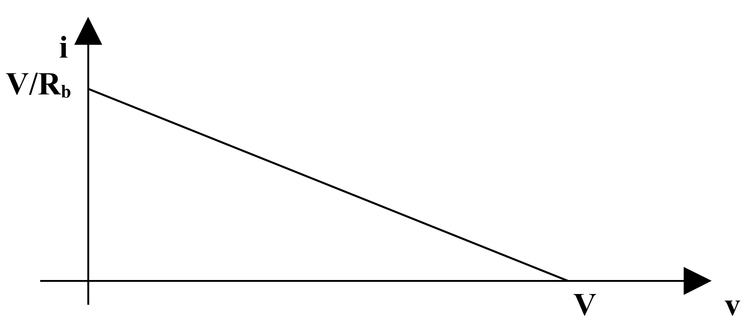
When $v = V$

$i = (V - V)/R_b$

$i = (0)/R_b$

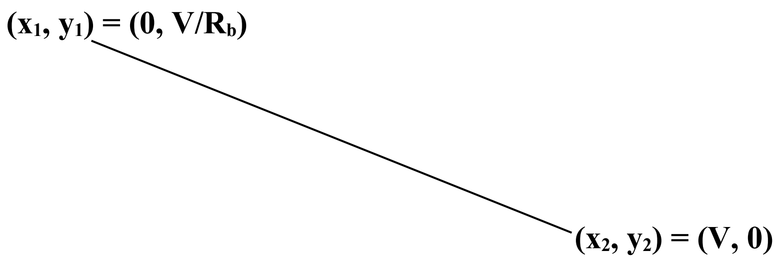
$i = 0$ **amp**

$v = 0$	$i = V/R$
$v = V$	$i = 0$



Slope:

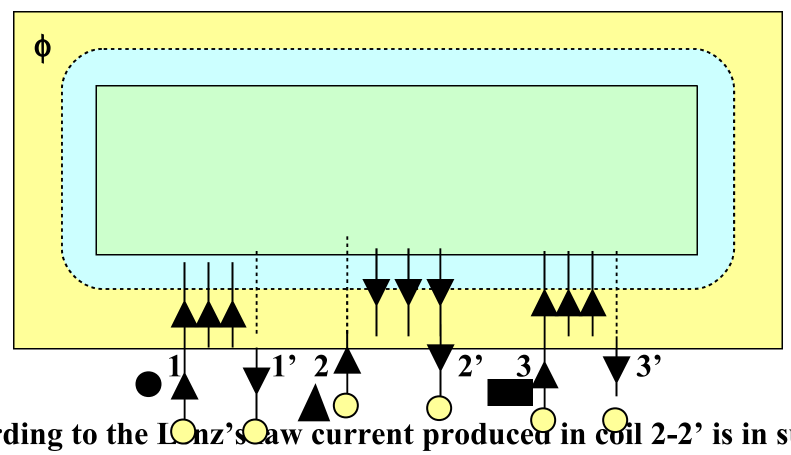
y = mx + c



$m = (y_2 - y_1)/(x_2 - x_1) = (0 - V/R_b)/(V - 0) = (-V/R_b)/V = (-V/Rb)(1/V) = -1/R_b$
y-intercept = V/R_b
x-intercept = V

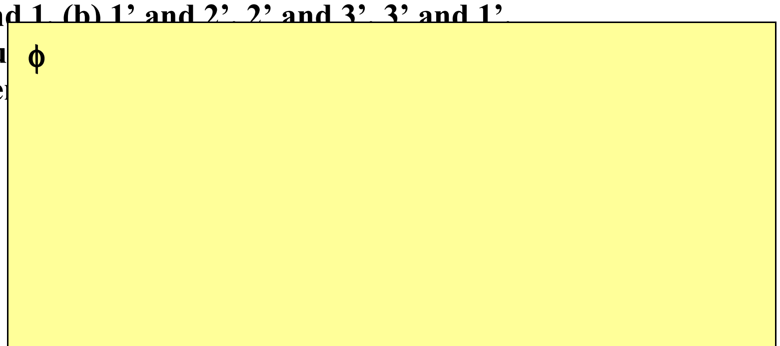
Slope	y-intercept	x-intercept
$-1/R_b$	V/R_b	V

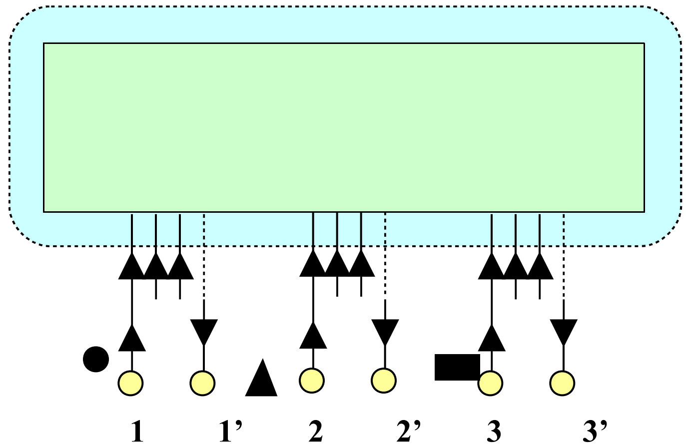
2-4. The magnetic system shown in the figure has three windings marked 1-1’, 2-2’, and 3-3’. Using three different forms of dots, establish polarity markings for these windings.
Solution:
Open your book & see the figure (P/46)
Lets assume current in coil 1-1’ has direction up at 1 (increasing). It produces flux ϕ (increasing) in that core in clockwise direction.



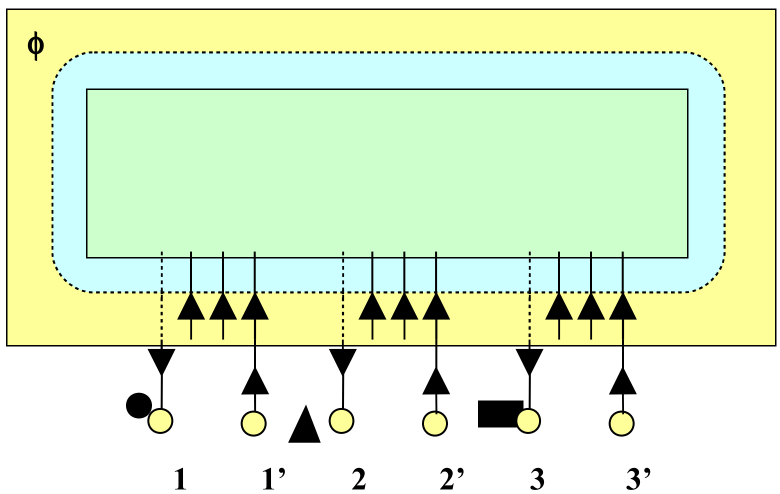
According to the Lenz’s law current produced in coil 2-2’ is in such a direction that it opposes the increasing flux ϕ . So direction of current in 2-2’ is down at 2’. Hence ends 1 & 2’ are of same polarity at any instant. Hence are marked with ○. Similarly assuming the direction of current in coil 2-2’, we can show at any instant 2 & 3’ have same polarities and also 1 & 3 have same polarities.

2-5. Place three windings on the core shown for Prob. 2-4 with winding senses selected such that the following terminals have the same mark: (a) 1 and 2, 2 and 3, 3 and 1. (b) 1’ and 2’, 2’ and 3’, 3’ and 1’.
Solu
Ope



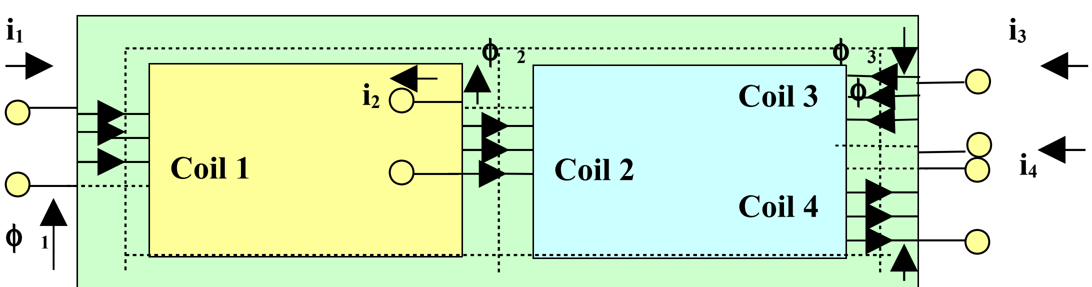


(a)



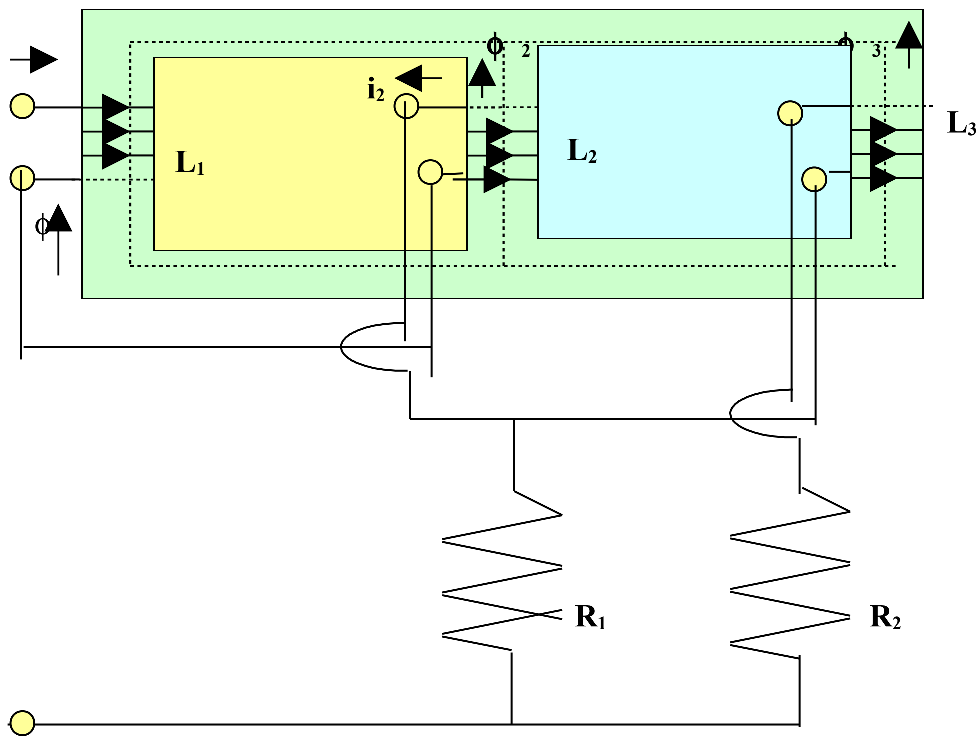
(b)

2-6. The figure shows four windings on a magnetic flux-conducting core. Using different shaped dots, establish polarity markings for the windings.
Solution:
Open your book & see the figure (P/47)

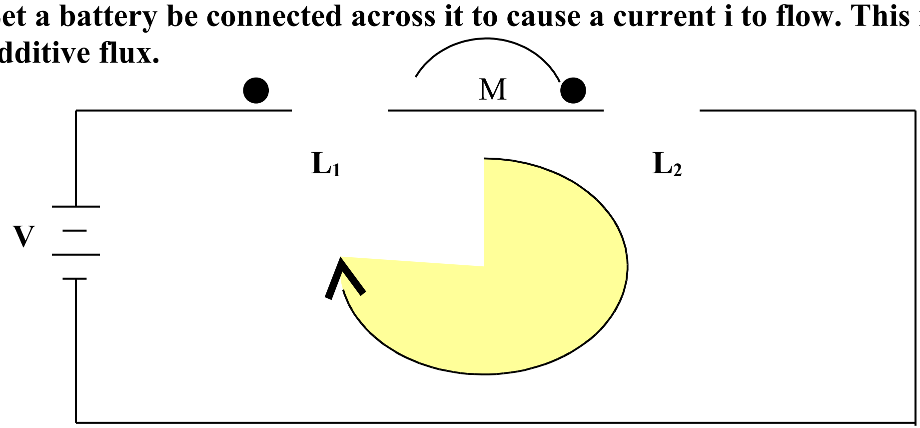


(Follow Fleming's right hand rule)
2-7. The accompanying schematic shows the equivalent circuit of a system with polarity marks on the three-coupled coils. Draw a transformer with a core similar to that shown for Prob. 2-6 and place windings on the legs of the core in such a way as

to be equivalent to the schematic. Show connections between the elements in the same drawing.
Solution:
Open your book & see the figure (P/47)



2-8. The accompanying schematics each show two inductors with coupling but with different dot markings. For each of the two systems, determine the equivalent inductance of the system at terminals 1-1' by combining inductances.
Solution:
Open your book & see the figure (P/47)
Let a battery be connected across it to cause a current i to flow. This is the case of additive flux.

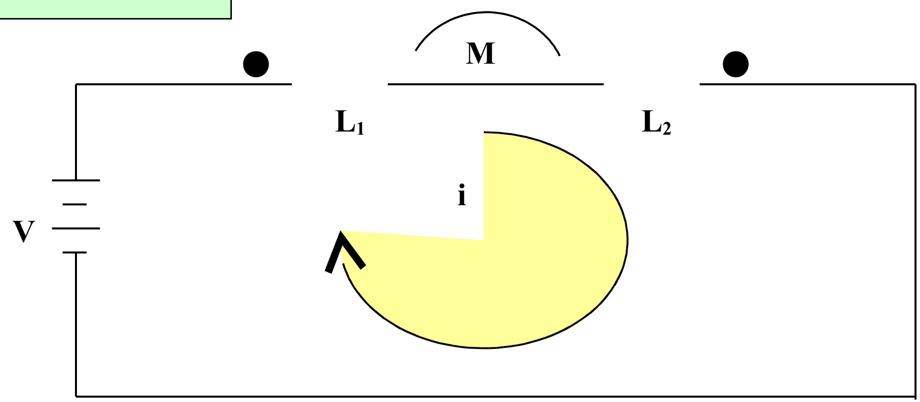


(a)

V = self induced e.m.f. (1) + self induced e.m.f. (2) + mutually induced e.m.f. (1) + mutually induced e.m.f. (2)
$V = L_1 di/dt + L_2 di/dt + M\ di/dt + M\ di/dt$
Let L_{eq} be the equivalent inductance then $V = L_{eq}\ di/dt$
$L_{eq}\ di/dt = (L_1 + L_2 + M + M)\ di/dt$

$\therefore$ $L_{eq} = L_1 + L_2 + M + M$

$L_{eq} = L_1 + L_2 + 2M$



(b)

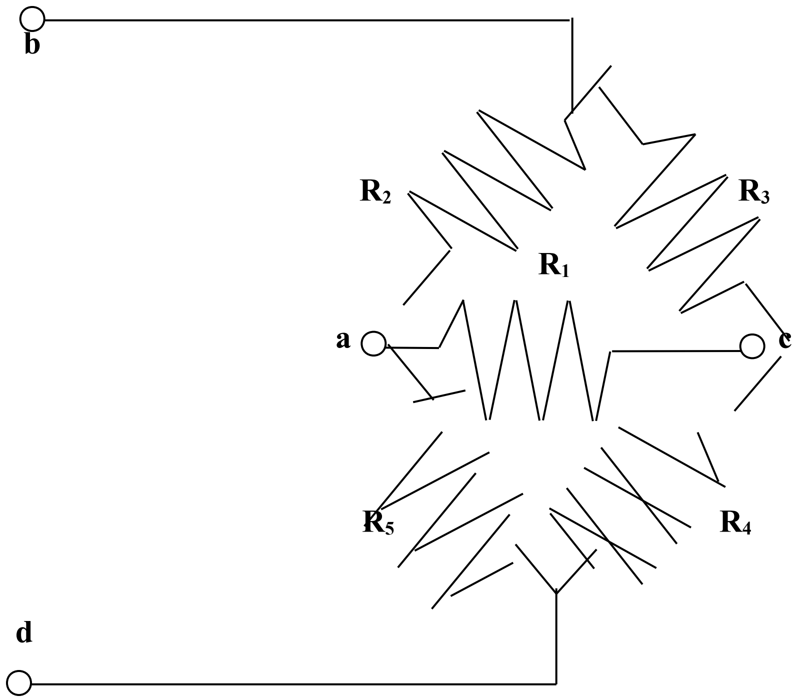
This is the case of subtractive flux.
$\therefore$ $V = L_1 di/dt + L_2 di/dt - M\, di/dt - M\, di/dt$
Let L_{eq} be the equivalent inductance then $V = L_{eq}\, di/dt$
$L_{eq}\, di/dt = (L_1 + L_2 - M - M)\, di/dt$
$\therefore$ $L_{eq} = L_1 + L_2 - M - M$

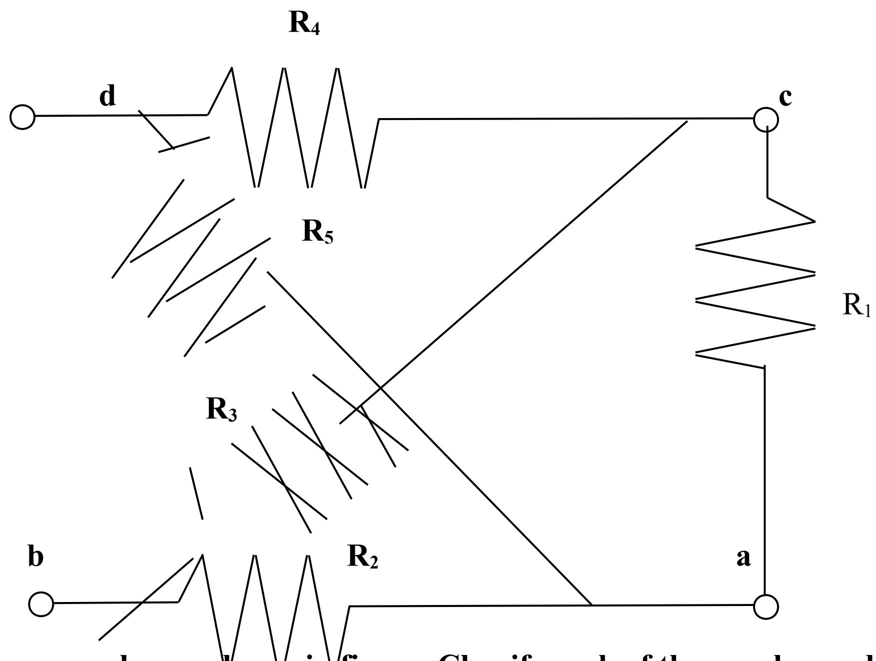
$L_{eq} = L_1 + L_2 - 2M$

2-9. A transformer has 100 turns on the primary (terminals 1-1') and 200 turns on the secondary (terminals 2-2'). A current in the primary causes a magnetic flux, which links all turns of both the primary and the secondary. The flux decreases according to the law $\phi = e^{-t}$ Weber, when $t \geq 0$. Find: (a) the flux linkages of the primary and secondary, (b) the voltage induced in the secondary.
Solution:
$N_1 = 100$
$N_2 = 200$
$\phi = e^{-t}\ (t \geq 0)$
Primary flux linkage $\psi_1 = N_1\phi = 100\, e^{-t}$
Secondary flux linkage $\psi_2 = N_2\phi = 200\, e^{-t}$
Magnitude of voltage induced in secondary $v_2 = d\psi_2/dt = d/dt(200\, e^{-t})$
$v_2 = -200\, e^{-t}$
Hence secondary induced voltage has magnitude

$v_2 = 200\, e^{-t}$

2-10. In (a) of the figure is shown a resistive network. In (b) and (c) are shown graphs with two of the four nodes identified. For these two graphs, assign resistors to the branches and identify the two remaining nodes such that the resulting networks are topologically identical to that shown in (a).
Solution:
Open your book & see the figure (P/48)





2-11. Three graphs are shown in figure. Classify each of the graphs as planar or nonplanar.

Solution:

Open your book & see the figure (P/48)

All are planar.

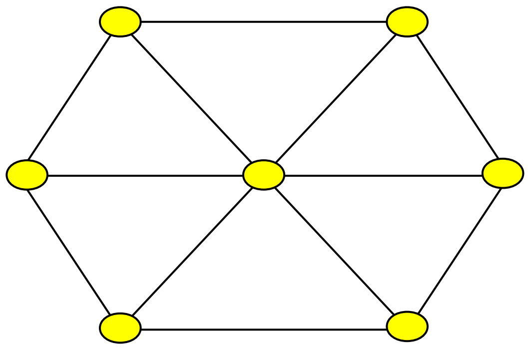
In that they may be drawn on a sheet of paper without crossing lines.

2-12. For the graph of figure, classify as planar or nonplanar, and determine the quantities specified in equations 2-13 & 2-14.
Solution:
Open your book & see the figure (P/48)
Classification:
Nonplanar
Number of branches in tree = number of nodes – 1 = 5 – 1 = 4
Number of chords = branches – nodes + 1 = 10 – 5 + 1 = 10 – 4 = 6
Chord means 'A straight line connecting two points on a curve'.

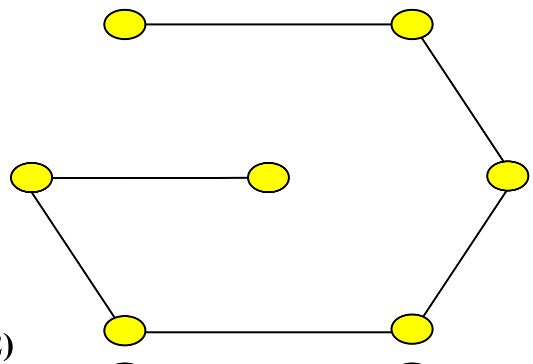
2-13. In (a) and (b) of the figure for Prob. 2-11 are shown two graphs, which may be equivalent. If they are equivalent, what must be the identification of nodes a, b, c, d in terms of nodes 1, 2, 3, 4 if a is identical with 1?
Solution:
Open your book & see the figure (P/48)
(b)
a is identical with 1
b is identical with 4
c is identical with 2
d is identical with 3

2-14. The figure shows a network with elements arranged along the edges of a cube. (a) Determine the number of nodes and branches in the network. (b) Can the graph of this network be drawn as a planar graph?
Solution:
Open your book & see the figure (P/48)
Number of nodes = 8
Number of branches = 11
(b) Yes it can be drawn.

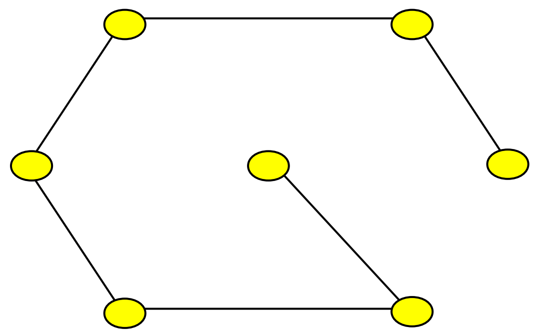
2-15. The figure shows a graph of six nodes and connecting branches. You are to add nonparallel branches to this basic structure in order to accomplish the following different objectives: (a) what is the minimum number of branches that may be added to make the resulting structure nonplanar? (b) What is the maximum number of branches you may add before the resulting structure becomes nonplanar?
Solution:
Open your book & see the figure (P/49)
Make the structure nonplanar
Minimum number of branches = 3
Maximum number of branches = 7
2-16. Display five different trees for the graph shown in the figure. Show branches with solid lines and chords with dotted lines. (b) Repeat (a) for the graph of (c) in Prob. 2-11.
Solution:
Open your book & see the figure (P/49)



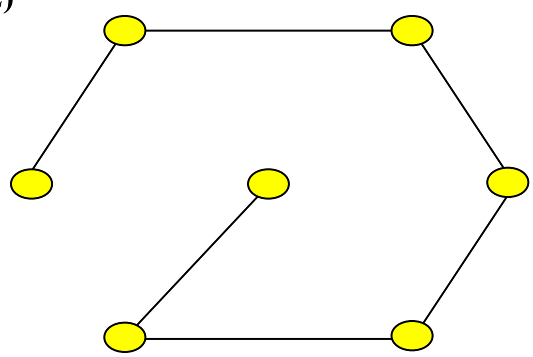
1)



3)



2)



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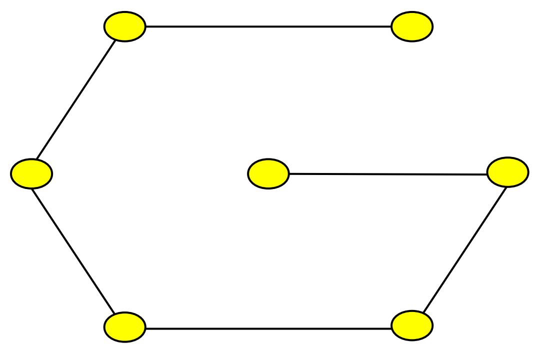
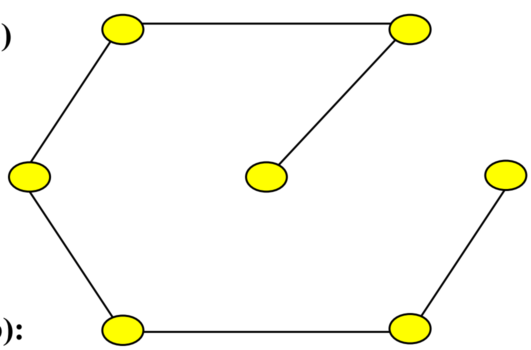
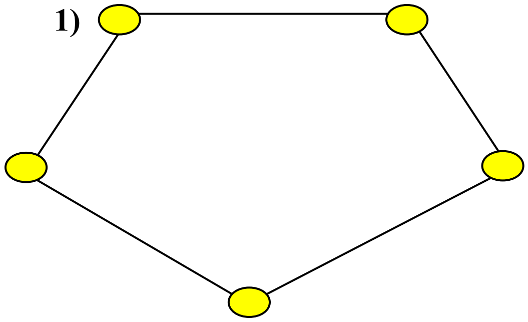


5)



b):

1)



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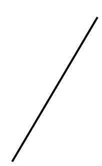

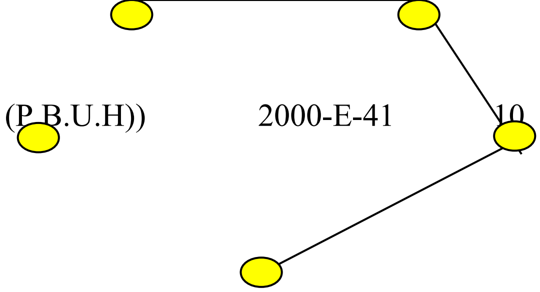

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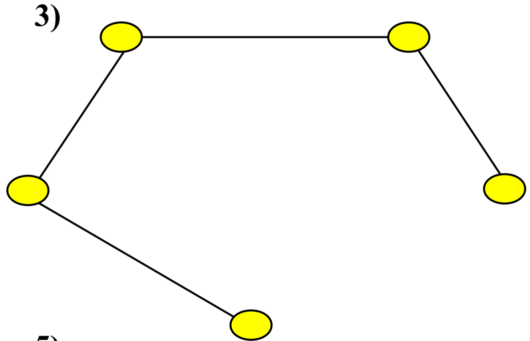

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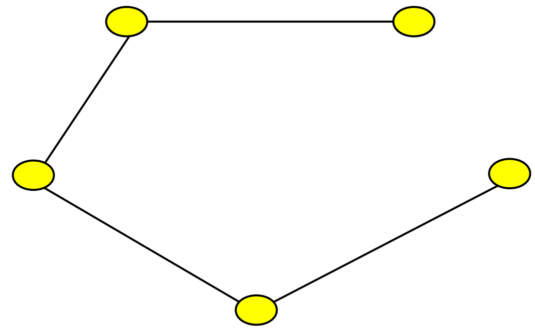

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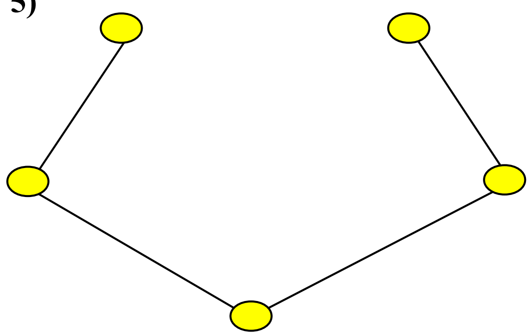


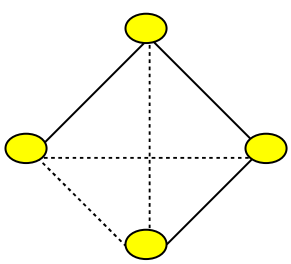
2-17. Determine all trees of the graphs shown in (a) of Prob. 2-11 and (b) of Prob. 2-10. Use solid lines for tree branches and dotted lines for chords.

Solution:

Open your book & see the figure (P/49)

<u>All trees:</u>

1)



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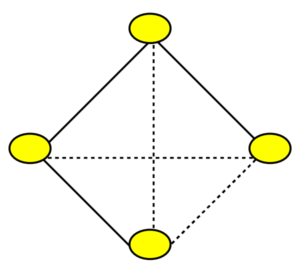

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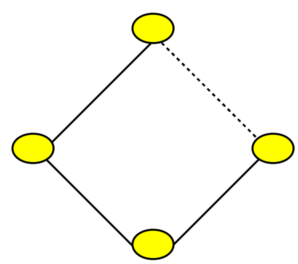

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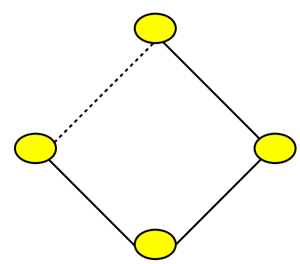
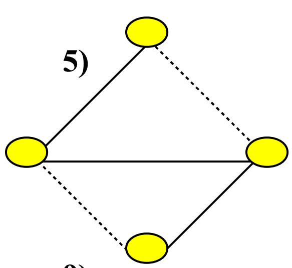


5)



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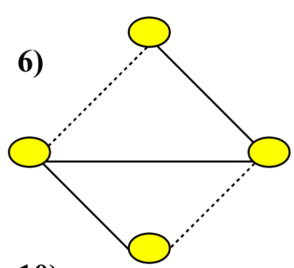

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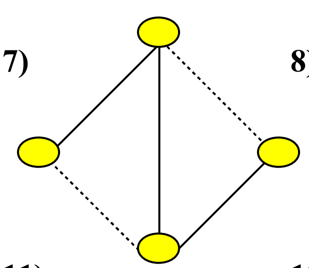

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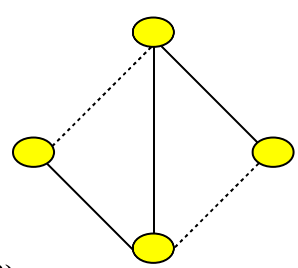

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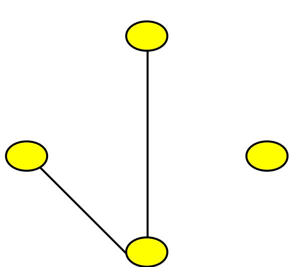
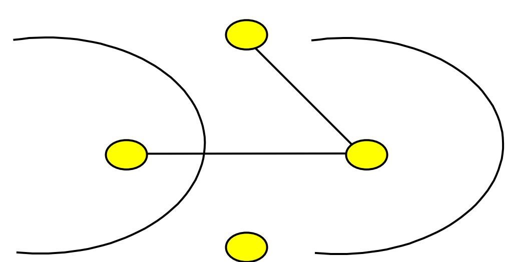
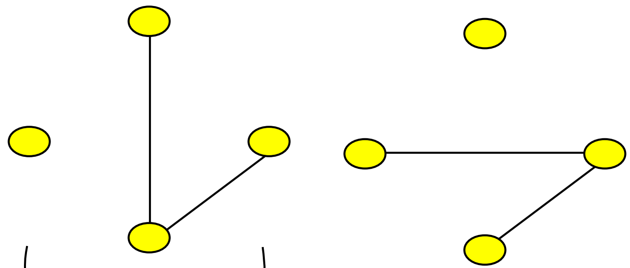


10)

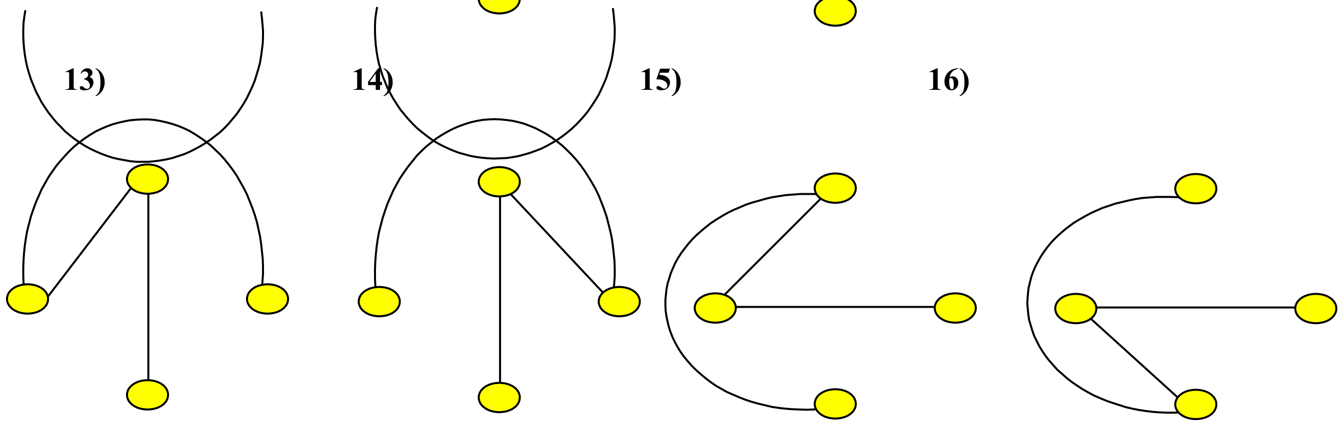
11)

12)

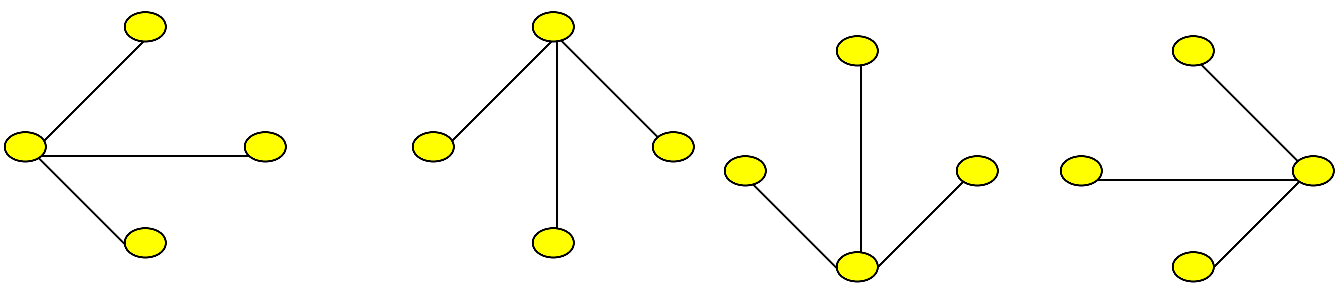




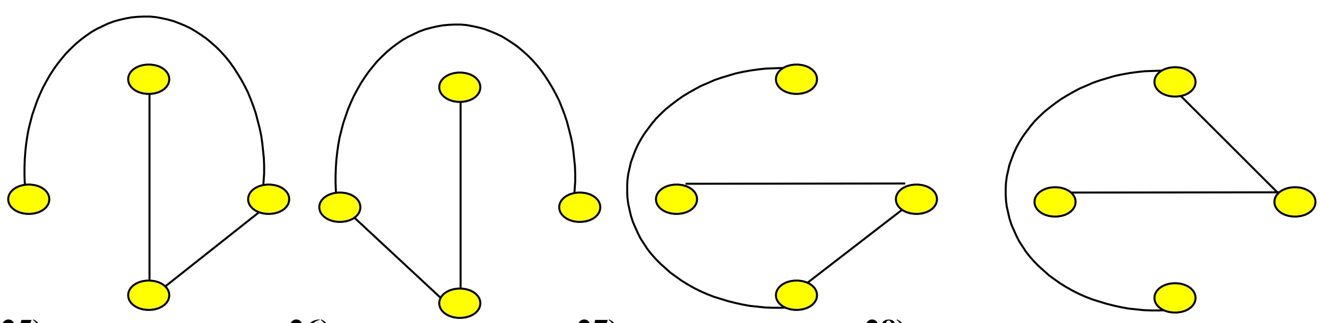
13) **14)** **15)** **16)**



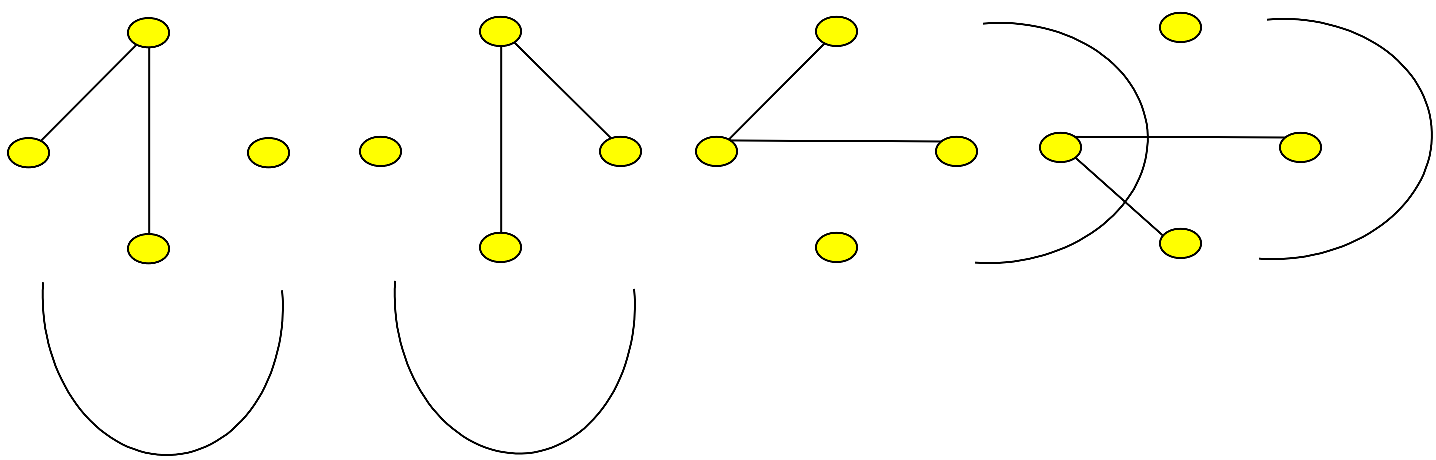
17) **18)** **19)** **20)**



21) **22)** **23)** **24)**



25) **26)** **27)** **28)**



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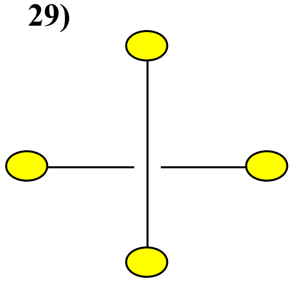

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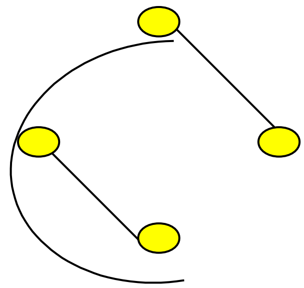
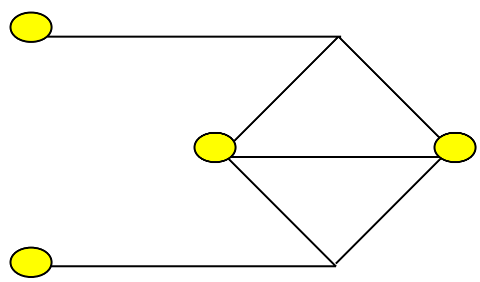
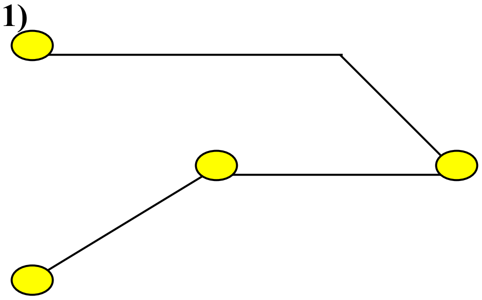


<u>All trees of</u>



Solution:

1)



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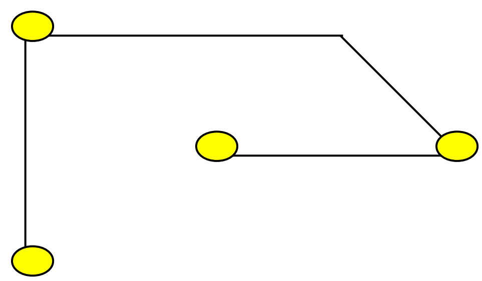
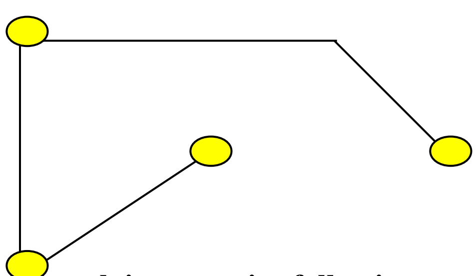


3)



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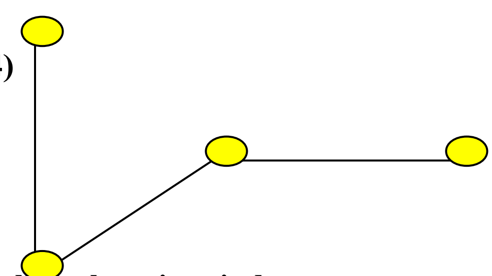


<u>Before solving exercise following terms should be kept in mind:</u>

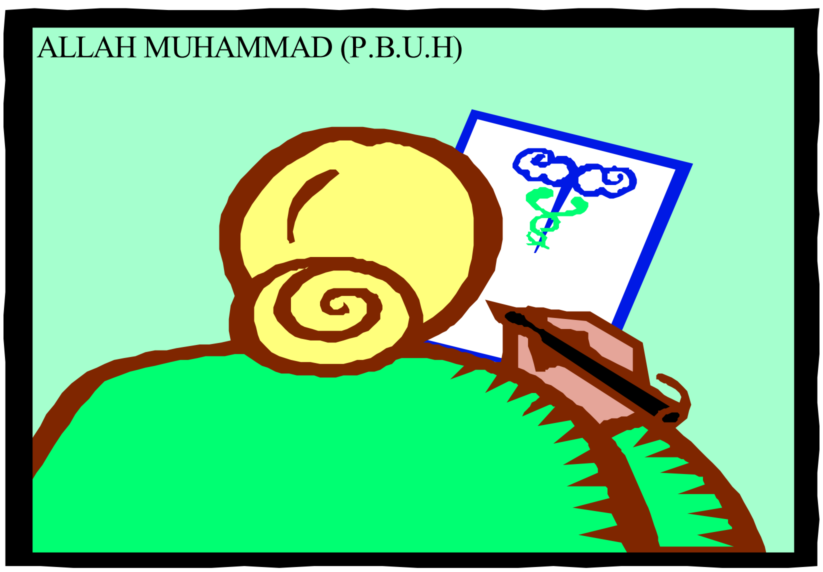
1. **Node**
2. **Branch**
3. **Tree**
4. **[illegible] theory**
5. [illegible]
6. **Straight line equation**
7. **Inte[illegible]**
8. **S[illegible]**
9. **M[illegible]l induction**
10. **[illegible]urrent controlled voltage source**

Chapter#2

Conventions for describing networks

MASHAALLAH BHAI [Peon of holy prophet(P.B.U.H)]

11. Voltage controlled current source
12. Coordinate system

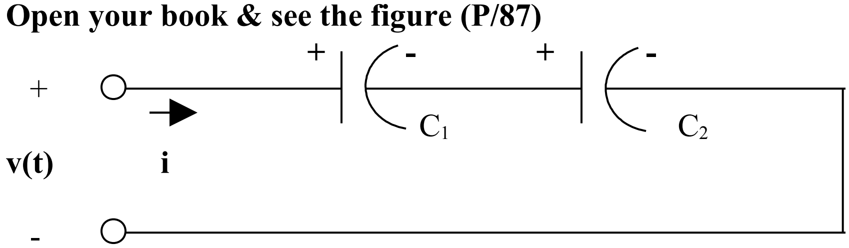


__Network equations__

3-1. What must be the relationship between C_{eq} and C_1 and C_2 in (a) of the figure of the networks if (a) and (c) are equivalent? Repeat for the network shown in (b).

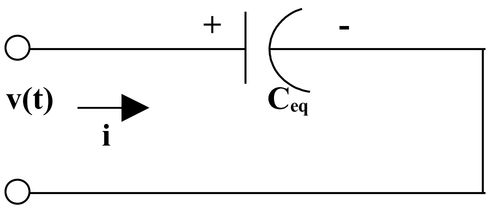
Solution:

Open your book & see the figure (P/87)



By kirchhoff's voltage law:

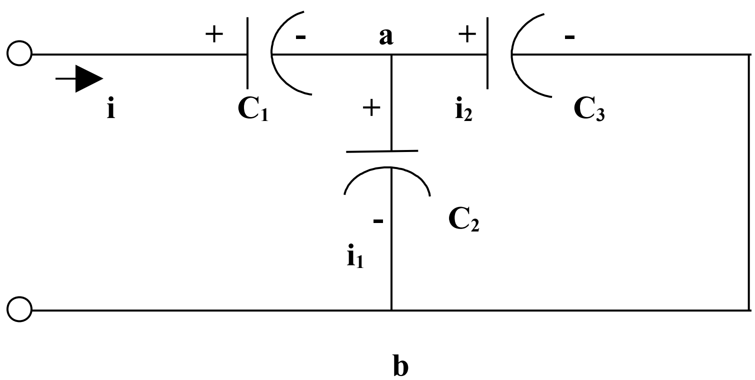
$v(t) = 1/C_1\int i\,dt + 1/C_2\int i\,dt$
$v(t) = (1/C_1 + 1/C_2)\int i\,dt$
In second case



$v(t) = 1/C_{eq}\int i\,dt$
If (a) & (c) are equivalent
$1/C_{eq}\int i\,dt = (1/C_1 + 1/C_2)\int i\,dt$

$$\boxed{1/C_{eq} = (1/C_1 + 1/C_2)}$$

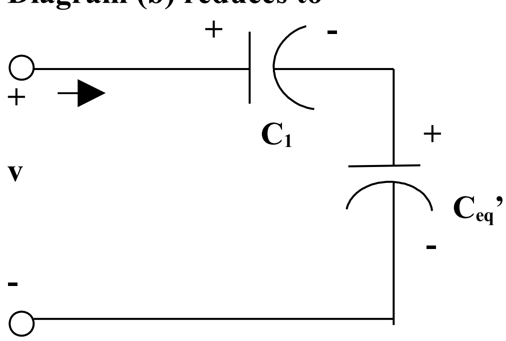
(b)



$i = i_1 + i_2$
$i = C_2dv_a/dt + C_3dv_a/dt$ when v_a is voltage across ab.
The equivalent capacitance between a & b be C_{eq}'
Then $i = C_{eq}'dv_a/dt$
$\therefore\; C_{eq}'dv_a/dt = C_2dv_a/dt + C_3dv_a/dt$

$$\boxed{C_{eq}' = C_2 + C_3}$$

Diagram (b) reduces to



From result obtained by (a)
$1/C_{eq} = (1/C_1 + 1/C_{eq}')$

$1/C_{eq} = (1/C_1 + 1/C_2 + C_3)$

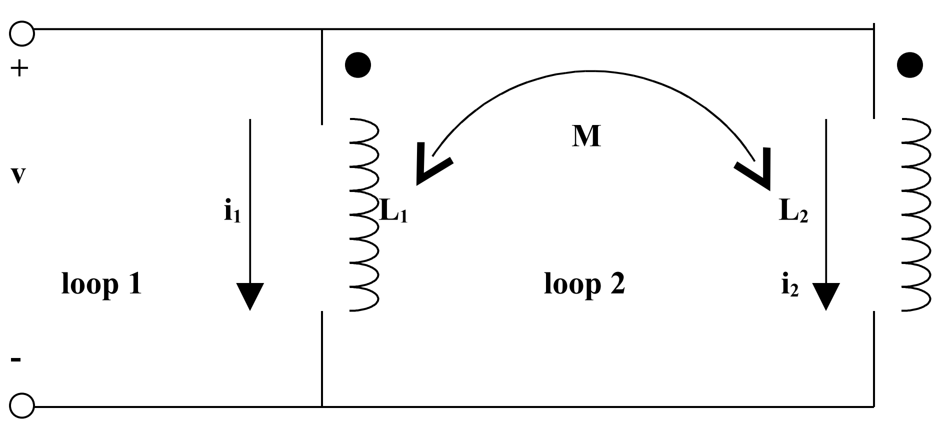
3-2. What must be the relationship between L_{eq} and L_1, L_2 and M for the networks of (a) and of (b) to be equivalent to that of (c)?
Solution:
Open your book & see the figure (P/87)
In network (a) applying KVL
$v = L_1 di/dt + L_2 di/dt + M di/dt + M di/dt$
$v = (L_1 + L_2 + M + M) di/dt$
$v = (L_1 + L_2 + 2M) di/dt$
In network (c)
$v = L_{eq} di/dt$
If (a) & (c) are equivalent
$(L_1 + L_2 + 2M) di/dt = L_{eq} di/dt$

$$\boxed{(L_1 + L_2 + 2M) = L_{eq}}$$

In network (b) applying KVL
$v = L_1 di/dt + L_2 di/dt - M di/dt - M di/dt$
$v = (L_1 + L_2 - M - M) di/dt$
$v = (L_1 + L_2 - 2M) di/dt$
In network (c)
$v = L_{eq} di/dt$
If (b) & (c) are equivalent
$(L_1 + L_2 - 2M) di/dt = L_{eq} di/dt$

$$\boxed{(L_1 + L_2 - 2M) = L_{eq}}$$

3-3. Repeat Prob. 3-2 for the three networks shown in the accompanying figure.
Solution:
Open your book & see the figure (P/87)



Applying KVL in loop 1

$v = L_1 d(i_1 - i_2)/dt + M di_2/dt$

$v = L_1 di_1/dt - L_1 di_2/dt + M di_2/dt$

$v = L_1 di_1/dt + M di_2/dt - L_1 di_2/dt$

$$\boxed{v = L_1 di_1/dt + (M - L_1) di_2/dt}$$

Applying KVL in loop 2

$0 = L_2 di_2/dt + L_1 d(i_2 - i_1)/dt + \{-M di_2/dt\} + \{-M d(i_2 - i_1)/dt\}$

$0 = L_2 di_2/dt + L_1 di_2/dt - L_1 di_1/dt - M di_2/dt - M d(i_2 - i_1)/dt$

$0 = L_2 di_2/dt + L_1 di_2/dt - L_1 di_1/dt - M di_2/dt - M di_2/dt + M di_1/dt$

$0 = L_2 di_2/dt + L_1 di_2/dt - L_1 di_1/dt - 2M di_2/dt + M di_1/dt$

$$\boxed{0 = (M - L_1)\, di_1/dt + (L_1 + L_2 - 2M)\, di_2/dt}$$

Writing in matrix form

$$\begin{pmatrix} L_1 & M - L_1 \\ M - L_1 & L_1 + L_2 - 2M \end{pmatrix} \begin{pmatrix} di_1/dt \\ di_2/dt \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$di_1/dt = \frac{\begin{vmatrix} v & M - L_1 \\ 0 & L_1 + L_2 - 2M \end{vmatrix}}{\begin{vmatrix} L_1 & M - L_1 \\ M - L_1 & L_1 + L_2 - 2M \end{vmatrix}}$$

$$\begin{vmatrix} v & M - L_1 \\ 0 & L_1 + L_2 - 2M \end{vmatrix}$$

$= (v)(L_1 + L_2 - 2M) - 0 = (v)(L_1 + L_2 - 2M)$

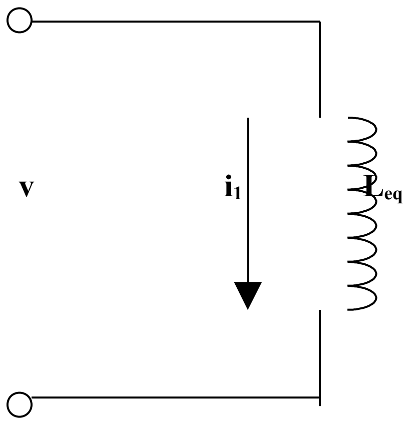
$$\begin{vmatrix} L_1 & M - L_1 \\ M - L_1 & L_1 + L_2 - 2M \end{vmatrix}$$

$= (L_1)(L_1 + L_2 - 2M) - (M - L_1)(M - L_1)$

$= (L_1)(L_1 + L_2 - 2M) - (M - L_1)^2$

$= (L_1^2 + L_1L_2 - 2L_1M) - M^2 - L_1^2 + 2ML_1$

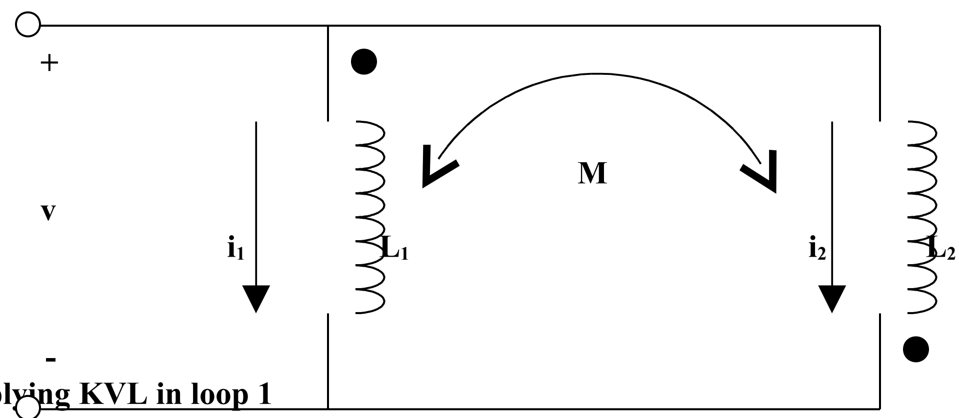
$= L_1^2 + L_1L_2 – 2L_1M – M^2 – L_1^2 + 2ML_1$
$= L_1L_2 – M^2$
$di_1/dt = (v)(L_1 + L_2 – 2M)/L_1L_2 – M^2$
$di_1/dt \{(L_1L_2 – M^2)/(L_1 + L_2 – 2M)\} = v$
In network (c)



$v = L_{eq}di_1/dt$
For (a) & (c) to be equal
$di_1/dt \{(L_1L_2 – M^2)/(L_1 + L_2 – 2M)\} = L_{eq}di_1/dt$

$$\boxed{(L_1L_2 – M^2)/(L_1 + L_2 – 2M) = L_{eq}}$$

(b)



Applying KVL in loop 1
$v = L_1d(i_1 – i_2)/dt - Mdi_2/dt$
$v = L_1di_1/dt - L_1di_2/dt - Mdi_2/dt$
$v = L_1di_1/dt + Mdi_2/dt - L_1di_2/dt$

$$\boxed{v = L_1di_1/dt - (L_1 + M)di_2/dt}$$

Applying KVL in loop 2
$0 = L_2di_2/dt + L_1d(i_2 – i_1)/dt + Mdi_2/dt + Md(i_2 – i_1)/dt$
$0 = L_2di_2/dt + L_1di_2/dt - L_1di_1/dt + Mdi_2/dt + Md(i_2 – i_1)/dt$
$0 = L_2di_2/dt + L_1di_2/dt - L_1di_1/dt + Mdi_2/dt + Mdi_2/dt - Mdi_1/dt$
$0 = L_2di_2/dt + L_1di_2/dt - L_1di_1/dt + 2Mdi_2/dt - Mdi_1/dt$

$$\boxed{0 = - (L_1 + M) di_1/dt + (L_1 + L_2 + 2M) di_2/dt}$$

Writing in matrix form

$$\begin{pmatrix} L_1 & -(L_1+M) \\ -(L_1+M) & L_1+L_2+2M \end{pmatrix} \begin{pmatrix} di_1/dt \\ di_2/dt \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$di_1/dt = \frac{\begin{vmatrix} v & -(L_1+M) \\ 0 & L_1+L_2+2M \end{vmatrix}}{\begin{vmatrix} L_1 & -(L_1+M) \\ -(L_1+M) & L_1+L_2+2M \end{vmatrix}}$$

$$\begin{vmatrix} v & -(L_1+M) \\ 0 & L_1+L_2+2M \end{vmatrix}$$

$= (v)(L_1 + L_2 + 2M) – 0 = (v)(L_1 + L_2 + 2M)$

$$\begin{vmatrix} L_1 & -(L_1+M) \\ -(L_1+M) & L_1+L_2+2M \end{vmatrix}$$

$= (L_1)(L_1 + L_2 + 2M) - (L_1 + M)(L_1 + M)$

$= (L_1)(L_1 + L_2 + 2M) - (L_1 + M)^2$

$= (L_1^2 + L_1L_2 + 2L_1M) - M^2 - L_1^2 - 2ML_1$

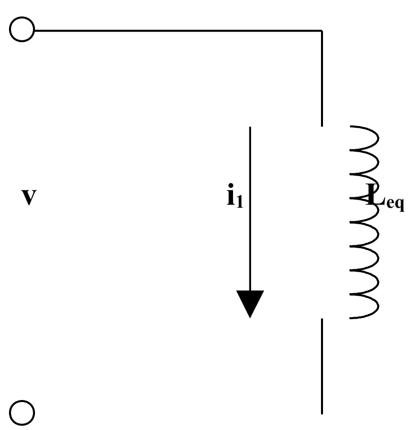
$= L_1^2 + L_1L_2 + 2L_1M - M^2 - L_1^2 - 2ML_1$

$= L_1L_2 – M^2$

$di_1/dt = (v)(L_1 + L_2 + 2M)/L_1L_2 – M^2$

$di_1/dt \{(L_1L_2 – M^2)/(L_1 + L_2 + 2M)\} = v$

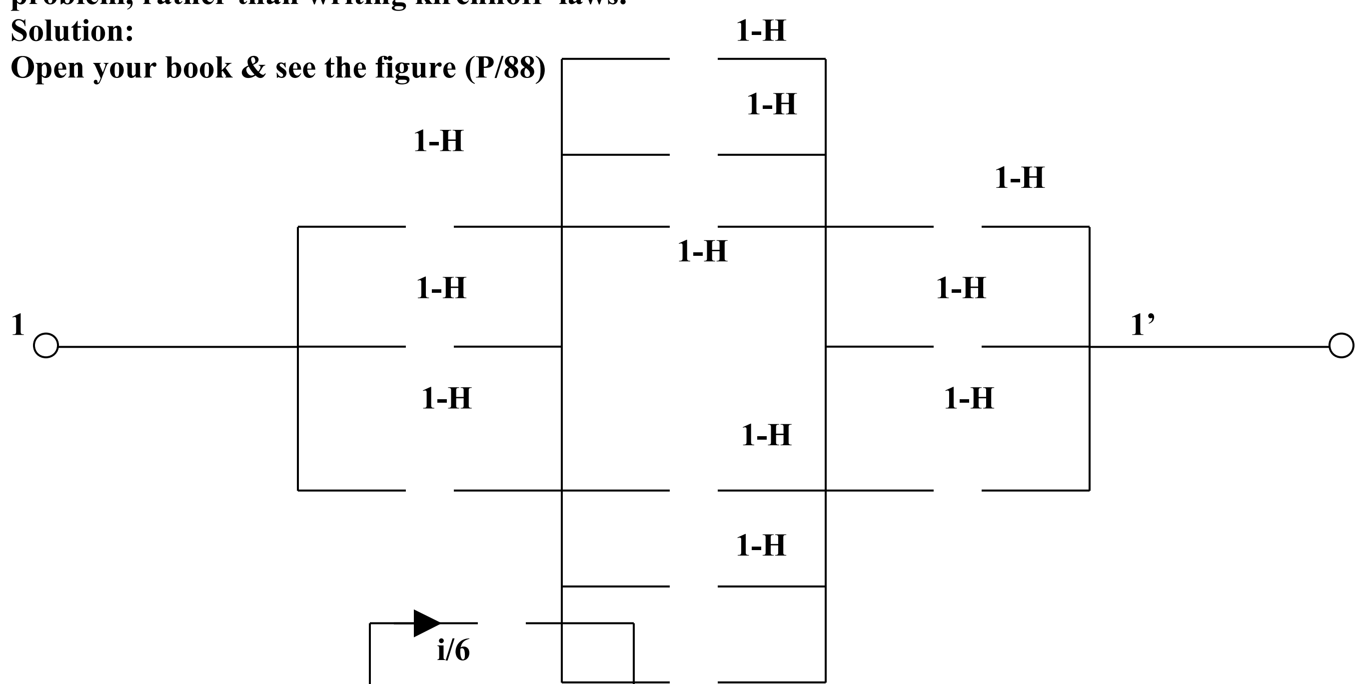
In network (c)

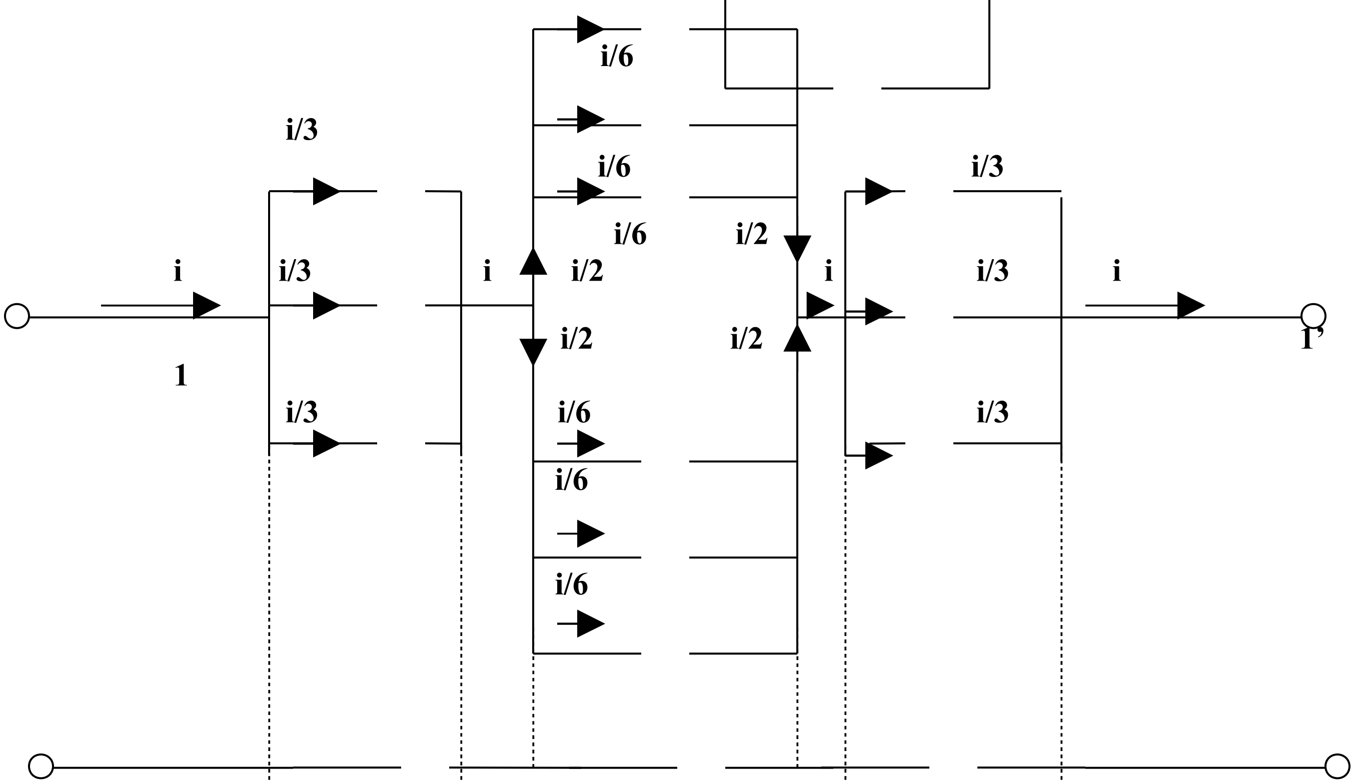


$v = L_{eq}di_1/dt$
For (a) & (c) to be equal
$di_1/dt\ \{(L_1L_2 - M^2)/(L_1 + L_2 + 2M)\} = L_{eq}di_1/dt$

$\boxed{(L_1L_2 - M^2)/(L_1 + L_2 + 2M) = L_{eq}}$

3-4. The network of inductors shown in the figure is composed of a 1-H inductor on each edge of a cube with the inductors connected to the vertices of the cube as shown. Show that, with respect to vertices a and b, the network is equivalent to that in (b) of the figure when Leq = 5/6 H. Make use of symmetry in working this problem, rather than writing kirchhoff laws.
Solution:
Open your book & see the figure (P/88)

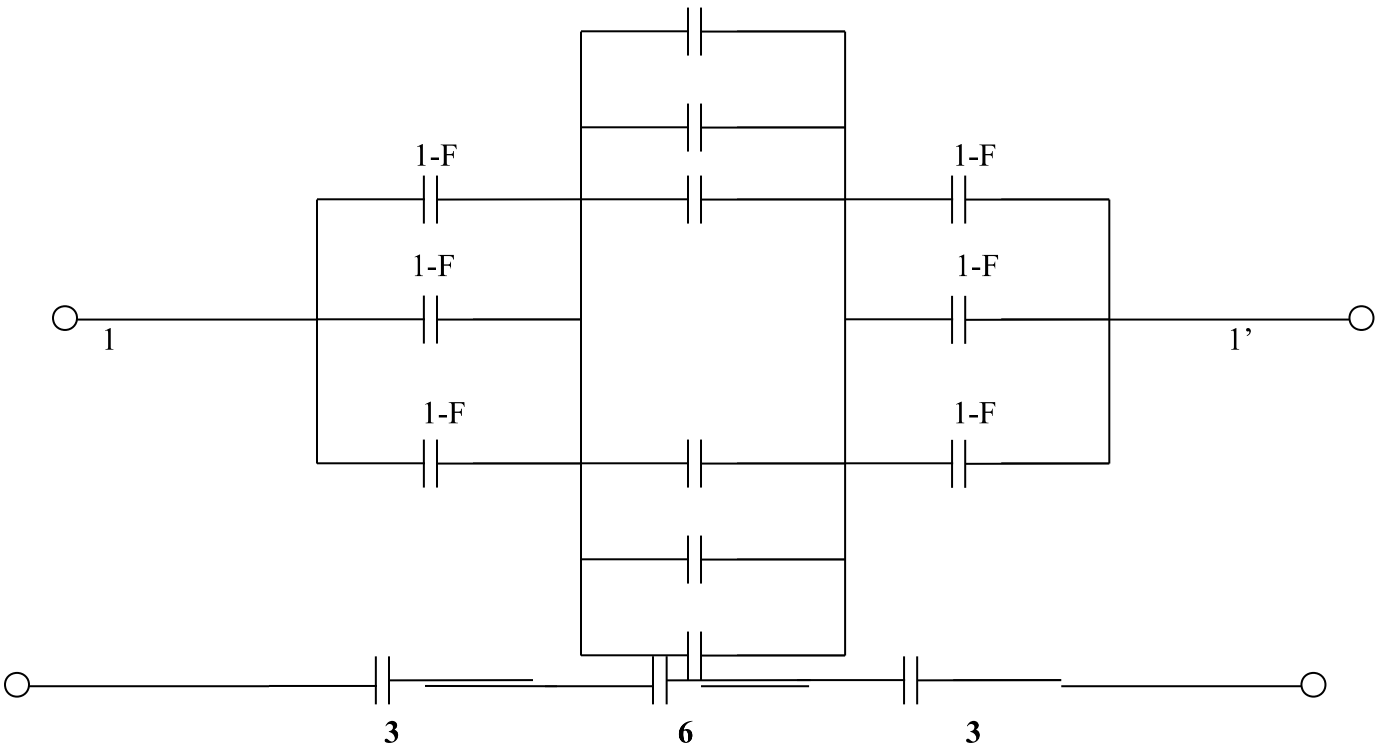




1/3-H 1/6-H 1/3-H

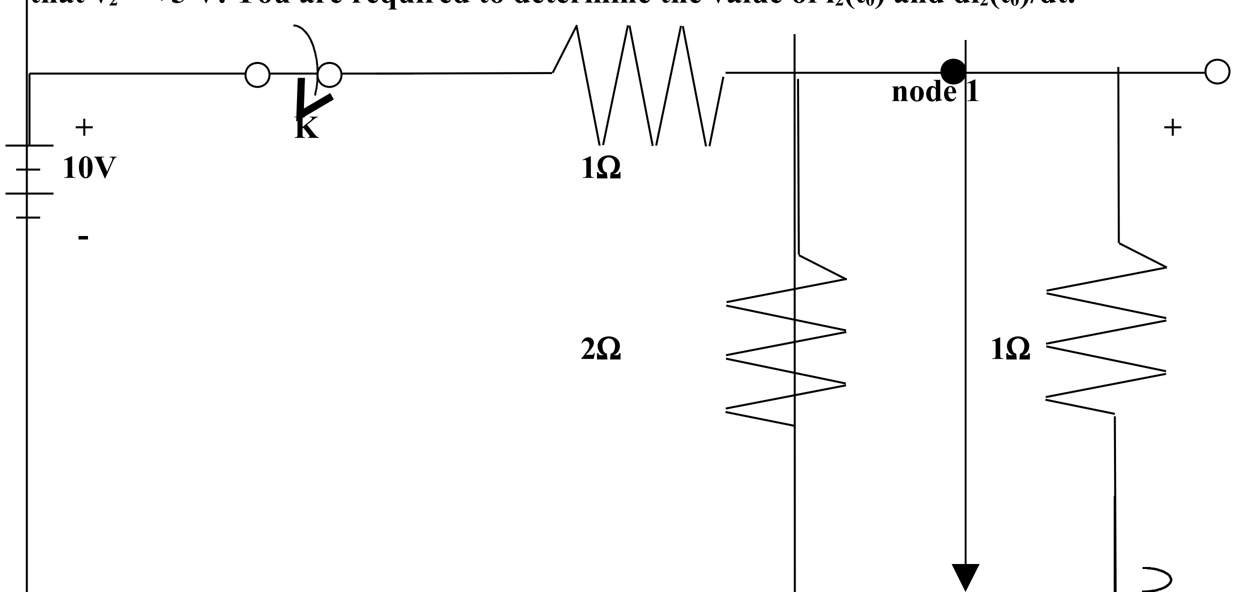
L_{eq} = 1/3-H + 1/6-H + 1/3-H = 5/6-H

3-5. In the networks of Prob. 3-4, each 1-H inductor is replaced by a 1-F capacitor, and L_{eq} is replaced by C_{eq}. What must be the value of C_{eq} for the two networks to be equivalent?
Solution:
Open your book & see the figure (P/88)



C_{eq} = 1/3 + 1/6 + 1/3 = 1.2 F

3-6. This problem may be solved using the two kirchoff laws and voltage current relationships for the elements. At time t_0 after the switch k was closed, it is found that v_2 = +5 V. You are required to determine the value of $i_2(t_0)$ and $di_2(t_0)/dt$.



i_2

v_2

1/2h

-

Using kirchhoff's current law at node 1

$v_2 - 10/1 + v_2/2 + i_2 = 0$

$v_2 - 10 + v_2/2 + i_2 = 0$

$3v_2/2 + i_2 = 10$

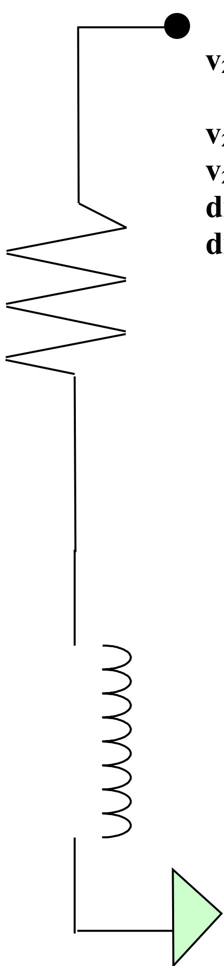
$i_2 = 10 - 3v_2/2$

at $t = t_0$

$i_2(t_0) = 10 - 3v_2(t_0)/2$

$i_2(t_0) = 10 - 3(5)/2 = 2.5$ **amp.**

Also



v_2

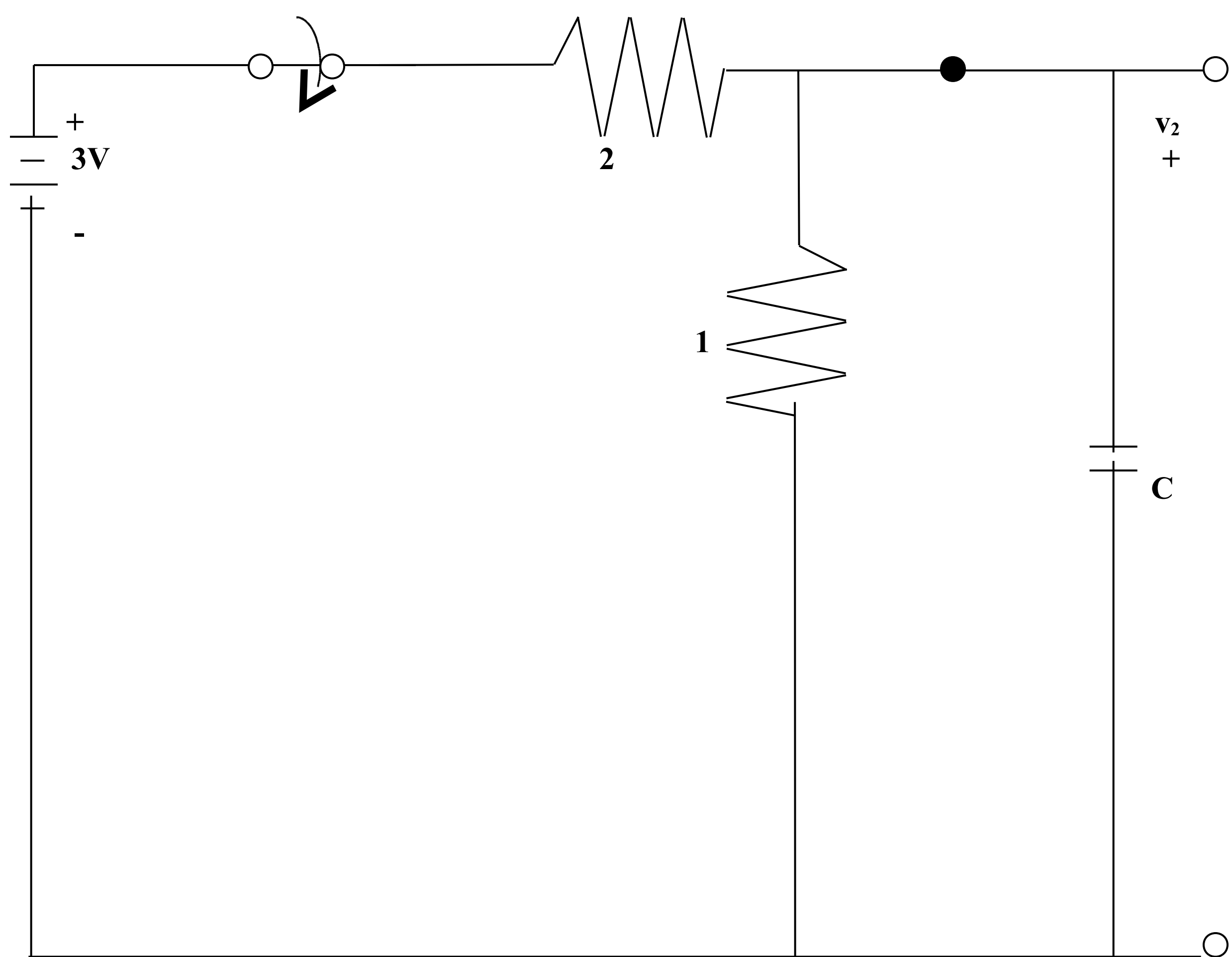
$v_2 = i_2(1) + L di_2/dt$

$v_2 = i_2(1) + (1/2) di_2/dt$

$di_2/dt = (v_2 - i_2)(2)$

$di_2(t_0)/dt = (v_2(t_0) - i_2(t_0))(2) = (5 - 2.5)(2) = (2.5)(2) = 5$ **amp/sec.**

3-7. This problem is similar to Prob. 3-6. In the network given in the figure, it is given that $v_2(t_0) = 2$ V, and $(dv_2/dt)(t_0) = -10$ V/sec, where t_0 is the time after the switch K was closed. Determine the value of C.
Solution:



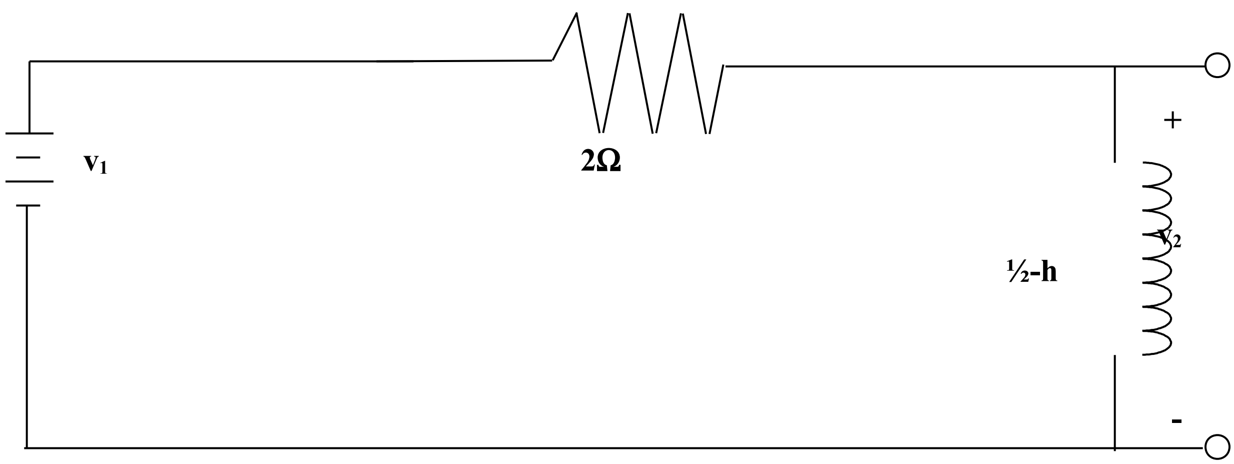
-

Using kirchhoff's current law at node
$v_2 - 3/2 + v_2/1 + i_c = 0$
$3v_2/2 + i_c = 3/2$
At $t = t_0$
$3v_2(t_0)/2 + i_c(t_0) = 3/2$
$3(2)/2 + i_c(t_0) = 3/2$
$i_c(t_0) = - 3/2$
also
at $t = t_0$
$i_c(t_0) = c\,dv_2(t_0)/dt$
$- 3/2 = c(- 10)$

$\boxed{c = 3/20\ 0.15\text{-F}}$

The series of problems described in the following table all pertain to the network of (g) of the figure with the network in A and B specified in the table.

3-8 (a)
Solution:



Open your book & see (P/89)

$v_2(t)$	0	$0<t<1$
$v_2(t)$	1	$1<t<2$
$v_2(t)$	0	$2<t<3$
$v_2(t)$	2	$3<t<4$

Applying KVL
$v_1 = 2(i) + (1/2)di/dt = 2(i) + v_2$

$v_2 = (1/2)di/dt$

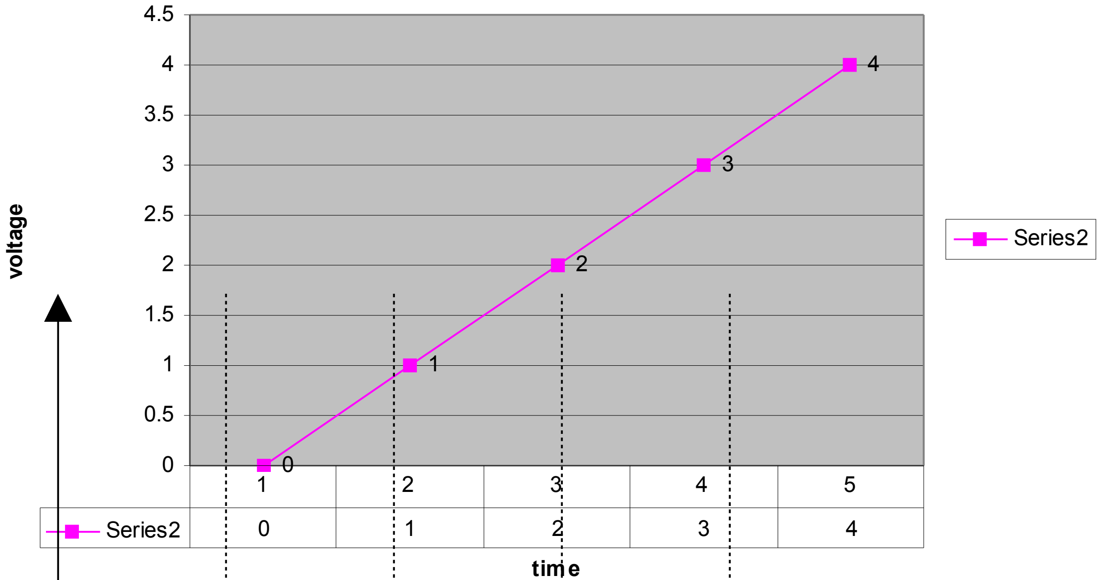
$i = 2\int_{-\infty}^{t} v_2 dt$

0<t<1	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + \int_{0}^{t} 0 dt = 0$ amp.	At t = 0 i(0) = 0 At t = 1 i(1) = 0
1<t<2	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{1} v_2 dt + 2\int_{1}^{t} v_2 dt$ $i(t) = i(1) + 2\int_{1}^{t} (1)dt = 0 + 2\Big\vert_{1}^{t} t\Big\vert$ i(t) = 2(t - 1) amp.	At t = 1 i(1) = 0 At t = 2 i(2) = 2

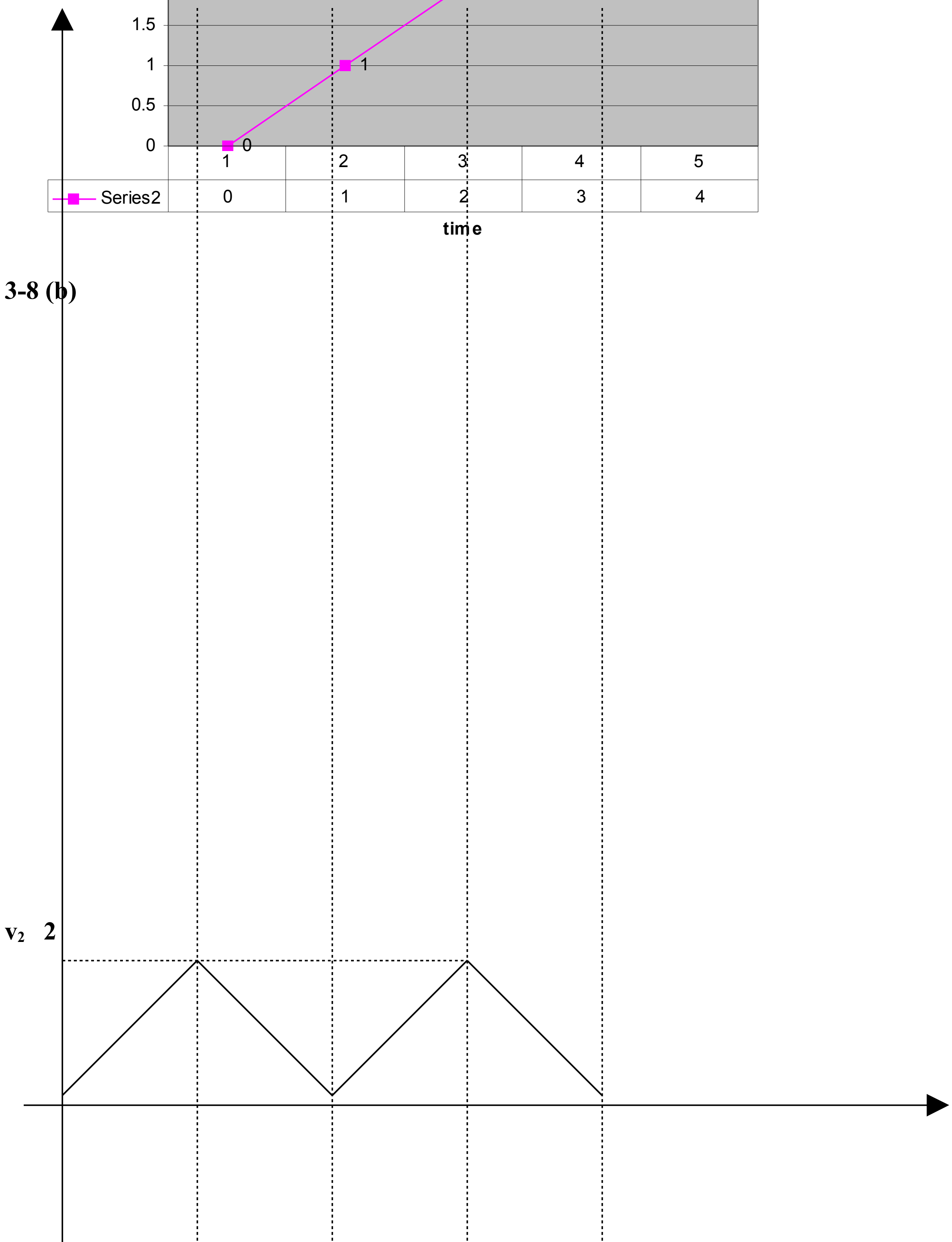
2<t<3	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{2} v_2 dt + 2\int_{2}^{t} v_2 dt$ $i(t) = i(2) + \int_{2}^{t} 0 dt = 2 + 0 = 2$ amp.	At t = 2 i(2) = 2 At t = 3 i(3) = 2
3<t<4	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{3} v_2 dt + 2\int_{3}^{t} v_2 dt$ $i(t) = i(3) + 2\int_{3}^{t} (2)dt = 2 + 4\Big\vert_{3}^{t} t\Big\vert$ i(t) = 2 + 4(t - 3) amp. i(4) = 2 + 4(4 - 3) amp. = 6 amp.	At t = 3 i(3) = 3 At t = 4 i(4) = 6

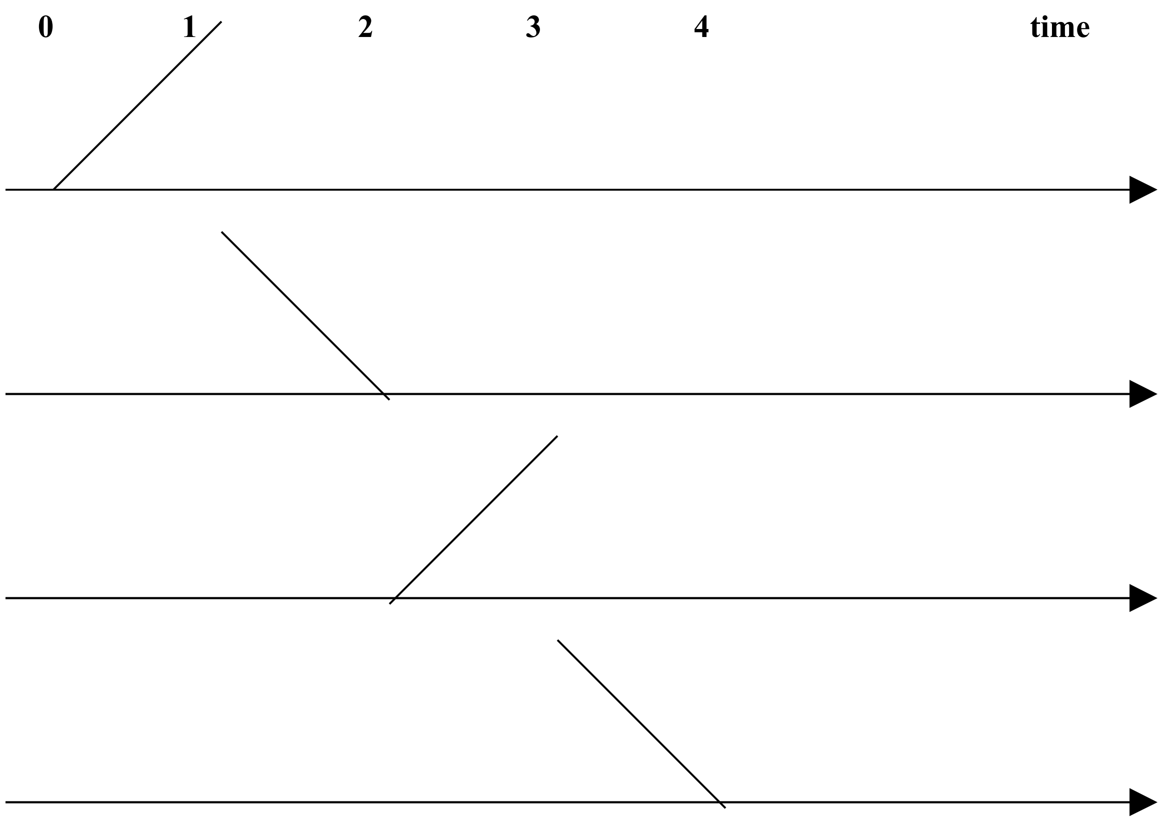
$0<t<1$	**$v_2(t) = 0$**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At $t = 0$** **$v_1(0) = 2(i(0)) + v_2(0)$** **$v_1(0) = 2(0) + 0 = 0$** **At $t = 1$** **$v_1(1) = 2(i(1)) + v_2(1)$** **$v_1(1) = 2(0) + 0 = 0$**
$1<t<2$	**1**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At $t = 2$** **$v_1(2) = 2(i(2)) + v_2(2)$** **$v_1(2) = 2(2) + 1 = 5$**
$2<t<3$	**0**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At $t = 3$** **$v_1(3) = 2(i(3)) + v_2(3)$** **$v_1(3) = 2(2) + 0 = 4$**
$3<t<4$	**2**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At $t = 4$** **$v_1(4) = 2(i(4)) + v_2(4)$** **$v_1(4) = 2(6) + 2 = 14$**

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**5**
$v_1(3)$	**4**
$v_1(4)$	**14**


voltage
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Series2
1 2 3 4 5
Series2 0 1 2 3 4
time

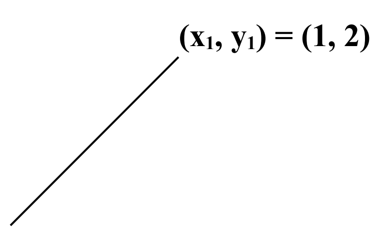
3-8 (b)


v_2 2



Interval	$v_2(t)$
$0<t<1$	$2t$
$1<t<2$	$-2(t-2)$
$2<t<3$	$2(t-2)$
$3<t<4$	$-2(t-4)$
$t>4$	0

$0<t<1$



$(x_0, y_0) = (0, 0)$

Straight-line equation

$y = mx + c$

$m = (y_1 – y_0)/(x_1 – x_0) = (2 - 0)/(1 - 0) = 2/1 = 2$

Slope = 2

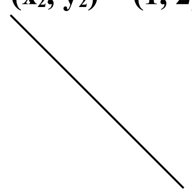
y-intercept = 0

$y = mx + c$

$v_2(t) = 2t + 0 = 2t$ Volts

$1<t<2$

$(x_2, y_2) = (1, 2)$



$(x_1, y_1) = (2, 0)$

Straight-line equation

$y = mx + c$

$m = (y_2 – y_1)/(x_2 – x_1) = (2 - 0)/(1 - 2) = 2/(-1) = - 2$

Slope = - 2

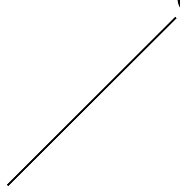
y-intercept = 4

$y = mx + c$

$v_2(t) = - 2t + 4 = - 2(t – 2)$ Volts

$2<t<3$

$(x_3, y_3) = (3, 2)$



$(x_2, y_2) = (2, 0)$

Straight-line equation

$y = mx + c$

$m = (y_3 - y_2)/(x_3 - x_1) = (2 - 0)/(3 - 2) = 2/1 = 2$

Slope = 2

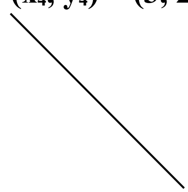
y-intercept = -4

$y = mx + c$

$v_2(t) = 2t + (-4) = 2t - 4 = 2(t - 2)$ Volts

$1<t<2$

$(x_4, y_4) = (3, 2)$



$(x_3, y_3) = (4, 0)$

Straight-line equation

$y = mx + c$

$m = (y_4 - y_3)/(x_4 - x_3) = (2 - 0)/(3 - 4) = 2/(-1) = - 2$

Slope = - 2

y-intercept = 8

$y = mx + c$

$v_2(t) = - 2t + 8 = - 2(t - 4)$ Volts

$v_1 = v_2 + 2i$

$v_2 = (1/2)di/dt$

$$i = 2\int_{-\infty}^{t} v_2 dt$$

$0<t<1$	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + 2\int_{0}^{t} 2t dt = 4\int_{0}^{t} t dt$ $= 4 \left. \frac{t^2}{2} \right\|_{0}^{t}$ $i(t) = 4[t^2/2 - 0] = 4[t^2/2] = 2t^2$ amp.	**At $t = 0$** **$i(0) = 0$** **At $t = 1$** **$i(1) = 2$**

1<t<2	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{1} v_2 dt + 2\int_{1}^{t} v_2 dt$ $i(t) = i(1) + 2\int_{1}^{t} -2(t-2)dt$ $i(t) = 2 + (-4)\int_{1}^{t} (t-2)dt$ $i(t) = 2 - 4\int_{1}^{t} (t-2)dt$ $i(t) = 2 - 4 \left. t^2/2 - 2t \right\vert_{1}^{t}$ $i(t) = 2 - 4[(t^2/2 - 2t) - (1/2 - 2)]$ $i(t) = 2 - 4[(t^2/2 - 2t) - (-3/2)]$ $i(t) = 2 - 4[t^2/2 - 2t + 3/2)]$ $i(t) = 2 - 2t^2 + 8t - 6$ $i(t) = -2t^2 + 8t - 4$	**At t = 2** **i(2) = 4** **amp.**

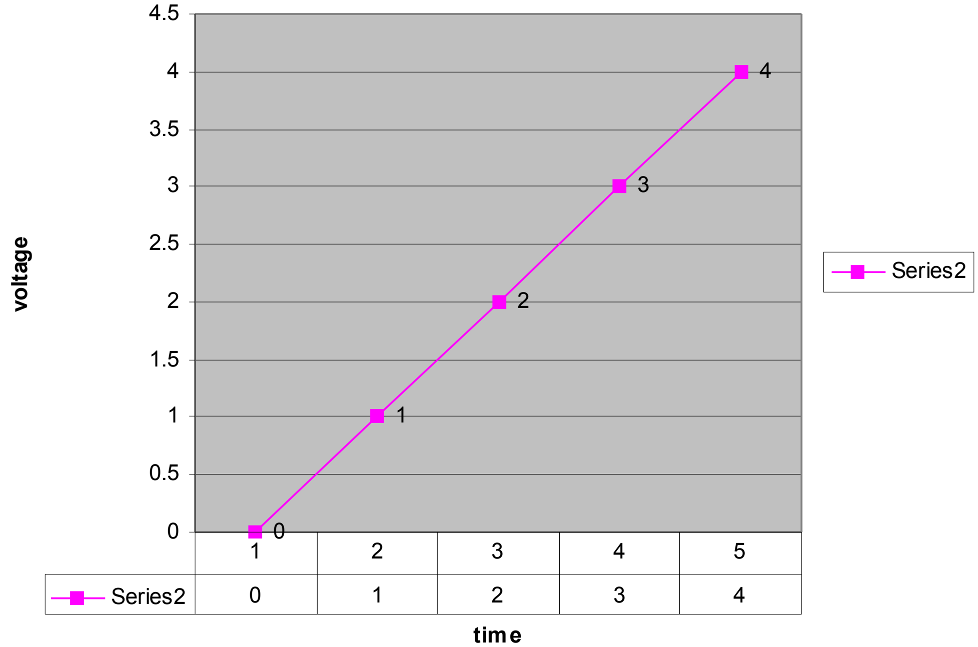
2<t<3	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{2} v_2 dt + 2\int_{2}^{t} v_2 dt$ $i(t) = i(2) + 2\int_{2}^{t} 2(t-2)dt$ $i(t) = 4 + 4\int_{2}^{t} (t-2)dt$	**At t = 3** **i(3) = 6** **amp.**

	$i(t) = 4 + 4\int_{1}^{t} (t - 2)dt$ $i(t) = 4 + 4 \left\| t^2/2 - 2t \right\|_{2}^{t}$ $i(t) = 4 + 4 [(t^2/2 - 2t) - (4/2 - 4)]$ $i(t) = 4 + 4 [(t^2/2 - 2t) - (- 2)]$ $i(t) = 4 + 4 [t^2/2 - 2t + 2)]$ $i(t) = 4 + 2t^2 - 8t + 8$ $i(t) = 2t^2 - 8t + 12$	
3<t<4	$i = 2\int_{-\infty}^{t} v_2dt = 2\int_{-\infty}^{3} v_2dt + 2\int_{3}^{t} v_2dt$ $i(t) = i(3) + 2 \int_{3}^{t} -2(t - 4)dt$ $i(t) = 6 - 4 \int_{3}^{t} (t - 4)dt$ $i(t) = 6 - 4\int_{3}^{t} (t - 4)dt$ $i(t) = 6 - 4 \left\| t^2/2 - 4t \right\|_{3}^{t}$ $i(t) = 6 - 4 [(t^2/2 - 4t) - (4.5 - 12)]$ $i(t) = 6 - 4 [(t^2/2 - 4t) - (- 7.5)]$ $i(t) = 6 - 4 [t^2/2 - 4t + 7.5)]$ $i(t) = 6 - 2t^2 + 16t - 30$ $i(t) = - 2t^2 + 16t - 24$	At t = 4 i(4) = 8 amp.

	$v_2(t) = 2t$	$v_1(t) = 2(i(t)) + v_2(t)$ At t = 0 $v_1(0) = 2(i(0)) + 2t$ $v_1(0) = 2(0) + 0 = 0$

0<t<1		**At t = 1** $v_1(1) = 2(i(1)) + 2t$ $v_1(1) = 2(0) + 2(1) = 2$
1<t<2	**1**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 2** $v_1(2) = 2(i(2)) – 2(t - 2)$ $v_1(2) = 2(4) - 0 = 8$
2<t<3	**0**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 3** $v_1(3) = 2(i(3)) + 2(t - 2)$ $v_1(3) = 2(6) + 2 = 14$
3<t<4	**2**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 4** $v_1(4) = 2(i(4)) – 2(t - 4)$ $v_1(4) = 2(8) - 0 = 16$

$v_1(0)$	**0**
$v_1(1)$	**2**
$v_1(2)$	**8**
$v_1(3)$	**14**
$v_1(4)$	**16**



0<t<1	$V_2 = 0$
1<t<2	**2**
2<t<3	**-3**
t>3	**0**

Applying KVL

$v_1 = 2(i) + (1/2)di/dt = 2(i) + v_2$

$v_2 = (1/2)di/dt$

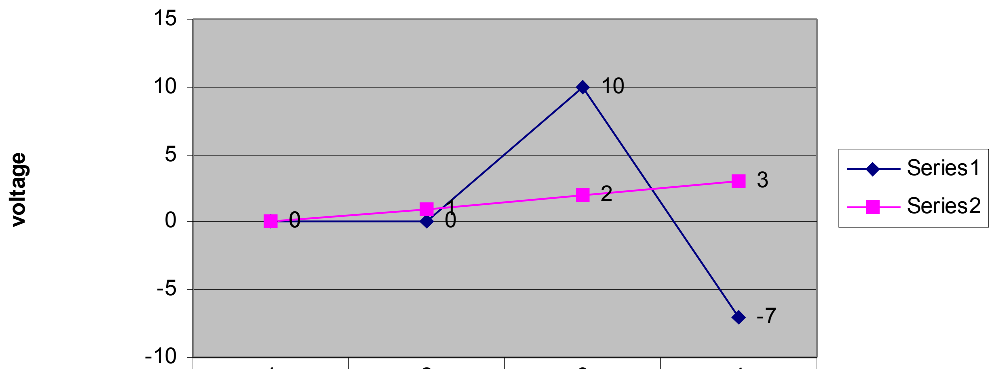
$$i = 2\int_{-\infty}^{t} v_2 dt$$

0<t<1	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + \int_{0}^{t} 0 dt = 0$ amp.	**At t = 0** **i(0) = 0** **At t = 1** **i(1) = 0**
1<t<2	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{1} v_2 dt + 2\int_{1}^{t} v_2 dt$ $i(t) = i(1) + 2\int_{1}^{t} (2)dt = 0 + 4\left. t \right\|_{1}^{t}$ $i(t) = 4(t - 1)$ amp.	**At t = 2** **i(2) = 4**

2<t<3	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{2} v_2 dt + 2\int_{2}^{t} v_2 dt$ $i(t) = i(2) + 2\int_{2}^{t} (-3)dt = 4 - 6\left. t \right\|_{2}^{t}$ $= 4 - 6(t - 2)$ amp.	**At t = 3** **i(3) = -2**

0<t<1	**$v_2(t) = 0$**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 0** **$v_1(0) = 2(i(0)) + v_2(0)$** **$v_1(0) = 2(0) + 0 = 0$** **At t = 1** **$v_1(1) = 2(i(1)) + v_2(1)$** **$v_1(1) = 2(0) + 0 = 0$**
1<t<2	**2**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 2** **$v_1(2) = 2(i(2)) + v_2(2)$** **$v_1(2) = 2(4) + 2 = 10$**
2<t<3	**-3**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 3** **$v_1(3) = 2(i(3)) + v_2(3)$** **$v_1(3) = 2(-2) - 3 = -7$**

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**10**
$v_1(3)$	**-7**



	1	2	3	4
Series1	0	0	10	-7
Series2	0	1	2	3

3-8 (d) $0<t<\pi$, $v_2 = \sin t$

Applying KVL

$v_1 = 2(i) + (1/2)di/dt = 2(i) + v_2$

$v_2 = (1/2)di/dt$

$$i = 2\int_{-\infty}^{t} v_2 dt$$

$0<t<\pi$	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + 2\int_{0}^{t} \sin t dt$ $i(t) = -2\left.\cos t\right\|_{0}^{t} = -2(\cos t - 1)$ amp. Because $\cos 0 = 1$	At $t = 0$ $i(0) = 0$ At $t = 1$ $i(1) = 0$

$v_1 = 2(i) + v_2$

$v_1(t) = 2(i(t)) + \sin t$

At $t = 0$

$v_1(0) = 2(i(0)) + \sin 0$

$$\boxed{v_1(0) = 2(0) + 0 = 0 \text{ Volt}}$$

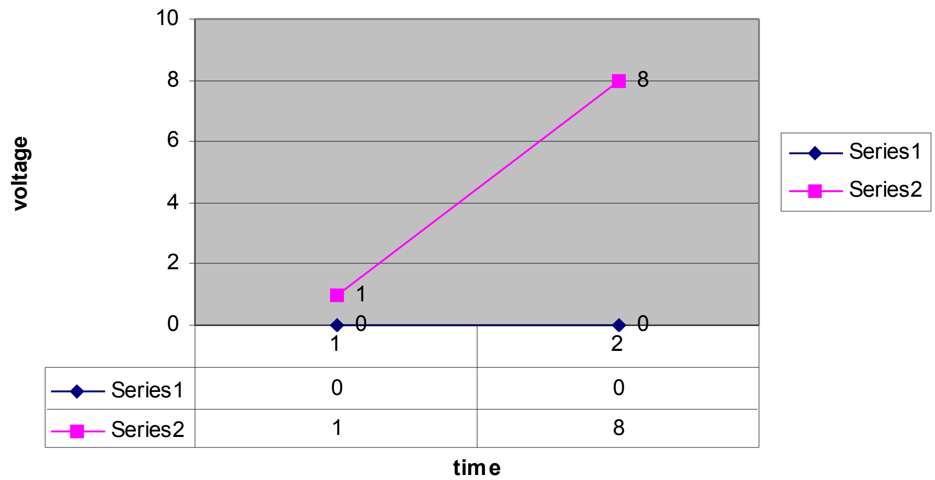
At $t = \pi$

$v_1(t) = 2(i(t)) + \sin t$

$v_1(\pi) = 2(i(\pi)) + \sin\pi$

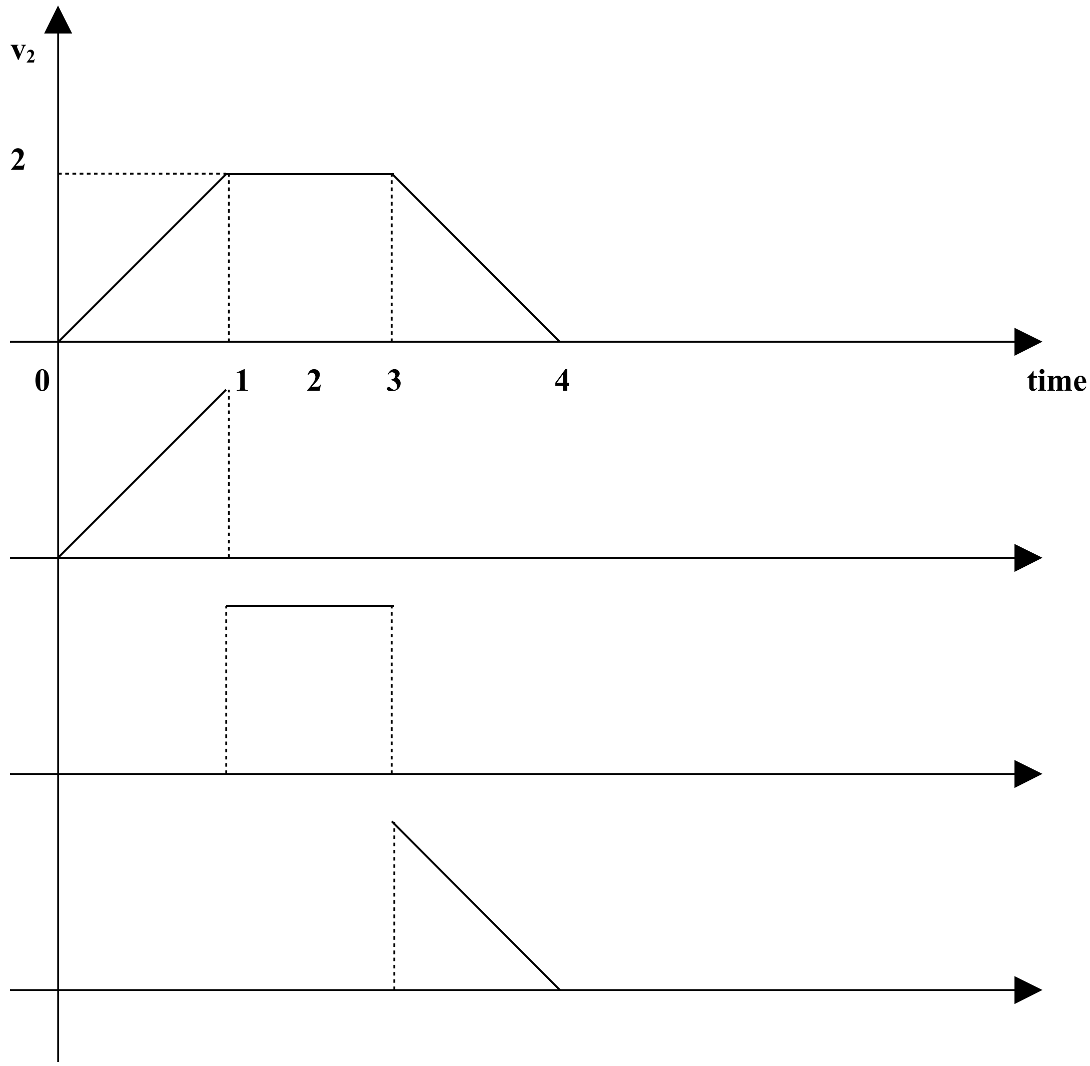
$$\boxed{v_1(\pi) = 2(4) + 0 = 8 \text{ Volt}}$$

$v_1(0)$	0
$v_1(\pi)$	8

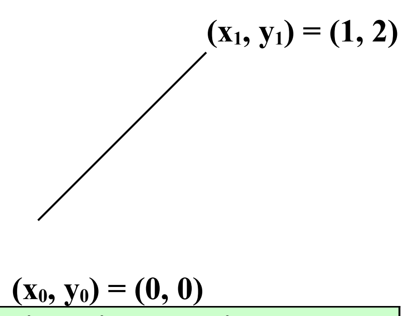


3-8 (f)

0<t<1	**2t**
1<t<3	**2**
3<t<4	**-2(t - 4)**



0<t<1



Straight-line equation

$y = mx + c$

$m = (y_1 - y_0)/(x_1 - x_0) = (2 - 0)/(1 - 0) = 2/1 = 2$

Slope = 2

y-intercept = 0

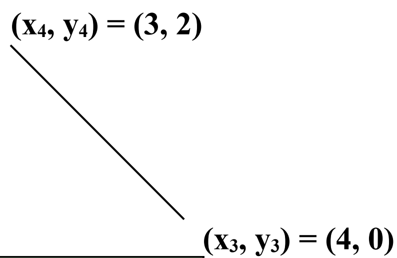
$y = mx + c$

$v_2(t) = 2t + 0 = 2t$ Volts

1<t<3

$v_2(t) = 2$ Volts

1<t<2



Straight-line equation

$y = mx + c$

$m = (y_4 - y_3)/(x_4 - x_3) = (2 - 0)/(3 - 4) = 2/(-1) = -2$

Slope = - 2

y-intercept = 8

$y = mx + c$

$v_2(t) = -2t + 8 = -2(t - 4)$ Volts

$v_1 = v_2 + 2i$

$v_2 = (1/2)di/dt$

$$i = 2\int_{-\infty}^{t} v_2 dt$$

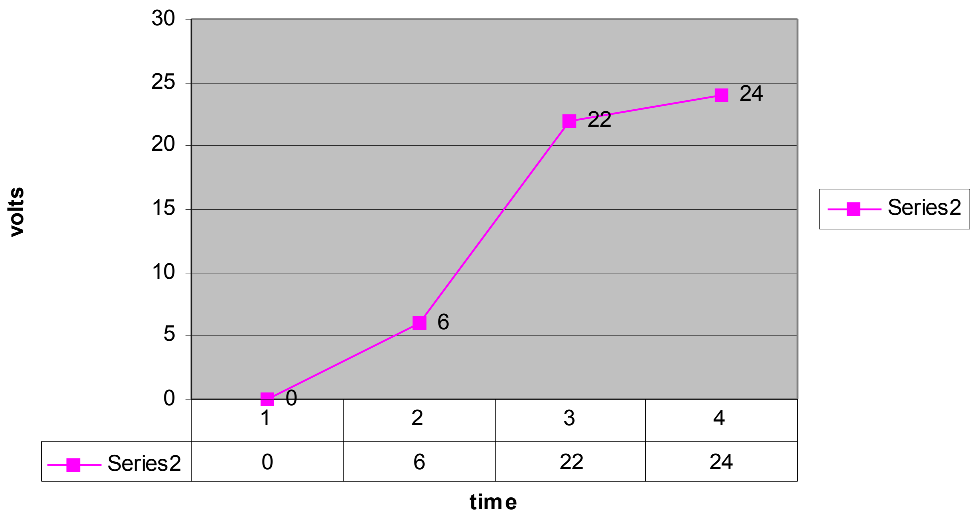
0<t<1	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + 2\int_{0}^{t} 2tdt = 4\int_{0}^{t} tdt$ $= 4 \left. t^2/2 \right\|_{0}^{t}$ $i(t) = 4[t^2/2 - 0] = 4[t^2/2] = 2t^2$ amp.	At t = 0 i(0) = 0 At t = 1 i(1) = 2
1<t<3	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{1} v_2 dt + 2\int_{1}^{t} v_2 dt$ $i(t) = i(1) + 2\int^{t} 2dt$	At t = 3 i(3) = 10 amp.

	1 $i(t) = 2 + 4\int_{1}^{t} dt$ $i(t) = 2 + 4 \left. t \right\|_{1}^{t}$ $i(t) = 2 + 4(t - 1) = 2 + 4t - 4 = -2 + 4t$	

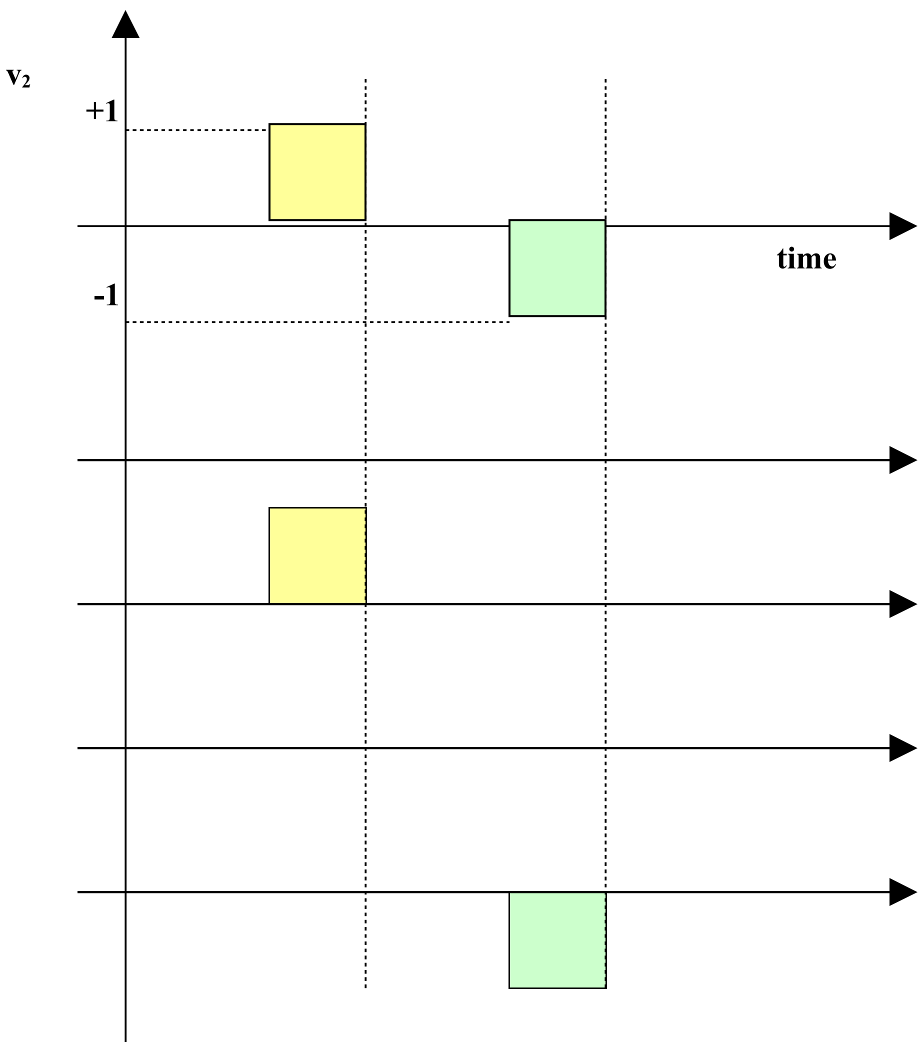
3<t<4	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{3} v_2 dt + 2\int_{3}^{t} v_2 dt$ $i(t) = i(3) + 2\int_{3}^{t} -2(t - 4)dt$ $i(t) = 10 - 4\int_{3}^{t} (t - 4)dt$ $i(t) = 10 - 4\int_{3}^{t} (t - 4)dt$ $i(t) = 10 - 4 \left. t^2/2 - 4t \right\|_{3}^{t}$ $i(t) = 10 - 4[(t^2/2 - 4t) - (4.5 - 12)]$ $i(t) = 10 - 4[(t^2/2 - 4t) - (-7.5)]$ $i(t) = 10 - 4[t^2/2 - 4t + 7.5)]$ $i(t) = 10 - 2t^2 + 16t - 30$ $i(t) = -2t^2 + 16t - 20$	**At t = 4** **i(4) = 12** **amp.**

0<t<1	**$v_2(t) = 2t$**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 0** **$v_1(0) = 2(i(0)) + 2t$** **$v_1(0) = 2(0) + 0 = 0$** **At t = 1** **$v_1(1) = 2(i(1)) + 2t$** **$v_1(1) = 2(2) + 2(1) = 6$ Volts**
1<t<3	**2**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 3** **$v_1(3) = 2(i(3)) + 2$** **$v_1(2) = 2(10) + 2 = 22$**
3<t<4	**-2(t - 4)**	**$v_1(t) = 2(i(t)) + v_2(t)$** **At t = 4** **$v_1(4) = 2(i(4)) - 2(t - 4)$** **$v_1(3) = 2(12) - 0 = 24$ Volts**

$v_1(0)$	**0**
$v_1(1)$	**6**
$v_1(3)$	**22**
$v_1(4)$	**24**


volts
30
25
20
15
10
5
0
0
6
22
24
Series2
1
2
3
4
Series2
0
6
22
24
time

3-8 (e)


v2
+1
-1
time

Interval	$v_2(t)$
0<t<1	0
1<t<2	1
2<t<3	0
3<t<4	-1

Applying KVL

$$v_1 = 2(i) + (1/2)di/dt = 2(i) + v_2$$

$$v_2 = (1/2)di/dt$$

$$i = 2\int_{-\infty}^{t} v_2 dt$$

0<t<1	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{0} v_2 dt + 2\int_{0}^{t} v_2 dt$ $i(t) = 0 + \int_{0}^{t} 0dt = 0$ amp.	At t = 0 i(0) = 0 At t = 1 i(1) = 0
1<t<2	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{1} v_2 dt + 2\int_{1}^{t} v_2 dt$ $i(t) = i(1) + 2\int_{2}^{t} (1)dt = 0 + 2\left. t \right\|_{1}^{t}$ $i(t) = 2(t - 1)$ amp.	At t = 2 i(2) = 2

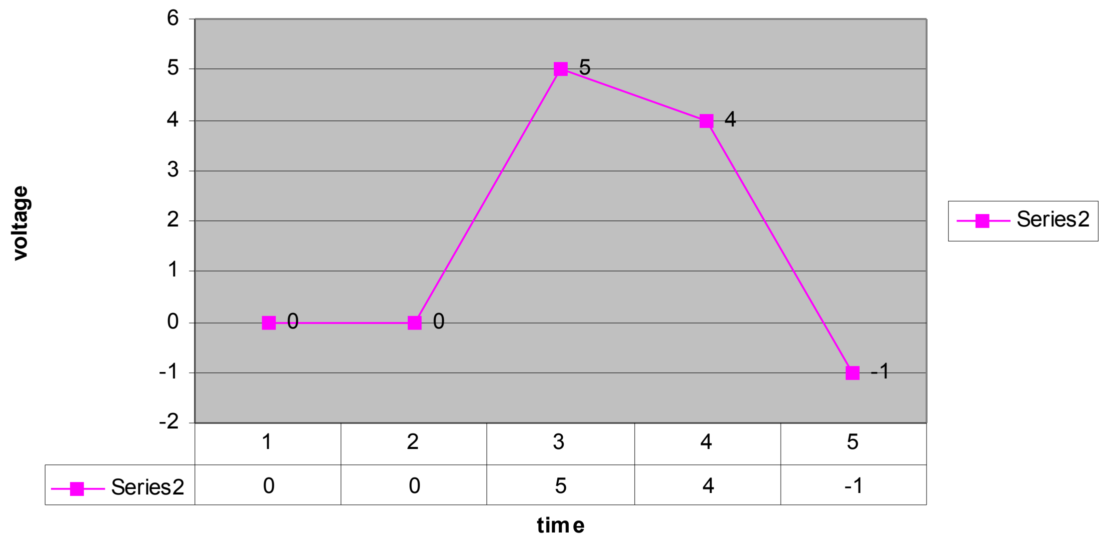
2<t<3	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{2} v_2 dt + 2\int_{2}^{t} v_2 dt$ $i(t) = i(2) + 2\int_{2}^{t} 0 dt = 2$ $= 2$ **amp.**	**At t = 3** $i(3) = 2$

3<t<4	$i = 2\int_{-\infty}^{t} v_2 dt = 2\int_{-\infty}^{3} v_2 dt + 2\int_{3}^{t} v_2 dt$ $i(t) = i(3) + 2\int_{3}^{t} (-1) dt$ $i(t) = 2 - 2\int_{3}^{t} (1) dt = 2 - 2\left. t \right\|_{3}^{t}$ $= 2 - 2(t - 3)$ **amp.**	**At t = 4** $i(4) = 0$

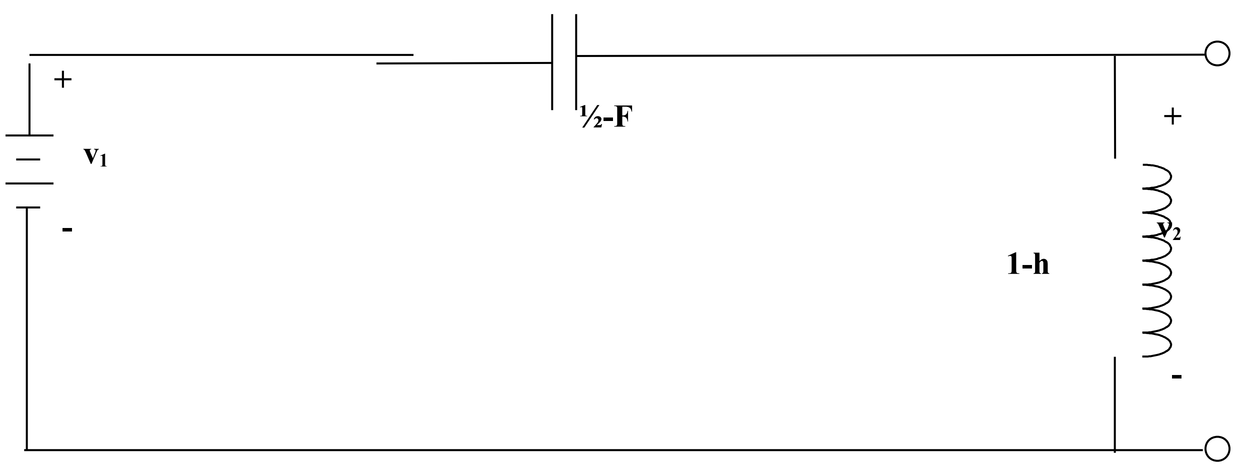
	$v_2(t) = 0$	$v_1(t) = 2(i(t)) + v_2(t)$

0<t<1		**At t = 0** $v_1(0) = 2(i(0)) + v_2(0)$ $v_1(0) = 2(0) + 0 = 0$ **At t = 1** $v_1(1) = 2(i(1)) + v_2(1)$ $v_1(1) = 2(0) + 0 = 0$
1<t<2	**1**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 2** $v_1(2) = 2(i(2)) + v_2(2)$ $v_1(2) = 2(2) + 1 = 5$
2<t<3	**0**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 3** $v_1(3) = 2(i(3)) + v_2(3)$ $v_1(3) = 2(2) + 0 = 4$
3<t<4	**-1**	$v_1(t) = 2(i(t)) + v_2(t)$ **At t = 4** $v_1(3) = 2(i(4)) + v_2(4)$ $v_1(3) = 2(0) - 1 = -1$

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**5**
$v_1(3)$	**4**
$v_1(4)$	**-1**



3-8 (a)
Solution:



$\therefore\ v_1 = v_c + v_2$

$$v_c = (1/c)\int_{-\infty}^{t} i(t)dt$$

$v_2(t)$	0	0<t<1
$v_2(t)$	1	1<t<2
$v_2(t)$	0	2<t<3
$v_2(t)$	2	3<t<4

i(0)	0<t<1	0
i(1)	0<t<1	0
i(2)	1<t<2	2
I(3)	2<t<3	2

I(4)	**3<t<4**	**6**

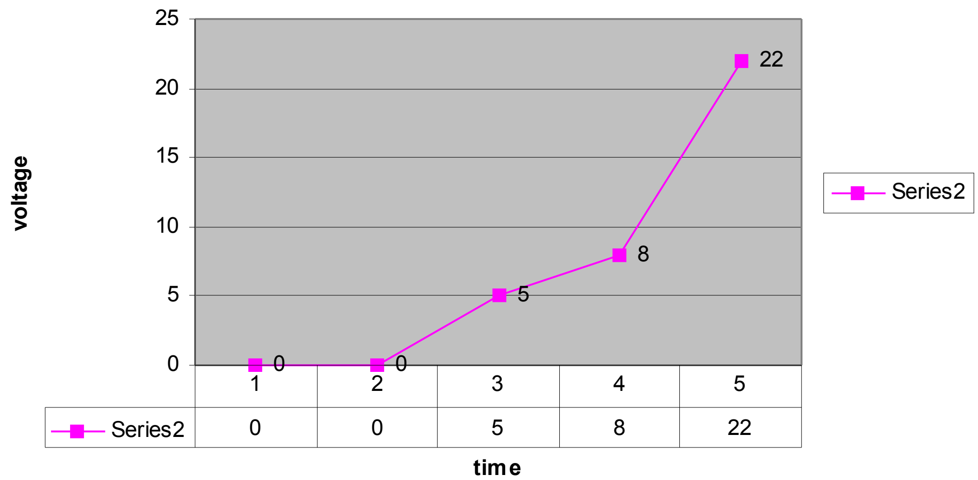
0<t<1	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{0} i\,dt + 2\int_{0}^{t} i\,dt$ $v_c = 0 + 2\int_{0}^{t} 0\,dt = 0$ **Volts.**	**At t = 0** $v_c(0) = 0$ **At t = 1** $v_c(1) = 0$
1<t<2	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{1} i\,dt + 2\int_{1}^{t} i\,dt$ $v_c(t) = v_c(1) + 2\int_{1}^{t} (2)dt = 0 + 4\left. t \right\vert_{1}^{t}$ $v_c(t) = 4(t - 1)$ **Volts.**	**At t = 2** $v_c(2) = 4$

2<t<3	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{2} i\,dt + 2\int_{2}^{t} i\,dt$ $v_c(t) = v_c(2) + 2\int_{2}^{t} (2)dt = 4 + 4\left. t \right\vert_{2}^{t}$ $v_c(t) = 4 + 4(t - 2)$ **Volts.**	**At t = 3** $v_c(3) = 8$
3<t<4	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{3} i\,dt + 2\int_{3}^{t} i\,dt$ $v_c(t) = v_c(3) + 2\int_{3}^{t} (6)dt = 8 + 12\left. t \right\vert_{3}^{t}$ $v_c(t) = 8 + 12(t - 3)$ **Volts.**	**At t = 4** $v_c(4) = 20$

t = 0	**$v_c(t)$ = 0**
1	**0**
2	**4**
3	**8**
4	**20**

0<t<1	**$v_2(t) = 0$**	**$v_1(t) = v_c(t) + v_2(t)$** **At t = 0** **$v_1(0) = v_c(0) + v_2(0)$** **$v_1(0) = (0) + 0 = 0$ Volts** **At t = 1** **$v_1(1) = v_c(1) + v_2(1)$** **$v_1(0) = (0) + 0 = 0$ Volts**
1<t<2	**1**	**At t = 2** **$v_1(2) = v_c(2) + v_2(2)$** **$v_1(0) = (4) + 1 = 5$ Volts**
2<t<3	**0**	**At t = 3** **$v_1(3) = v_c(3) + v_2(3)$** **$v_1(0) = (8) + 0 = 8$ Volts**
3<t<4	**2**	**At t = 4** **$v_1(4) = v_c(4) + v_2(4)$** **$v_1(0) = (20) + 2 = 22$ Volts**

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**5**
$v_1(3)$	**8**
$v_1(4)$	**22**



3-9 (b)

$\therefore\ v_1 = v_c + v_2$

$$v_c = (1/c)\int_{-\infty}^{t} i(t)dt$$

$v_2(t)$	$2t$	$0<t<1$
$v_2(t)$	$-2(t-2)$	$1<t<2$
$v_2(t)$	$2(t-2)$	$2<t<3$
$v_2(t)$	$-2(t-4)$	$3<t<4$

i(0)	$0<t<1$	0
i(1)	$0<t<1$	2
i(2)	$1<t<2$	4
i(3)	$2<t<3$	6
i(4)	$3<t<4$	8

$0<t<1$	$v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} idt$ $v_c = 0 + 2\int_{0}^{t} 0dt = 0$ Volts. $v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} idt$ $v_c = 0 + 2\int_{0}^{t} 2dt = 4\Big\vert_{0}^{t} t\Big\vert_{0}^{t} = 4t$	At t = 0 $v_c(0) = 0$ At t = 1 $v_c(1) = 4$

1<t<2	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{1} i\,dt + 2\int_{1}^{t} i\,dt$ $v_c(t) = v_c(1) + 2\int_{1}^{t} (4)dt = 4 + 8\,t\Big\vert_{1}^{t}$ $v_c(t) = 4 + 8(t - 1)$ **Volts.**	**At t = 2** $v_c(2) = 12$

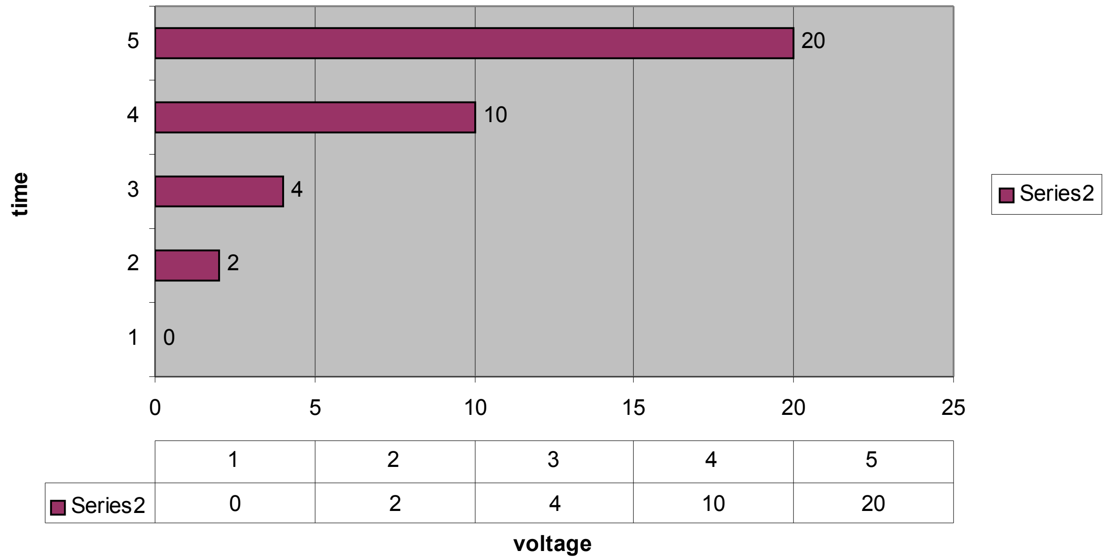
2<t<3	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{2} i\,dt + 2\int_{2}^{t} i\,dt$ $v_c(t) = v_c(2) + 2\int_{2}^{t} (6)dt = 12 + 12\,t\Big\vert_{2}^{t}$ $v_c(t) = 12 + 12(t - 2)$ **Volts.**	**At t = 3** $v_c(3) = 24$
3<t<4	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{3} i\,dt + 2\int_{3}^{t} i\,dt$ $v_c(t) = v_c(3) + 2\int_{3}^{t} (8)dt = 24 + 16\,t\Big\vert_{3}^{t}$ $v_c(t) = 24 + 16(t - 3)$ **Volts.**	**At t = 4** $v_c(4) = 40$

t = 0	**$v_c(t)$ = 0**
1	**0**
2	**4**
3	**8**
4	**20**

0<t<1	**$v_2(t) = 2t$**	**$v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 0 + 2t = 2t$ At t = 0 $v_1(0) = 0 + 2t = 2(0) = 0$ Volts. $v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 0 + 2t = 2t$ At t = 1 $v_1(1) = 0 + 2t = 2(1) = 2$ Volts.**
1<t<2	**-2(t – 2)**	**$v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 4 – 2(t – 2)$ At t = 2 $v_1(2) = 4$ Volts.**
2<t<3	**2(t – 2)**	**At t = 3 $v_1(3) = v_c(3) + v_2(3)$ $v_1(3) = (8) + 2(t - 2) = 10$ Volts**
3<t<4	**-2(t - 4)**	**At t = 4 $v_1(4) = v_c(4) + v_2(4)$ $v_1(0) = (20) – 2(t - 4) = 20$ Volts**

$v_1(0)$	**0**
$v_1(1)$	**2**
$v_1(2)$	**4**

$v_1(3)$	**10**
$v_1(4)$	**20**



3-9 (c)

$\therefore\ v_1 = v_c + v_2$

$$v_c = (1/c)\int_{-\infty}^{t} i(t)dt$$

$v_2(t)$	**0**	**$0<t<1$**
$v_2(t)$	**2**	**$1<t<2$**
$v_2(t)$	**-3**	**$2<t<3$**

$i(0)$	**$0<t<1$**	**0**
$i(1)$	**$0<t<1$**	**0**
$i(2)$	**$1<t<2$**	**4**
$i(3)$	**$2<t<3$**	**2**

$0<t<1$	$v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} idt$ $v_c = 0 + 2\int_{0}^{t} 0dt = 0$ **Volts.** $v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} idt$	**At $t = 0$** $v_c(0) = 0$ **At $t = 1$** $v_c(1) = 0$

	$v_c = 0 + 2\int_0^t 0dt = 0$ Volts	
1<t<2	$v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{1} idt + 2\int_{1}^{t} idt$ $v_c(t) = v_c(1) + 2\int_1^t (4)dt = 0 + 8\Big\|_1^t t$ $v_c(t) = 0 + 8(t - 1)$ Volts.	At t = 2 $v_c(2) = 8$

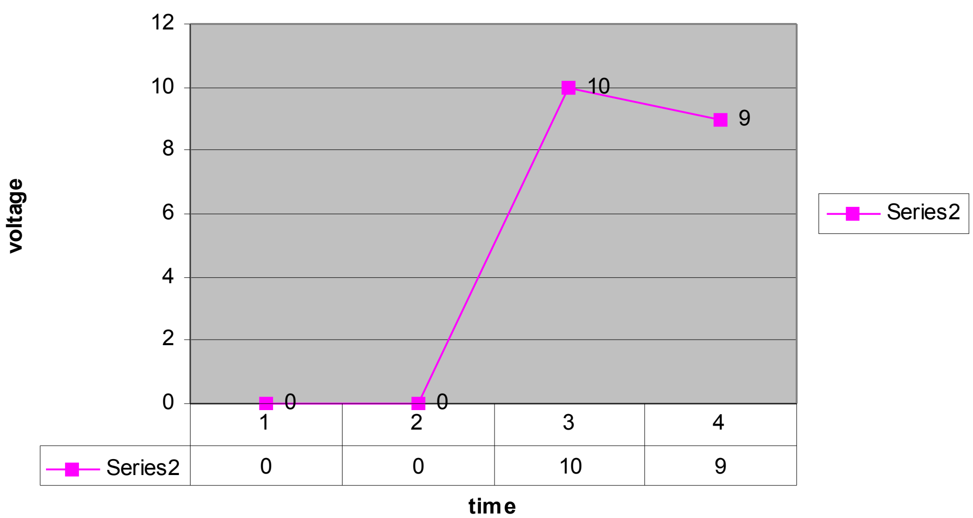
2<t<3	$v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{2} idt + 2\int_{2}^{t} idt$ $v_c(t) = v_c(2) + 2\int_2^t (2)dt = 8 + 4\Big\|_2^t t$ $v_c(t) = 8 + 4(t - 2)$ Volts.	At t = 3 $v_c(3) = 12$

t = 0	$v_c(t)$ = 0
1	0
2	8

3	**12**

0<t<1	**$v_2(t) = 0$**	**$v_1(t) = v_c(t) + v_2(t)$** **$v_1(t) = 0 + 0 = 0$** **At t = 0** **$v_1(0) = 0$ Volts.** **$v_1(t) = v_c(t) + v_2(t)$** **$v_1(t) = 0 + 0 = 0$** **At t = 1** **$v_1(1) = 0$ Volts.**
1<t<2	**2**	**$v_1(t) = v_c(t) + v_2(t)$** **$v_1(t) = 8 + 2 = 10$** **At t = 2** **$v_1(2) = 10$ Volts.**
2<t<3	**-3**	**At t = 3** **$v_1(3) = v_c(3) + v_2(3)$** **$v_1(3) = (12) - 3 = 9$ Volts**

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**10**
$v_1(3)$	**9**



3-9 (d)
$\therefore$ $v_1 = v_c + v_2$

t

$$v_c = (1/c)\int_{-\infty} i(t)dt$$

$v_2(t)$	sint	$0<t<\pi$

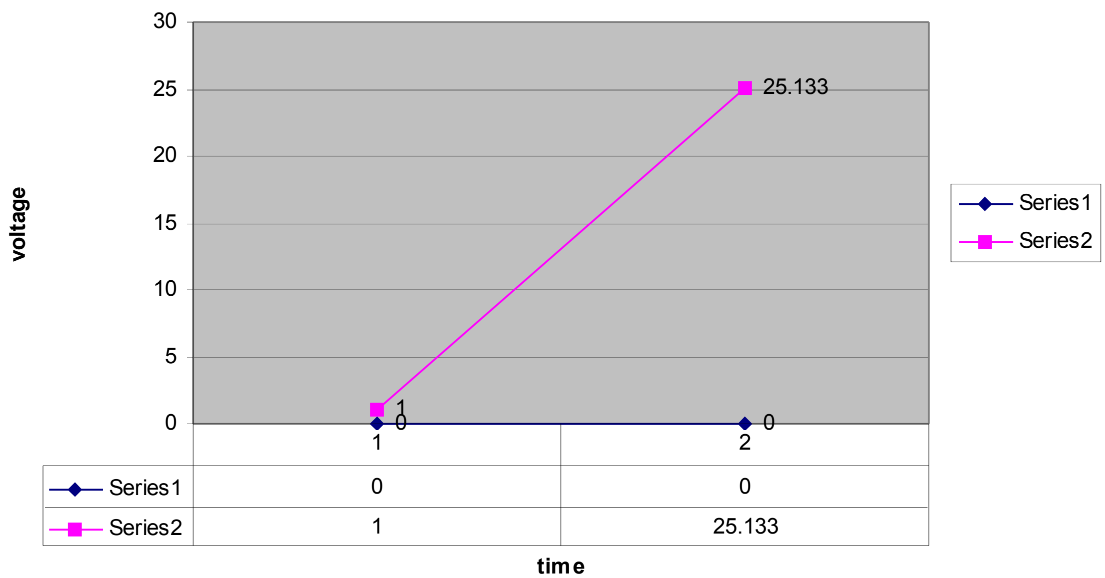
i(0)	$0<t<\pi$	0
$i(\pi\ )$	$0<t<\pi$	4

$0<t<\pi$	$v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} idt$ $v_c = 0 + 2\int_{0}^{t} 0dt = 0$ Volts. $v_c = 2\int_{-\infty}^{t} idt = 2\int_{-\infty}^{0} idt + 2\int_{0}^{t} 4dt$ $v_c = 0 + 8\int_{0}^{t} dt = 8\Big\vert_{0}^{t} t\Big\vert = 8t$ Volts	At t = 0 $v_c(0) = 0$ At $t = \pi$ $v_c(1) =$ 8π Volts

t = 0	$v_c(t)\quad = 0$
π	8π

$0<t<\pi$	$v_2(t) = \sin t$	$v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = v_c(t) + \sin t = 0$ At $t = 0$ $v_1(0) = 0 + \sin 0 = 0$ Volts. $v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 8\pi + \sin t$ At $t = \pi$ $v_1(\pi) = 8\pi + \sin\pi =$ 8π Volts

$v_1(0)$	0
$v_1(\pi)$	8π



3-9 (e)

$\therefore\ v_1 = v_c + v_2$

$$v_c = (1/c)\int_{-\infty}^{t} i(t)dt$$

$v_2(t)$	0	$0<t<1$
$v_2(t)$	1	$1<t<2$

$v_2(t)$	0	$2<t<3$
$v_2(t)$	-1	$3<t<4$

i(0)	$0<t<1$	0
i(1)	$0<t<1$	0
i(2)	$1<t<2$	2
i(3)	$2<t<3$	2
i(4)	$3<t<4$	0

$0<t<1$	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{0} i\,dt + 2\int_{0}^{t} i\,dt$ $v_c = 0 + 2\int_{0}^{t} 0\,dt = 0$ Volts. $v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{0} i\,dt + 2\int_{0}^{t} i\,dt$ $v_c = 0 + 2\int_{0}^{t} 0\,dt = 0$ Volts.	At $t = 0$ $v_c(0) = 0$ At $t = 1$ $v_c(1) = 0$
$1<t<2$	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{1} i\,dt + 2\int_{1}^{t} i\,dt$ $v_c(t) = v_c(1) + 2\int_{1}^{t} (2)dt = 0 + 4\left. t \right\|_{1}^{t}$ $v_c(t) = 0 + 4(t - 1)$ Volts.	At $t = 2$ $v_c(2) = 4$

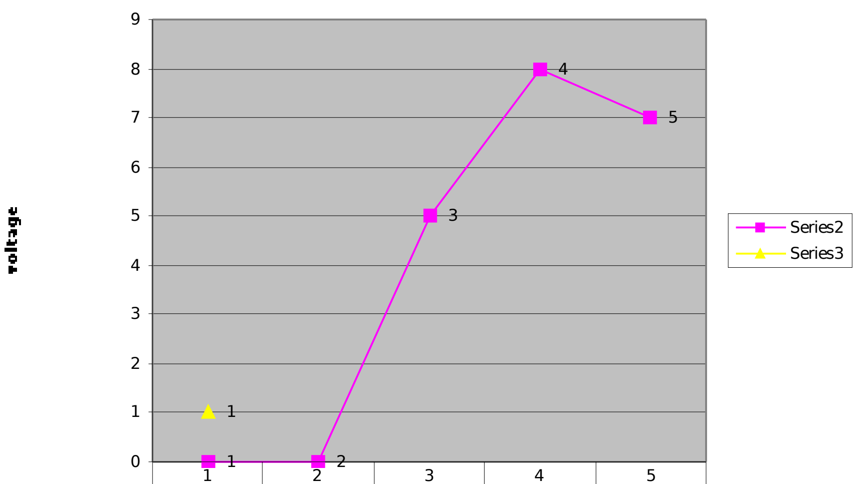
2<t<3	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{2} i\,dt + 2\int_{2}^{t} i\,dt$ $v_c(t) = v_c(2) + 2\int_{2}^{t}(2)dt = 4 + 4\Big\|_{2}^{t}$ $v_c(t) = 4 + 4(t - 2)$ **Volts.**	**At t = 3** $v_c(3) = 8$
3<t<4	$v_c = 2\int_{-\infty}^{t} i\,dt = 2\int_{-\infty}^{3} i\,dt + 2\int_{3}^{t} i\,dt$ $v_c(t) = v_c(3) + 2\int_{3}^{t}(0)dt = 8$ $v_c(t) = 8$ **Volts.**	**At t = 4** $v_c(4) = 8$ **Volts**

t = 0	$v_c(t) = 0$
1	**0**
2	**4**
3	**8**
4	**8**

0<t<1	$v_2(t) = 0$	$v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 0 + 0 = 0$ **Volts** **At t = 0** $v_1(0) = 0$ **Volts.** $v_1(t) = v_c(t) + v_2(t)$ $v_1(t) = 0 + 0 = 0$ **At t = 1** $v_1(1) = 0$ **Volts.**

1<t<2	**1**	**$v_1(t) = v_c(t) + v_2(t)$** **$v_1(t) = 4 + 1 = 5$** **At t = 2** **$v_1(2) = 5$ Volts.**
2<t<3	**0**	**At t = 3** **$v_1(3) = v_c(3) + v_2(3)$** **$v_1(3) = (8) + 0 = 8$ Volts**
3<t<4	**-1**	**At t = 4** **$v_1(4) = v_c(4) + v_2(4)$** **$v_1(4) = 8 - 1 = 7$ Volts**

$v_1(0)$	**0**
$v_1(1)$	**0**
$v_1(2)$	**5**
$v_1(3)$	**8**
$v_1(4)$	**7**



	1	2	3	4	5
Series2	0	0	5	8	7
Series3	1				

3-17. For each of the four networks shown in the figure, determine the number of independent loop currents, and the number of independent node-to-node voltages that may be used in writing equilibrium equations using the kirchhoff laws.
Solution:
Open your book & see (P/90)
- **(a) Number of independent loops = 2**
  Node-to-node voltages = 4
- **(b) Number of independent loops = 2**
  Node-to-node voltages = 3
- **(c) Number of independent loops = 2**
  Node-to-node voltages = 3
- **(d) Number of independent loops = 4**
  Node-to-node voltages = 7

3-18. Repeat Prob. 3-17 for each of the four networks shown in the figure on page 91.
- **(e) Number of independent loops = 7**
  Node-to-node voltages = 4
- **(f) Number of independent loops = 3**
  Node-to-node voltages = 5
- **(g) Number of independent loops = 4**
  Node-to-node voltages = 5
- **(h) Number of independent loops = 5**
  Node-to-node voltages = 6

3-19. Demonstrate the equivalence of the networks shown in figure 3-17 and so establish a rule for converting a voltage source in series with an inductor into an equivalent network containing a current source.
Solution:
Open your book & read article source transformation (P/57).
3-20. Demonstrate that the two networks shown in figure 3-18 are equivalent.
Solution:
Open your book & read (P/60).
3-21. Write a set of equations using the kirchhoff voltage law in terms of appropriate loop-current variables for the four networks of Prob. 3-17.
(a)
i_1:
$R_2 i_1 + 1/c\int (i_1 - i_2)\, dt = 0$
i_2:
$v(t) = i_2 R_1 + 1/c\int (i_2 - i_1)\, dt + L di_2/dt + R_3 i_2$
(b)
i_1:
$R_1 i_1 + L d(i_1 - i_2)/dt = v(t)$
i_2:
$0 = i_2 R_2 + 1/c\int i_2\, dt + L d(i_2 - i_1)/dt$
(c)
i_1:
$R(i_1 - i_2) + L di_1/dt = v(t)$

i_2:

$$0 = (i_2 - i_1)R + 1/c\int i_2\, dt$$

(d)

i_1:

$$L_1 d(i_1 - i_3)/dt + 1/c_1\int i_1 dt = 0$$

i_2:

$$R_1 i_2 + L_2 d(i_2 - i_3)/dt + 1/c_2\int (i_2 - i_4)dt = 0$$

i_3:

$$L_1 d(i_3 - i_1)/dt + L_2 d(i_3 - i_2)/dt + R_3(i_3 - i_4) = v(t)$$

i_4:

$$R_2 i_4 + R_3(i_4 - i_3) + 1/c_2\int (i_4 - i_2)dt = 0$$

3-22. Make use of the KVL to write equations on the loop basis for the four networks of Prob. 3-18.

Solution:

Open your book & see (P/91).

(a)

i_1:

$$R_{p1} i_1 + 1/c_3\int (i_1 - i_2)dt = - v(t)$$

i_2:

$$1/c_3\int (i_2 - i_1)dt + R_1(i_2 - i_3) = 0$$

i_3:

$$1/c_1\int i_3 dt + R_1(i_3 - i_2) + R_3(i_3 - i_4) = 0$$

i_4:

$$1/c_4\int (i_4 - i_5)dt + R_2(i_4 - i_3) = 0$$

i_5:

$$R_{p2} i_5 + 1/c_4\int (i_5 - i_4)dt + 1/c_2\int i_5 dt = - v(t)$$

i_6:

$$R_{p2}(i_6 - i_5) + R_3(i_6 - i_7) = - v(t)$$

i_7:

$$1/c_5\int i_7 dt + R_3(i_7 - i_6) = 0$$

(b)

i_1:

$$L_2 di_1/dt + 1/c_1\int (i_1 - i_2)dt + 1/c_3\int (i_1 - i_3)dt + L_4 d(i_1 - i_3)/dt = v(t)$$

i_2:

$$L_1 di_2/dt + 1/c_2\int (i_2 - i_3)dt + 1/c_1\int (i_2 - i_1)dt = 0$$

i_3:

$$L_3 di_3/dt + 1/c_2\int (i_3 - i_2)dt + 1/c_3\int (i_3 - i_1)dt + L_4 d(i_3 - i_1)/dt + Ri_3 = 0$$

(c)

i_1:

$$1/c\int (i_1 - i_3)dt + R_1(i_1 - i_2) = v(t)$$

i_2:

$$1/c\int (i_2 - i_3)dt + R_1(i_2 - i_1) + R_L(i_2 - i_4) = 0$$

i_3:

$$Ri_3 + R(i_3 - i_4) + 1/c\int (i_3 - i_2)dt + 1/c\int (i_3 - i_1)dt = 0$$

i_4:

$R_L(i_4 - i_2) + R(i_4 - i_3) + 1/c_1\int i_4dt = 0$

(d)

i_1:

$1/c_a\int (i_1 - i_2)dt + 2L_1di_1/dt + L_bd(i_1 - i_3)/dt + 1/c_b\int (i_1 - i_3)dt = v(t)$

i_2:

$L_ad(i_2 - i_4)/dt + 1/c_a\int (i_2 - i_1)dt = 0$

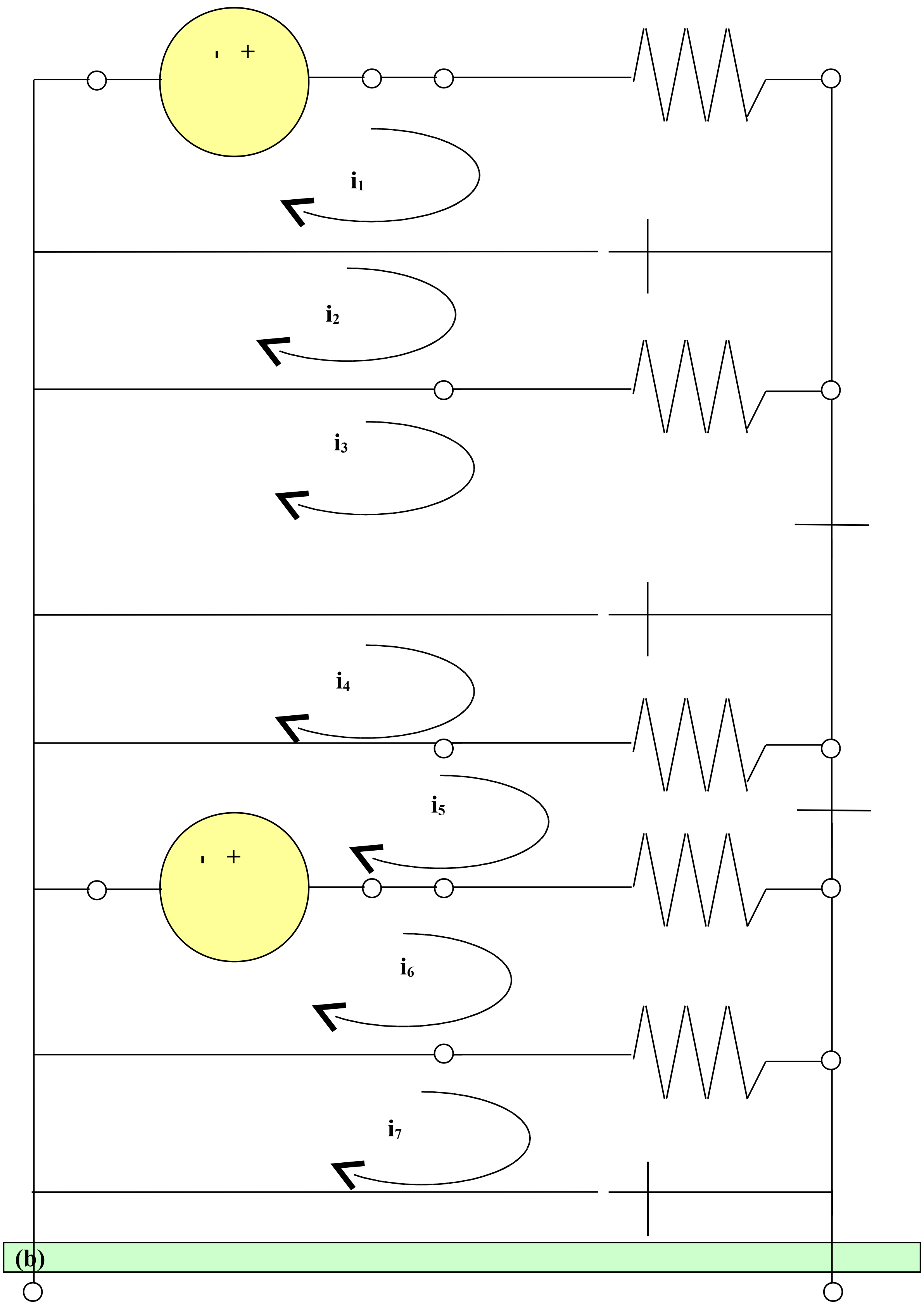
i_3:

$2L_2d(i_3 - i_4)/dt + R(i_3 - i_4) + 1/c_a\int (i_3 - i_5)dt + L_bd(i_3 - i_1)/dt + 1/c_b\int (i_3 - i_1)dt = 0$

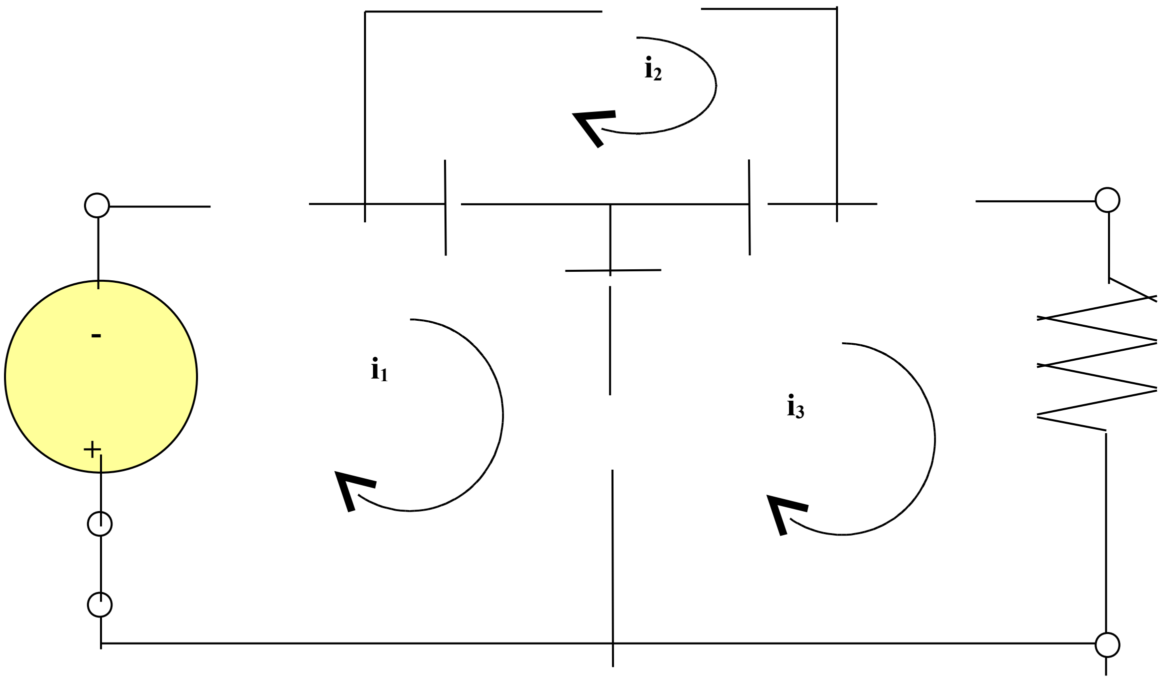
i_4:

$L_ad(i_4 - i_2)/dt + L_bdi_4/dt + 1/c_b\int i_4dt + 2L_2d(i_4 - i_3)/dt + R(i_4 - i_3) = 0$

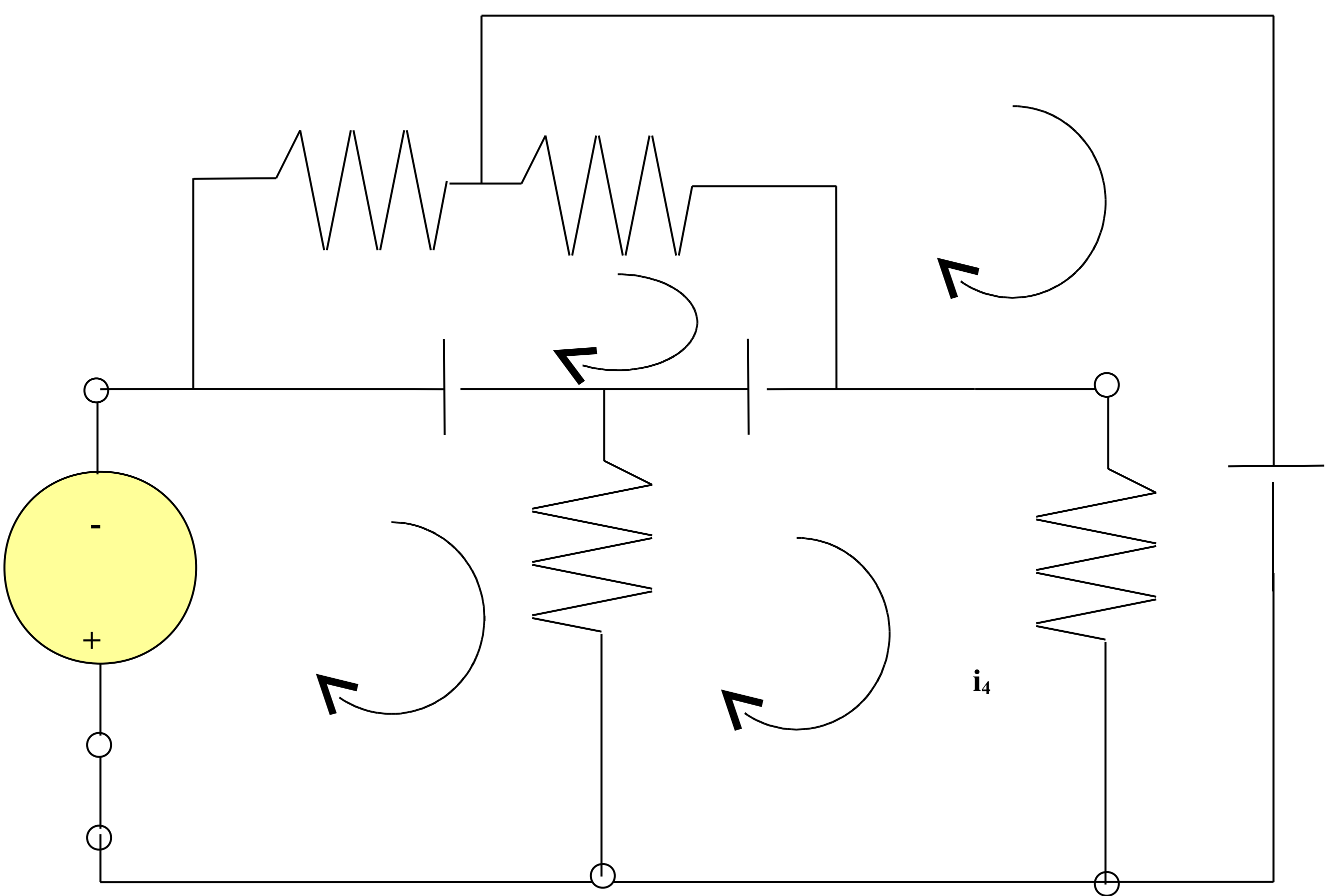
(a)



(b)



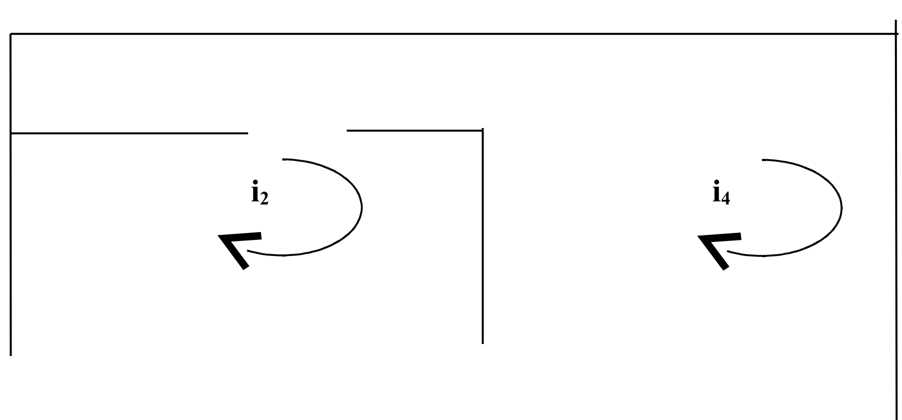
(c)

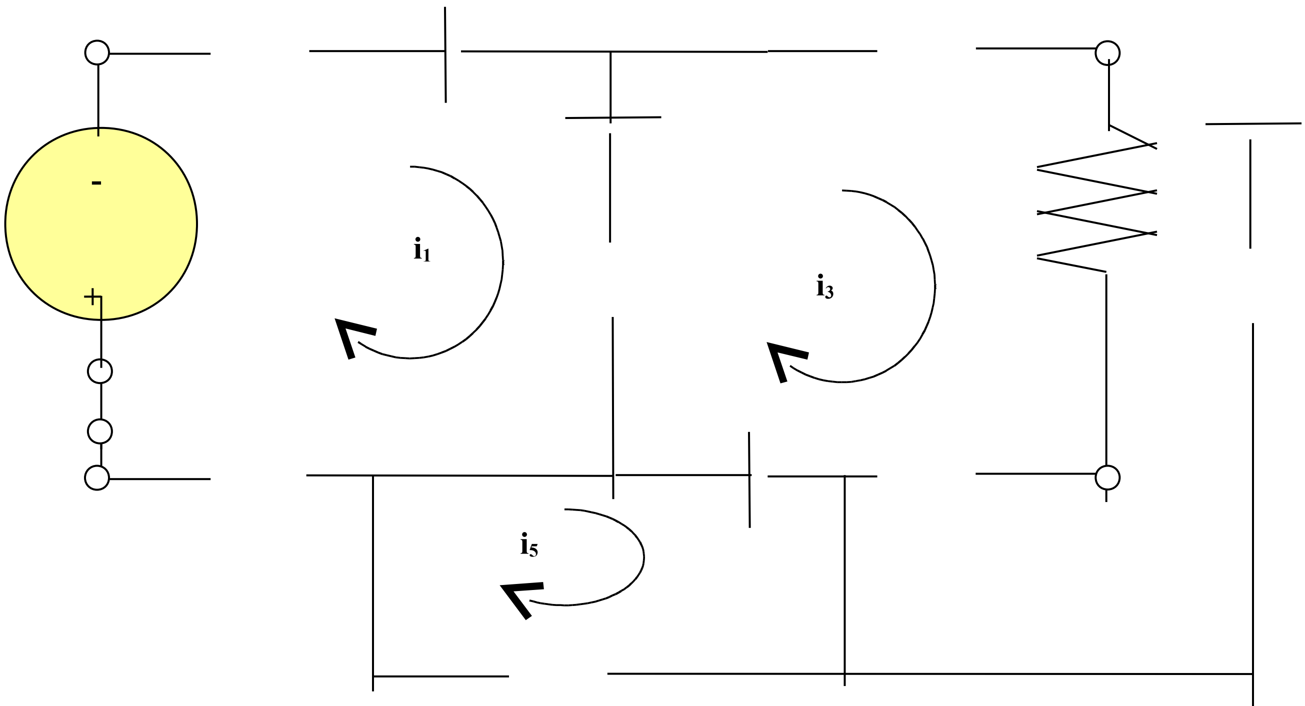


i_3

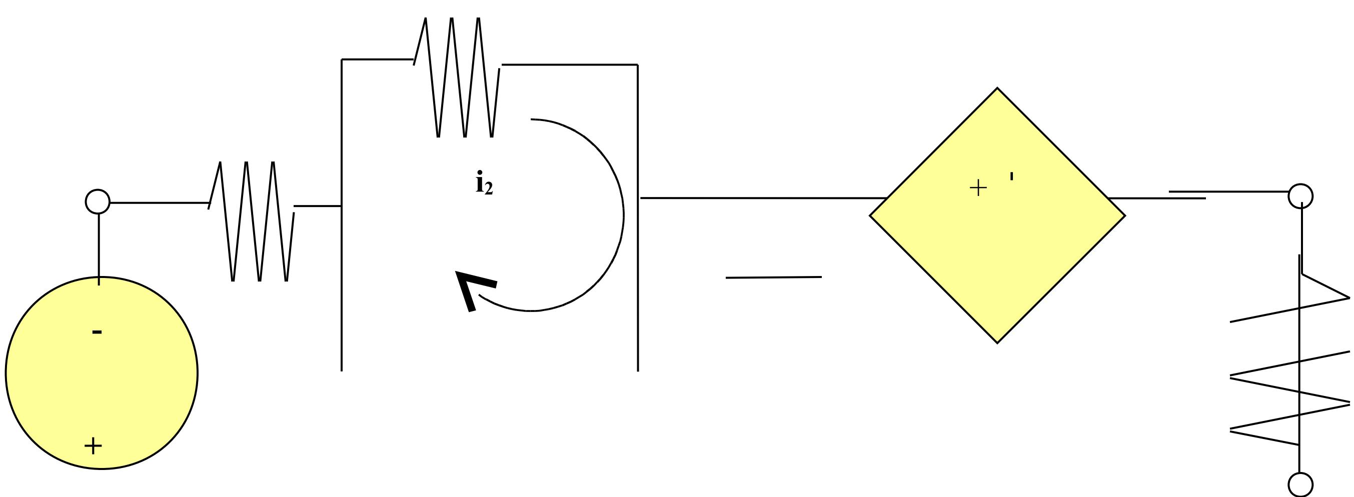
i_1 i_2

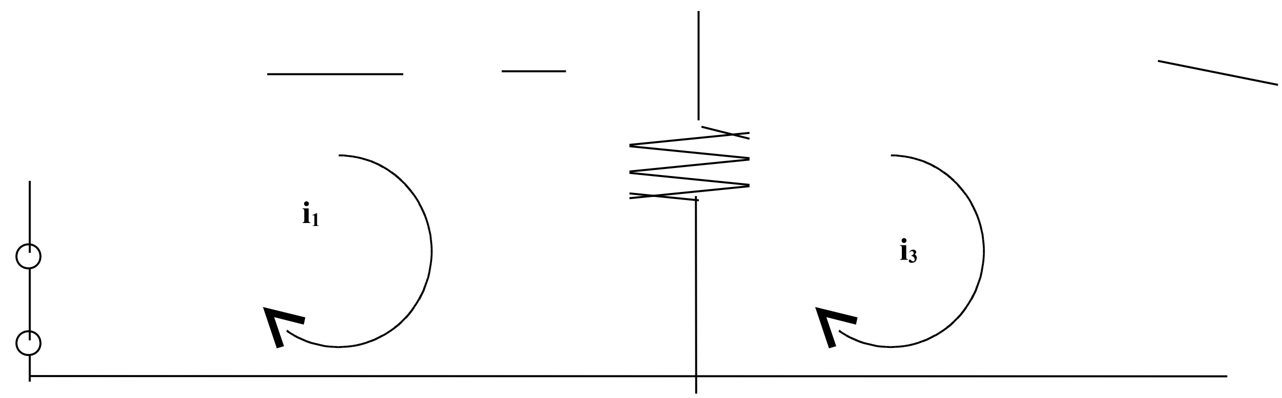
(d)





3-23.
Write a set of equilibrium equations on the loop basis to describe the network in the accompanying figure. Note that the network contains one controlled source. Collect terms in your formulation so that your equations have the general form of Eqs. (3-47).





i_1:

$i_1 + (i_1 - i_2) + (i_1 - i_3) + 1\int (i_1 - i_3)dt = v_1(t)$

i_2:

$1(i_2 - i_1) + Ldi_2/dt = 0$

i_3:

$i_3 + (i_3 - i_1) + 1\int (i_3 - i_1)dt - k_1i_1= 0$

3-24. For the coupled network of the figure, write loop equations using the KVL. In your formulation, use the three loop currents, which are identified.

Solution:

Open your book & see (P/92).

i_1:

$R_1i_1 + (L_1 + L_2)di_1/dt + Mdi_2/dt = v_1$

i_2:

$L_3di_2/dt + Mdi_1/dt + 1/c\int (i_2 - i_3)dt = v_2$

i_3:

$R_2i_3 + 1/c\int (i_3 - i_2)dt = 0$

3-25. Using the specified currents, write the KVL equations for this network.

Solution:

Open your book & see (P/92).

i_1:

$R_1(i_1 + i_2 + i_3) + L_1di_1/dt + M_{12}di_2/dt + R_2i_1 - M_{13}di_2/dt = v_1(t)$

i_2:

$R_1(i_1 + i_2 + i_3) + L_2di_2/dt + M_{12}di_1/dt + M_{23}di_2/dt = v_1(t)$

i_3:

$R_1(i_1 + i_2 + i_3) + L_3di_3/dt - M_{13}di_1/dt + M_{23}di_2/dt + 1/c\int i_3dt= v_1(t)$

3-26. A network with magnetic coupling is shown in figure. For the network, M_{12} = 0. Formulate the loop equations for this network using the KVL.

i_1:

$R_1i_1 + L_1di_1/dt + M_{13}d(i_1 - i_2)/dt + L_3d(i_1 - i_2)/dt + M_{23}d(-i_2)/dt + M_{13}di_1/dt + R_2(i_1 - i_2) = v_1(t)$

i_2:

$R_3i_2 + L_2di_2/dt + M_{23}d(i_2 - i_1)/dt + L_3d(i_2 - i_1)/dt + M_{23}d(i_2)/dt + M_{13}d(-i_1)/dt + R_2(i_2 - i_1) = 0$

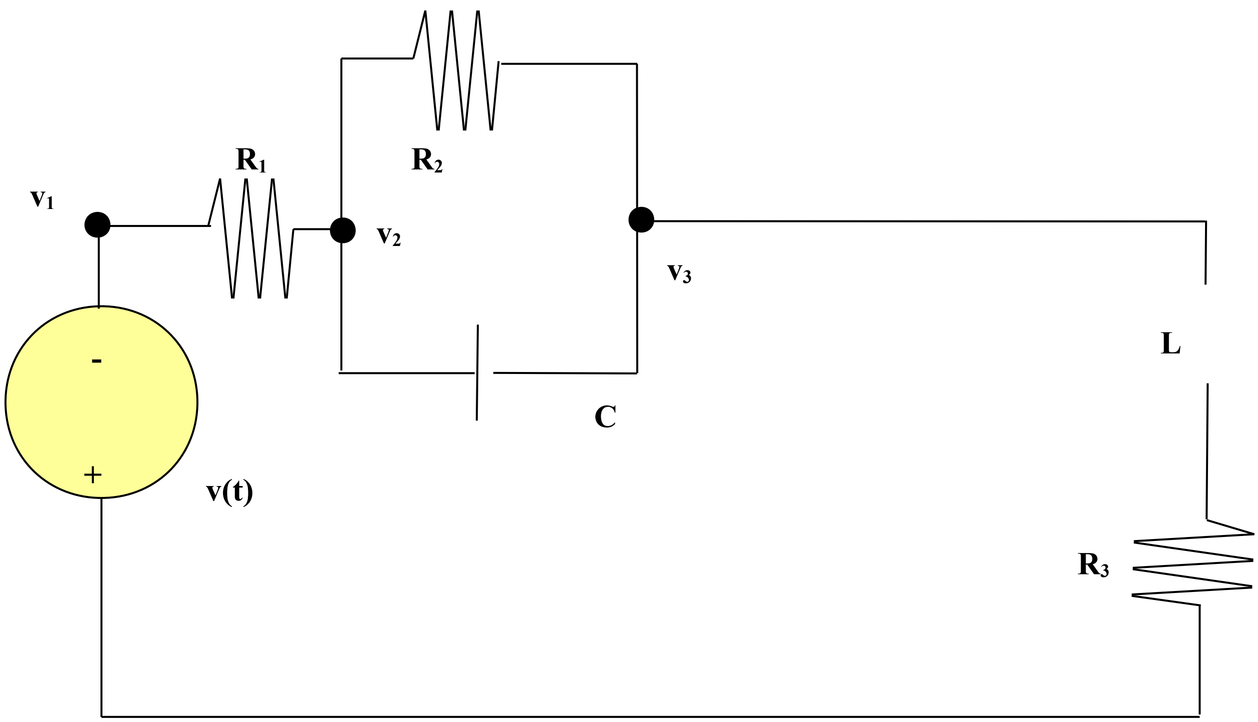
3-27. Write the loop-basis voltage equations for the magnetically coupled network with k closed.

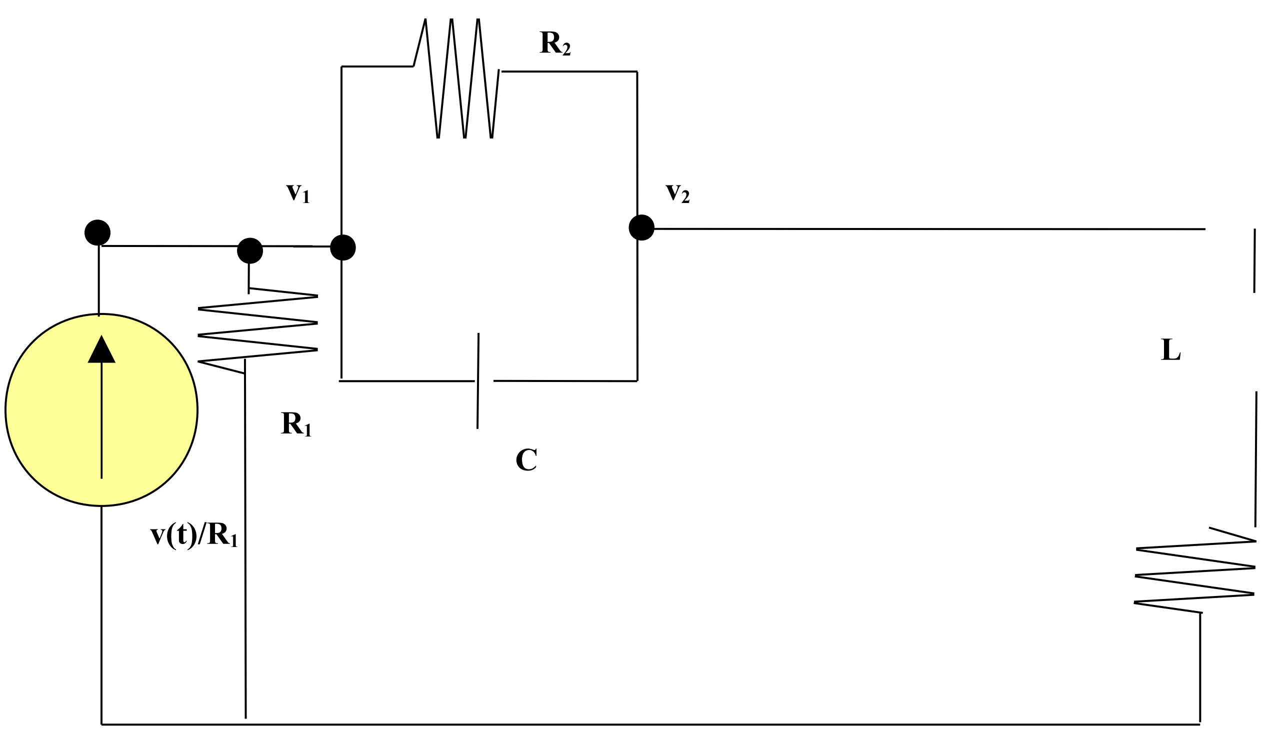
Solution:

Same as 3.26.

3-28. Write equations using the KCL in terms of node-to-datum voltage variables for the four networks of Prob. 3-17.

(a)





R_3

<u>Node-v_1</u>

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

$$v(t)/R_1 = v_1/R_1 + (v_1 - v_2)/R_2 + cd(v_1 - v_2)/dt$$
$$v(t)/R_1 = v_1/R_1 + v_1/R_2 - v_2/R_2 + cdv_1/dt - cdv_2/dt$$
$$v(t)/R_1 = v_1/R_1 + v_1/R_2 + cdv_1/dt - v_2/R_2 - cdv_2/dt$$
$$v(t)/R_1 = v_1(1/R_1 + 1/R_2 + cd/dt) + (-1/R_2 - cd/dt)v_2$$

$$\boxed{v(t)/R_1 = v_1(G_1 + G_2 + cd/dt) + (-G_2 - cd/dt)v_2} \quad \text{Because } G = 1/R$$

<u>Node-v_1</u>

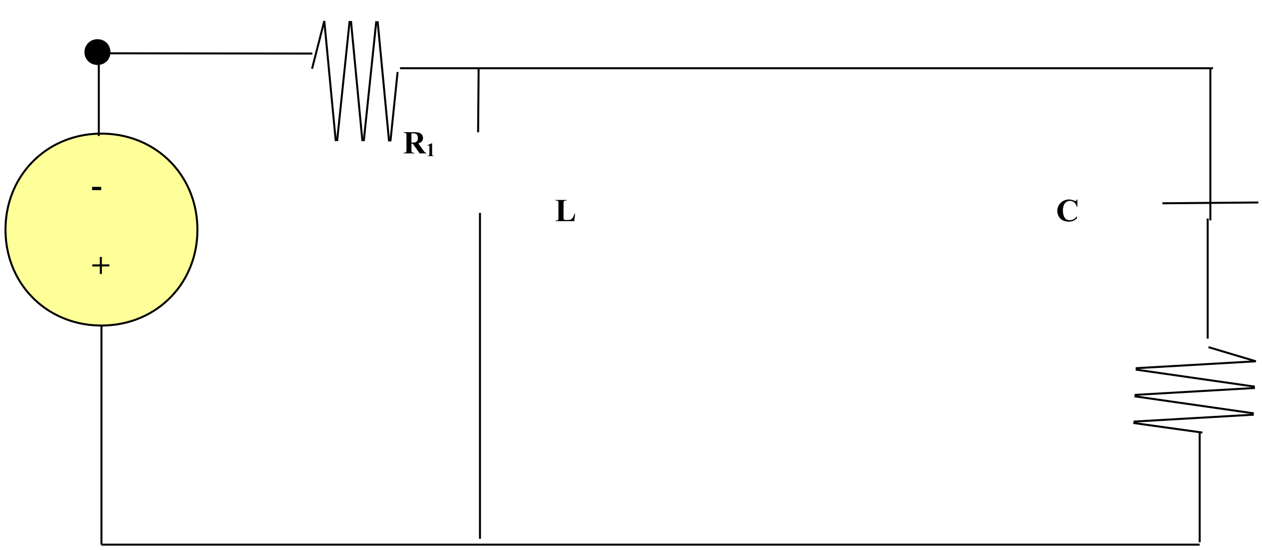
According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

$$0 = (v_2 - v_1)/R_2 + cd(v_2 - v_1)/dt + 1/L\int v_2 dt + v_2/R_3$$
$$0 = (v_2 - v_1)/R_2 + cd(v_2 - v_1)/dt + X\int v_2 dt + v_2/R_3$$
$$0 = v_2/R_2 - v_1/R_2 + cdv_2/dt - cdv_1/dt + X\int v_2 dt + v_2/R_3$$
$$0 = v_2/R_2 + cdv_2/dt + v_2/R_3 + X\int v_2 dt - v_1/R_2 - cdv_1/dt$$
$$0 = v_2(1/R_2 + cd/dt + 1/R_3 + X\int dt) + v_1(-1/R_2 - cd/dt)$$

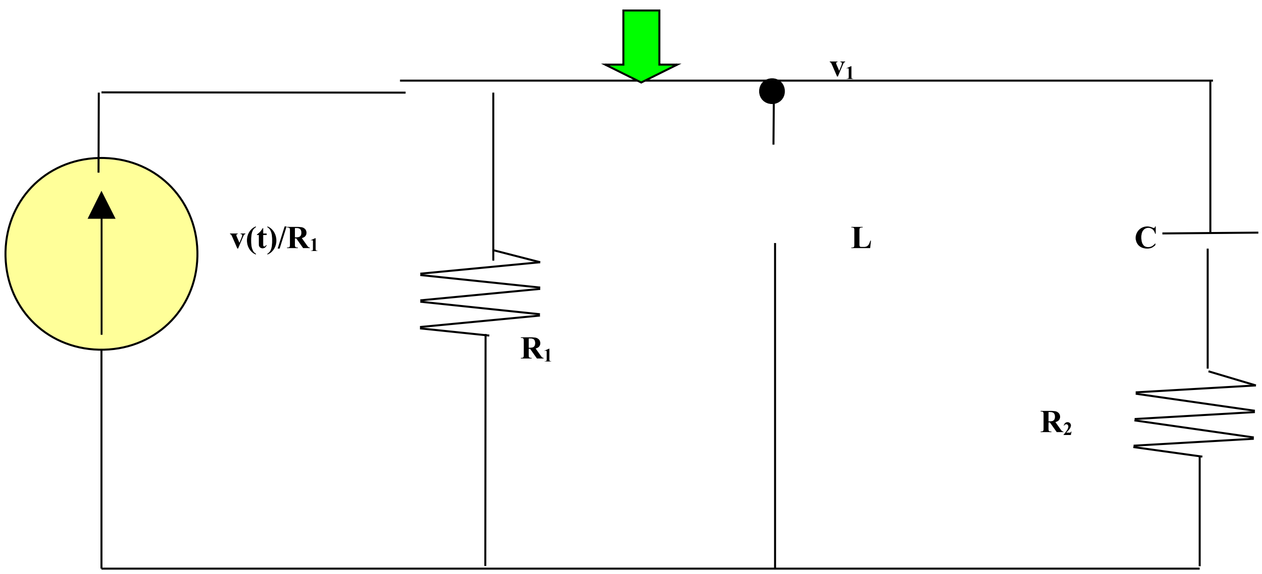
$$\boxed{0 = v_2(G_2 + cd/dt + G_3 + X\int dt) + v_1(-G_2 - cd/dt)} \quad \text{Because } G = 1/R,\ X = 1/L$$

(b)



v(t)

R_2



<u>Node-v_1</u>

According to KCL

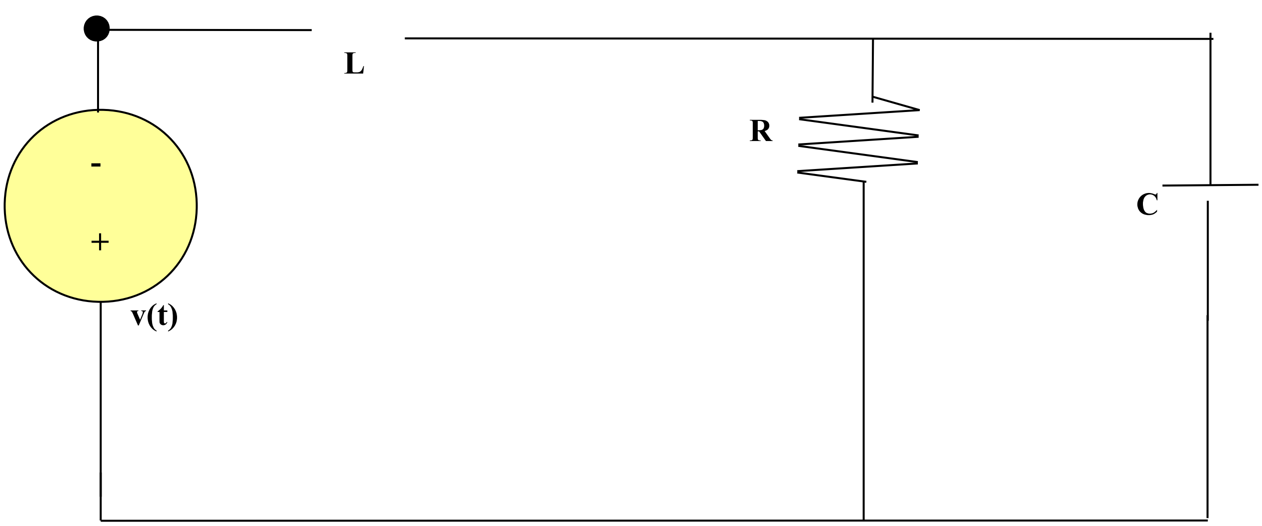
Sum of currents entering into the junction = Sum of currents leaving the junction

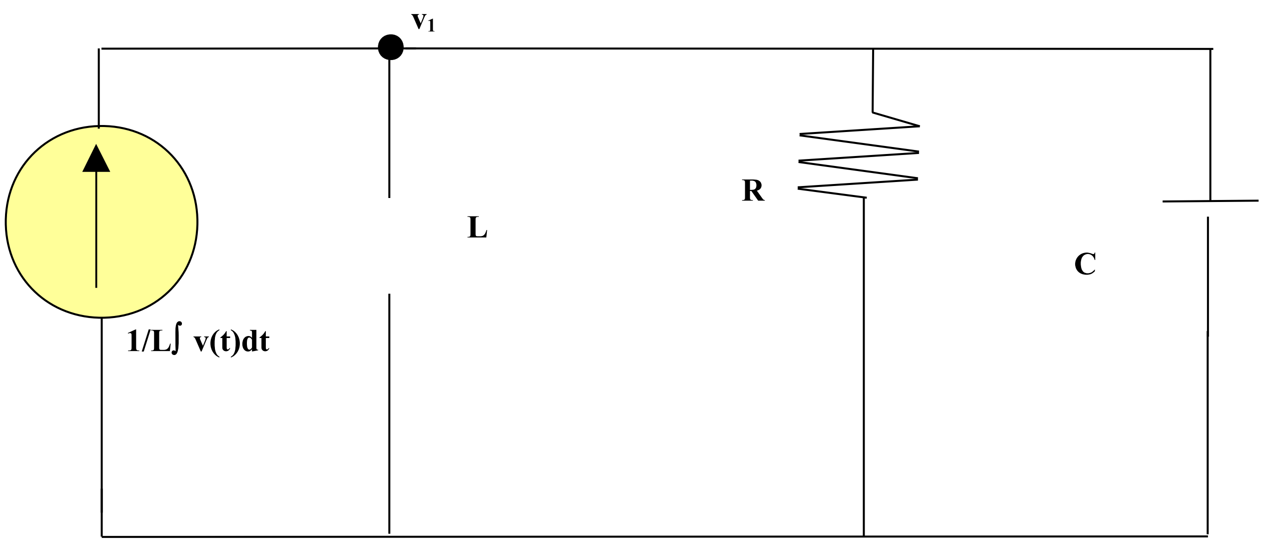
$$v(t)/R_1 = v_1/R_1 + v_1/R_2 + c\,dv_1/dt + 1/L\int v_1\,dt$$

$$v(t)/R_1 = v_1/R_1 + v_1/R_2 + c\,dv_1/dt + X\int v_1\,dt \quad \{\text{Because } 1/L = X\}$$

$$\boxed{v(t)/R_1 = v_1(1/R_1 + 1/R_2 + c\,d/dt + X\int dt)}$$

(c)





<u>Node-v_1</u>

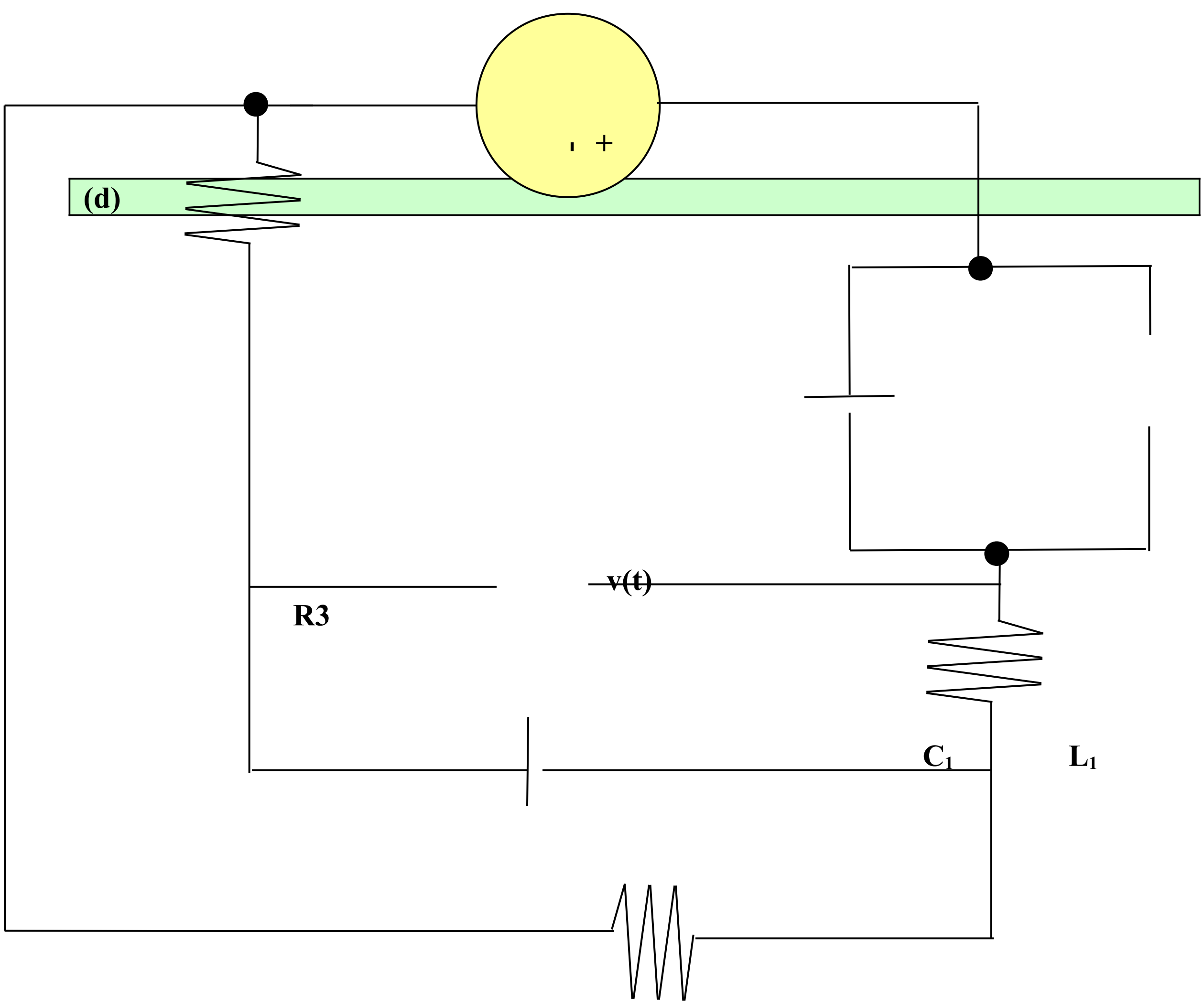
According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

$$1/L\int v_1dt + v_1/R + cdv_1/dt = 1/L\int v(t)dt$$

$$\boxed{v_1(1/L\int dt + 1/R + cd/dt) = 1/L\int v(t)dt}$$

(d)

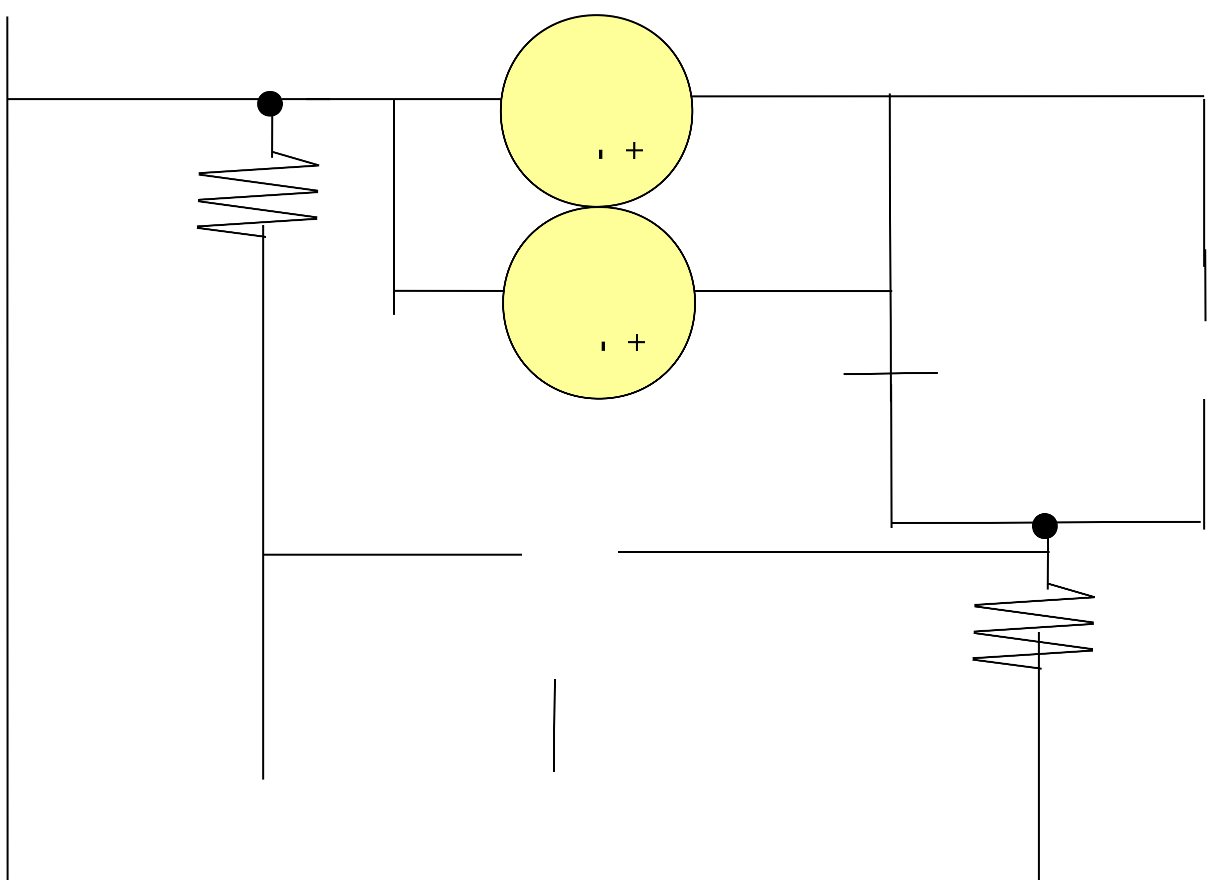


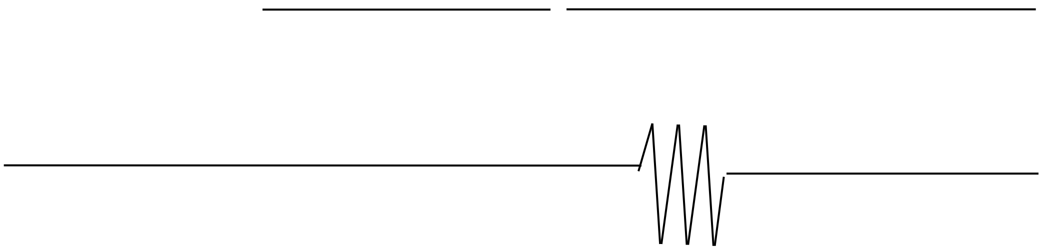
L_2

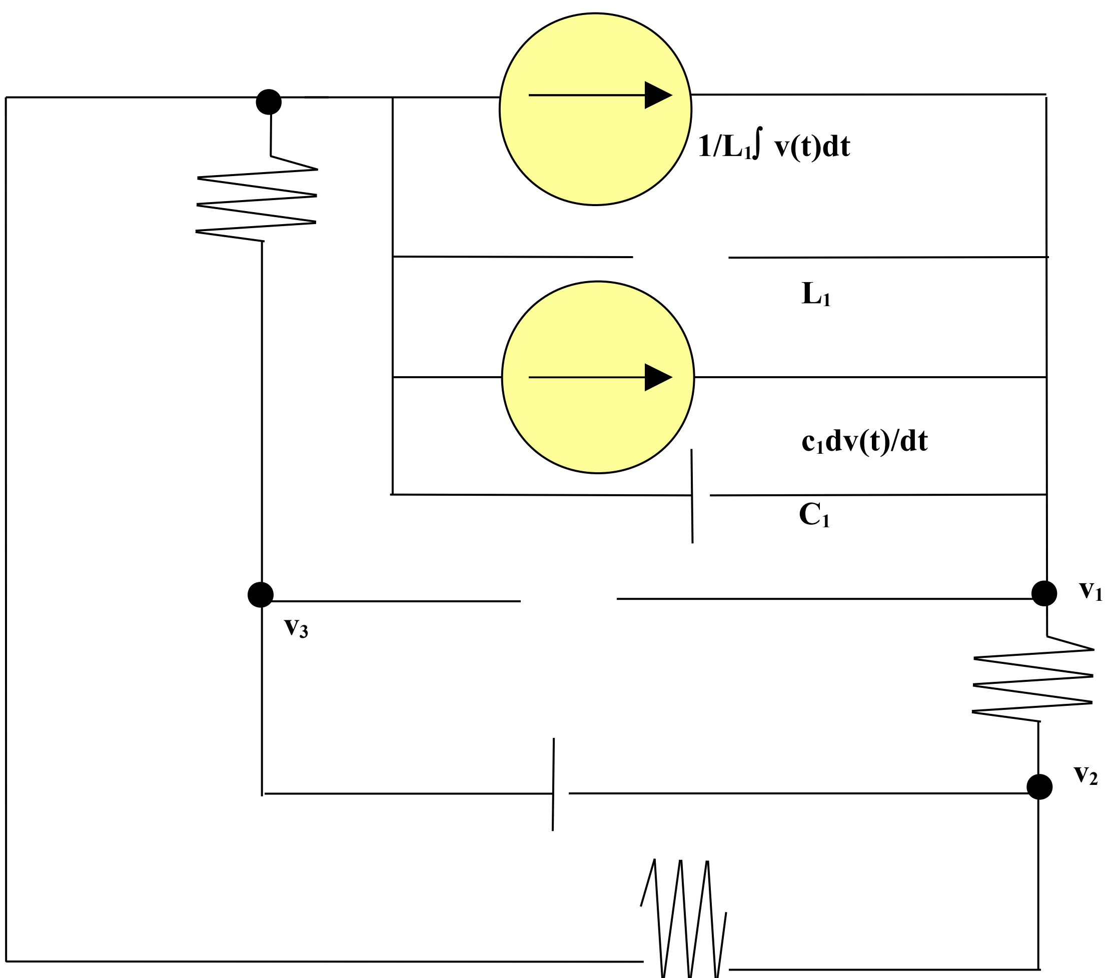
R_1

C_2

R_2






1/L1∫ v(t)dt
L1
c1dv(t)/dt
C1
v1
v3
v2

Node-v_1:

$c_1dv(t)/dt + 1/L_1\int v(t)dt = c_1dv_1/dt + 1/L_1\int v_1dt + 1/L_2\int (v_1 - v_3)dt + (v_1 - v_2)/R_1$

$(c_1d/dt + 1/L_1\int dt)v(t) = c_1dv_1/dt + 1/L_1\int v_1dt + 1/L_2\int v_1dt - 1/L_2\int v_3dt + v_1/R_1 - v_2/R_1$

$(c_1d/dt + 1/L_1\int dt)v(t) = c_1dv_1/dt + 1/L_1\int v_1dt + 1/L_2\int v_1dt + v_1/R_1 - v_2/R_1 - 1/L_2\int v_3dt$

$$\boxed{(c_1d/dt + 1/L_1\int dt)v(t) = (c_1d/dt + 1/L_1\int dt + 1/L_2\int dt + 1/R_1)\, v_1 - v_2/R_1 - 1/L_2\int v_3dt}$$

Node-v_2:

$c_2d(v_2 - v_3)/dt + (v_2 - v_1)/R_1 + v_2/R_2 = 0$

$c_2dv_2/dt - c_2dv_3/dt + v_2/R_1 - v_1/R_1 + v_2/R_2 = 0$

$- v_1/R_1 + v_2/R_2 + c_2dv_2/dt + v_2/R_1 - c_2dv_3/dt = 0$

$$\boxed{- v_1/R_1 + (1/R_2 + c_2d/dt + 1/R_1)v_2 - c_2dv_3/dt = 0}$$

Node-v_3:

$1/L_2\int (v_3 - v_1)dt + c_2d(v_3 - v_2)/dt + v_3/R_3 = 0$

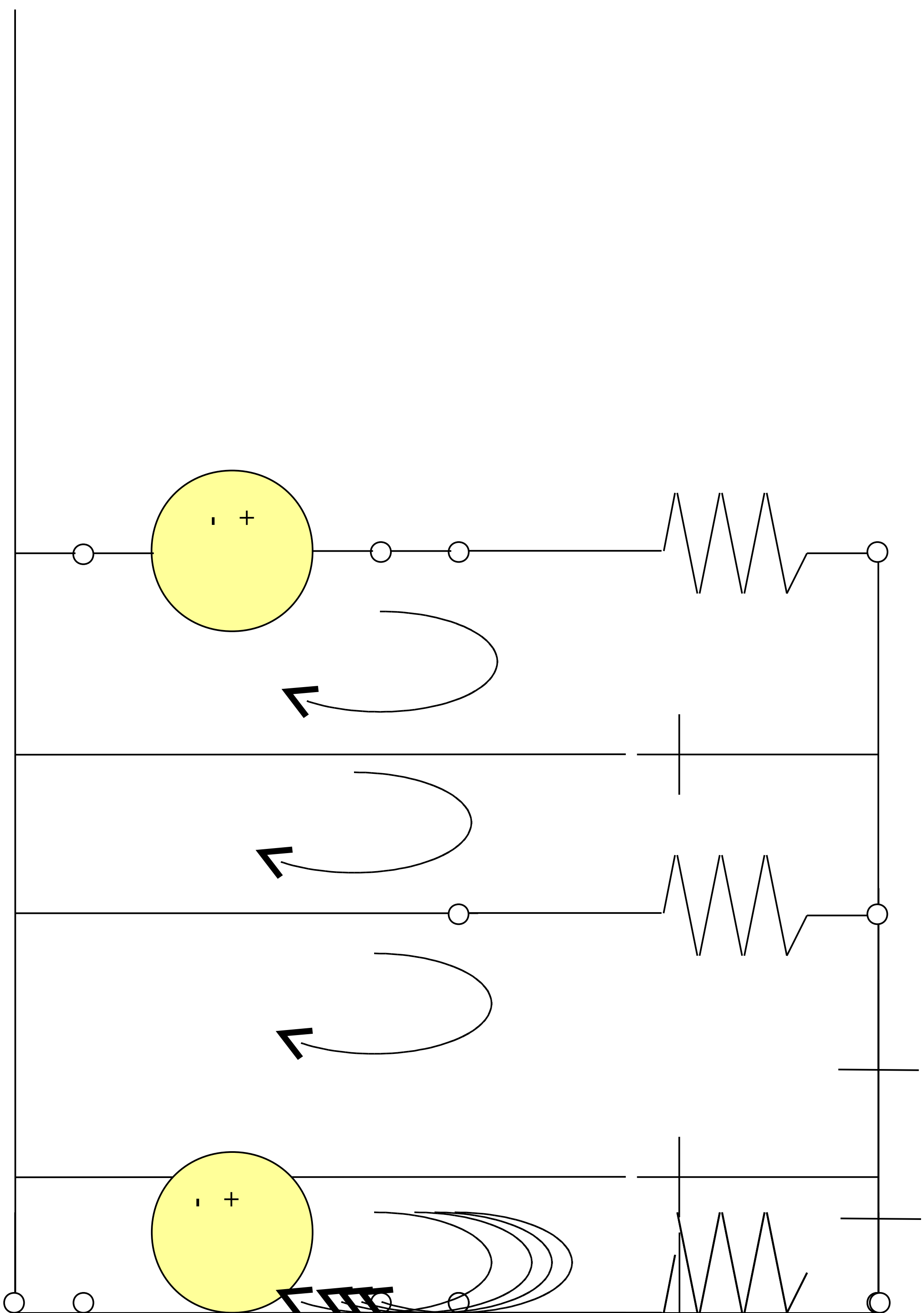
$1/L_2\int v_3dt - 1/L_2\int v_1dt + c_2dv_3/dt - c_2dv_2/dt + v_3/R_3 = 0$

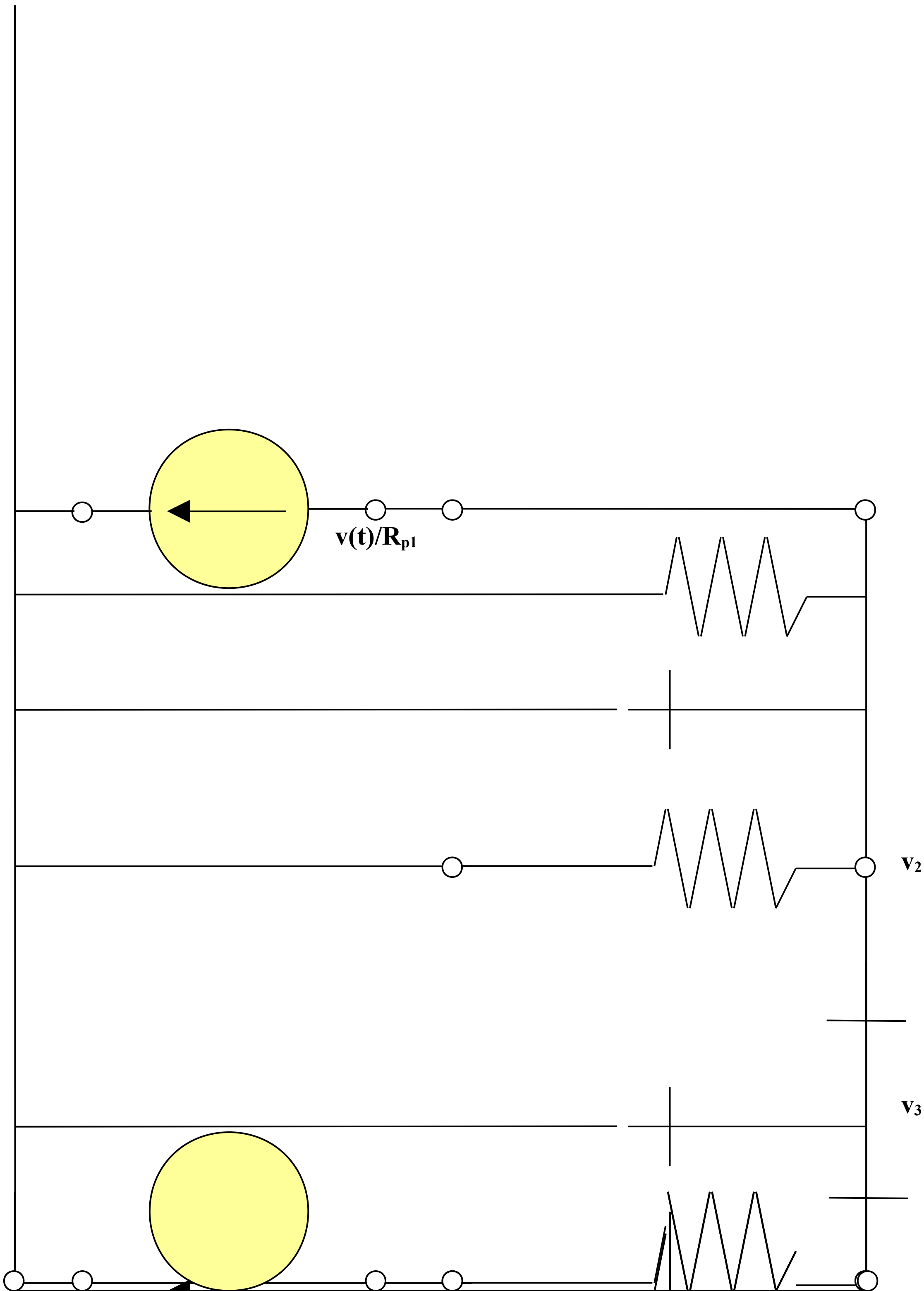
$- 1/L_2\int v_1dt - c_2dv_2/dt + 1/L_2\int v_3dt + v_3/R_3 + c_2dv_3/dt = 0$

$$\boxed{- 1/L_2\int v_1dt - c_2dv_2/dt + (1/L_2\int dt + 1/R_3 + c_2d/dt)v_3 = 0}$$

3-29. Making use of the KCL, write equations on the node basis for the four networks of Prob. 3-18.

(a)




v(t)/Rp1
v2
v3

$\mathbf{v_4}$

$\mathbf{v_1(t)/R_{p2}}$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_2:

$v_2/R_1 + v_2/R_{p1} + c_3 dv_2/dt + c_1 d(v_2 - v_3)/dt = v(t)/R_{p1}$

$v_2/R_1 + v_2/R_{p1} + c_3 dv_2/dt + c_1 dv_2/dt - c_1 dv_3/dt = v(t)/R_{p1}$

$$v_2(1/R_1 + 1/R_{p1} + c_3 d/dt + c_1 d/dt) - c_1 dv_3/dt = v(t)/R_{p1}$$

Node-v_3:

$V_3/R_2 + c_4 dv_3/dt + c_1 d(v_3 - v_2)/dt + c_2 d(v_3 - v_4)/dt = 0$

$V_3/R_2 + c_4 dv_3/dt + c_1 dv_3/dt - c_1 dv_2/dt + c_2 dv_3/dt - c_2 dv_4/dt = 0$

$- c_1 dv_2/dt + V_3/R_2 + c_4 dv_3/dt + c_1 dv_3/dt + c_2 dv_3/dt - c_2 dv_4/dt = 0$

$$- c_1 dv_2/dt + V_3(1/R_2 + c_4 d/dt + c_1 d/dt + c_2 d/dt) - c_2 dv_4/dt = 0$$

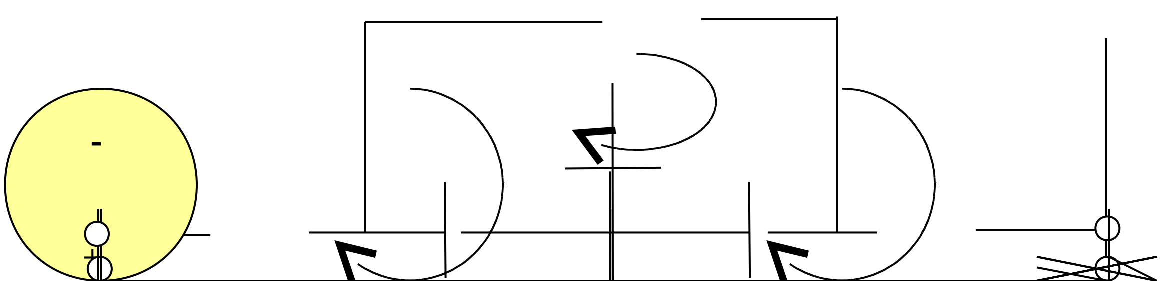
Node-v_4:

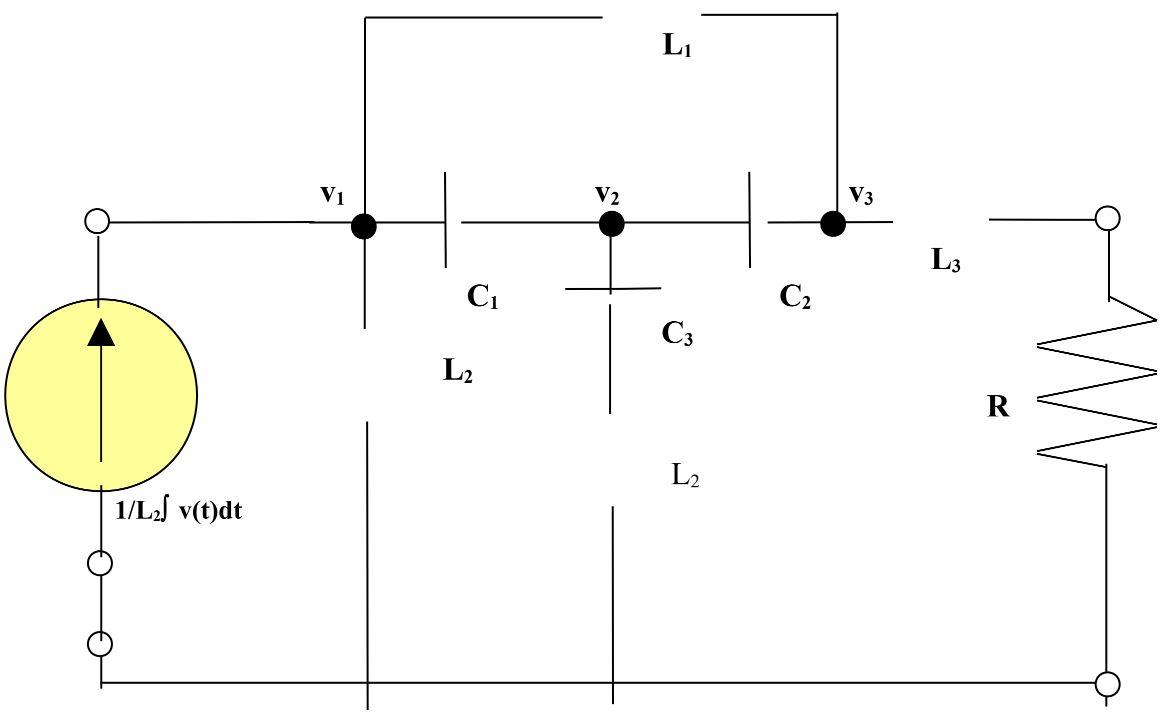
$v_4/R_3 + c_2 d(v_4 - v_3)/dt + c_5 dv_4/dt + v_4/R_{p2} = v_1(t)/R_{p2}$

$v_4/R_3 + c_2 dv_4/dt - c_2 dv_3/dt + c_5 dv_4/dt + v_4/R_{p2} = v_1(t)/R_{p2}$

$$- c_2 dv_3/dt + (c_5 d/dt + 1/R_{p2} + 1/R_3 + c_2 d/dt)v_4 = v_1(t)/R_{p2}$$

(b)





According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_1:

$1/L_2\int v(t)dt = 1/L_2\int v_1 d\,t + 1/L_1\int (v_1 - v_3)d\,t + c_1 d(v_1 - v_2)/dt$

$1/L_2\int v(t)dt = 1/L_2\int v_1 d\,t + 1/L_1\int v_1 d\,t - 1/L_1\int v_3 d\,t + c_1 dv_1/dt - c_1 dv_2/dt$

$1/L_2\int v(t)dt = 1/L_2\int v_1 d\,t + 1/L_1\int v_1 d\,t + c_1 dv_1/dt - c_1 dv_2/dt - 1/L_1\int v_3 d\,t$

$$1/L_2\int v(t)dt = v_1(1/L_2\int dt + 1/L_1\int dt + c_1 d/dt) - c_1 dv_2/dt - 1/L_1\int v_3 dt$$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_2:

$c_1 d(v_2 - v_1)/dt + c_1 d(v_2 - v_3)/dt + c_3 dv_2/dt + 1/L_2\int v_2 dt = 0$

$c_1 dv_2/dt - c_1 dv_1/dt + c_1 dv_2/dt - c_1 dv_3/dt + c_3 dv_2/dt + 1/L_2\int v_2 dt = 0$

$- c_1 dv_1/dt + c_1 dv_2/dt + c_1 dv_2/dt + c_3 dv_2/dt + 1/L_2\int v_2 dt - c_1 dv_3/dt = 0$

$$- c_1 dv_1/dt + v_2(c_1 d/dt + c_1 d/dt + c_3 d/dt + 1/L_2\int dt) - c_1 dv_3/dt = 0$$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_3:

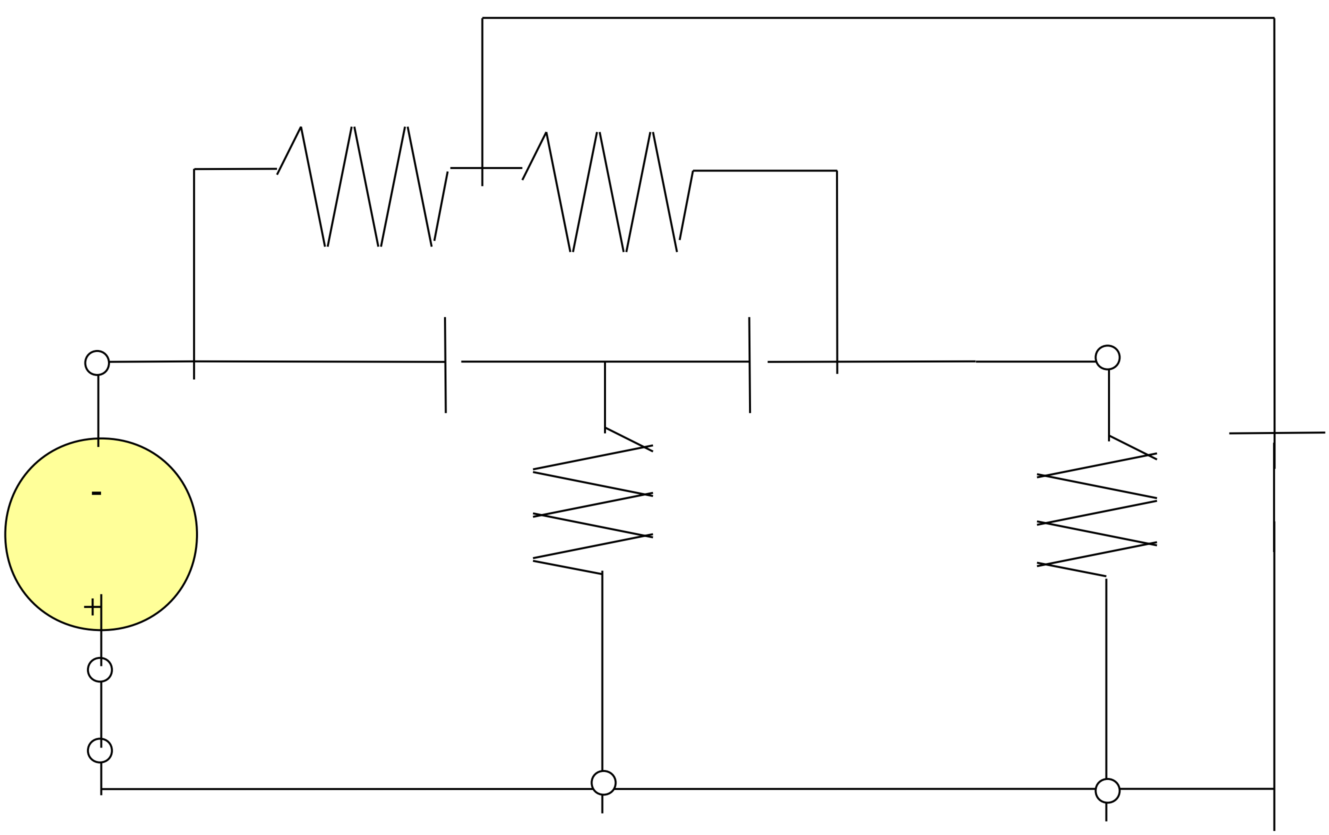
$1/L_1\int (v_3 - v_1)dt + c_1 d(v_3 - v_2)/dt + 1/L_3\int v_3 dt + v_3/R_3 = 0$

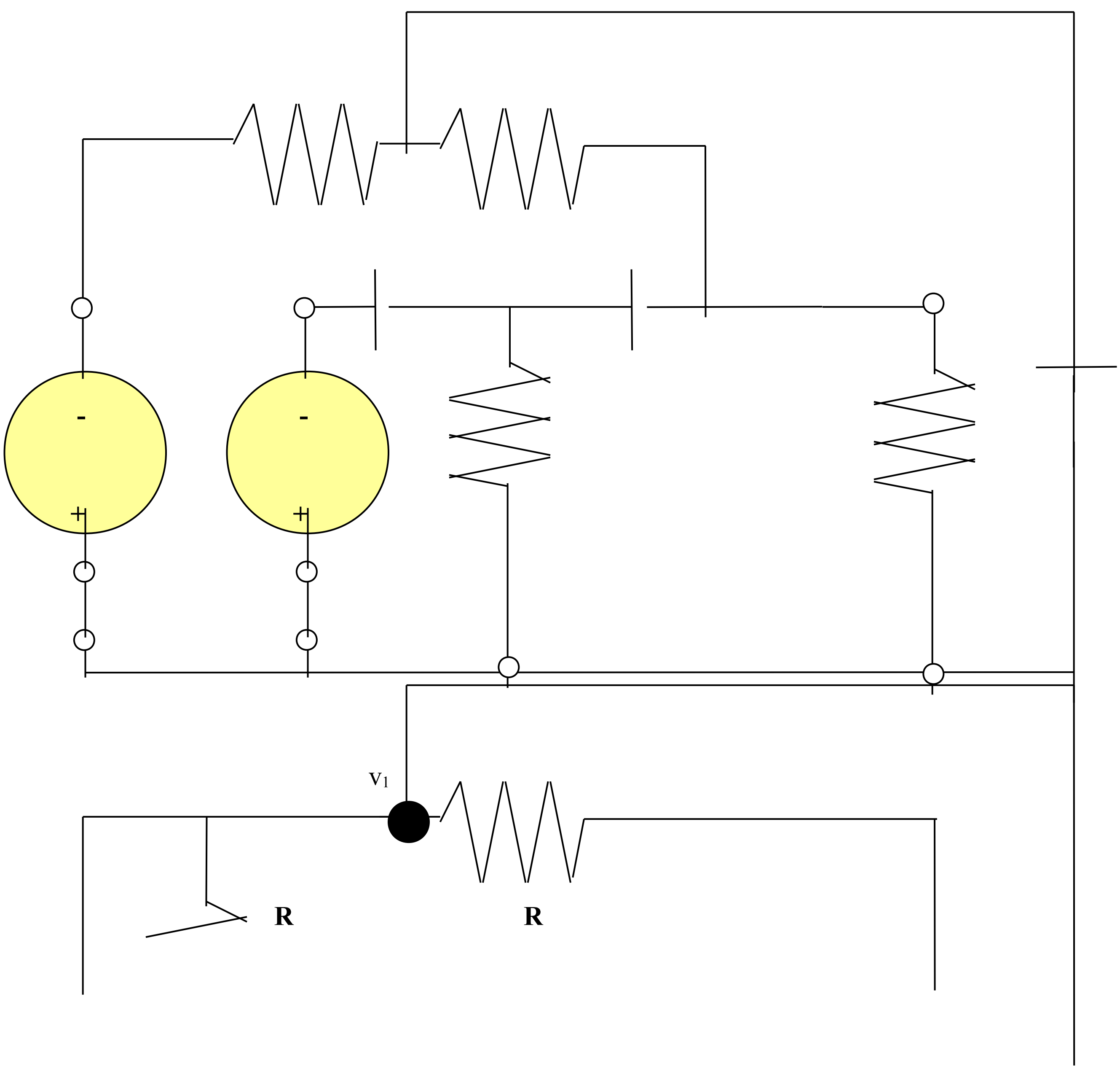
$1/L_1\int v_3 dt - 1/L_1\int v_1 dt + c_1 dv_3/dt - c_1 dv_2/dt + 1/L_3\int v_3 dt + v_3/R_3 = 0$

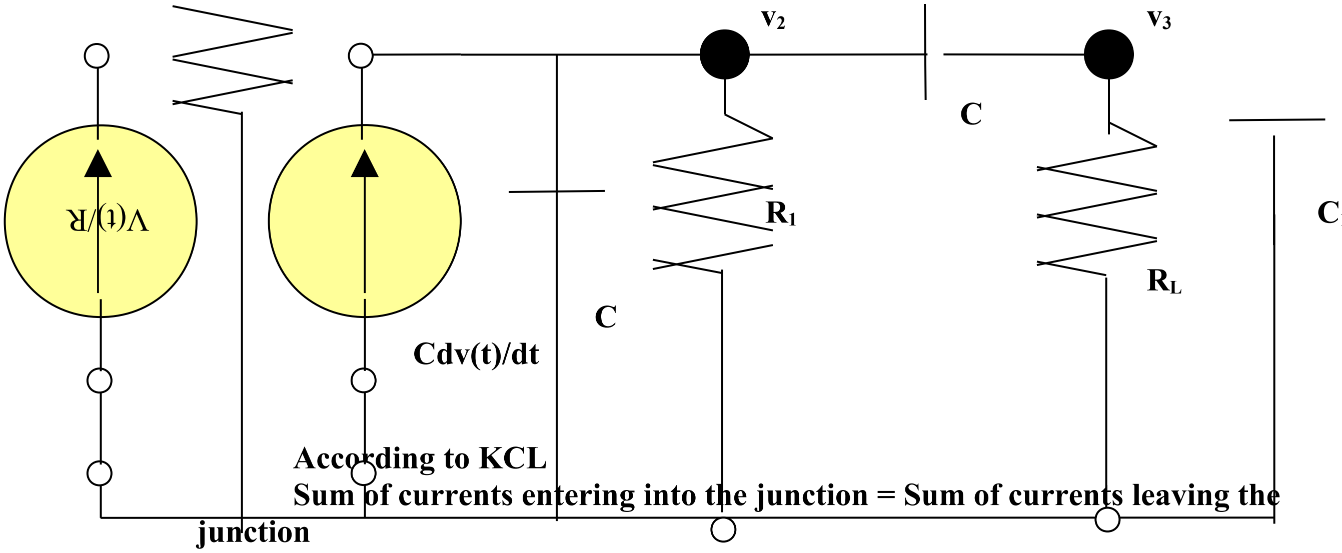
$- 1/L_1\int v_1 dt - c_1 dv_2/dt + c_1 dv_3/dt + 1/L_3\int v_3 dt + v_3/R_3 + 1/L_1\int v_3 dt = 0$

$$- 1/L_1\int v_1 dt - c_1 dv_2/dt + v_3(c_1 d/dt + 1/L_3\int dt + 1/R_3 + 1/L_1\int dt) = 0$$

(c)




-
-
+
+
V1
R
R



According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_1:

$v(t)/R = v_1/R + (v_1 - v_3)/R + c_1 dv_1/dt$

$v(t)/R = v_1/R + v_1/R - v_3/R + c_1 dv_1/dt$

$$v(t)/R = v_1(1/R + 1/R + c_1 d/dt) - v_3/R$$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_2:

$cdv(t)/dt = cdv_2/dt + v_2/R_1 + cd(v_2 - v_3)/dt$

$cdv(t)/dt = cdv_2/dt + v_2/R_1 + cdv_2/dt - cdv_3/dt$

$$cdv(t)/dt = v_2(cd/dt + 1/R_1 + cd/dt) - cdv_3/dt$$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_3:

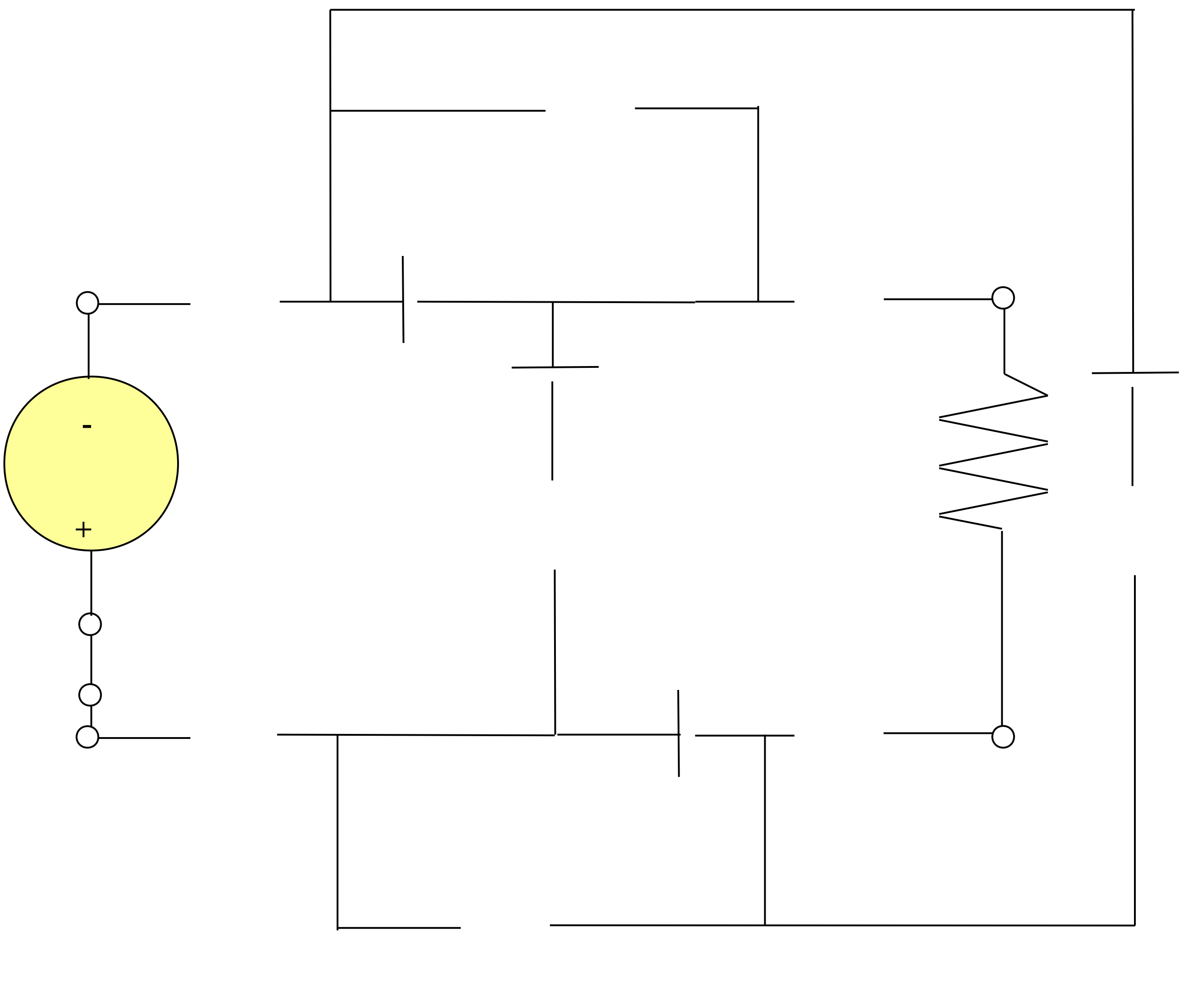
$0 = v_3/R_L + (v_3 - v_1)/R + cd(v_3 - v_2)/dt$

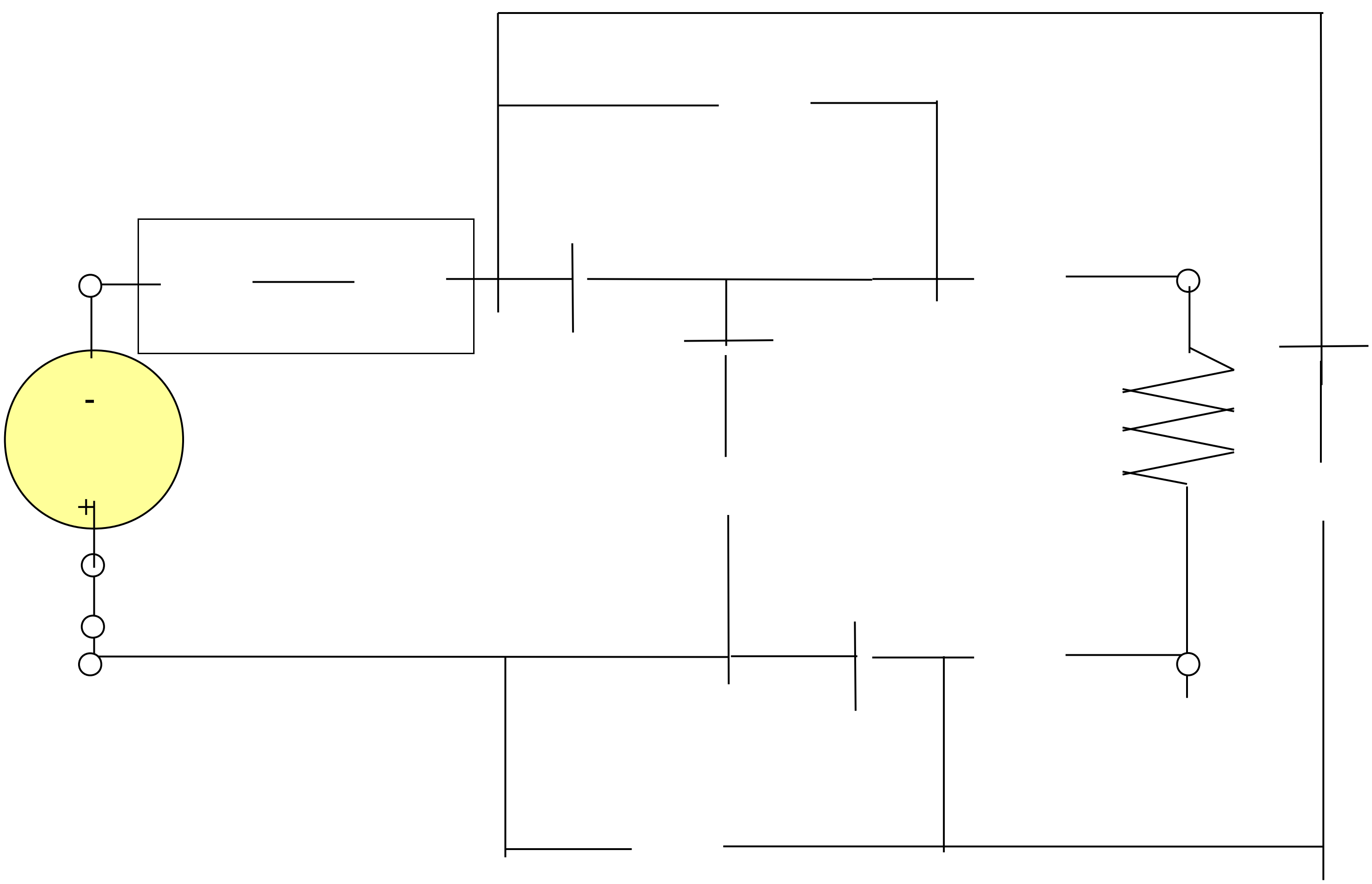
$0 = v_3/R_L + v_3/R - v_1/R + cdv_3/dt - cdv_2/dt$

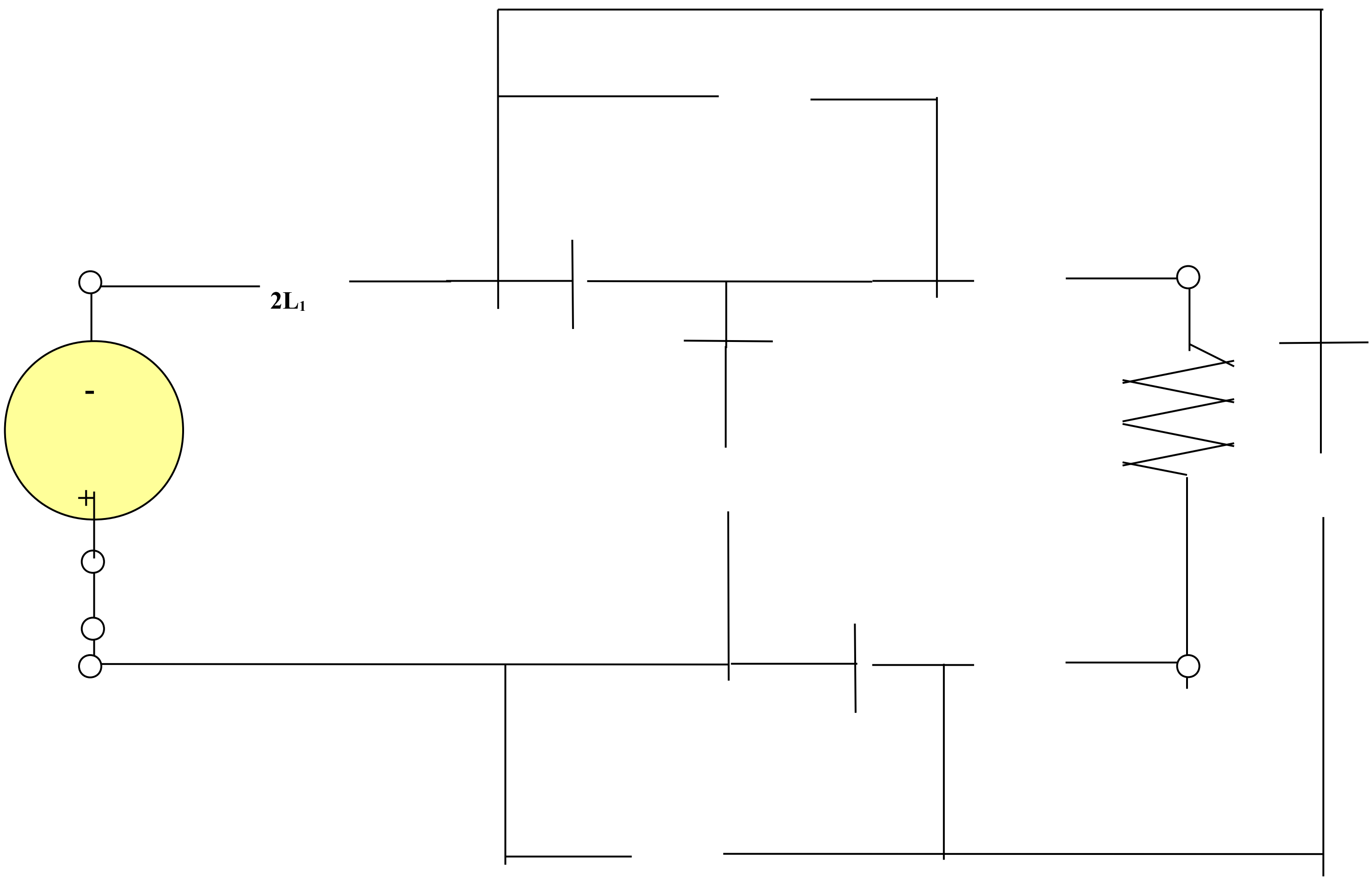
$0 = - v_1/R - cdv_2/dt + cdv_3/dt + v_3/R_L + v_3/R$

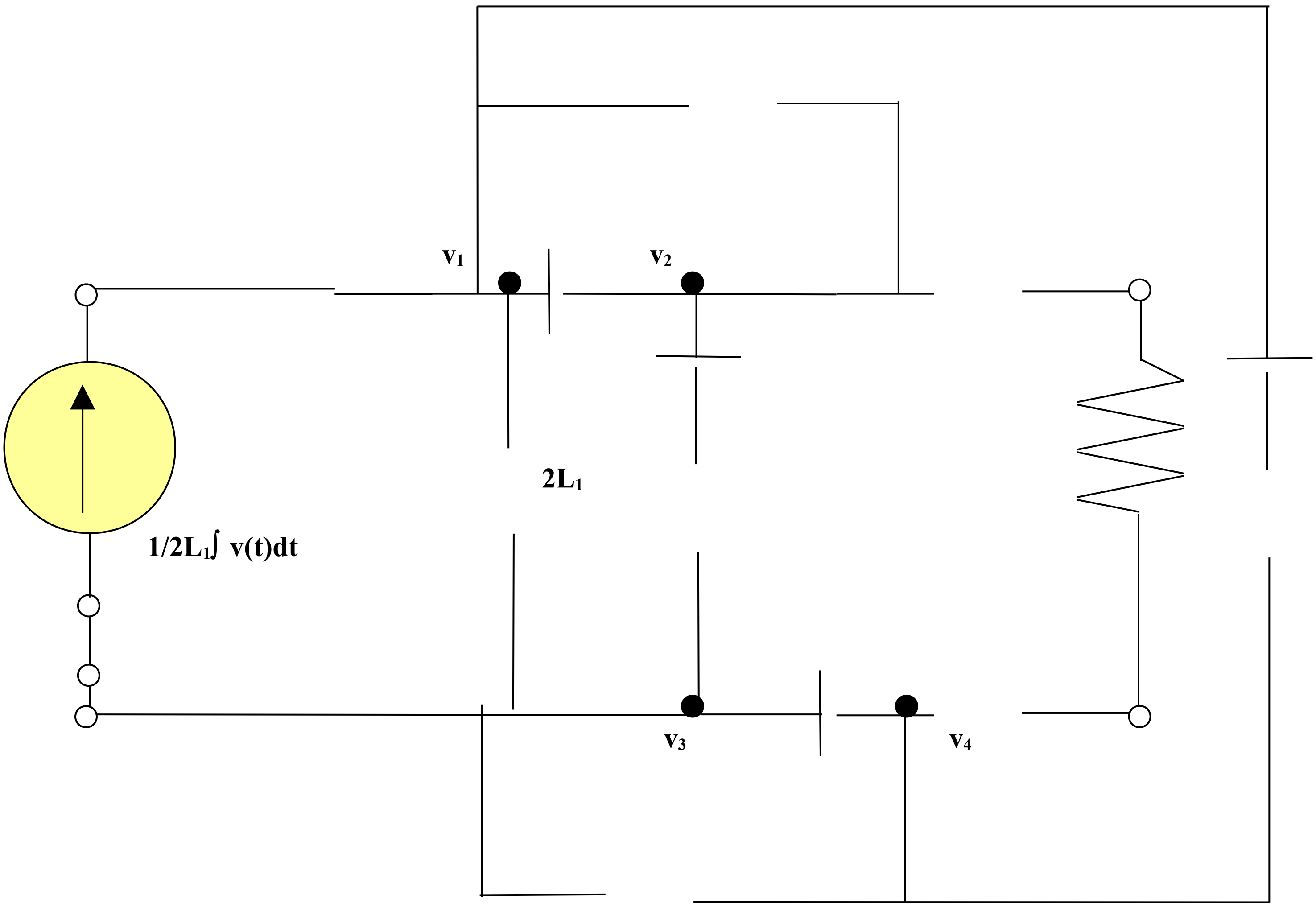
$$0 = - v_1/R - cdv_2/dt + v_3(cd/dt + 1/R_L + 1/R)$$

(d)






2L1
-
+



According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_1:

$1/2L_1\int v(t)dt = 1/2L_1\int (v_1 - v_3)dt + c_a d(v_1 - v_2)/dt + 1/L_a\int (v_1 - v_2)dt + 1/L_b\int (v_1 - v_4)dt + c_b d(v_1 - v_4)/dt$

According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

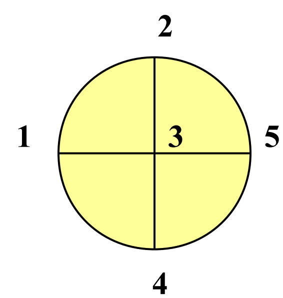
Node-v_2:

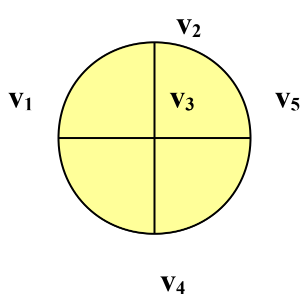
$c_a d(v_2 - v_1)/dt + c_b d(v_2 - v_3)/dt + 1/L_b \int (v_2 - v_3)dt + 1/2L \int (v_2 - v_4)dt + R(v_2 - v_4) = 0$

According to KCL
Sum of currents entering into the junction = Sum of currents leaving the junction
Node-v_3:
$c_a d(v_3 - v_4)/dt + 1/L_b \int (v_3 - v_2)dt + cd(v_3 - v_2)/dt + 1/2L_1 \int (v_3 - v_1)dt = 0$
Node-v_4:

$1/L_a \int (v_4 - v_3)dt + c_a d(v_4 - v_3)/dt + 1/2L_2 \int (v_4 - v_2)dt + 1/L_b \int (v_4 - v_1)dt + c_b d(v_4 - v_1)/dt = 0$

3-30. For the given network, write the node-basis equations using the node-to-datum voltages as variables.





According to KCL
Sum of currents entering into the junction = Sum of currents leaving the junction
Node-v_1:
$(v_1 - v_2)/(1/2) + (1/2)d(v_1 - v_3)/dt + (v_1 - v_4)/(1/2) = 0$

$(v_1 - v_2)/(2) + (2)d(v_1 - v_3)/dt + (v_1 - v_4)/(2) = 0$

Node-v_2:
$i_2 = (v_2 - v_1)/(1/2) + (v_2 - 0)/(1/2)$

$i_2 = (v_2 - v_1)/2 + v_2/2$

Node-v_3:
$i_2 = (1/2)d(v_3 - v_4)/dt + (1/2)d(v_3 - v_1)/dt + (1/2)d(v_3 - 0)/dt$

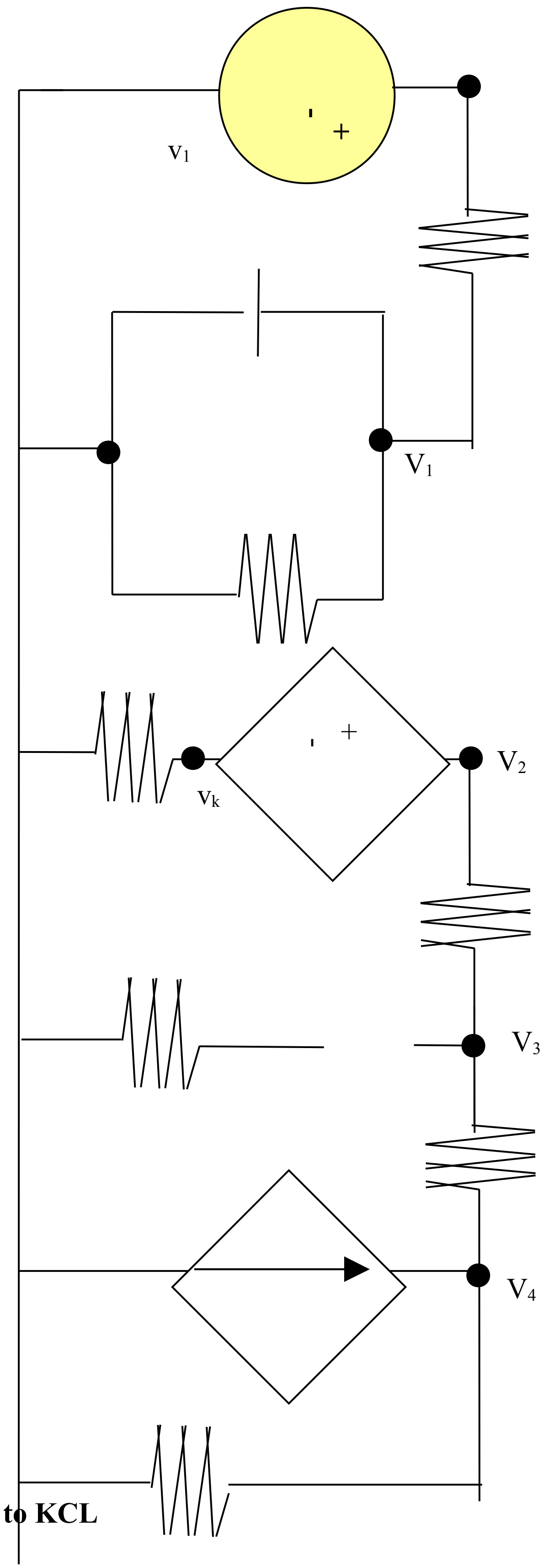
$$i_2 = (2)d(v_3 – v_4)/dt + (2)d(v_3 – v_1)/dt + (2)dv_3 /dt$$

Node-v_4:

$$0 = (1/2)d(v_4 – v_3)/dt + (v_4 – 0)/(1/2) + (v_4 – v_1)/(1/2)$$

$$0 = (2)d(v_4 – v_3)/dt + (v_4)/(2) + (v_4 – v_1)/(2)$$

3-31. The network in the figure contains one independent voltage source and two controlled sources. Using the KCL, write node-basis equations.



According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-V_1:

$$(V_1 - v_1)/R_1 + C_1 dV_1/dt + V_1/R_2 = 0$$

Node-V_2 & v_k:

$$v_k - V_2 = \mu\ (v_1 - v_k)$$

Node-V_2:

$$(V_3 - V_2)/R_3 + V_3/R + 1/L\int v_4 dt + (V_3 - V_4)/R_5 = 0$$

Node-V_4:

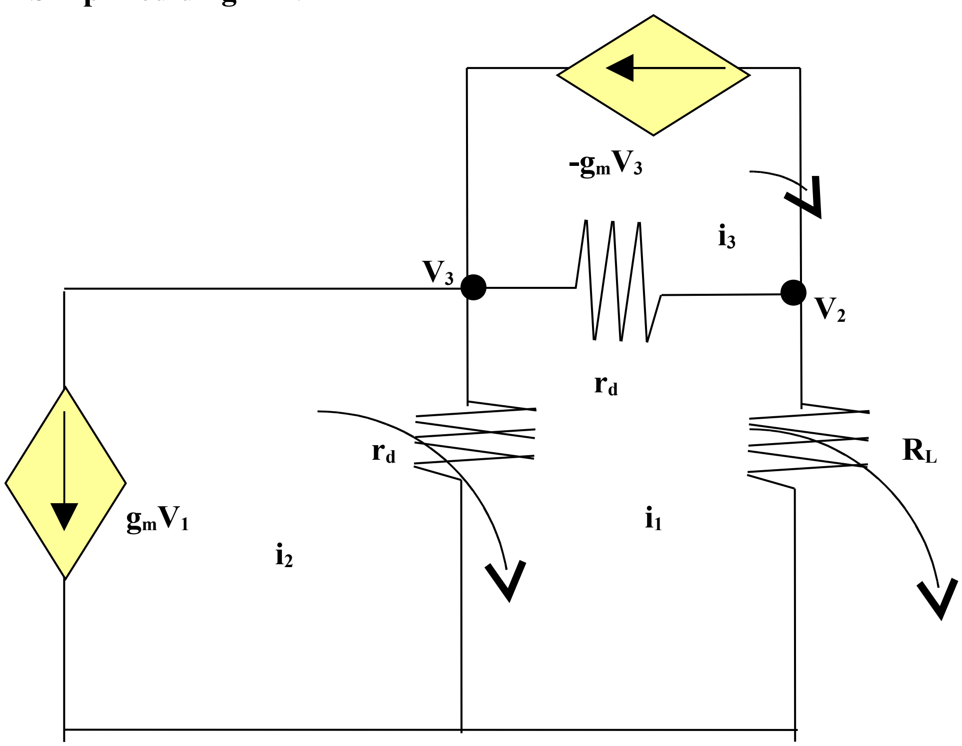
$$(V_4 - V_3)/R_5 + V_4/R_6 = \alpha\ i_2 \{\text{where } i_2 = V_4/R_6\}$$

3-32. The network of the figure is a model suitable for "midband" operation of the "cascode-connected" MOS transistor amplifier.

Solution:

Open your book & see (P/93).

Simplified diagram:



Loop-basis:

$i_2 = -g_m V_1$

$i_3 = g_m V_3$

i_1: $(i_1 - i_3)r_d + i_1 R_L - V_3 = 0$

$(i_1 - i_3)r_d + i_1R_L = V_3$
$i_1r_d - i_3r_d + i_1R_L = V_3$

$$\begin{array}{ccc} i_1 & i_2 & i_3 \end{array}$$
$$\begin{pmatrix} (r_d + R_L) & 0 & -r_d \\ 0 & -g_mV_1 & 0 \\ 0 & 0 & g_mV_3 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} V_3 \\ 0 \\ 0 \end{pmatrix}$$

Node-basis:

Node-V_3:

$g_mV_1 = V_3/r_d - g_mV_3 + (V_3 - V_2)/r_d$; $g_mV_1 + g_mV_3 = V_3/r_d + V_3/r_d - V_2/r_d$

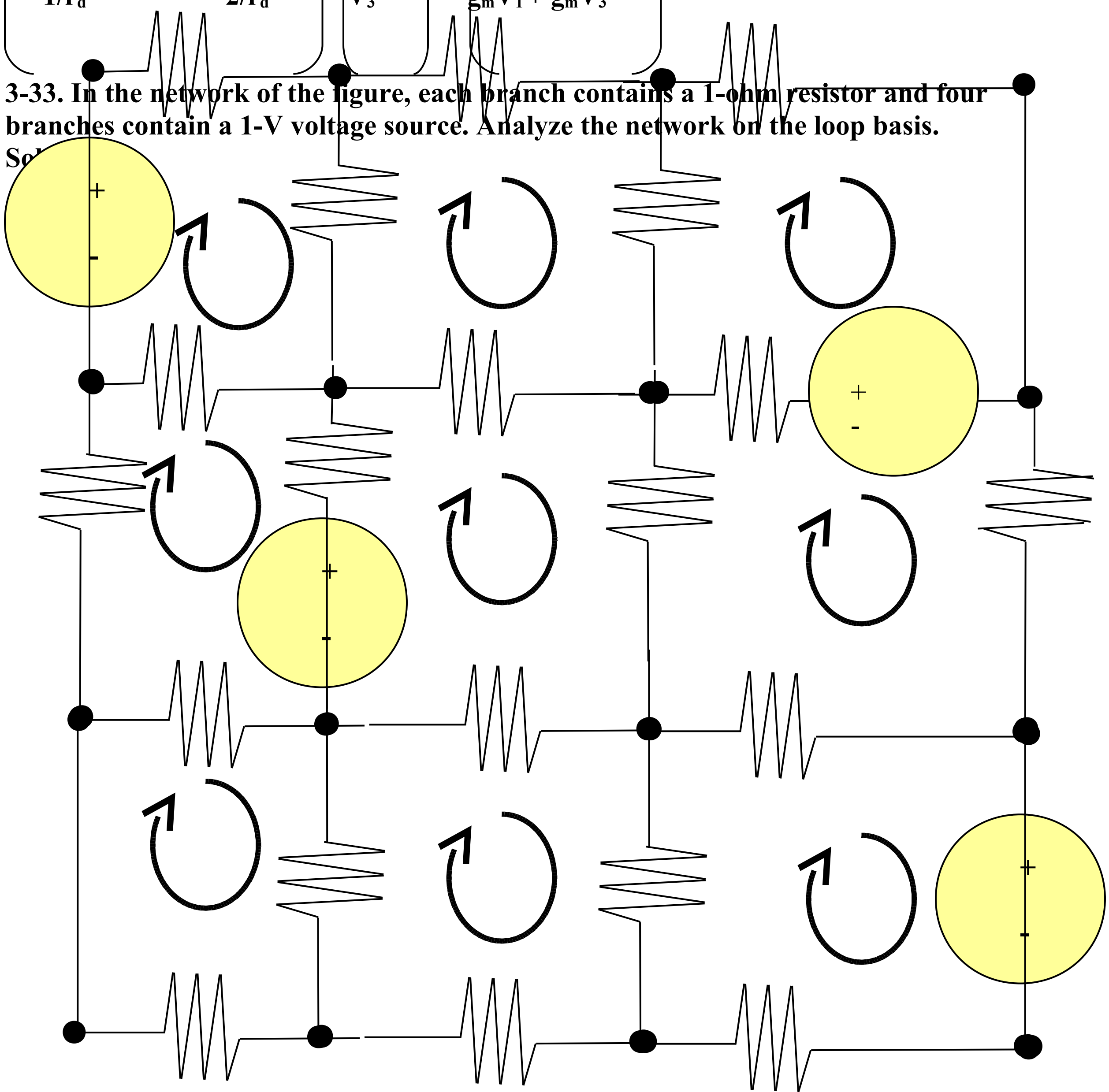
Node-V_2:

$- g_mV_3 = V_2/R_L + (V_2 - V_3)/r_d$; $- g_mV_3 = V_2/R_L + V_2/r_d - V_3/r_d$

$$\begin{array}{cc} V_2 & V_3 \end{array}$$
$$\begin{pmatrix} (1/R_L + 1/r_d) & -1/r_d \\ -1/r_d & 2/r_d \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} \begin{pmatrix} - g_mV_3 \\ g_mV_1 + g_mV_3 \end{pmatrix}$$

3-33. In the network of the figure, each branch contains a 1-ohm resistor and four branches contain a 1-V voltage source. Analyze the network on the loop basis.

So



1 **4** **7**

2 5 8

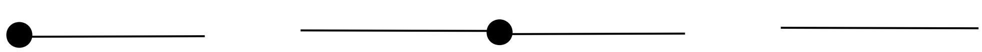
3 6 9

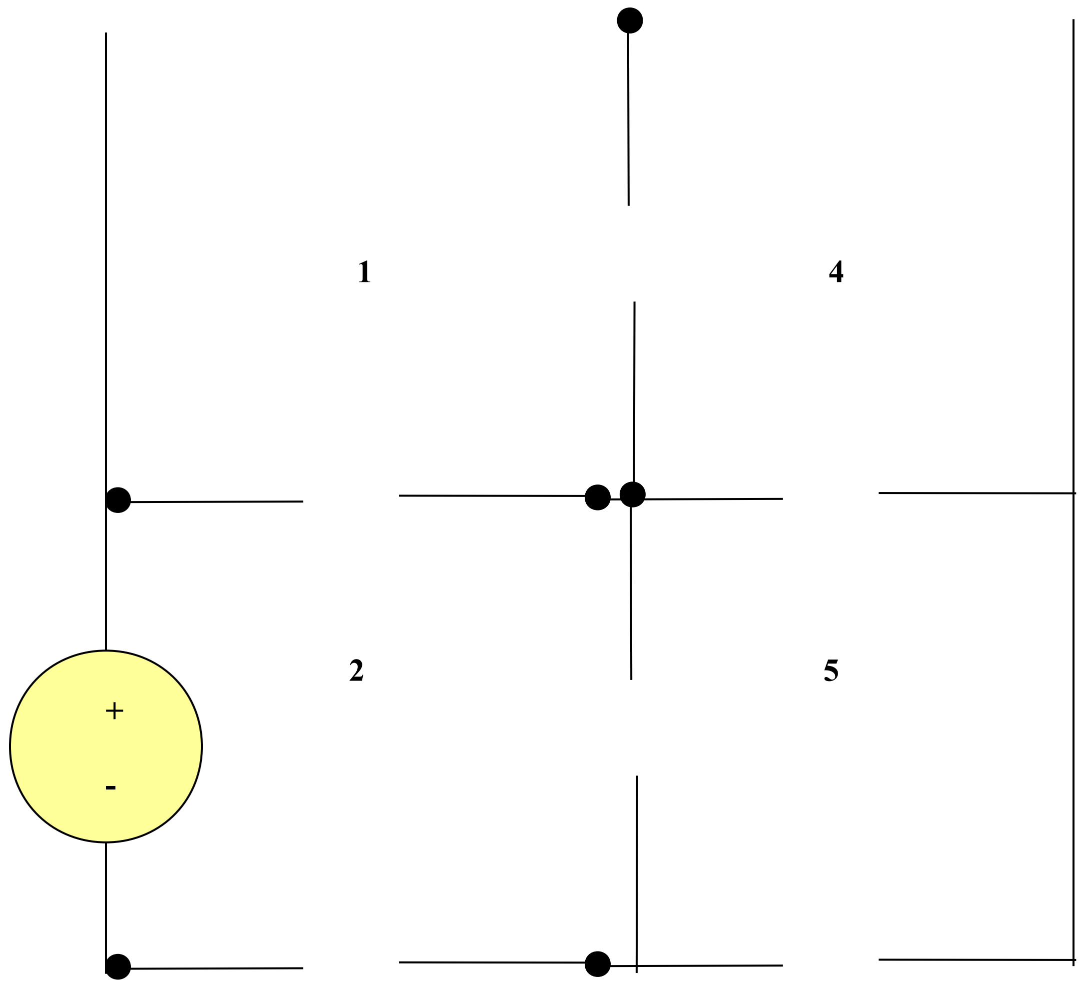
Eq.	**Voltage**	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9

1	**1**	**3**	**-1**	**0**	**-1**	**0**	**0**	**0**	**0**	**0**
2	**-1**	**-1**	**4**	**-1**	**0**	**-1**	**0**	**0**	**0**	**0**
3	**0**	**0**	**-1**	**3**	**0**	**0**	**-1**	**0**	**0**	**0**
4	**0**	**-1**	**0**	**0**	**4**	**-1**	**0**	**-1**	**0**	**0**
5	**1**	**0**	**-1**	**0**	**-1**	**4**	**-1**	**0**	**-1**	**0**
6	**0**	**0**	**0**	**-1**	**0**	**-1**	**4**	**0**	**0**	**-1**
7	**1**	**0**	**0**	**0**	**-1**	**0**	**0**	**3**	**-1**	**0**
8	**-1**	**0**	**0**	**0**	**0**	**-1**	**0**	**-1**	**4**	**-1**
9	**-1**	**0**	**0**	**0**	**0**	**0**	**-1**	**0**	**-1**	**3**

3-34. Write equations on the node basis.

<u>Repeat Prob. 3-33 for the network.</u>

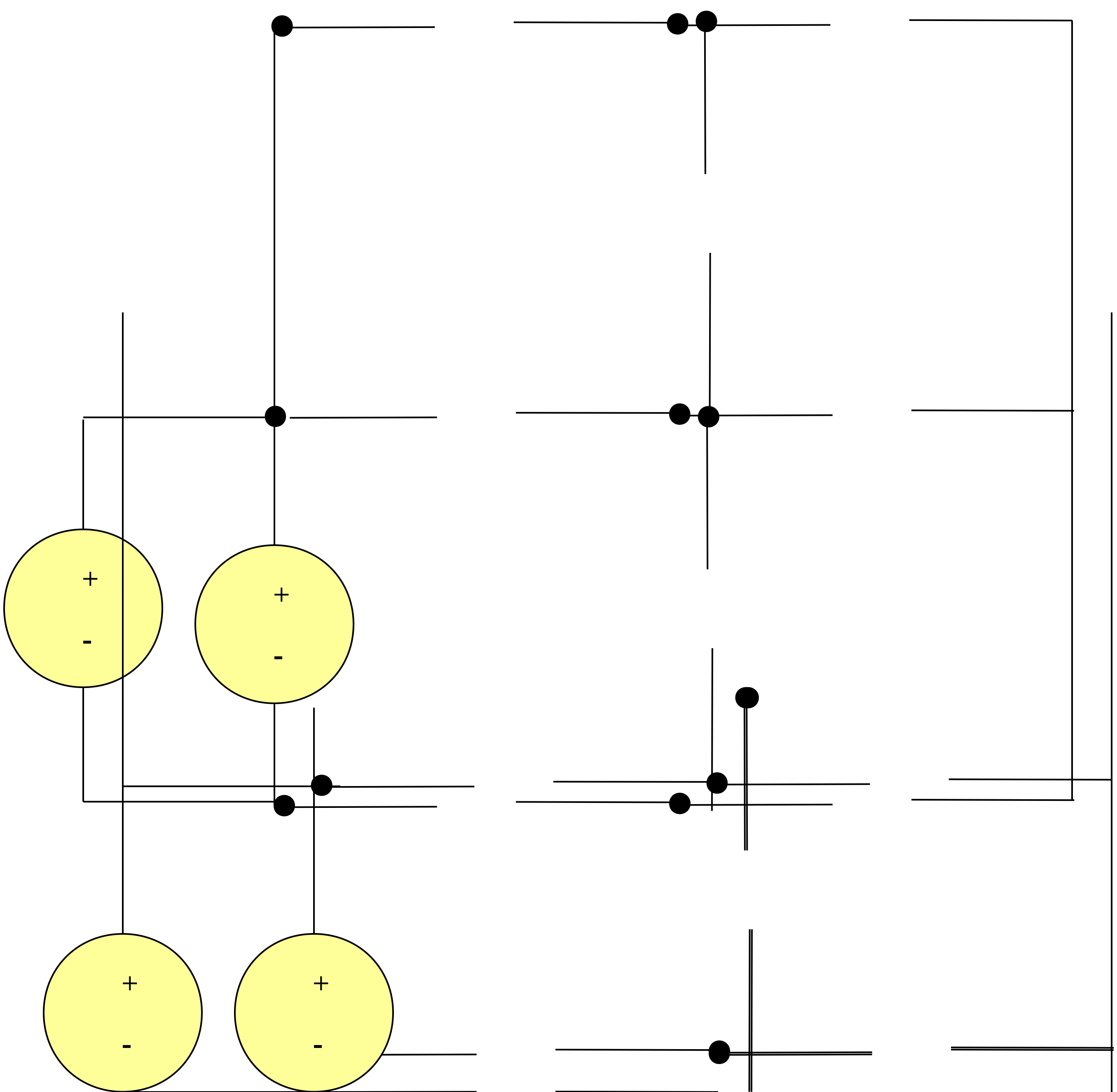


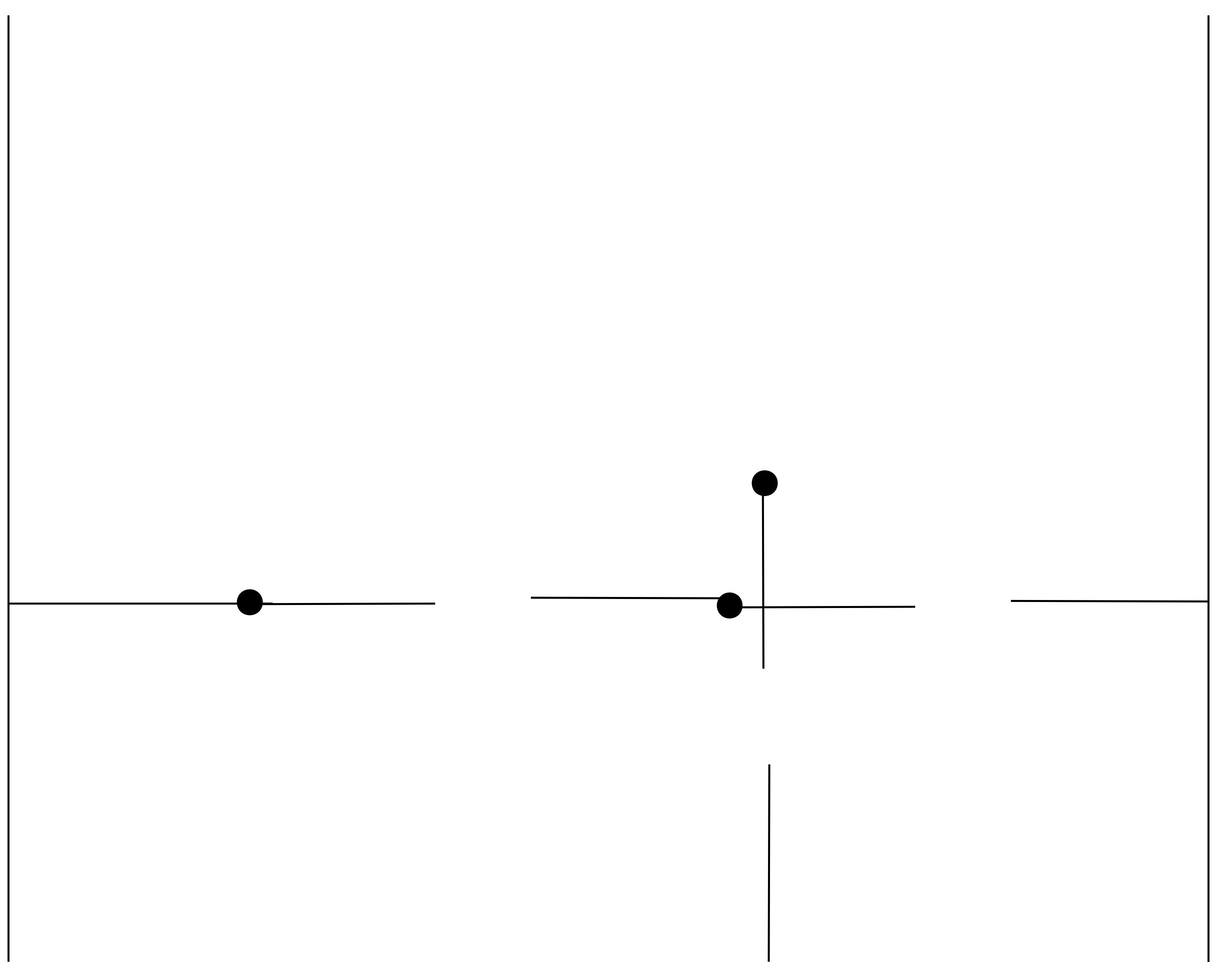


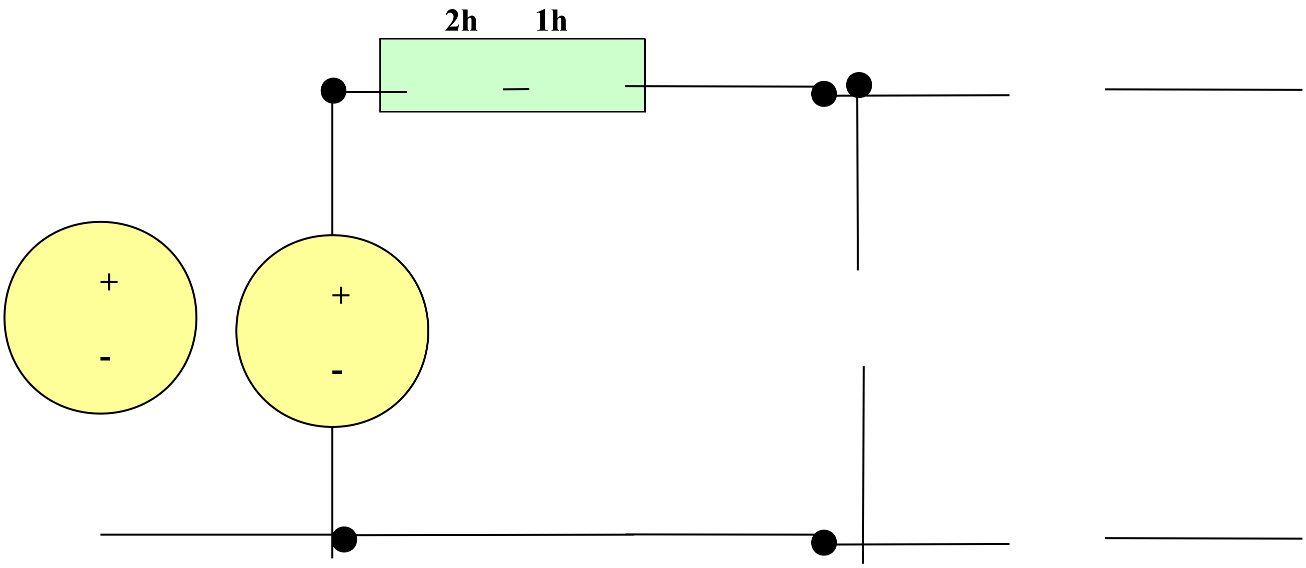
Coefficients of

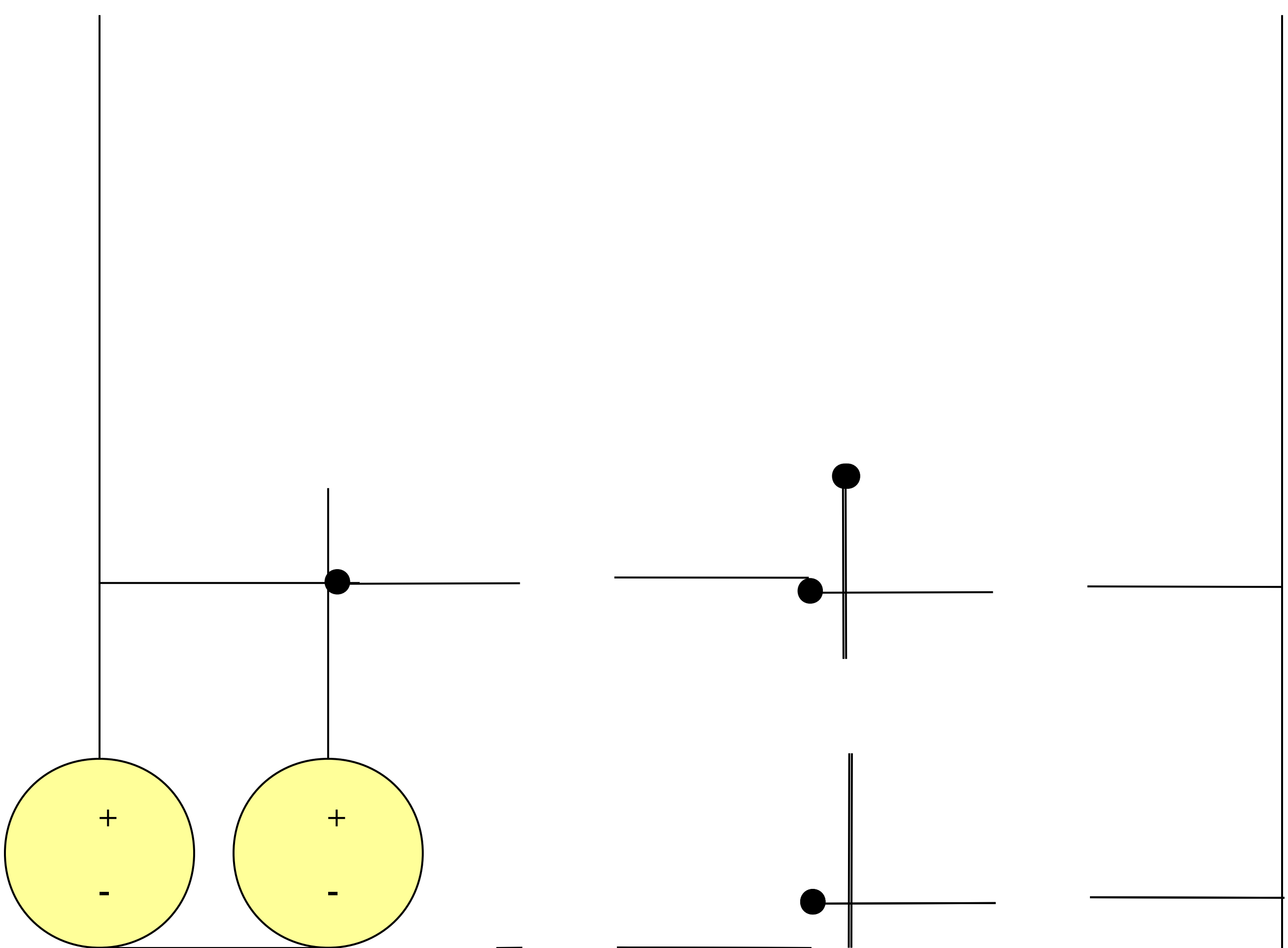
Eq.	Voltage	di_1/dt	di_2/dt	di_3/dt	di_4/dt

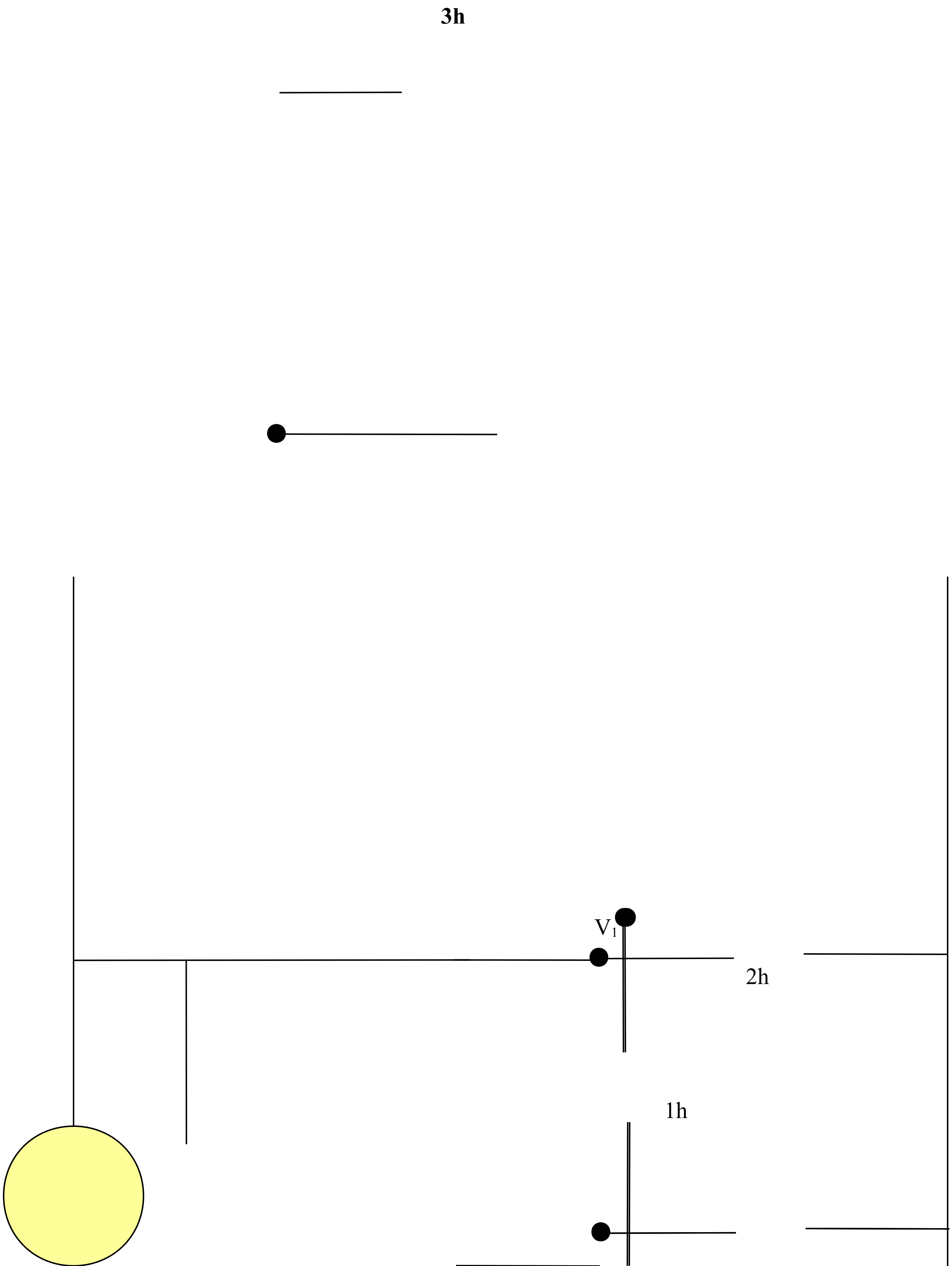
1	**0**	**4**	**-1**	**-1**	**0**
2	**1**	**-1**	**4**	**0**	**-1**
3	**0**	**-1**	**0**	**4**	**-1**
4	**0**	**0**	**-1**	**-1**	**4**

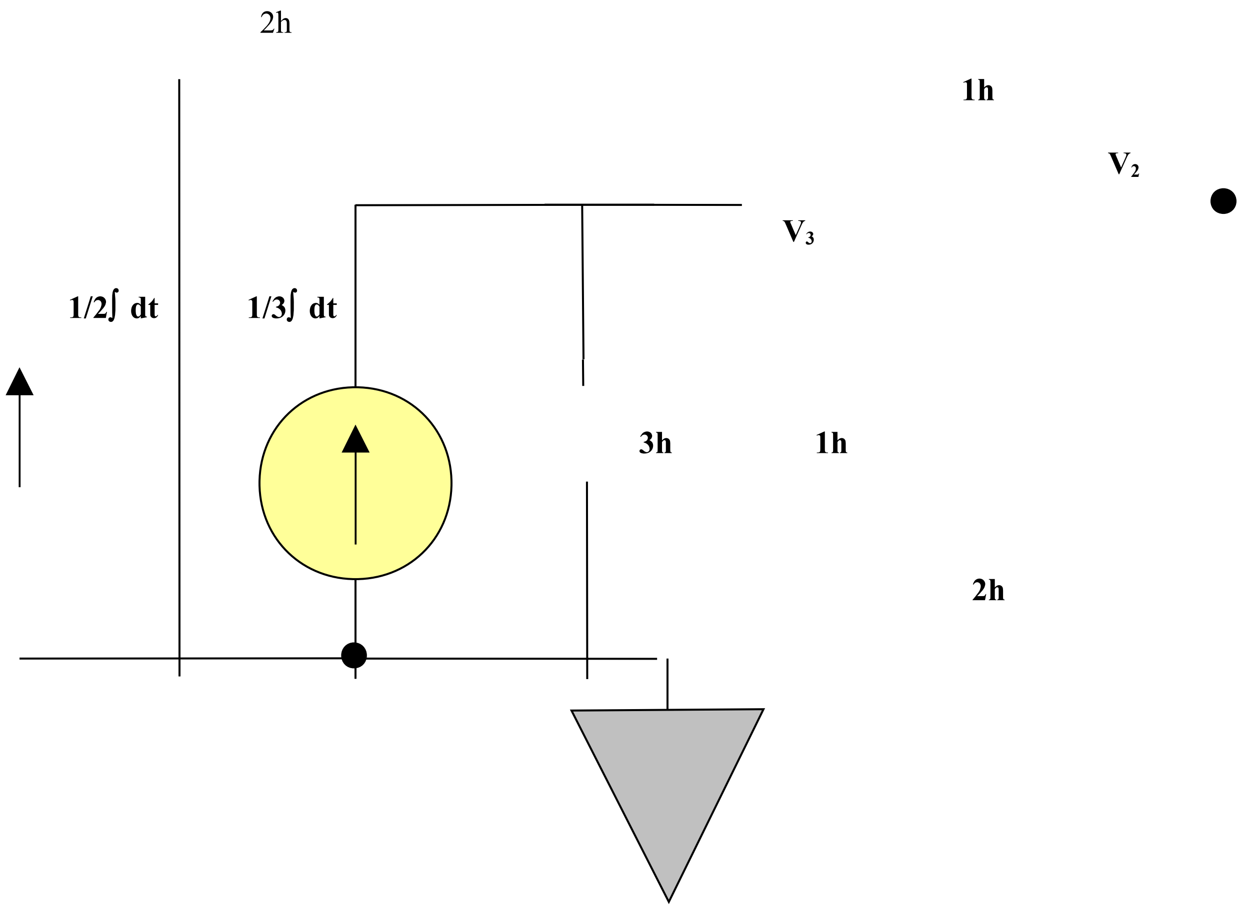





2h
1h
+
-
+
-


+
-
+
-


3h
V_1
2h
1h



Node-V_1

$1/2\int dt = 1/2\int V_1dt + \int (V_1 - V_3)dt + 1/2\int (V_1 - V_2)dt$

$1/2\int dt = 1/2\int V_1dt + \int V_1dt - \int V_3dt + 1/2\int V_1dt - 1/2\int V_2dt$

Node-V_2

$0 = 1/2\int V_2dt + \int (V_2 - V_3)dt + 1/2\int (V_2 - V_1)dt$

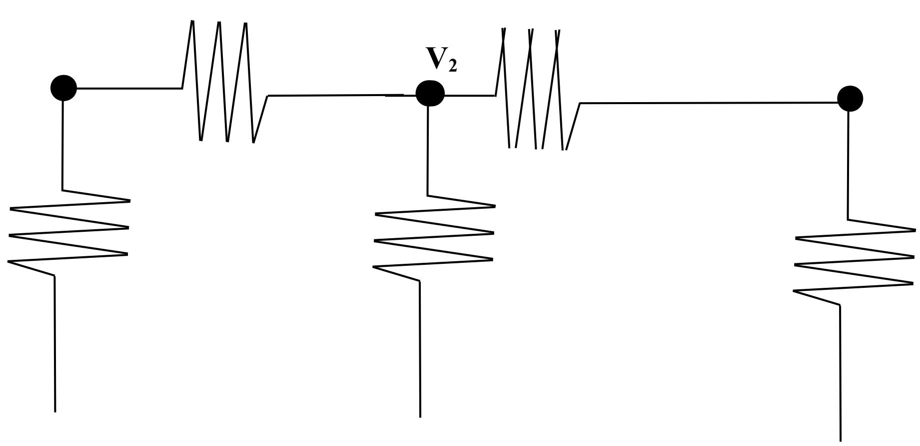
$0 = 1/2\int V_2dt + \int V_2dt - \int V_3dt + 1/2\int V_2dt - 1/2\int V_1dt$

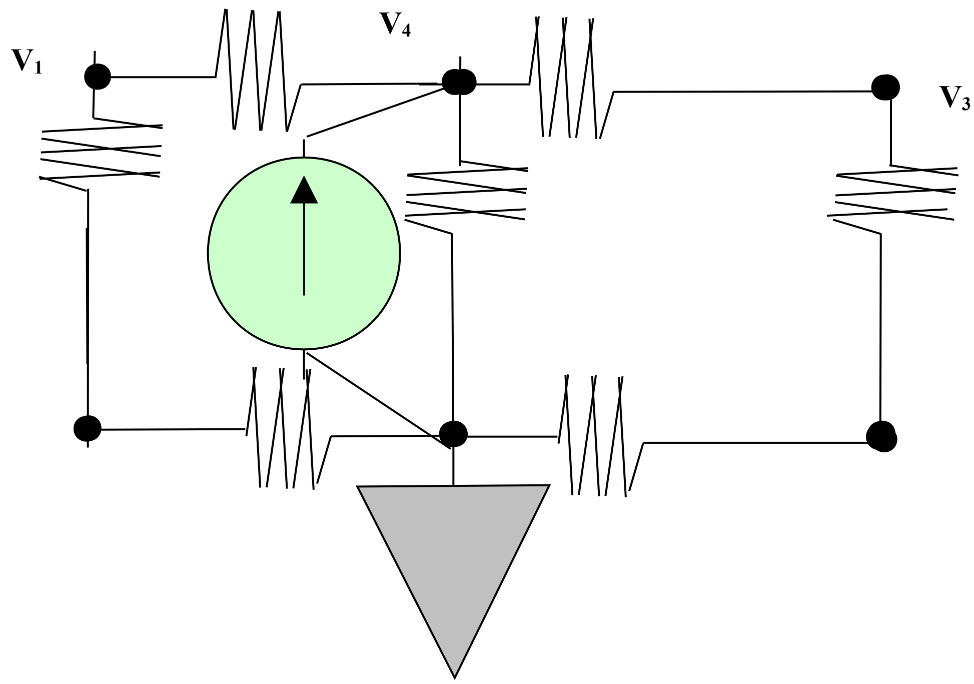
$1/3\int dt = 1/3\int V_3dt + \int V_3dt + \int (V_3 - V_1)dt + \int (V_3 - V_2)dt$

$1/3\int dt = 1/3\int V_3dt + \int V_3dt + \int V_3dt - \int V_1dt + \int V_3dt - \int V_2dt$

Eq.	Current	V_1	V_2	V_3

1	**½∫ dt**	**2∫ dt**	**-1/2∫ dt**	**-∫ dt**
2	**0**	**-1/2∫ dt**	**2∫ dt**	**-∫ dt**
3	**1/3∫ dt**	**-∫ dt**	**-∫ dt**	**4∫ dt**





<u>Node-V1:</u>

$V_1/4 + (V_1 - V_4)/1 + (V_1 - V_2)/4 = 0$
$V_1/4 + (V_1 - V_4) + (V_1 - V_2)/4 = 0$
$V_1/4 + V_1 - V_4 + V_1/4 - V_2/4 = 0$
$2V_1/4 + V_1 - V_4 - V_2/4 = 0$
$V_1/2 + V_1 - V_4 - V_2/4 = 0$

$$\boxed{1.5V_1 - V_4 - V_2/4 = 0}$$

<u>Node-V2:</u>

$(V_2 - V_4)/1 + (V_2 - V_3)/4 + (V_2 - V_1)/4 = 0$
$V_2 - V_4 + V_2/4 - V_3/4 + V_2/4 - V_1/4 = 0$

$$\boxed{1.5V_2 - V_4 - V_3/4 - V_1/4 = 0}$$

<u>Node-V3:</u>

$(V_3 - V_2)/4 + (V_3 - 0)/4 + (V_3 - V_4)/1 = 0$
$V_3/4 - V_2/4 + V_3/4 + V_3 - V_4 = 0$

$$\boxed{1.5V_3 - V_2/4 - V_4 = 0}$$

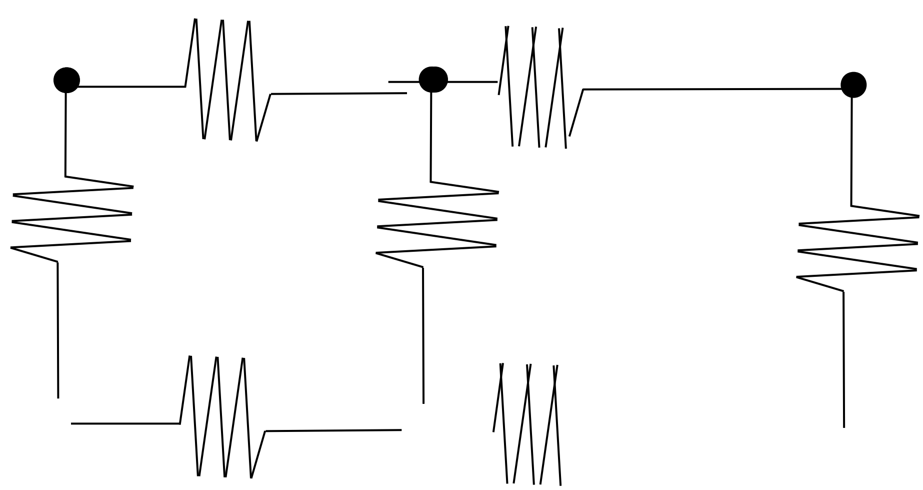
<u>Node-V4:</u>

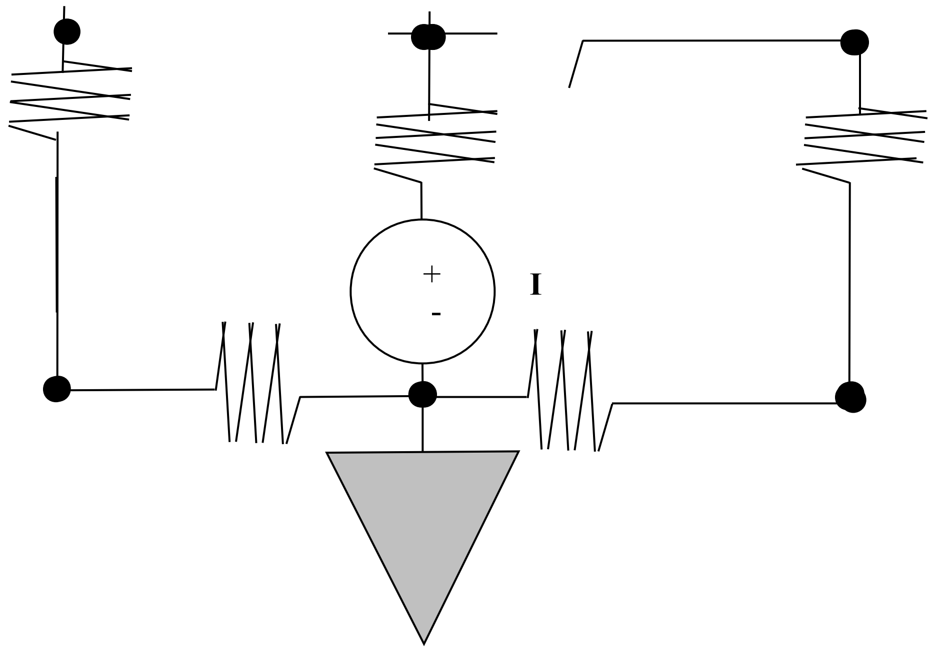
$(V_4 - V_1)/1 + (V_4 - 0)/1 + (V_4 - V_3)/1 + (V_4 - V_2)/1 = I$
$V_4 - V_1 + V_4 + V_4 - V_3 + V_4 - V_2 = I$

$$\boxed{4V_4 - V_1 - V_2 - V_3 = I}$$

Eq.	**Current**	**V_1**	**V_2**	**V_3**	**V_4**
1	**0**	**1.5**	**-0.25**	**0**	**-1**
2	**0**	**-0.25**	**1.5**	**-0.25**	**-1**
3	**0**	**0**	**-0.25**	**1.5**	**-1**
4	**I**	**-1**	**-1**	**-1**	**4**

Loop-basis:

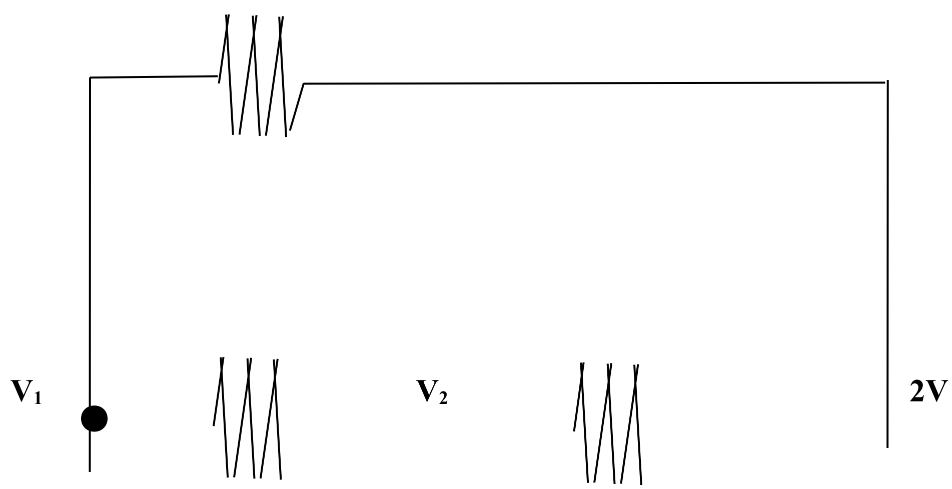


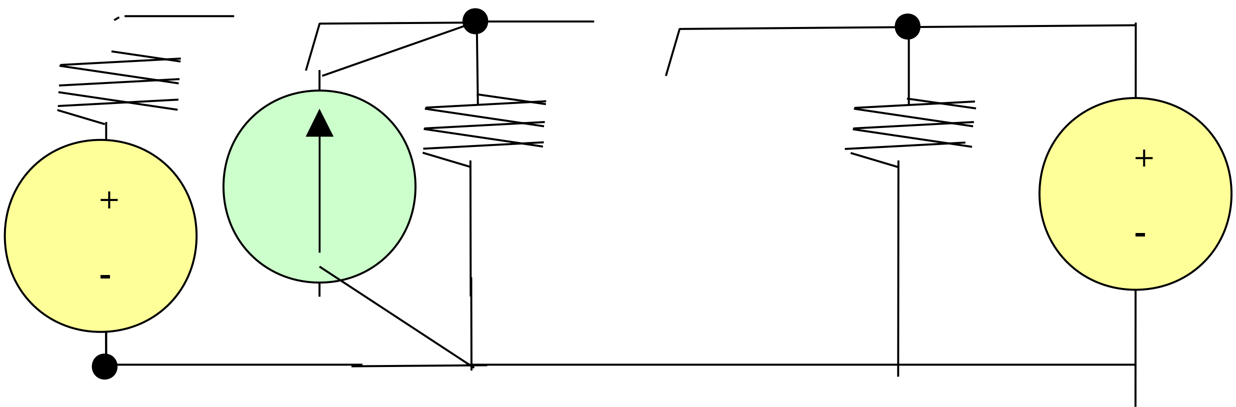


Eq.	**Voltage**	**V_1**	**V_2**	**V_3**	**V_4**
1	**0**	**6**	**-1**	**-1**	**0**
2	**0**	**-1**	**6**	**0**	**-1**

3	**-I**	**-1**	**0**	**6**	**-1**
4	**I**	**0**	**-1**	**-1**	**6**

3-36. For the network shown in the figure, determine the numerical value of the branch current i_1. All sources in the network are time invariant.





$$(V_1 - 2)/(1/2) + (V_1 - V_2)/1 + (V_1 - 2)/2 = 0$$
$$(V_1 - 2)2 + (V_1 - V_2) + (V_1 - 2)/2 = 0$$
$$2V_1 - 4 + V_1 - V_2 + V_1/2 - 1 = 0$$

$$3.5V_1 - V_2 - 5 = 0 \qquad \text{(i)}$$

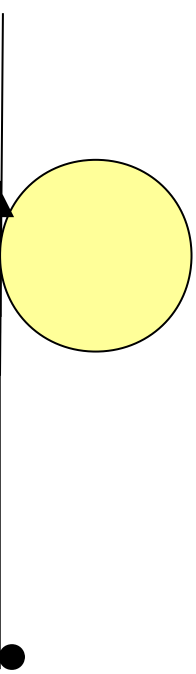
$$V_2/(1/2) + (V_2 - V_1)/1 + (V_2 - 2)/1 = 1$$
$$2V_2 + V_2 - V_1 + V_2 - 2 = 1$$
$$4V_2 - V_1 = 3$$

$$V_1 = -3 + 4V_2 \qquad \text{(ii)}$$

Put V_1 in (i)

$$3.5V_1 - V_2 - 5 = 0$$
$$3.5(-3 + 4V_2) - V_2 - 5 = 0$$
$$-10.5 + 14V_2 - V_2 - 5 = 0$$
$$13V_2 - 15.5 = 0$$
$$13V_2 = 15.5$$

$$V_2 = 15.5/13 = 1.192\text{Volts}$$



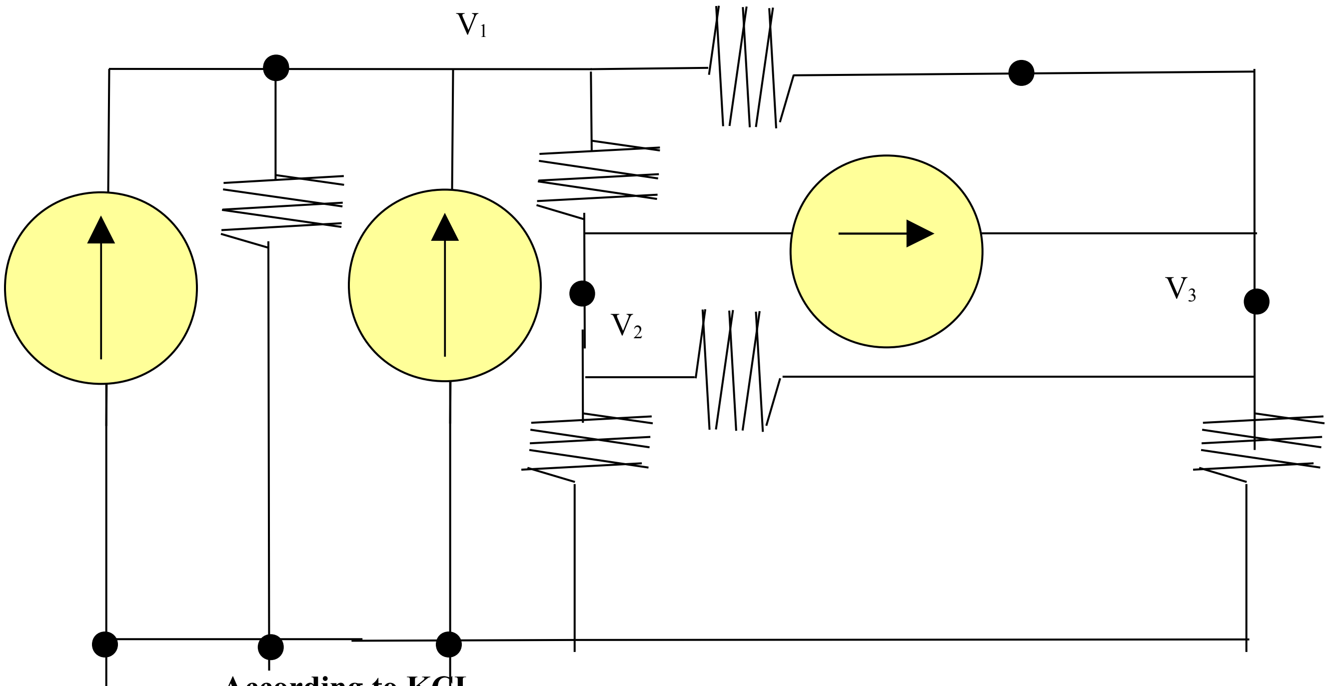
Put value of V_2 in (ii)

$$V_1 = -3 + 4V_2 \qquad \text{(ii)}$$
$$V_1 = -3 + 4(1.192)$$
$$V_1 = -3 + 4(1.192)$$

$$V_1 = 1.768\text{Volts}$$

$$i_1 = (V_1 - V_2)/1 = V_1 - V_2 = 1.768 \text{ Volts} - 1.192 \text{ Volts} = 0.576 \text{ amperes.}$$

3-37. In the network of the figure, all sources are time invariant. Determine the numerical value of i_2.



According to KCL

Sum of currents entering into the junction = Sum of currents leaving the junction

Node-v_1:

$2 + 1 = V_1/1 + (V_1 - V_2)/1 + (V_1 - V_3)/1$

$\mathbf{3 = V_1 + (V_1 - V_2) + (V_1 - V_3)}$

$\mathbf{3 = V_1 + V_1 - V_2 + V_1 - V_3}$

$\boxed{\mathbf{3 = 3V_1 - V_2 - V_3}}$ **(i)**

<u>Node-v_2:</u>

$\mathbf{(V_2 - V_1)/1 + (V_2 - V_3)/2 + (V_2 - 0)/1 = 1}$

$\mathbf{V_2 - V_1 + V_2/2 - V_3/2 + V_2 = 1}$

$\mathbf{V_2 - V_1 + V_2/2 - V_3/2 + V_2 = 1}$

$\boxed{\mathbf{2.5V_2 - V_1 - 0.5V_3 = 1}}$ **(ii)**

<u>Node-v_3:</u>

$\mathbf{(V_3 - V_1)/1 + (V_3 - V_2)/2 + V_3/1 = 1}$

$\mathbf{V_3 - V_1 + V_3/2 - V_2/2 + V_3 = 1}$

$\boxed{\mathbf{2.5V_3 - V_1 - V_2/2 = 1}}$ **(iii)**

$3 = 3V_1 - V_2 - V_3$
$3V_1 = 3 + V_2 + V_3$
$V_1 = (3 + V_2 + V_3)/3$
$2.5V_3 - V_1 - V_2/2 = 1$
$2.5V_3 - ((3 + V_2 + V_3)/3) - V_2/2 = 1$
$2.5V_3 - (3/3 + V_2/3 + V_3/3) - V_2/2 = 1$
$2.5V_3 - 1 - V_2/3 - V_3/3 - V_2/2 = 1$
$2.5V_3 - V_2/3 - V_3/3 - V_2/2 = 2$
$2.5V_3 - 0.334V_2 - 0.334V_3 - 0.5V_2 = 2$
$2.166V_3 - 0.834V_2 = 2$
Subtracting (ii) & (iii)
$2.5V_2 - V_1 - 0.5V_3 = 1$ (ii)
$2.5V_3 - V_1 - V_2/2 = 1$ (iii)

$2.5V_2 - 2.5V_3 - 0.5V_3 + V_2/2 = 0$
$3V_2 - 3V_3 = 0$
$3V_2 = 3V_3$

$$\boxed{V_2 = V_3}$$

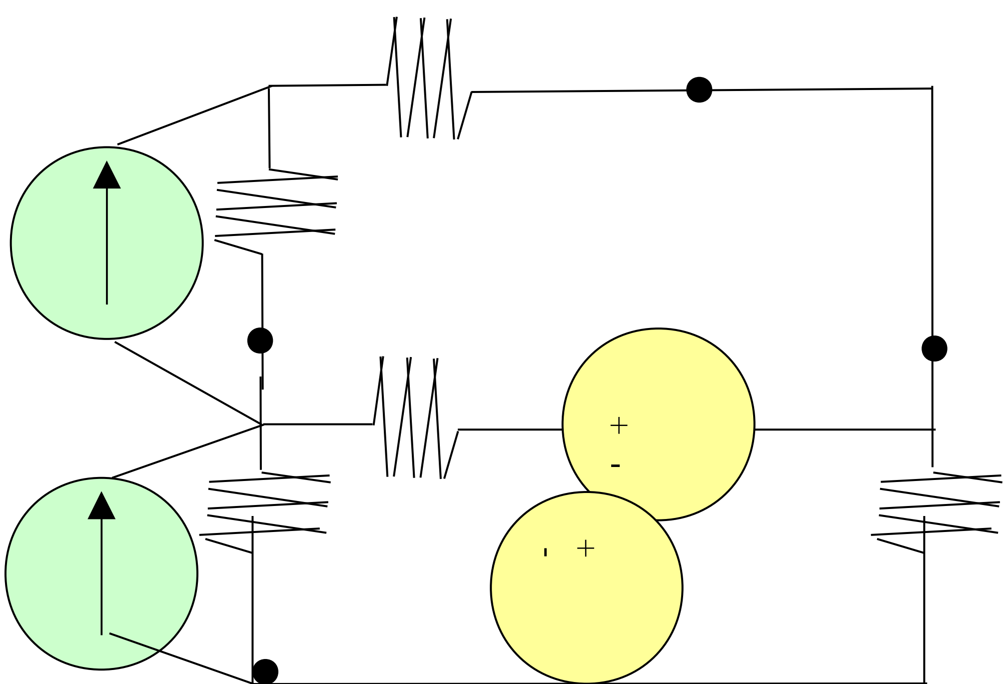
$2.166V_3 - 0.834V_2 = 2$
By putting $V_2 = V_3$
$2.166V_3 - 0.834V_3 = 2$
$1.332V_3 = 2$
$V_3 = 2/1.332 = 1.501$ V

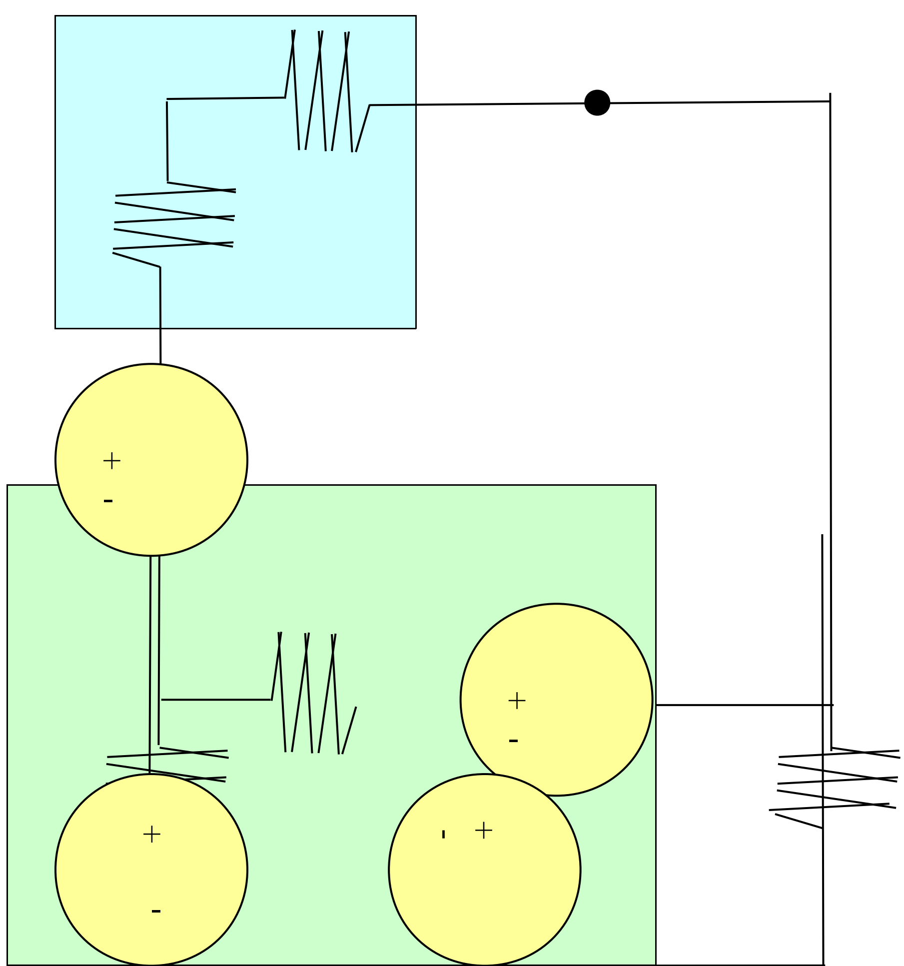
$$\boxed{V_3 = 1.501 \text{ V}}$$

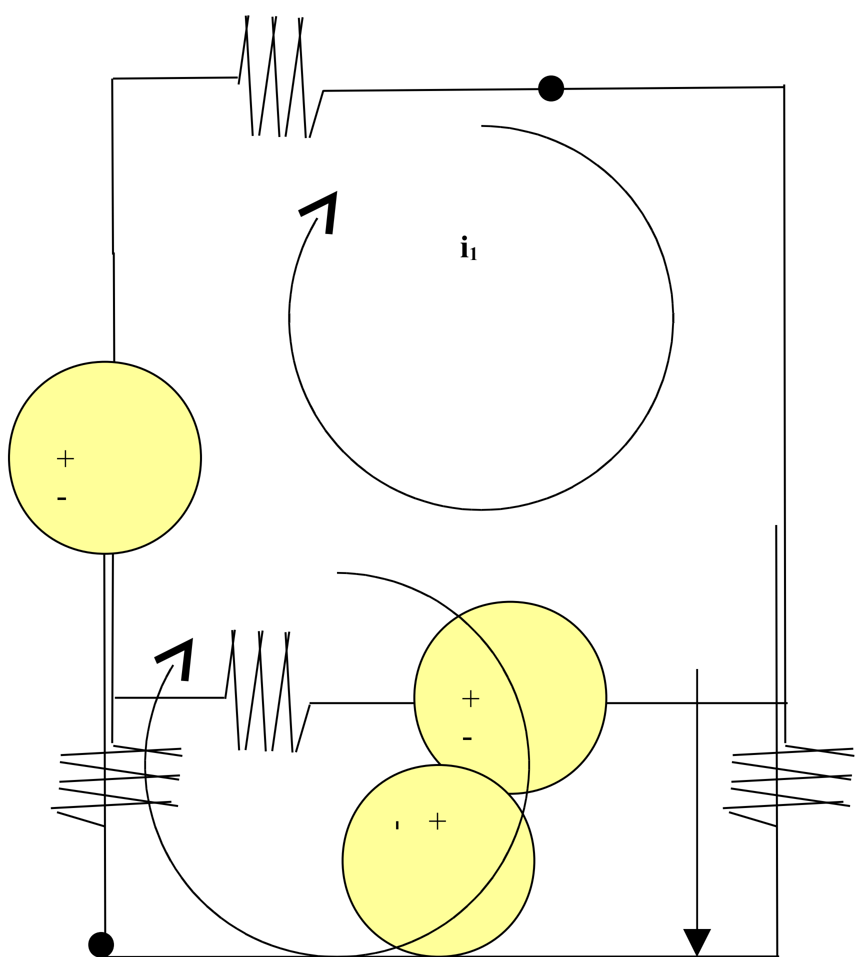
$i_2 = (2 - V_3)/2 = (2 - 1.501)/2 = 0.2495$ amperes.

$$\boxed{i_2 = 0.2495 \text{ amperes.}}$$

3-38. In the given network, all sources are time invariant. Determine the branch current in the 2 ohm resistor.







i_1
+
-
+
-
+

i_2

<u>Loop-basis:</u>

i_1:

According to kirchhoff's voltage law

Sum of potential rise = sum of potential drop

$(3/2)i_1 + 1(i_1 - i_2) = 2$

$(3/2)i_1 + i_1 - i_2 = 2$

$$\boxed{(5/2)i_1 - i_2 = 2}$$

i_2:

$(i_2 - i_1)1 + 2i_2 + i_2(1/2) = 2$

$i_2 - i_1 + 2i_2 + i_2(1/2) = 2$

$$\boxed{3.5i_2 - i_1 = 2}$$

$$\begin{pmatrix} 5/2 & -1 \\ -1 & 7/2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{vmatrix} 5/2 \quad (+) \quad (-) & -1 \\ -1 & 7/2 \end{vmatrix} = [(5/2)(7/2) - 1] = 7.75$$

$$\boxed{\text{Determinant} = 7.75}$$

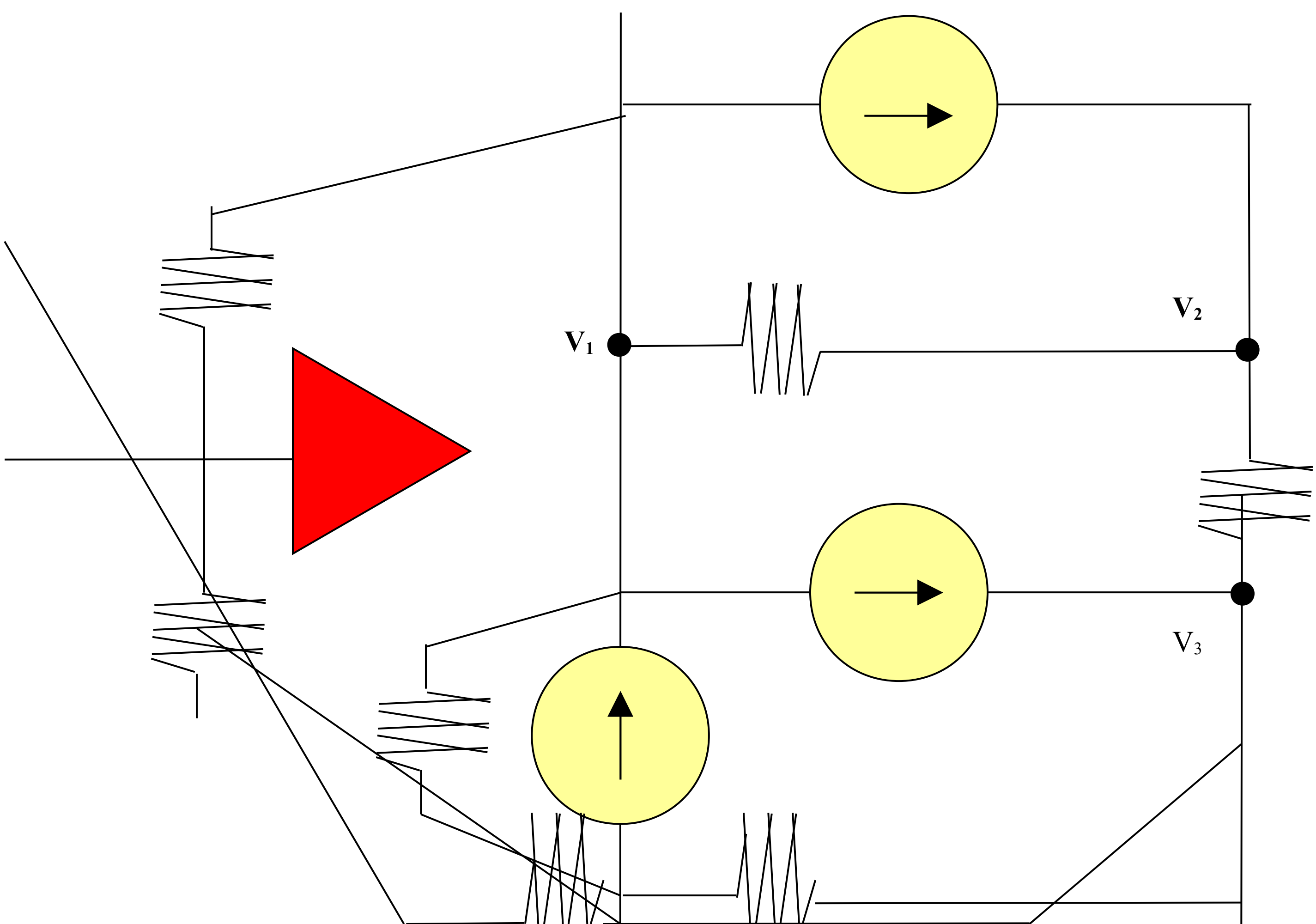
$$\begin{vmatrix} 5/2 \quad (+) \quad (-) & 2 \\ & \end{vmatrix}$$

= [(5/2)(2) + 2] = 7

-1 **2**

$\boxed{i_2 = 7/7.75 = 0.904 \text{ amperes.}}$ **Ans.**

3-39. Solve for the four node-to-datum voltages.



V_4

Node-V_1:

$(V_1 – V_2)/(1/2) + (V_1 – V_4)/(1/2) + (V_1 - 0)/(1/2) + 2 = 8$

$2(V_1 – V_2) + 2(V_1 – V_4) + 2V_1 + 2 = 8$

Node-V_2:

$(V_2 – V_1)/(1/2) + (V_2 – V_3)/(1/2) = 6$

$2(V_2 – V_1) + 2(V_2 – V_3) = 6$

Node-V_3:

$(V_3 – V_4)/(1/2) – (V_3 - 0)/(1/2) + (V_3 – V_2)/(1/2) = 2$

$2(V_3 – V_4) – 2V_3 + 2(V_3 – V_2) = 2$

Node-V_4:

$(V_4 - 0)/(1/2) + (V_4 – V_1)/(1/2) + (V_4 – V_3)/(1/2) = 2$

$2V_4 + 2(V_4 – V_1) + 2(V_4 – V_3) = 2$

3-41 – 3-48, 3-54 – 3-57 (Do yourself).

3-60. Find the equivalent inductance.

Solution:

See Q#3-2. for reference.

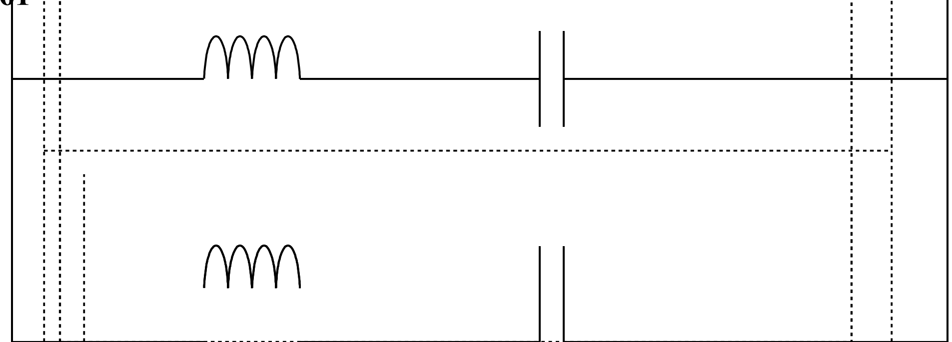
3-61. It is intended that the two networks of the figure be equivalent with respect to the pair of terminals, which are identified. What must be the values for C_1, L_2, and L_3?

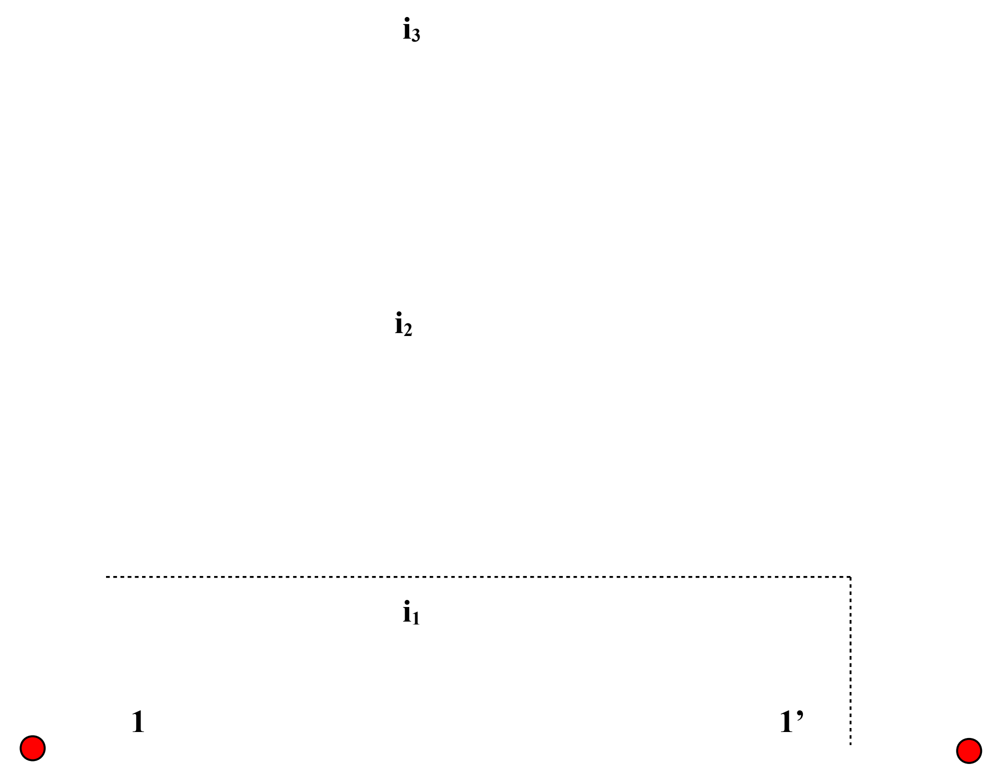
Solution:

Do yourself.

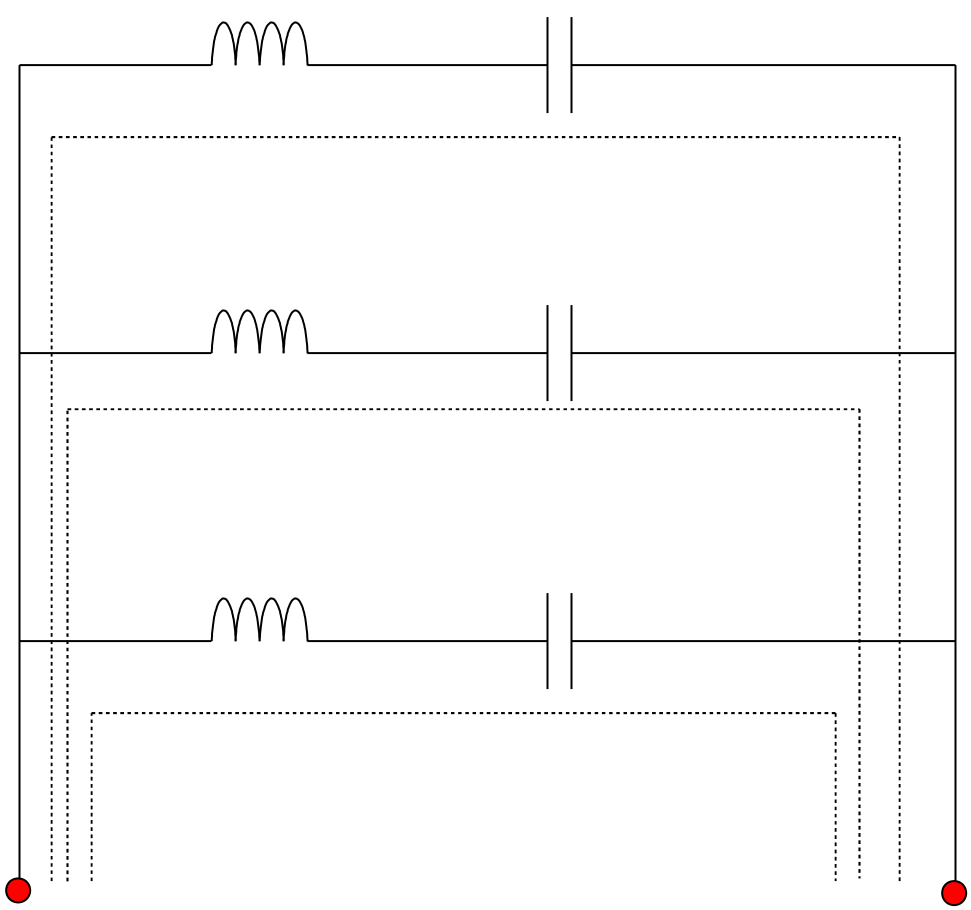
Hint:

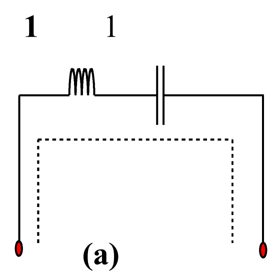
Fig. P3-61



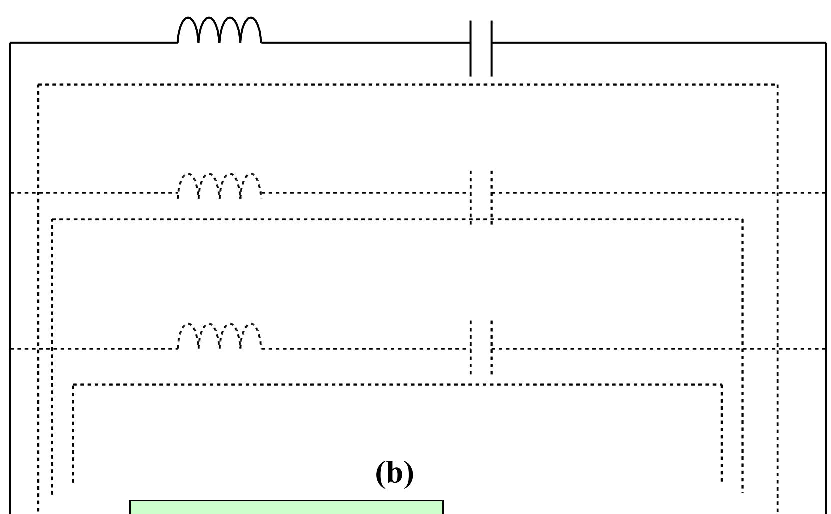


Be equivalent with respect to the pair of terminals



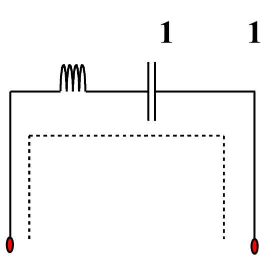


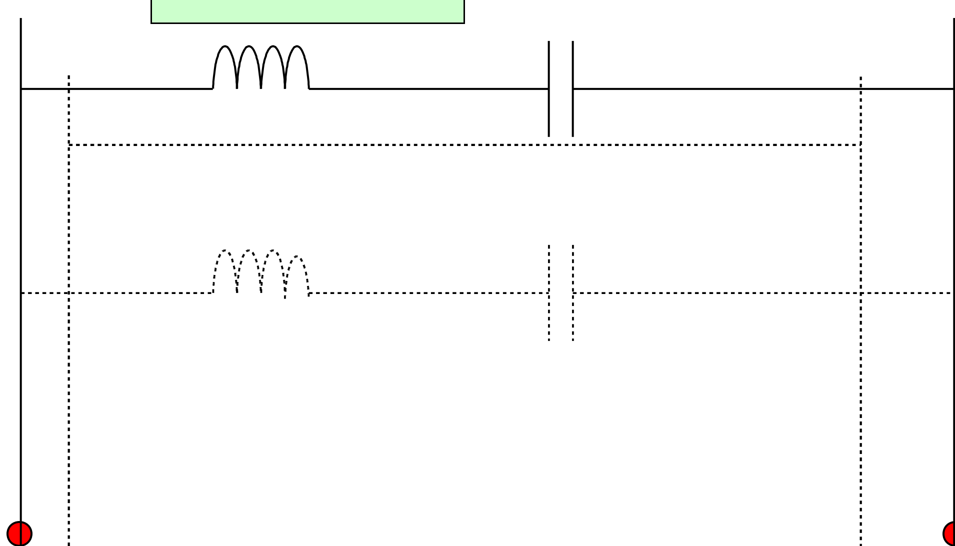
(a)



(b)

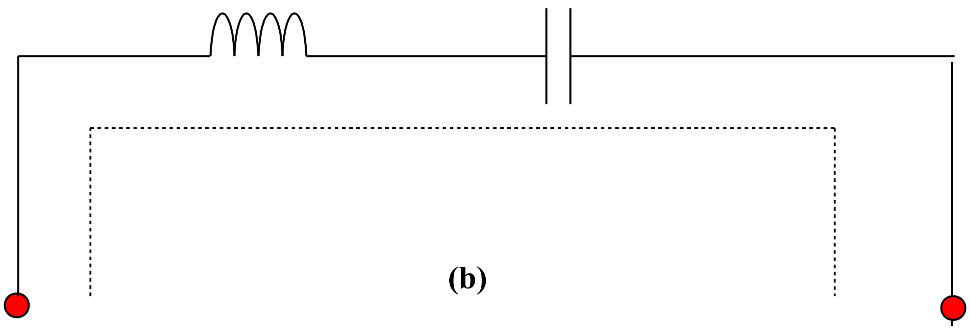
Equating (a) & (b)

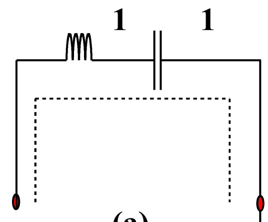




(a) **(b)**

Equating (a) & (b)



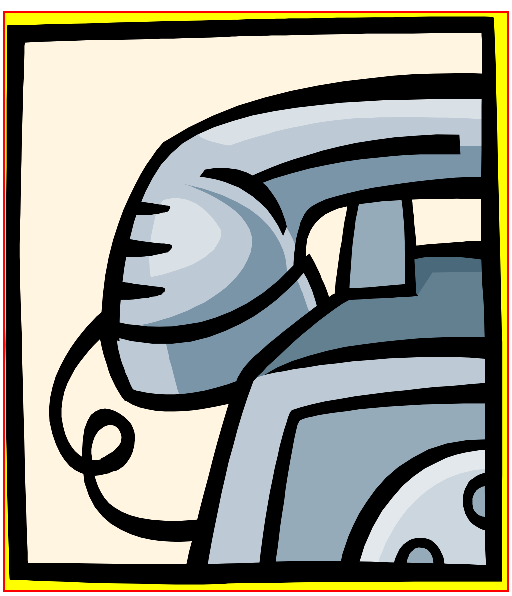


(a)

Equating (a) & (b)

3-62.
Solution:
See 3-61 for reference.
<u>Before solving exercise following terms should be kept in mind:</u>

1. **kirchhoffs current law**
2. **kirchhoffs voltage law**
3. **Loop analysis**
4. **Node analysis**
5. **Determinant**
6. **State variable analysis**
7. **Source transformation**



1-5. Solution:

$v = V_0\sin\omega t$

$C = C_0(1 - \cos\omega t)$

$Q = I \times t$

$Q = CV$

$\boxed{i = d(q)/dt = d(Cv)/dt = Cdv/dt + vdC/dt}$

$i = Cdv/dt + vdC/dt$

$i = C_0(1 - \cos\omega t)d(V_0\sin\omega t)/dt + V_0\sin\omega t\, dC_0(1 - \cos\omega t)/dt$

$\boxed{i = C_0(1 - \cos\omega t)\,\omega V_0\cos\omega t + V_0\sin\omega t\{\omega C_0\sin\omega t\}}$

1-10.

$$w = \int_{-\infty}^{t} vi\,dt$$

For an inductor

$\boxed{v_L = Ldi/dt}$

By putting the value of voltage

$$w = \int_{-\infty}^{t} vi\,dt$$

$$w = \int_{-\infty}^{t} (Ldi/dt)i\,dt$$

$$w = L\int_{-\infty}^{t} idi$$

$$w = L\left| i^2/2 \right|_{-\infty}^{t}$$

$w = L[i^2(t)/2 - i^2(-\infty)/2]$

$w = L[i^2(t)/2 - (i(-\infty))^2/2]$

$w = L[i^2(t)/2 - (0)^2/2]$

$\boxed{w = L[i^2(t)/2]}$ **{Because $i(-\infty) = 0$ for an inductor}**

As we know

$\psi = Li$
$\psi^2 = L^2i^2$
$\psi^2/L = Li^2$
$w = L[i^2(t)/2]$
$w = Li^2/2$
By putting the value of Li^2
$w = (\psi^2/L)/2$

$\boxed{w = \psi^2/2L}$ **{where ψ = flux linkage}**

1-11.

$$w = \int_{-\infty}^{t} vi\,dt$$

For a capacitor

$\boxed{i = Cdv/dt}$

By putting the value of current

$$w = \int_{-\infty}^{t} vi\,dt$$

$$w = \int_{-\infty}^{t} (Cdv/dt)v\,dt$$

$$w = C\int_{-\infty}^{t} vdv$$

$$w = C\left|\, v^2/2 \,\right|_{-\infty}^{t}$$

$w = C[v^2(t)/2 - v^2(-\infty)/2]$
$w = C[v^2(t)/2 – (v(-\infty))^2/2]$
$w = C[v^2(t)/2 – (0)^2/2]$

$\boxed{w = C[v^2(t)/2]}$ **{Because $v(-\infty) = 0$ for an inductor}**

As we know
$Q = CV$
$V = Q/C$
$w = C[v^2(t)/2]$
$w = C[(q/C)^2/2]$
$w = C[q^2/2C^2]$
$w = q^2/2C$

$w = q^2D/2$ Ans.

1-12. $w_L = (1/2)Li^2$

$P = vi$

$P = dw_L/dt$

By putting values of P & w_L

$vi = d((1/2)Li^2)/dt$

$vi = (1/2)dLi^2/dt$

$vi = (1/2)Ldi^2/dt$

$vi = (1/2)L2i\{di/dt\}$

$$\boxed{v = L\{di/dt\}}$$

1-13.

$w_c = (1/2)Dq^2$

$P = vi$

$P = dw_L/dt$

By putting values of P & w_L

$vi = d((1/2)Dq^2)/dt$

$vi = (1/2)dDq^2/dt$

$vi = (1/2)Ddq^2/dt$

$vi = (1/2)D2q\{dq/dt\}$

$$\boxed{vi = Dq\{dq/dt\}}$$

As we know

$i = dq/dt$

$vi = Dq\{dq/dt\}$

$vi = Dq\{i\}$

$v = Dq$

$$q = \int_{-\infty}^{t} i\,dt$$

$v = Dq$

$$\boxed{v = D \int_{-\infty}^{t} i\,dt}$$

1-17.

$V = 12$ V

$C = 1\mu$ F

$w = ?$

$w = (1/2)CV^2$

$= (1/2)(1\times 10^{-6})(12)^2$

$$\boxed{w = 72\mu \text{ J}}$$

1-18.

v_c = 200 V

C = 1μ F

mass = 100 lb = 45.3 kg

work done = Fd = mgd = (45.3)(9.8)d

work done = energy = $(1/2)C(v_c)^2 = (1/2)(1\times 10^{-6})(200)^2 = 0.02$ joule

work done = (45.3)(9.8)d

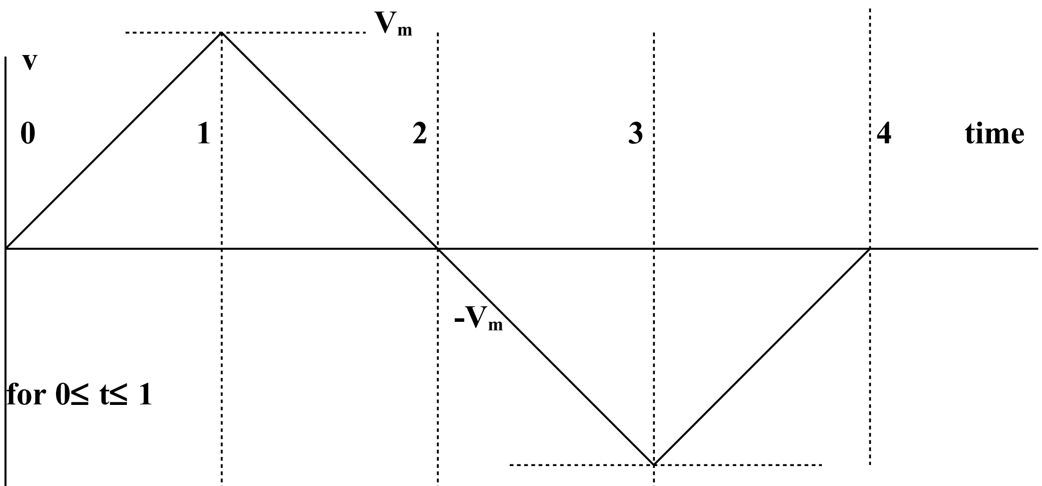
0.02 = (45.3)(9.8)d

d = 0.02/(45.3)(9.8) = 0.02/443.94

$\boxed{d = 4.505\times 10^{-5} m}$ Ans.

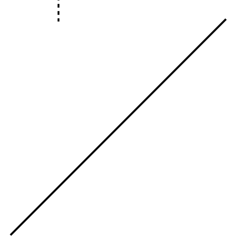
1-19.

Solution:



for 0≤ t≤ 1

$(x_1, y_1) = (1, V_m)$



$(x_0, y_0) = (0, 0)$

Slope = m

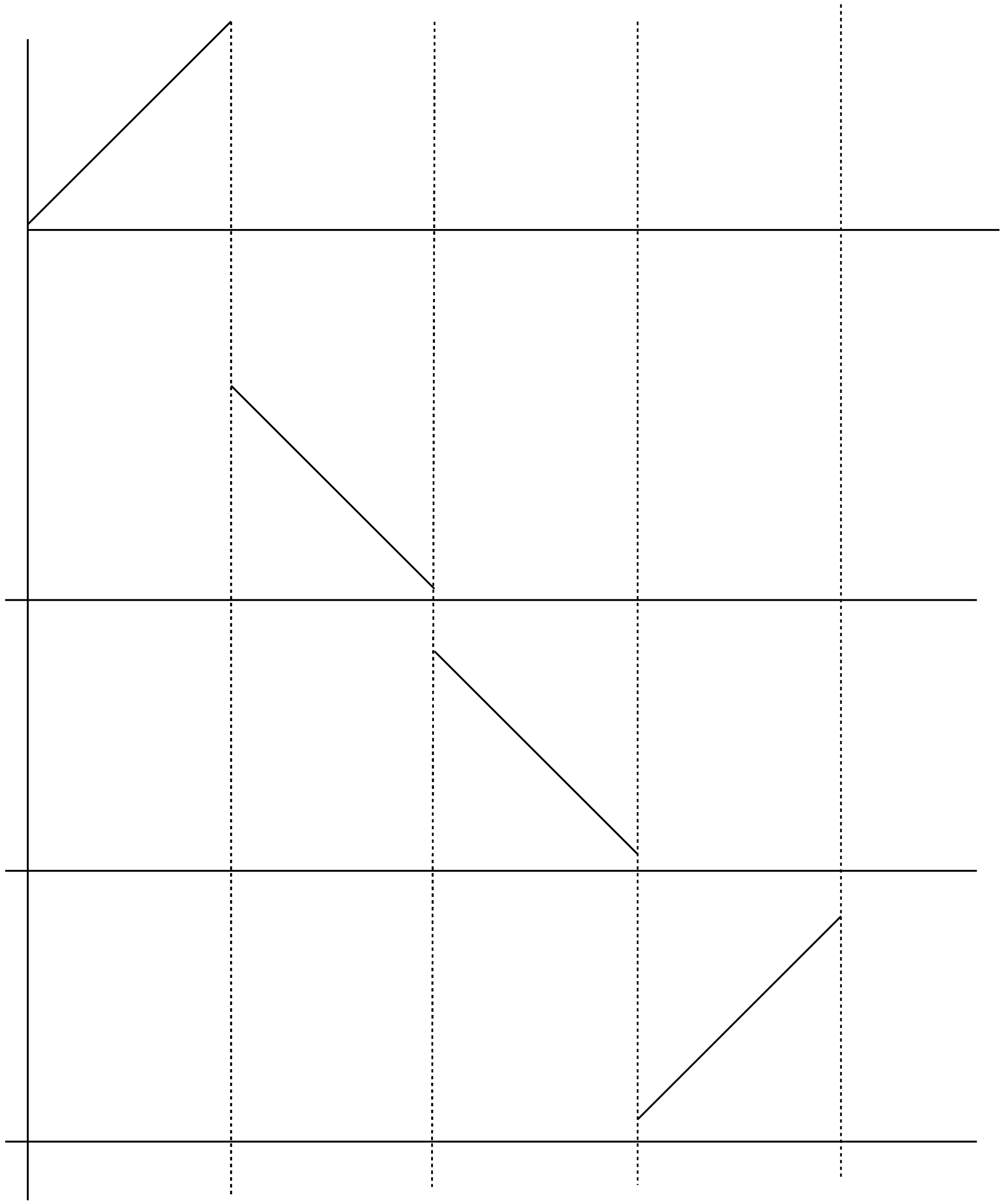
$m = (y_1 - y_0)/(x_1 - x_0) = (V_m - 0)/(1 - 0)$

$\boxed{m = V_m}$

$y = mx + c = V_m(t) + 0 = V_m t$

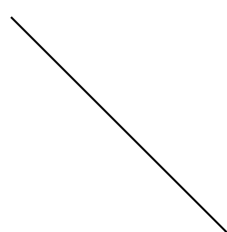
Straight-line equation

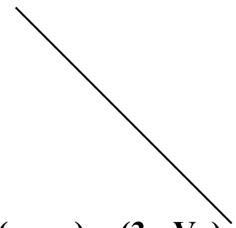
y-intercept = c = 0



for 1≤ t≤ 3

$(x_1, y_1) = (1, V_m)$





$(x_3, y_3) = (3, -V_m)$

Slope = m

$m = (y_3 – y_1)/(x_3 – x_1) = (-V_m – V_m)/(3 - 1) = -2V_m/2 = -V_m$

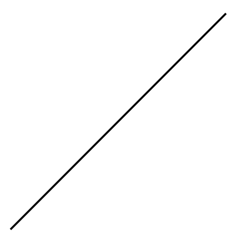
$m = -V_m$

Straight-line equation

$y = mx + c = -V_m(t) + 2V_m = -V_mt + 2V_m$

y-intercept = c = $2V_m$

for $3 \leq t \leq 4$



$(x_4, y_4) = (4, 0)$

$(x_3, y_3) = (3, -V_m)$

Slope = m

$m = (y_4 – y_3)/(x_4 – x_3) = (0 – (-V_m))/(4 – 3)$

$m = V_m$

Straight-line equation

$y = mx + c = V_m(t) - 3V_m = V_mt – 3V_m$

y-intercept = c = $-3V_m$

Let capacitance be C
for $0 \leq t \leq 1$
$i = Cdv/dt = Cd(V_mt)/dt = CV_m$

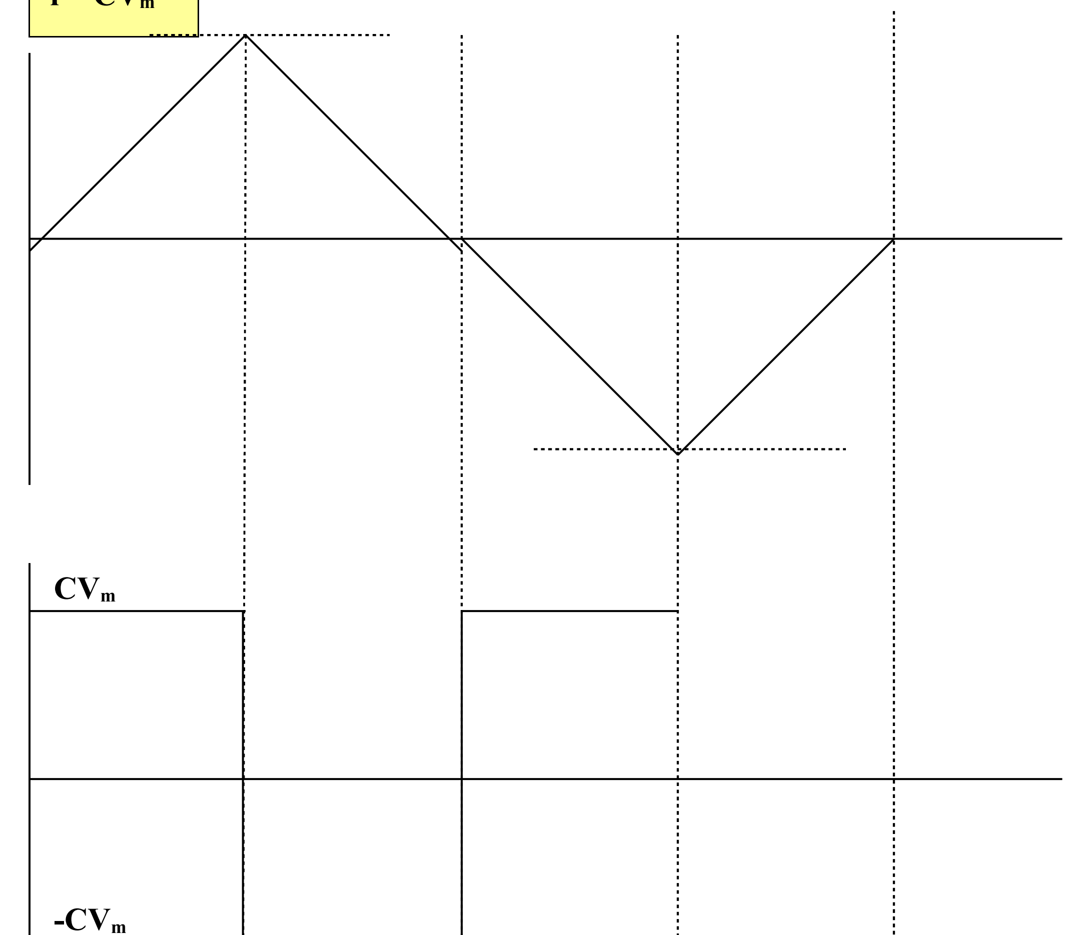
$i = CV_m$

for $1 \leq t \leq 3$
$i = Cdv/dt = Cd(-V_m t + 2V_m)/dt = -CV_m$

$i = -CV_m$

for $3 \leq t \leq 4$
$i = Cdv/dt = Cd(V_m t - 3V_m)/dt = CV_m$

$i = CV_m$



for $0 \leq t \leq 1$
$q = CV$

$q = CV_m t$

for $1 \leq t \leq 3$
$q = CV$

$q = CV_m(2 - t)$

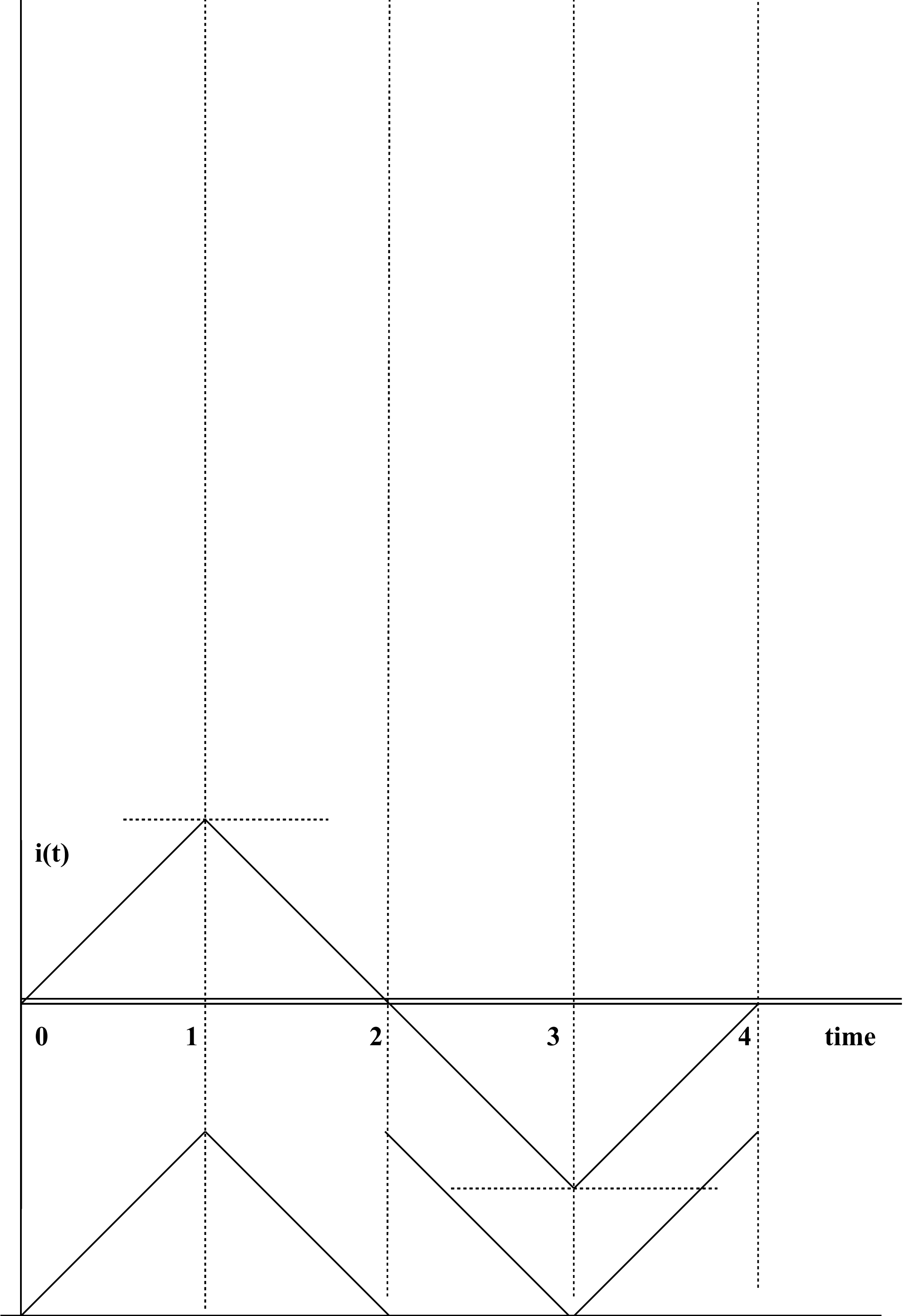
for $3 \leq t \leq 4$

$q = CV$

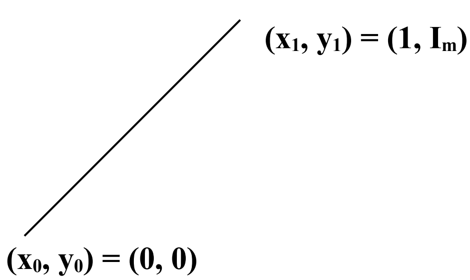
$$q = CV_m(t - 4)$$

for $0 \le t \le 1$	$q = CV_m t$	$t = 0,\ q = 0$	$t = 1,\ q = CV_m$
for $1 \le t \le 3$	$q = CV_m(2 - t)$	$t = 3,\ q = -CV_m$	
for $3 \le t \le 4$	$q = CV_m(t - 4)$	$t = 4,\ q = 0$	

Charge waveform same as voltage waveform.
(b)



for $0 \leq t \leq 1$



Slope = m

$m = (y_1 - y_0)/(x_1 - x_0) = (I_m - 0)/(1 - 0)$

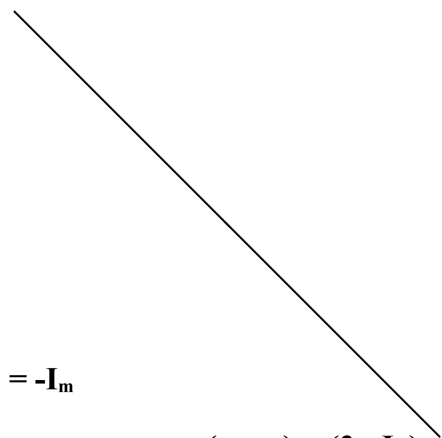
$\boxed{m = I_m}$

$y = mx + c = I_m(t) + 0 = I_m t$

Straight-line equation

y-intercept = c = 0

for $1 \leq t \leq 3$



Slope = m

$m = (y_3 - y_1)/(x_3 - x_1) = (-I_m - I_m)/(3 - 1) = -2I_m/2 = -I_m$

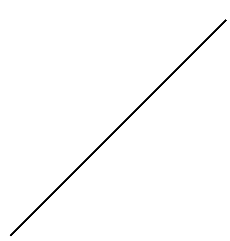
$\boxed{m = -I_m}$

Straight-line equation

$y = mx + c = -I_m(t) + 2I_m = -I_m t + 2I_m$

y-intercept = c = $2I_m$

for $3 \leq t \leq 4$



Slope = m

$$m = (y_4 - y_3)/(x_4 - x_3) = (0 - (-I_m))/(4 - 3)$$

$$m = I_m$$

Straight-line equation

$$y = mx + c = I_m(t) - 3I_m = I_mt - 3I_m$$

y-intercept = c = $-3I_m$

for $0 \le t \le 1$

$$v(t) = (1/C)\int_{t_1}^{t} id(t) + v(t_1)$$

$$v(t) = (1/C)\int_{0}^{t} I_mtd(t) + 0$$

$$v(t) = (1/C)\int_{0}^{t} I_mtd(t)$$

$$v(t) = (1/C)I_m \left| t^2/2 \right|_{0}^{t}$$

$$v(t) = (1/C)I_m[(t^2/2) - ((0)^2/2)]$$

$$v(t) = (1/C)I_m(t^2/2)$$

$$v(1) = (1/C)I_m((1)^2/2) = (1/C)I_m(1/2) = I_m/2C$$

for $1 \le t \le 3$

$$v(t) = (1/C)\int_{t_1}^{t} id(t) + v(t_1)$$

$$v(t) = (1/C)\int_{1}^{t} I_m(2 - t)d(t) + I_m/2C$$

$$v(t) = (1/C)\left| 2t - t^2/2 \right|_{1}^{t} + I_m/2C$$

$$v(t) = (1/C)[(2t - t^2/2) - (2(1) - 1^2/2)] + I_m/2C$$
$$v(t) = (1/C)[(2t - t^2/2) - (2 - 1/2)] + I_m/2C$$

$\boxed{v(t) = (1/C)[(2t - t^2/2) - (3/2)] + I_m/2C}$

at time t = 3

$\boxed{v(3) = (1/C)[(2(3) - (3)^2/2) - (3/2)] + I_m/2C}$

$v(3) = (1/C)[6 - 4.5 - 1.5] + I_m/2C$

$\boxed{v(3) = I_m/2C}$

for 3≤ t≤ 4

$$v(t) = (1/C)\int_{t_1}^{t} id(t) + v(3)$$

$$v(t) = (1/C)\int_{1}^{t} I_m(t - 3)d(t) + I_m/2C$$

$$v(t) = (1/C)I_m \left| \; t^2/2 - 3t \right|_{1}^{t} + I_m/2C$$

$v(t) = (1/C)I_m[(t^2/2 - 3t) - (1/2 - 3)] + I_m/2C$

$\boxed{v(t) = (1/C)I_m[(t^2/2 - 3t) + 2.5] + I_m/2C}$

at time t = 4

$v(4) = (1/C)I_m[((4)^2/2 - 3(4)) + 2.5] + I_m/2C$

$v(4) = (1/C)I_m[16/2 - 12 + 2.5] + I_m/2C$

$v(4) = (1/C)I_m[8 - 12 + 2.5] + I_m/2C$

$v(4) = (1/C)I_m[-1.5] + I_m/2C$

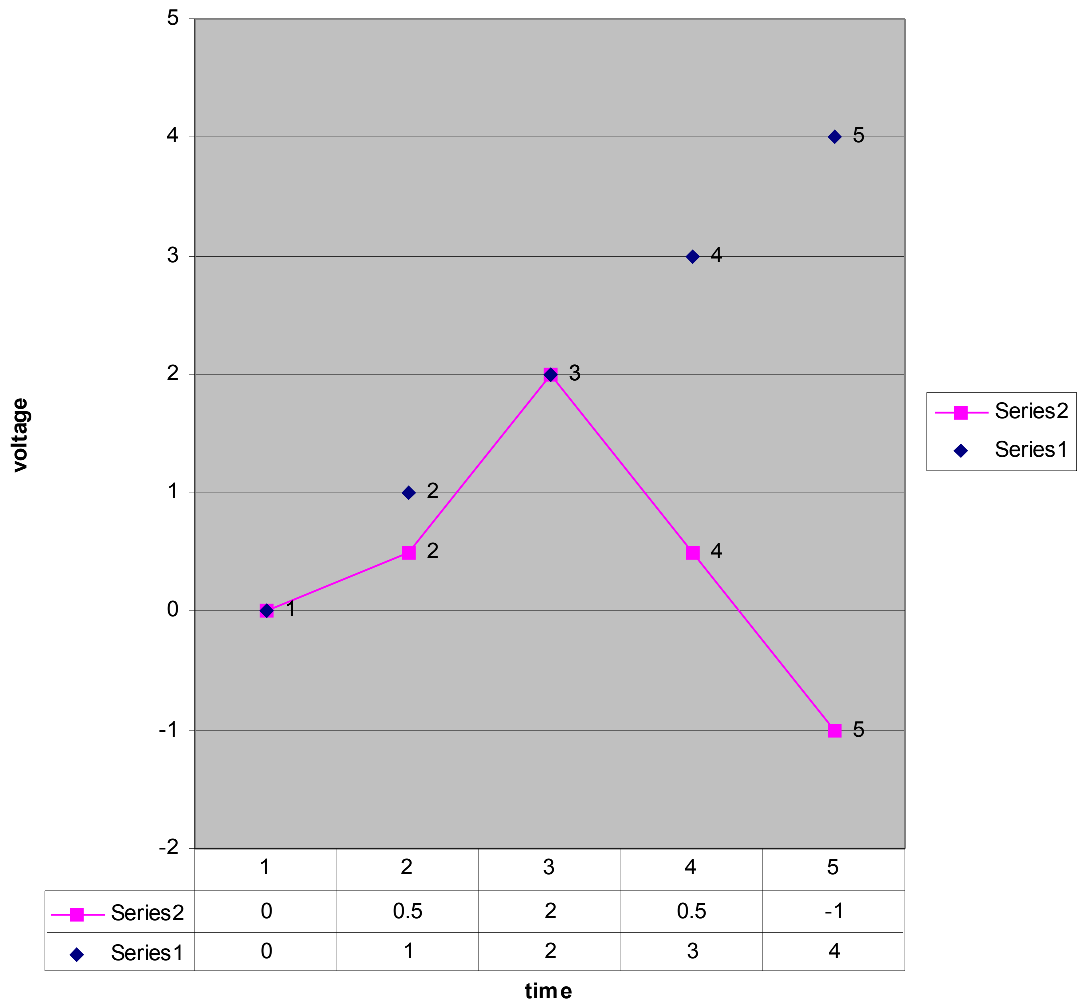
$\boxed{v(4) = -I_m/C}$

v(0)	0
v(1)	$I_m/2C = 0.5(I_m/C)$
v(2)	$I_m2/C = 2(I_m/C)$
v(3)	$I_m/2C = 0.5(I_m/C)$
v(4)	$-I_m/C = -1(I_m/C)$

$v(t) = (1/C)I_m(t^2/2)$

at time t = 2

$\boxed{v(2) = (1/C)I_m((2)^2/2) = I_m(2)/C}$



for $0 \le t \le 1$

$q = CV$

$$q = C(I_m t^2/2C) = I_m t^2/2$$

for $1 \le t \le 3$

$q = CV$

$$q = CI_m(4t - t^2 - 2)/2C$$

for $3 \le t \le 4$

$q = CV$

$$q = C(1/C)[I_m[(t^2/2 - 3t) + 2.5] + I_m/2C] = I_m[(t^2/2 - 3t) + 2.5] + I_m/2C$$

At time t = 0

$$q = C(I_m t^2/2C) = I_m t^2/2 = I_m(0)^2/2 = 0\ C$$

At time t = 1

$$q = C(I_m t^2/2C) = I_m t^2/2 = I_m 1^2/2 = I_m/2\ C$$

At time t = 2

$$q = C(I_m t^2/2C) = I_m t^2/2 = I_m 2^2/2 = 2I_m\ C$$

At time t = 3

$$q = CI_m(4t - t^2 - 2)/2C = CI_m(4(3) - 3^2 - 2)/2C = (0.5I_m/C)\ C$$

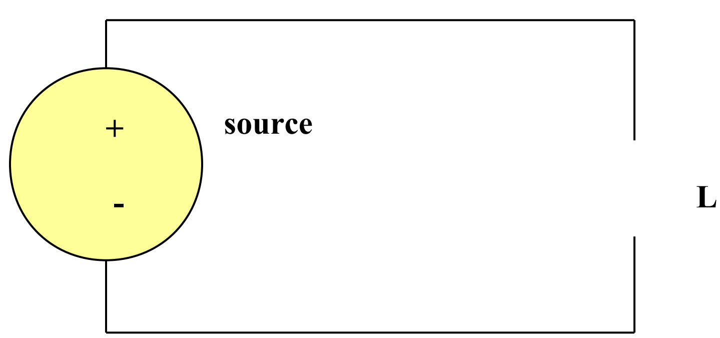
At time t = 4

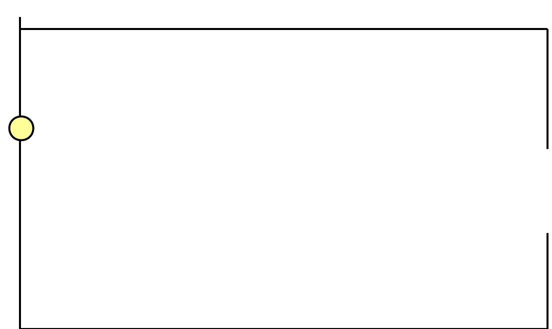
$$q = C(1/C)[I_m[(t^2/2 - 3t) + 2.5] + I_m/2C] = I_m[(t^2/2 - 3t) + 2.5] + I_m/2C$$

$$q = I_m[(4^2/2 - 3(4)) + 2.5] + I_m/2C = -I_m/C$$

Charge waveform same as voltage waveform.
1-20.
Solution:
I = 1A
L = ½ H
$w_L = (1/2)LI^2 = (1/2)(1/2)(1)^2 = 0.25\ J$

As we know energy in an inductor = (1/2)LI² J





Short circuit

L

Energy will be lost after short-circuiting.
1-21.
Solution:
L = 1H

(a) **ψ (flux linkage) at t = 1sec.**

ψ (flux linkage) = LI

I = t {Because y = mx + c; m = 1 = slope}

ψ (flux linkage) = Lt

at t = 1sec.

ψ (flux linkage) = Lt = (1)(1) = 1 H(henry)A(ampere).

(b) $d\psi/dt = Ld(t)/dt = L = 1$

(c)

$$q = \int_{-\infty}^{t} i\,dt$$

$$q = \int_{-\infty}^{t} t\,dt$$

$$q = \left| t^2/2 \right|_{-\infty}^{t} = [t^2/2 - (-\infty)^2/2] =$$

$\boxed{q = t^2/2}$

At time t = 1sec $q = t^2/2 = q = (1)^2/2 = ½$ Coulomb

1-24.
Solution:
K is closed at t = 0
$i(t) = 1 - e^{-t}, t>0$
$i(t_0) = 0.63$ A
$1 - e^{-t0} = 0.63$ A
$-e^{-t0} = -1 + 0.63$ A
$-e^{-t0} = -0.37$
$e^{-t0} = 0.37$
Taking logarithm of both the sides
$\log e^{-t0} = \log 0.37$
$-t_0 \log e = -0.432$
$t_0(0.434) = 0.432$ {Because e = 2.718}
$t_0 = 0.432/0.434 = 0.995$ sec = 1 sec.

$\boxed{t_0 = 1 \text{ sec.}}$

(a) $di(t_0)/dt = ?$
$di(t)/dt = d(1 – e^{-t})/dt$
$di(t)/dt = d(1)/dt - d(e^{-t})/dt$
$di(t)/dt = 0 - e^{-t}\{d(-t)/dt\}$
$di(t)/dt = 0 - e^{-t}(-1)$
$di(t)/dt = e^{-t}$
$di(t_0)/dt = e^{-t0}$
$t_0 = 1$ sec.
$di(1)/dt = e^{-1}$
$di(1)/dt = 1/e = ½.718 = 0.37$ Ampere per second

$di(1)/dt = 1/e = ½.718 = 0.37$ Ampere per second

(b) $\psi = Li$
$i(t) = 1 – e^{-t}$
$\psi = Li(t)$
$\psi = L(1 – e^{-t})$
$\psi(t_0) = L(1 – e^{-t0})$
$t_0 = 1$ sec
$\psi(1) = L(1 – e^{-1})$
$\psi(1) = L(1 – 1/e)$
As $L = 1H$ & $1/e = 0.37$

$\psi(1) = (1)(1 – 0.37) = 0.63$ weber

(c) $d\psi/dt = ?$
$\psi = L(1 – e^{-t}) = \psi = (1 – e^{-t})$
$d\psi/dt = d(1 – e^{-t})/dt = d1/dt - de^{-t}/dt = 0 + e^{-t} = e^{-t}$

$d\psi/dt = e^{-t}$
$d\psi(t_0)/dt = e^{-t0}$
$t_0 = 1$ sec.

$d\psi(1)/dt = e^{-1} = 1/e = 0.37$ weber per sec.

(d)
$v(t) = Ldi(t)/dt$
$i(t) = 1 – e^{-t}$ & $L = 1$
$v(t) = (1)d(1 – e^{-t})/dt$
$v(t) = d(1 – e^{-t})/dt = e^{-t}$
$v(t_0) = e^{-t0}$
$t_0 = 1$ sec.

$v(1) = e^{-1} = 1/e = 0.37$ V

(e)
$w = (1/2)Li^2 = (1/2)(1)(1 – e^{-t})^2$

$w = (1/2)(1 – e^{-t})^2$
$w = (1/2)(1 + 2e^{-2t} – 2e^{-t})$
$w(t_0) = (1/2)(1 + 2e^{-2t0} – 2e^{-t0})$
$t_0 = 1$ sec.
$w(1) = (1/2)(1 + 2e^{-2(1)} – 2e^{-1})$
$w(1) = (1/2)(1 + 2e^{-2} – 2e^{-1})$
$w(1) = (1/2)(1 + 2(1/e^2) – 2(1/e))$ $\{1/e = 0.37; 1/e^2 = 0.135\}$
$w(1) = (1/2)(1 + 2(0.135) – 2(0.37))$
$w(1) = (1/2)(1 + 0.27 – 0.74)$

$w(1) = 0.265$ Joule

(f)
$v_R = ?$
$v_R = iR = (1 – e^{-t})(1) = (1 – e^{-t})$
$v_R = iR = (1 – e^{-t})$
$v_R(t_0) = (1 – e^{-t0})$
at time $t_0 = 1$ sec.

$v_R(1) = (1 – e^{-1}) = 0.63$ V

(g)
$w = (1/2)(1 + 2e^{-2t} – 2e^{-t})$
$dw/dt = d((1/2)(1 + 2e^{-2t} – 2e^{-t}))/dt$
$dw/dt = (1/2)d(1 + 2e^{-2t} – 2e^{-t})/dt$
$dw/dt = (1/2)\{d(1)/dt + d(2e^{-2t})/dt - d(2e^{-t})/dt\}$
$dw/dt = (1/2)\{0 + 2e^{-2t})(-2) - 2e^{-t})(-1)\}$
$dw/dt = (1/2)\{-4e^{-2t} + 2e^{-t})\}$
$dw(t_0)/dt = (1/2)\{-4e^{-2t0} + 2e^{-t0})\}$
$dw(1)/dt = (1/2)\{-4e^{-2} + 2e^{-1})\}$
$dw(1)/dt = (1/2)\{-4(1/e^2) + 2(1/e)\}$
$dw(1)/dt = (1/2)\{-4(0.135) + 2(0.37)\}\{1/e = 0.37; 1/e^2 = 0.135\}$

$dw(1)/dt = (1/2)\{-0.54 + 0.74\} = 0.1$ watts

(h)
$P_R = i^2R = (1 + e^{-2t} – 2e^{-t})(1)$
$P_R = i^2R = (1 + e^{-2t} – 2e^{-t})$
$P_R(t_0) = i^2R = (1 + e^{-2t0} – 2e^{-t0})$
At time $t_0 = 1$ sec.
$P_R(1) = i^2R = (1 + e^{-2(1)} – 2e^{-(1)})$
$P_R(1) = i^2R = (1 + e^{-2} – 2e^{-1})$
$P_R(1) = i^2R = (1 + 1/e^2 – 2(1/e))$
$P_R(1) = i^2R = (1 + 0.135 – 2(0.37))$
$P_R(1) = i^2R = (1 + 0.135 – 0.74)$

$P_R(1) = i^2R = (0.395)$ watts

(i)
$P_{total} = vi = (1)(1 - e^{-t}) = (1 - e^{-t})$
At time $t_0 = 1$ sec.
$P_{total}(t_0) = vi = (1)(1 - e^{-t}) = (1 - e^{-t0})$
$P_{total}(1) = (1 - e^{-1})$

$P_{total}(1) = (1 - e^{-1}) = 0.63$ watts.

1-25.
Voltage across the capacitor at time t = 0
$v_c(0) = 1$ Volt
k is closed at t = 0
$i(t) = e^{-t}, t>0$
$i(t_0) = 0.37$ A
$0.37 = e^{-t0}$
Taking logarithm of both the sides
$\log 0.37 = \log e^{-t0}$
$-t_0 \log e = -0.432$
$t_0(0.434) = 0.432$ {Because e = 2.718}

$t_0 = 1$ sec.

(a) $dv_c(t_0)/dt = ?$
Using loop equation
$v_c(t) = iR = e^{-t}(1) = e^{-t}$ Volts
$dv_c(t)/dt = -e^{-t}$ Volts
$dv_c(t_0)/dt = -e^{-t0}$ Volts
$t_0 = 1$ sec.
$dv_c(t_0)/dt = -e^{-1}$ Volts

$dv_c(t_0)/dt = -0.37$ V/sec

(b)

Charge on the capacitor = $q = Cv = (1)(e^{-t}) = e^{-t}$ coulomb

Charge on the capacitor = $q(t_0) = Cv = (1)(e^{-t}) = e^{-t0}$ coulomb
$t_0 = 1$ sec.

Charge on the capacitor = $q(1) = Cv = (1)(e^{-t}) = e^{-1}$ coulomb = 0.37 coulomb

(c) $d(Cv)/dt = Cdv/dt = Cde^{-t}/dt = -Ce^{-t}$
$d(Cv(t_0))/dt = -Ce^{-t0}$
$t_0 = 1$ sec.
$d(Cv(t_0))/dt = -Ce^{-1}$
As C = 1F

$d(Cv(t_0))/dt = -e^{-1} = -0.37$ coulomb/sec.

(d) $v_c(t) = e^{-t}$
$t_0 = 1$ sec.
$v_c(t_0) = e^{-t0}$

$v_c(1) = e^{-1} = 0.37$ Volt

(e) $w_c = ?$
$w_c = (1/2)Cv^2 = (1/2)(1)(e^{-t})^2 = (1/2)e^{-2t}$
$w_c(t_0) = (1/2)Cv^2 = (1/2)(1)(e^{-t0})^2 = (1/2)e^{-2t0}$
$t_0 = 1$ sec.
$w_c(1) = (1/2)e^{-2(1)}$
$w_c(1) = (1/2)e^{-2}$
$w_c(1) = (1/2)(1/e^2)$ $\{1/e^2 = 0.135\}$
$w_c(1) = (1/2)(0.135)$

$w_c(1) = (1/2)(0.135) = 0.067$ Joules

(f)
$v_R(t) = iR = e^{-t}(1) = e^{-t}$ Volts
$v_R(t_0) = iR = e^{-t}(1) = e^{-t0}$ Volts
$t_0 = 1$ sec.

$v_R(1) = iR = e^{-t}(1) = e^{-1}$ Volts = 0.37 Volts

(g) $dw_c/dt = ?$
$w_c = (1/2)e^{-2t}$
$dw_c/dt = d(1/2)e^{-2t}/dt$
$dw_c/dt = (1/2)e^{-2t}(-2) = -e^{-2t}$
$dw_c(t_0)/dt = (1/2)e^{-2t}(-2) = -e^{-2t0}$
$t_0 = 1$ sec.
$dw_c(1)/dt = (1/2)e^{-2t}(-2) = -e^{-2(1)}$
$dw_c(1)/dt = (1/2)e^{-2t}(-2) = -e^{-2}$

$dw_c(1)/dt = (1/2)e^{-2t}(-2) = -e^{-2} = -0.135$ watts.

(h) $P = i^2R = (e^{-t})^2(1) = e^{-2t}$
$P(t_0) = i^2R = (e^{-t})^2(1) = e^{-2t0}$
$t_0 = 1$ sec.
$P(1) = i^2R = (e^{-t})^2(1) = e^{-2(1)}$
$P(1) = i^2R = (e^{-t})^2(1) = e^{-2}$

$P(1) = i^2R = (e^{-t})^2(1) = e^{-2} = 0.135$ watts.

1-26.
Solution:

(a) $RC = (1/I)(q) = q/(q/t) = t$
(b) L/R
$V = Ldi/dt$
$L = V/(di/dt)$
$L = Vdt/di$
$R = V/I$

$L/R = (Vdt/di)/(V/I) = V^2dt/Idi$

(c)

$\sqrt{LC} = \sqrt{(Vdt/di)(q/V)} = \sqrt{(dt/di)(q)}$
(d)

$R^2C = (V^2/I^2)(q/V) = Vq/I^2 = V(I\times t)/I^2 = V\times t/I$

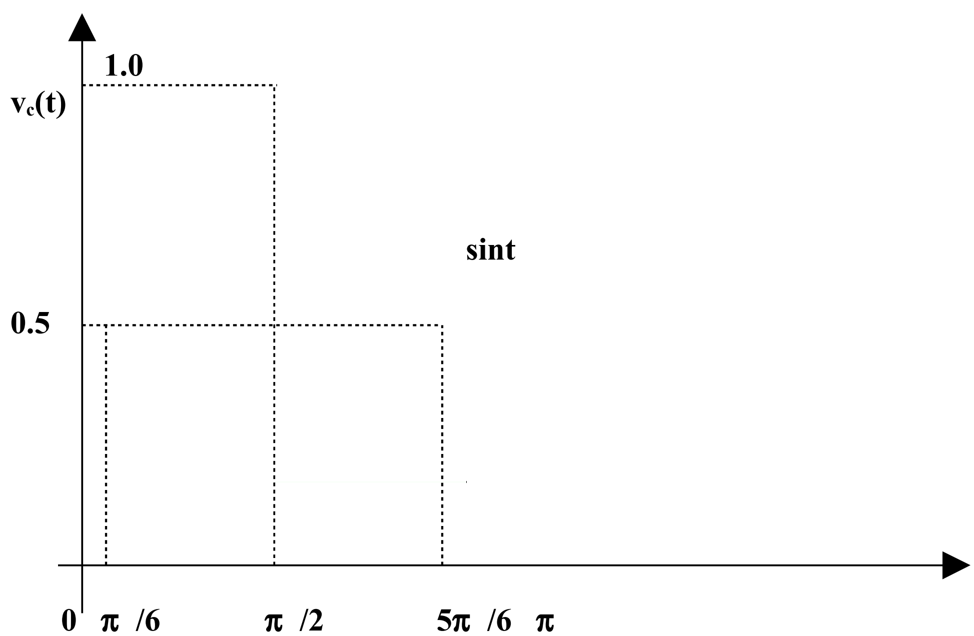
(e)

$\sqrt{LC} = \sqrt{(Vdt/di)/(q/V)} = \sqrt{(V^2dt)/(qdi)} = \sqrt{(V^2/di)/(1/(q/t))}$

(f)

$L/R^2 = = (Vdt/di)/(V^2/I^2) = Idt/V = q/V = C$

1-39.



for $v_c \leq$ -0.5Volt

C = (-1.0 + 0.5)/(-1.5 + 0.5) = -0.5/-1 = 0.5 F

for –0.5 $v_c \leq$ 0.5

C = (0.5 + 0.5)/(0.5 + 0.5) = 1/1 = 1F

for 0.5 $v_c \leq$ 1.5

C = (1.0 - 0.5)/(1.5 - 0.5) = 0.5/1 = 0.5F

for 0 $\leq v_c \leq$ 0.5 **for 0 $\leq t \leq \pi/6$**

for 0.5 $\leq v_c \leq$ 1 **for $\pi/6 \leq t \leq 5\pi/6$**

for 0.5 $v_c \leq$ 0 **for $5\pi/6 \leq t \leq \pi$**

$i_c(t)$ = d(Cv)/dt = Cdv/dt + vdC/dt
$i_c(t)$ = Cdv/dt + vdC/dt
for 0 $\leq t \leq \pi/6$
C = 1F
V = sint
$i_c(t)$ = (1)dsint/dt + sintd(1)/dt

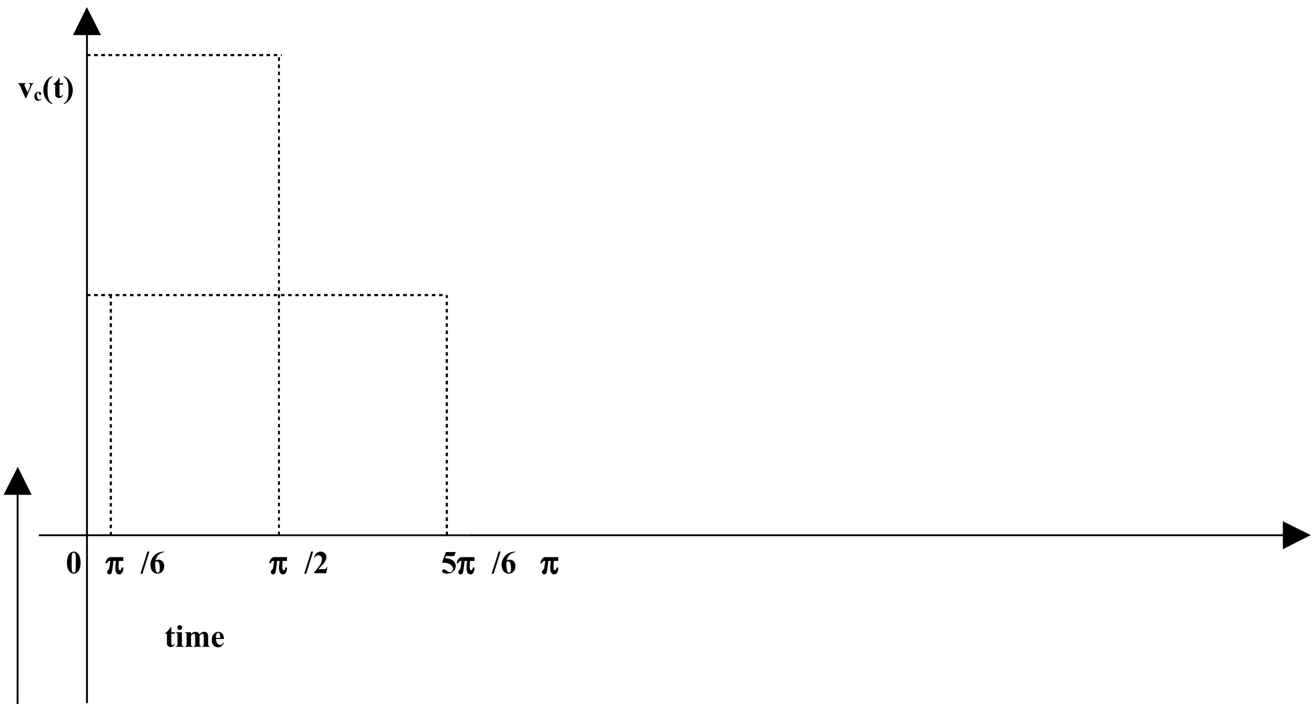
$i_c(t)$ = cost

$i_c(t)$ = d(Cv)/dt = Cdv/dt + vdC/dt
$i_c(t)$ = Cdv/dt + vdC/dt
for $\pi/6 \leq t \leq 5\pi/6$
C = 0.5F
v = sint
$i_c(t)$ = (0.5)dsint/dt + sintd(0.5)/dt

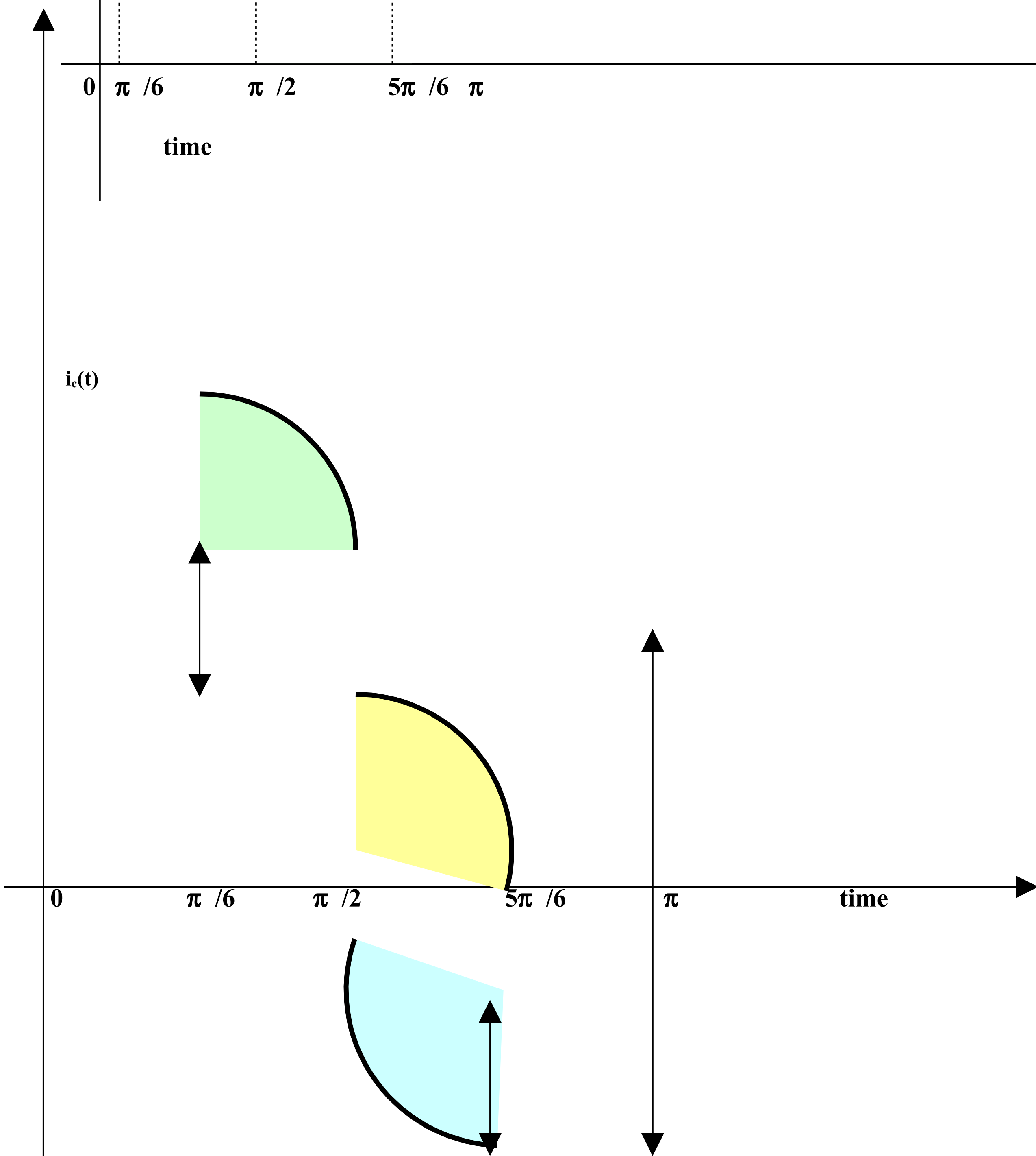
$i_c(t)$ = (0.5)cost

$i_c(t)$ = d(Cv)/dt = Cdv/dt + vdC/dt
$i_c(t)$ = Cdv/dt + vdC/dt
for $5\pi/6 \leq t \leq \pi$
C = 1F
v = sint
$i_c(t)$ = (1)dsint/dt + sintd(1)/dt

$i_c(t)$ = cost

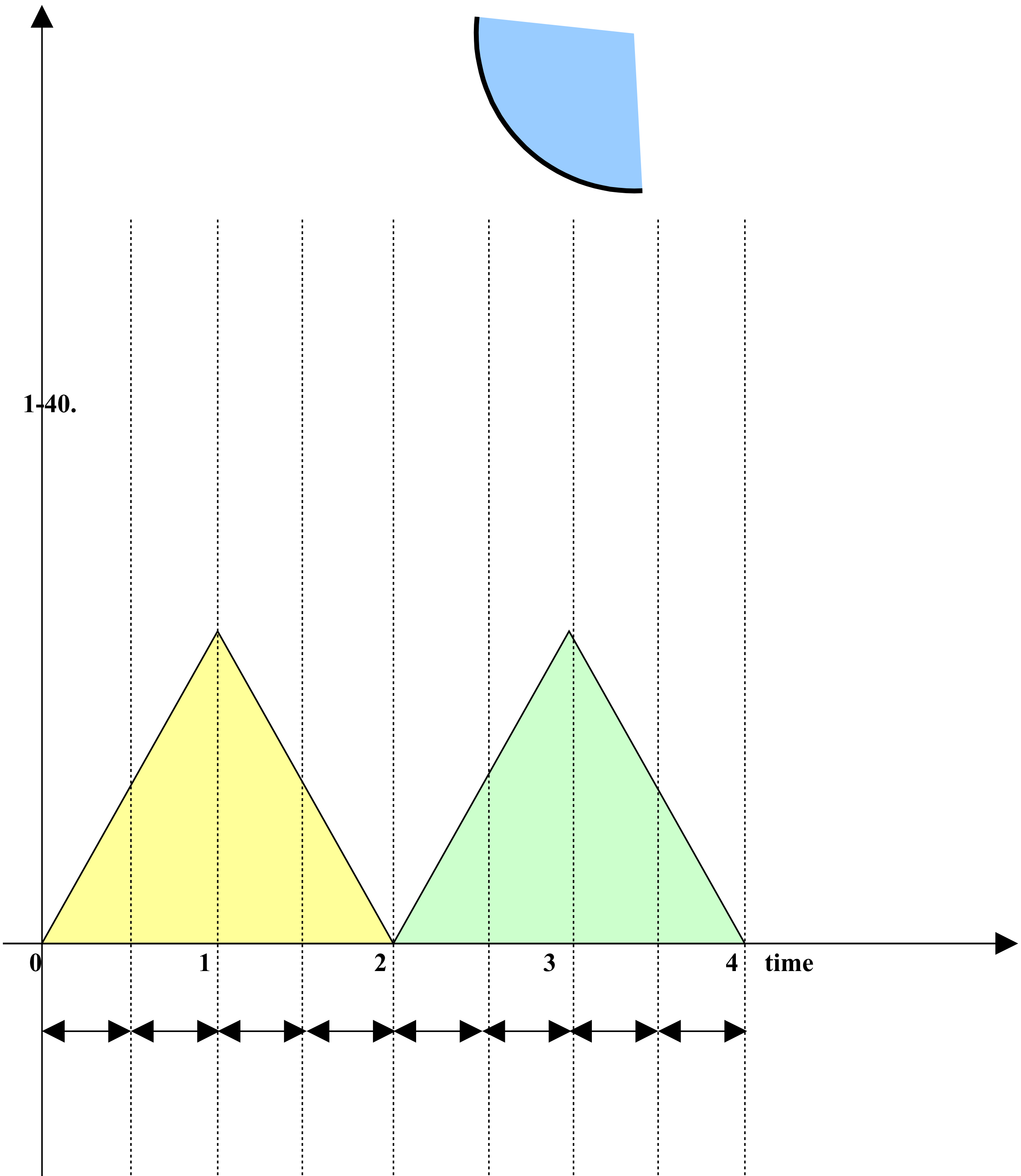


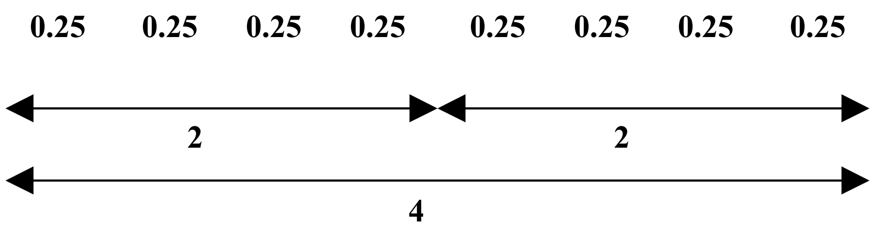
$v_c(t)$
0
π /6
π /2
5π /6
π
time



$i_c(t)$
0
π /6
π /2
5π /6
π
time

1-40.





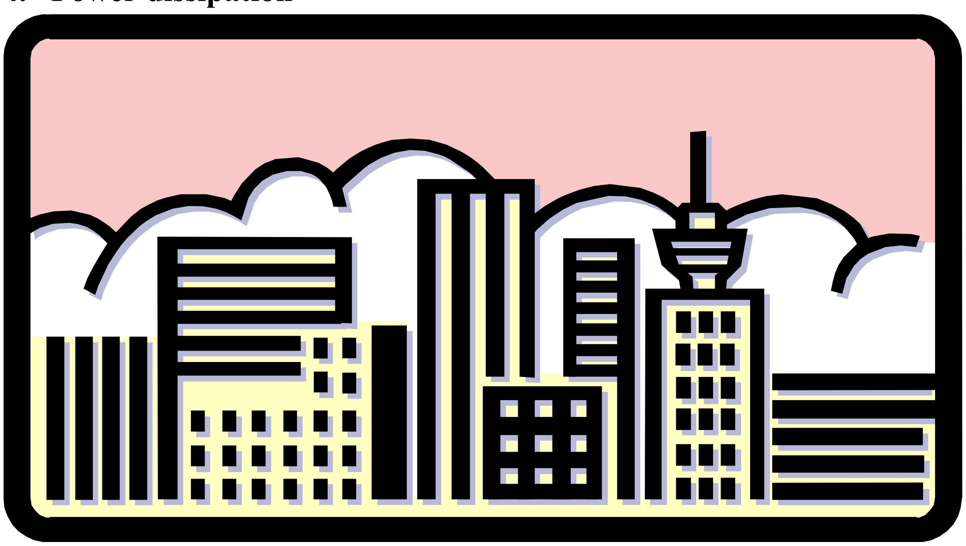
$v_c(t)$	interval
2t	**for $0 \leq t \leq 1$**
-2t + 4	**for $1 \leq t \leq 2$**
2t – 4	**for $02 \leq t \leq 3$**
-2t + 8	**for $3 \leq t \leq 4$**
0	**for $t \geq 4$**

$v_c(t)$	interval	Capacitor(value)
2t	**for $0 \leq t \leq 0.25$**	**1F**
2t	**for $0.25 \leq t \leq 1$**	**0.5F**
-2t + 4	**for $1 \leq t \leq 1.75$**	**0.5F**
-2t + 4	**for $1.75 \leq t \leq 2$**	**1F**
2t – 4	**for $2 \leq t \leq 2.25$**	**1F**

2t – 4	**for 2.25 ≤ t ≤ 3**	**0.5F**
-2t + 8	**for 3 ≤ t ≤ 3.75**	**0.5F**
-2t + 8	**for 3.75 ≤ t ≤ 4**	**1F**
0	**for t ≥ 4**	**1F**

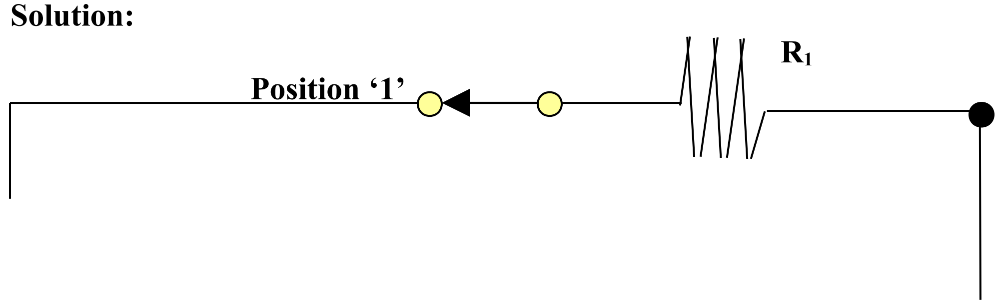
For the remaining part see 1-39 for reference.
1-27 – 1-38. (See chapter#3 for reference)
Before solving chapter#1 following points should be kept in mind:

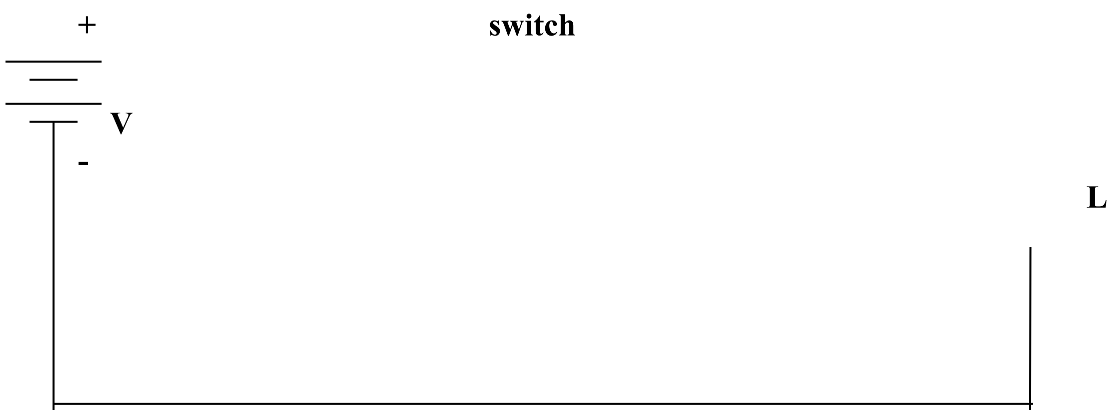
1. **Voltage across an inductor**
2. **Current through the capacitor**
3. **Graphical analysis**
4. **Power dissipation**



MASHAALLAH BHAI(Peon of holy prophet(P.B.U.H))

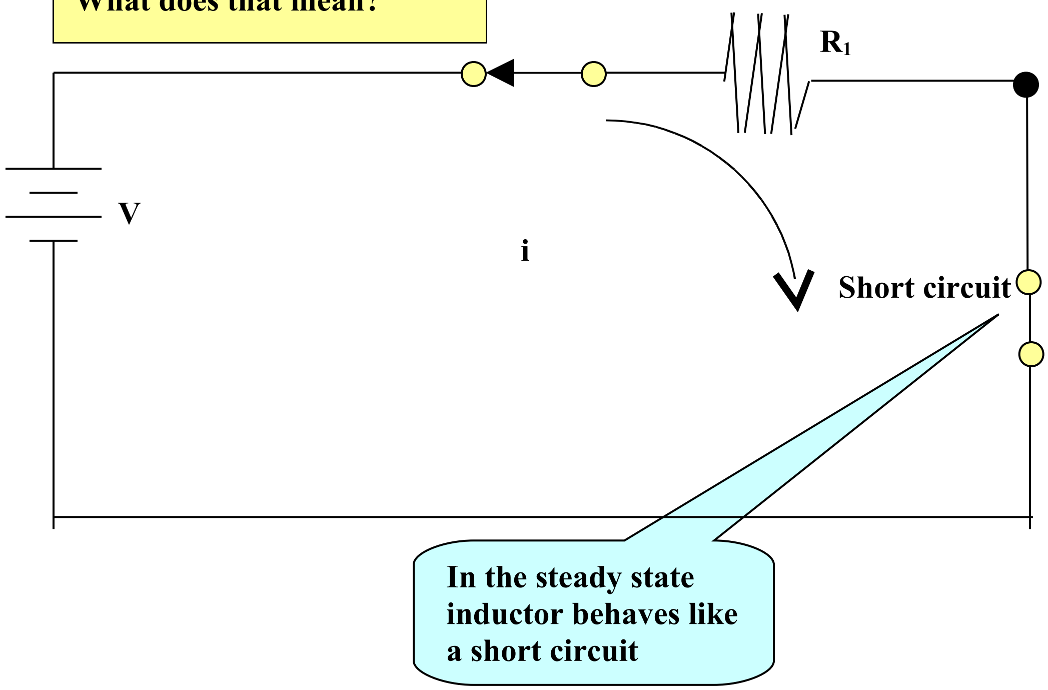
4-1.
Solution:





A steady state current having previously been established in the RL circuit.

What does that mean?

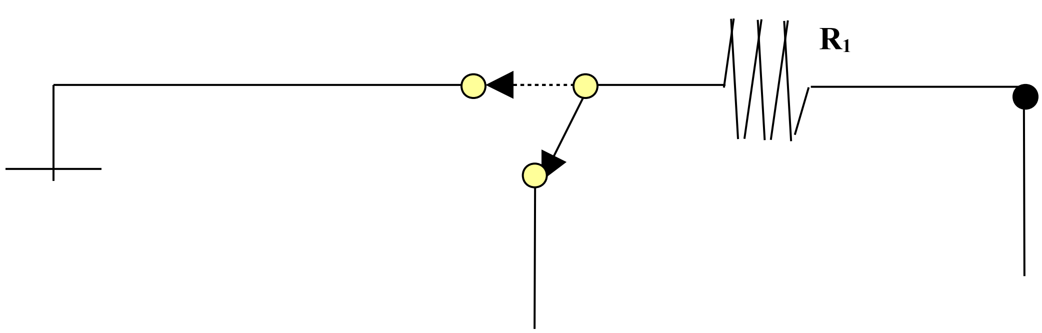


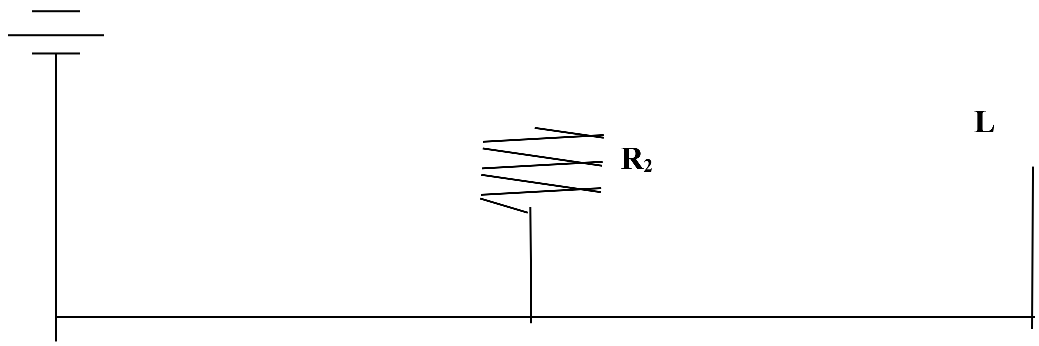
$i(0-) = V/R_1$ (current in RL circuit before switch 'k' is closed)

In an inductor $i(0-) = i(0+) = 0$

It means that $i(0-) = i(0+) = V/R_1$

K is moved from position 1 to position 2 at t = 0.

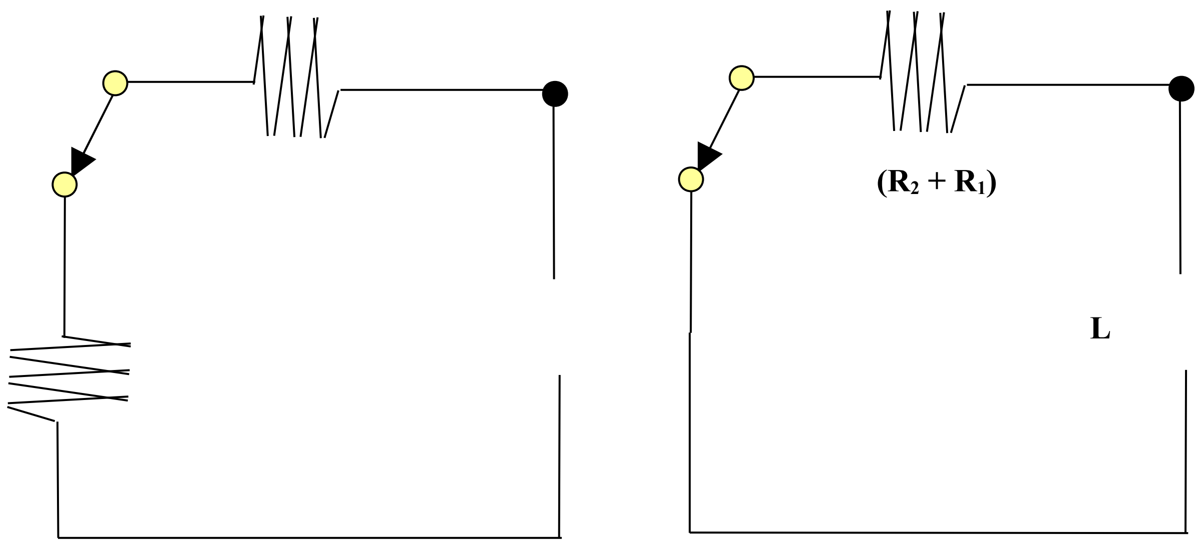




for t ≥ 0
According to kirchhoffs voltage law

Sum of voltage rise = sum of voltage drop

Circuit simplification:



(a) (b)

$Ldi/dt + (R_1 + R_2)i = 0$
$Ldi/dt = -(R_1 + R_2)i$
$di/dt = -(R_1 + R_2)i/L$
$di/i = -(R_1 + R_2)dt/L$
Integrating both the sides,
$\int di/i = \int -(R_1 + R_2)dt/L$
$\int di/i = -(R_1 + R_2)/L\int dt$
$\ln i = -(R_1 + R_2)t/L + C$
$i = e^{-(R1 + R2)t/L + C}$
$i = e^{-(R1 + R2)t/L}e^{C}$
$i = ke^{-(R1 + R2)t/L}$
Applying initial condition
$i(0+) = V/R_1$
$i(0+) = ke^{-(R1 + R2)(0)/L}$
$i(0+) = ke^{0}$

$i(0+) = k$

$i(0+) = V/R_1$
$i(0+) = k$
Equating

$K = V/R_1$

$i = ke^{-(R1 + R2)t/L}$

$i = (V/R_1)e^{-(R1 + R2)t/L}$ is the particular solution.

4-2.
Solution:
Switch is closed to b at t = 0
Initial conditions $v_2(0+) = 0$ & $i(0+) = V_0/R_1$
for $t > 0$
$(1/C_1)\int idt + (1/C_2)\int idt + R_1i = 0$
Differentiating both sides with respect to 't'
$(1/C_1)i + (1/C_2)i + R_1di/dt = 0$
$(1/C_1 + 1/C_2)i + R_1di/dt = 0$
$i/C_{eq} + R_1di/dt = 0$
$i/C_{eq} = -R_1di/dt$
$di/i = (-1/C_{eq}R_1)dt$
Integrating both the sides
$\int di/i = \int (-1/C_{eq}R_1)dt$
$\int di/i = (-1/C_{eq}R_1)\int dt$
$\ln i = (-1/C_{eq}R_1)t + k_1$
$i = e^{(-1/CeqR1)t + k1}$
$i = e^{(-1/CeqR1)t}e^{k1}$
$i = ke^{(-1/CeqR1)t}$
Applying initial condition
$i(0+) = ke^{(-1/CeqR1)(0)}$
$i(0+) = ke^0$
$i(0+) = k(1)$
$i(0+) = k$
$i(0+) = V_0/R_1$
Equating

$K = V_0/R_1$

Therefore
$i = ke^{(-1/CeqR1)t}$

$i = (V_0/R_1)e^{(-1/CeqR1)t}$

$$v_2(t) = (1/C_2)\int_{-\infty}^{t} i\,dt$$

$$v_2(t) = (1/C_2)\int_{-\infty}^{0} i\,dt + (1/C_2)\int_{0}^{t} i\,dt$$

$$= v_2(0+) + (1/C_2)\int_{0}^{t} (V_0/R_1)e^{(-1/CeqR1)t}dt$$

$$= 0 + (1/C_2)\,(V_0/R_1)(-CeqR_1)\Big|\; e^{(-1/CeqR1)t}\Big|_{0}^{t}$$

$$= (1/C_2)\,(V_0)(-Ceq)\Big|\; e^{(-1/CeqR1)t}\Big|_{0}^{t}$$

$$= (1/C_2)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - e^{(-1/CeqR1)(0)}]$$

$$= (1/C_2)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - e^{0}]$$

$$= (1/C_2)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - 1]$$

$$\boxed{v_2(t) = (1/C_2)\,(V_0)(Ceq)[1 - e^{(-1/CeqR1)t}]}$$

$$v_1(t) = (1/C_1)\int_{-\infty}^{t} i\,dt$$

$$v_1(t) = (1/C_1)\int_{-\infty}^{0} i\,dt + (1/C_1)\int_{0}^{t} i\,dt$$

$$= v_1(0+) + (1/C_1)\int_{0}^{t} (V_0/R_1)e^{(-1/CeqR1)t}dt$$

$$= -V_0 + (1/C_1)\,(V_0/R_1)(-CeqR_1)\Big|\; e^{(-1/CeqR1)t}\Big|_{0}^{t}$$

$$= (1/C_1)\,(V_0)(-Ceq)\Big|\; e^{(-1/CeqR1)t}\Big|_{0}^{t}$$

$$= (1/C_1)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - e^{(-1/CeqR1)(0)}]$$

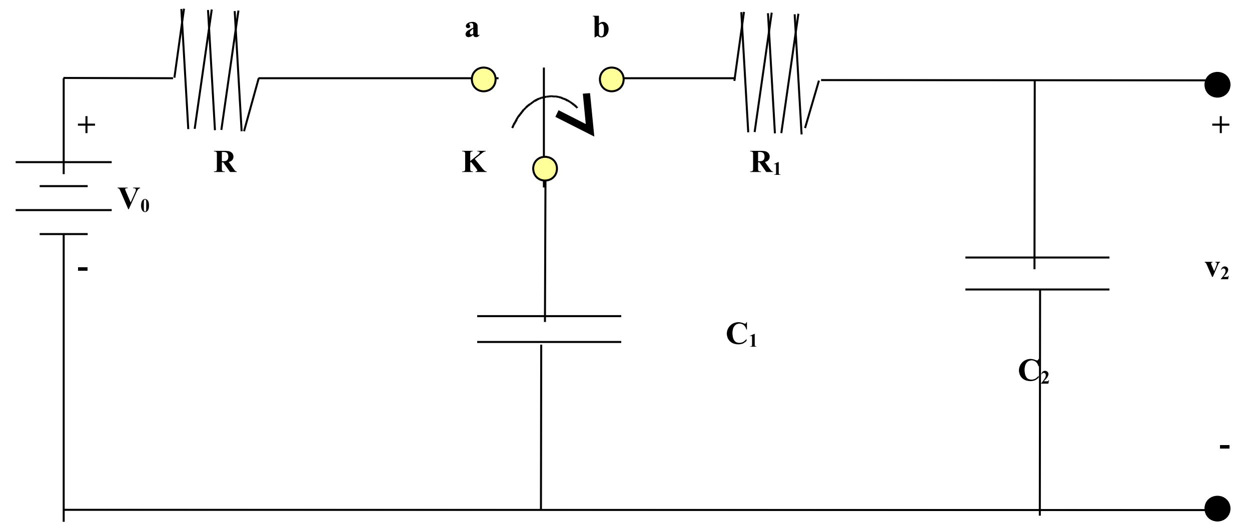
$$= (1/C_1)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - e^{0}]$$

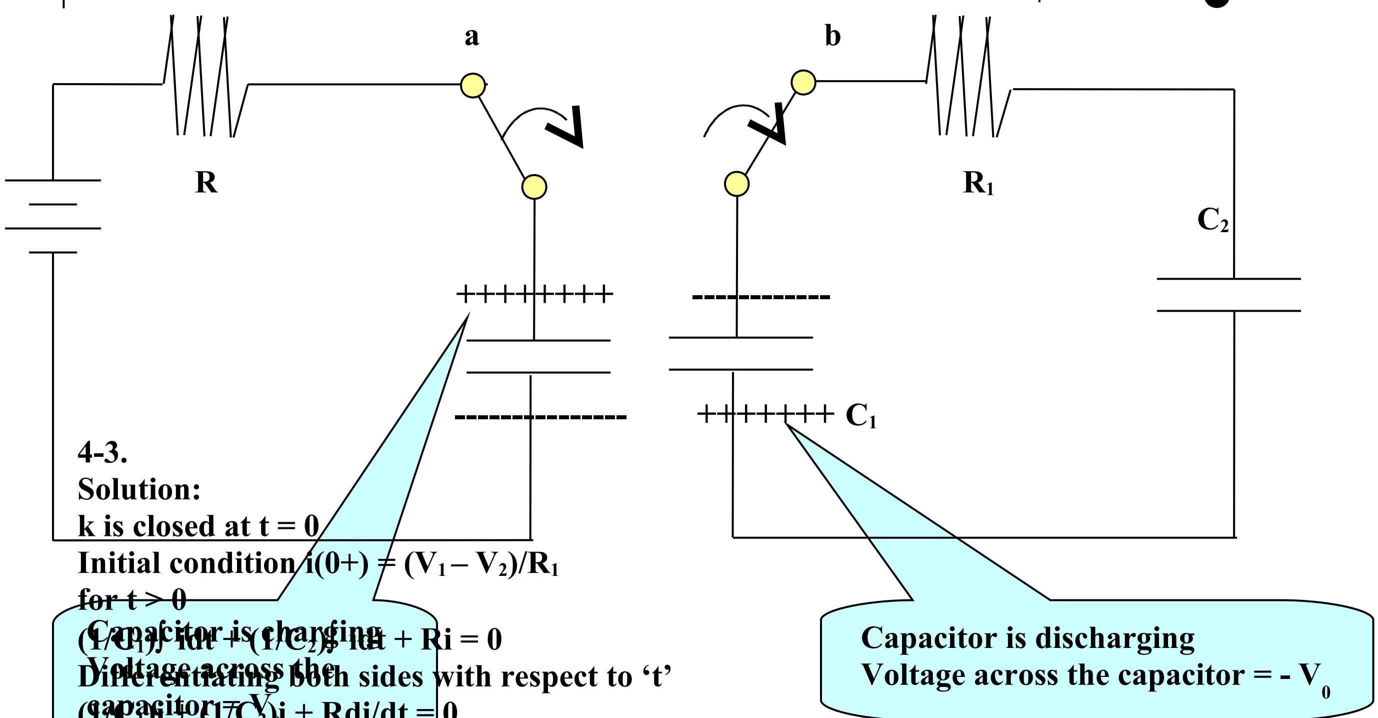
$$= (1/C_1)\,(V_0)(-Ceq)[e^{(-1/CeqR1)t} - 1]$$

$$\boxed{v_1(t) = (1/C_1)\,(V_0)(Ceq)[1 - e^{(-1/CeqR1)t}]}$$

$$V_R(t) = iR_1 = ((V_0/R_1)e^{(-1/CeqR1)t})R_1$$

$V_R(t) = iR_1 = (V_0)e^{(-1/C_{eq}R_1)t}$





4-3.
Solution:
k is closed at t = 0
Initial condition $i(0+) = (V_1 - V_2)/R_1$
for $t > 0$
$(1/C_1)\int idt + (1/C_2)\int idt + Ri = 0$
Differentiating both sides with respect to 't'
$(1/C_1)i + (1/C_2)i + Rdi/dt = 0$
$(1/C_1 + 1/C_2)i + Rdi/dt = 0$
$i/C_{eq} + Rdi/dt = 0$
$i/C_{eq} = -Rdi/dt$
$di/i = (-1/C_{eq}R)dt$
Integrating both the sides
$\int di/i = \int (-1/C_{eq}R)dt$
$\int di/i = (-1/C_{eq}R)\int dt$
$\ln i = (-1/C_{eq}R)t + k_1$
$i = e^{(-1/C_{eq}R)t + k_1}$
$i = e^{(-1/C_{eq}R)t}e^{k_1}$
$i = ke^{(-1/C_{eq}R)t}$

Applying initial condition

$$i(0+) = ke^{(-1/CeqR)(0)}$$
$$i(0+) = ke^{0}$$
$$i(0+) = k(1)$$
$$i(0+) = k$$
$$i(0+) = (V_1 - V_2)/R$$

Equating

$$\boxed{K = (V_1 - V_2)/R}$$

Therefore

$$i = ke^{(-1/CeqR1)t}$$

$$\boxed{i = (V_1 - V_2)/R)e^{(-1/CeqR)t}}$$

$$v_2(t) = (1/C_2)\int_{-\infty}^{t} idt$$

$$v_2(t) = (1/C_2)\int_{-\infty}^{0} idt + (1/C_2)\int_{0}^{t} idt$$

$$= v_2(0+) + (1/C_2)\int_{0}^{t} ((V_1 - V_2)/R)e^{(-1/CeqR)t}dt$$

$$= V_2 + (1/C_2)\,((V_1 - V_2)/R)(-CeqR)\Big|\; e^{(-1/CeqR)t}\Big|_{0}^{t}$$

$$= (1/C_2)\,(V_1 - V_2))(-Ceq)\Big|\; e^{(-1/CeqR)t}\Big|_{0}^{t} + V_2$$

$$= (1/C_2)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - e^{(-1/CeqR)(0)}] + V_2$$
$$= (1/C_2)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - e^{0}] + V_2$$
$$= (1/C_2)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - 1]$$

$$\boxed{v_2(t) = (1/C_2)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)t}] + V_2} \quad (i)$$

$$v_1(t) = (1/C_1)\int_{-\infty}^{t} idt$$

$$v_1(t) = (1/C_1)\int_{-\infty}^{0} idt + (1/C_1)\int_{0}^{t} idt$$

$$= v_1(0+) + (1/C_1)\int_{0}^{t} ((V_1 - V_2)/R)e^{(-1/CeqR)t}dt$$

t

$$= -V_1 + (1/C_1)\,((V_1 - V_2)/R)(-CeqR)\Big|\; e^{(-1/CeqR)t}\Big|_0$$

$$= (1/C_1)\,(V_1 - V_2)(-Ceq)\Big|\; e^{(-1/CeqR)t}\Big|_0^t \; - V_1$$

$$= (1/C_1)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - e^{(-1/CeqR)(0)}] - V_1$$

$$= (1/C_1)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - e^{0}] - V_1$$

$$= (1/C_1)\,(V_1 - V_2)(-Ceq)[e^{(-1/CeqR)t} - 1] - V_1$$

$$\boxed{v_1(t) = (1/C_1)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)t}] - V_1} \qquad (ii)$$

from (i)

$$v_2(t) = (1/C_2)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)t}] + V_2$$
$$v_2(\infty) = (1/C_2)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)(\infty)}] + V_2$$
$$v_2(\infty) = (1/C_2)\,(V_1 - V_2)(Ceq)[1 - e^{-(\infty)}] + V_2$$
$$v_2(\infty) = (1/C_2)\,(V_1 - V_2)(Ceq)[1 - 0] + V_2$$
$$v_2(\infty) = (1/C_2)\,(V_1 - V_2)(Ceq) + V_2$$

$$\boxed{v_2(\infty) = (1/C_2)\,(V_1 - V_2)(Ceq) + V_2}$$

from (ii)

$$v_1(t) = (1/C_1)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)t}] - V_1$$
$$v_1(\infty) = (1/C_1)\,(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)(\infty)}] - V_1$$
$$v_1(\infty) = (1/C_1)\,(V_1 - V_2)(Ceq)[1 - e^{-(\infty)}] - V_1$$
$$v_1(\infty) = (1/C_1)\,(V_1 - V_2)(Ceq)[1 - 0] - V_1 \quad \{e^{-(\infty)} = 0\}$$
$$v_1(\infty) = (1/C_1)\,(V_1 - V_2)(Ceq) - V_1$$

$$\boxed{v_1(\infty) = (1/C_1)\,(V_1 - V_2)(Ceq) - V_1}$$

Hence numerically

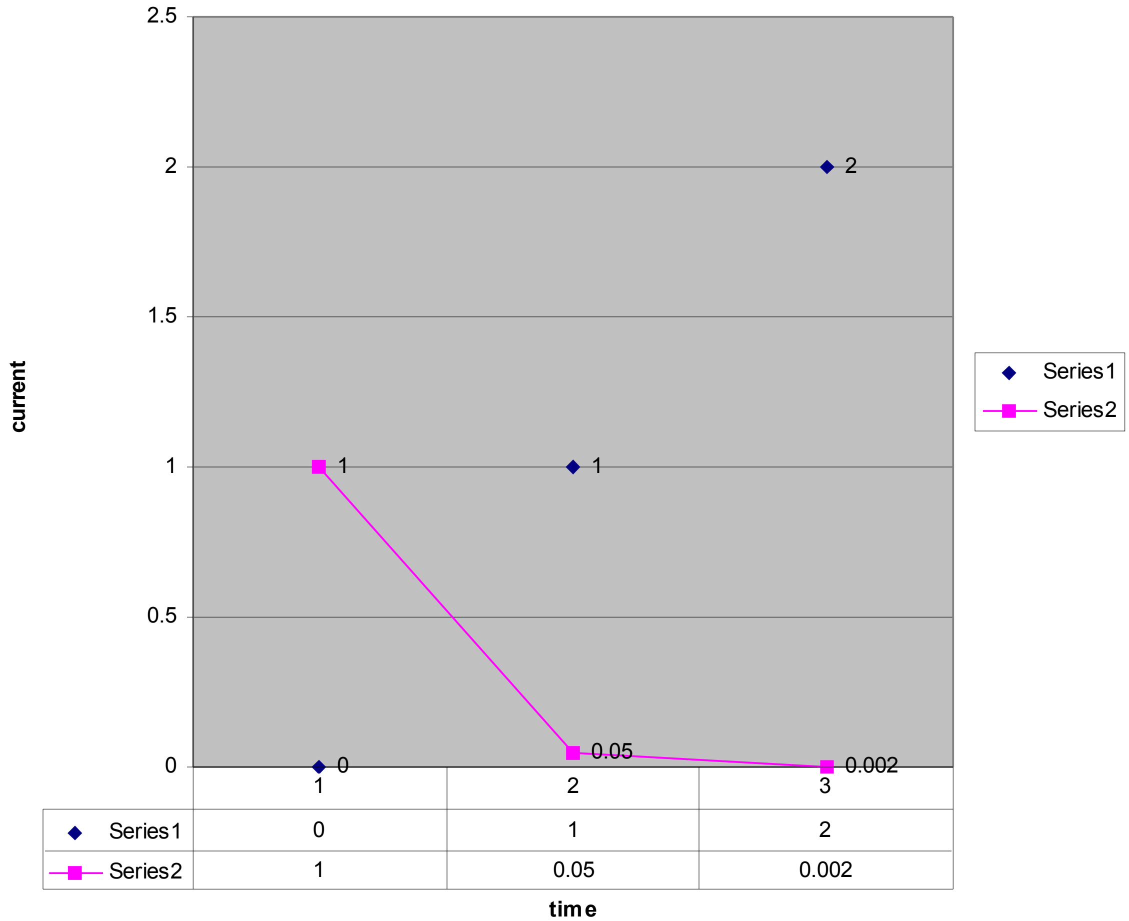
$$\boxed{v_1(\infty) = v_2(\infty)}$$

for $R = 1$-ohm, $C_1 = 1F$, $C_2 = 1/2F$, $V_1 = 2V$, $V_2 = 1V$

$$C_{eq} = C_1C_2/C_1 + C_2 = (1)(1/2)/(1) + (1/2) = (1/2)(2/3) = 2/6 = 1/3F$$
$$i = (V_1 - V_2)/R)e^{(-1/CeqR)t}$$
$$i = (2 - 1)/1)e^{(-t/(1/3)(1)}$$
$$i = e^{-3t}$$

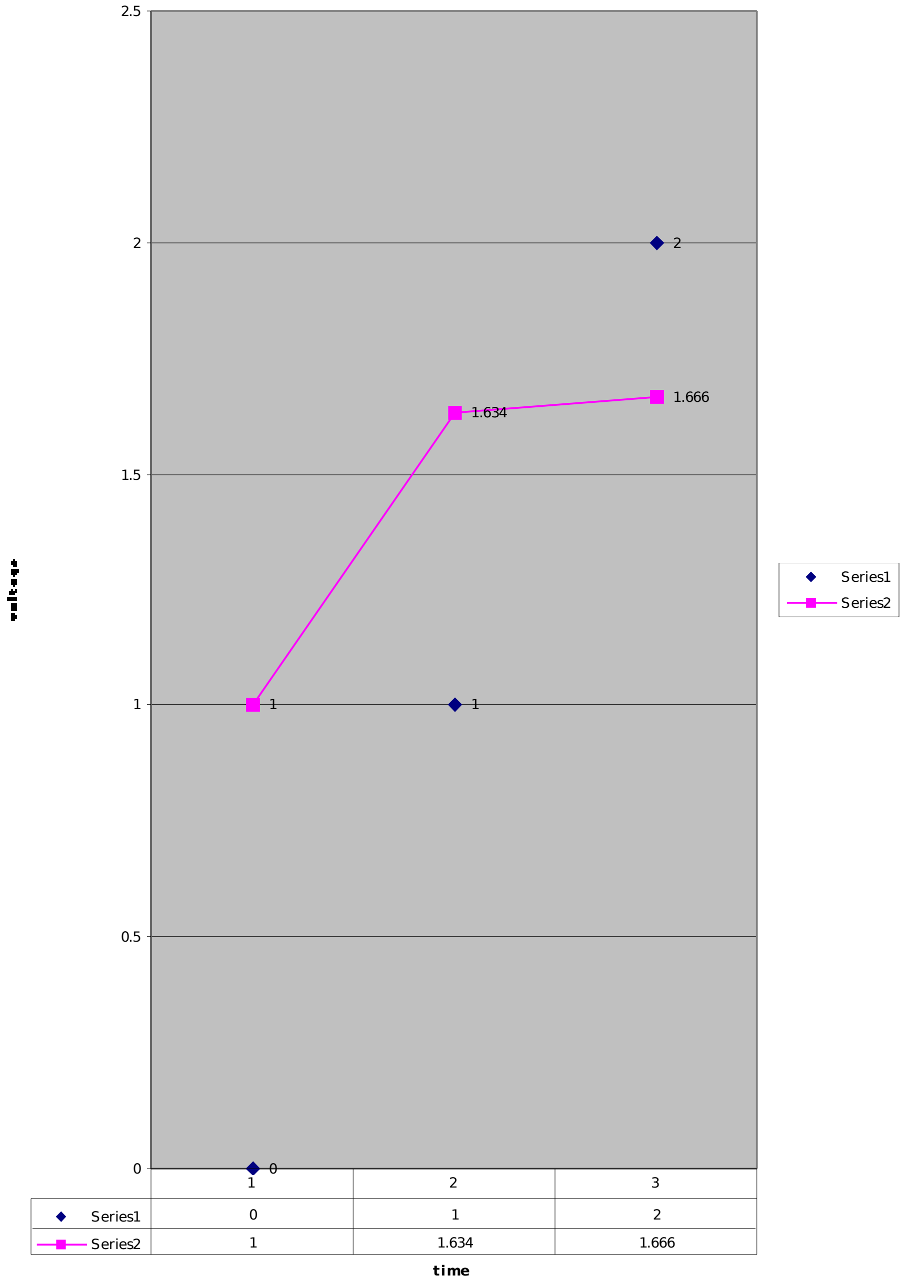
$$\boxed{\text{Time constant} = T = C_{eq}R = (1)(1/3) = 1/3 \text{ secs.}}$$

<u>Sketch i(t)</u>



$$\mathbf{v_2(t) = (1/C_2)(V_1 - V_2)(Ceq)[1 - e^{(-1/CeqR)t}] + V_2}$$
$$\mathbf{v_2(t) = (1/(1/2))(2 - 1)(Ceq)[1 - e^{(-1/CeqR)t}] + 1}$$
$$\mathbf{v_2(t) = 2(1/3)[1 - e^{(-1/(1/3))t}] + 1}$$

$$\boxed{\mathbf{v_2(t) = (2/3)[1 - e^{-3t}] + 1}}$$



At t = 0 switch is moved to position b.
Initial condition $i_{L1}(0-) = i_{L1}(0+) = V/R = 1/1 = 1A$.
$V_2(0+) = (-1)(1/2) = -0.5$ volts
for $t \geq 0$, KCL
$(1/1)\int v_2 dt + v_2/(1/2) + (1/2)\int v_2 dt = 0$

$(1 + 1/2)\int v_2 dt + 2v_2 = 0$
$(3/2)\int v_2 dt + 2v_2 = 0$
Differentiating both sides with respect to 't'
$(3/2)v_2 + 2dv_2/dt = 0$
Dividing both the sides by 2
$\{(3/2)/2\}v_2 + (2/2)dv_2/dt = 0$
$(3/4)v_2 + dv_2/dt = 0$
Solving by method of integrating factor
$P = 3/4, Q = 0$

$$\boxed{v_2(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}}$$

$v_2(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$v_2(t) = e^{-(3/4)t}\int e^{(3/4)t}.(0)dt + ke^{-(3/4)t}$

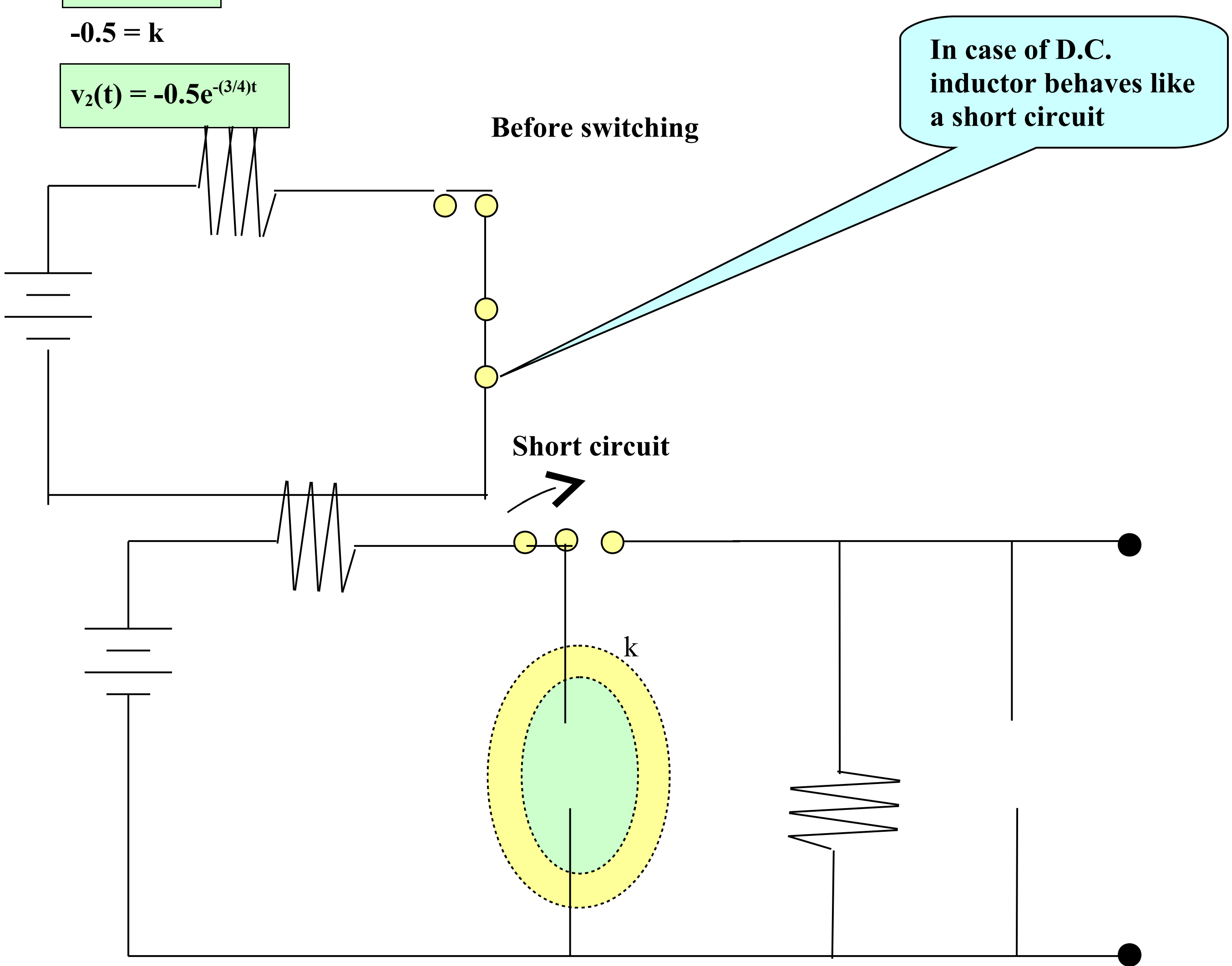
$$\boxed{v_2(t) = ke^{-(3/4)t}}$$

Applying initial condition
$v_2(0+) = ke^{-(3/4)(0+)}$
$v_2(0+) = ke^{0}$
$v_2(0+) = k(1)$

$$\boxed{v_2(0+) = k}$$

$-0.5 = k$

$$\boxed{v_2(t) = -0.5e^{-(3/4)t}}$$

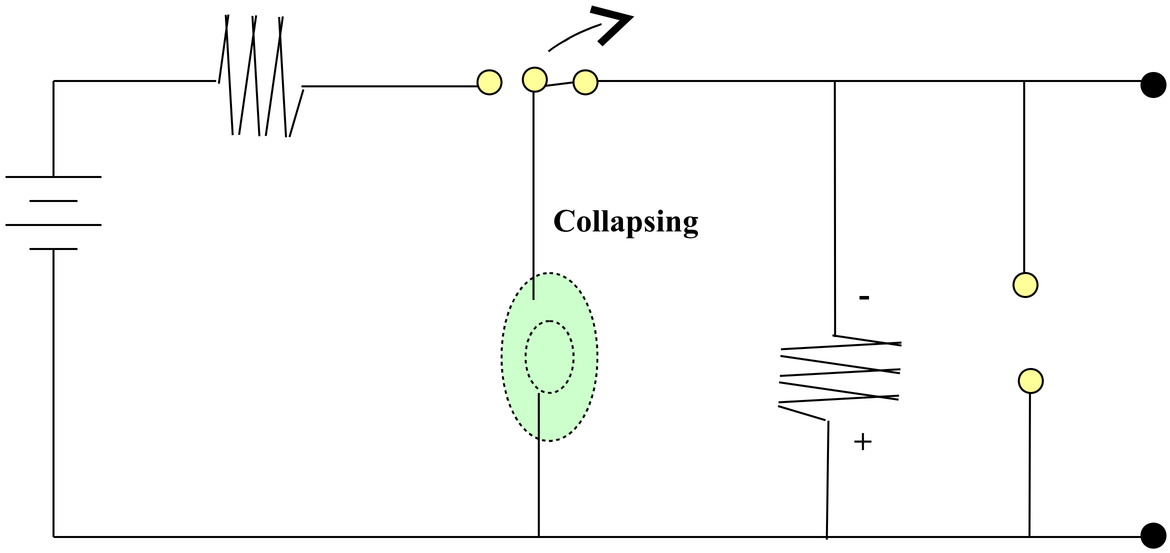


1V **expanding**

+

-

Equivalent network at t = 0+



4-5.
Solution:
Switch is closed at t = 0
Initial condition:-
i(0-) = i(0+) = (20 + 10)/(30 + 20) = 30/50 = 3/5 A
for t ≥ 0,
According to KVL
Sum of voltage rise = sum of voltage drop
20i + (1/2)di/dt = 10
Multiplying both the sides by ‘2’
2(20i) + 2(1/2)di/dt = 10(2)

40i + di/dt = 20
di/dt + 40i = 20
Solving by the method of integrating factor

P = 40	**Q = 20**

i(t) = $e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$

$i(t) = e^{-Pt}\int e^{Pt}.Q dt + ke^{-Pt}$
$i(t) = e^{-40t}\int e^{40t}.(20)dt + ke^{-40t}$
$i(t) = 20e^{-40t}\int e^{40t}dt + ke^{-40t}$
$i(t) = 20e^{-40t}(e^{40t}/40) + ke^{-40t}$

$\boxed{i(t) = 1/2 + ke^{-40t}}$

Applying initial condition
$i(0+) = 1/2 + ke^{-40(0+)}$
$i(0+) = 1/2 + ke^{0}$
$i(0+) = 1/2 + k(1)$
$3/5 = 1/2 + k$
$3/5 - 1/2 = k$
$0.6 - 0.5 = k$

$\boxed{0.1 = k}$

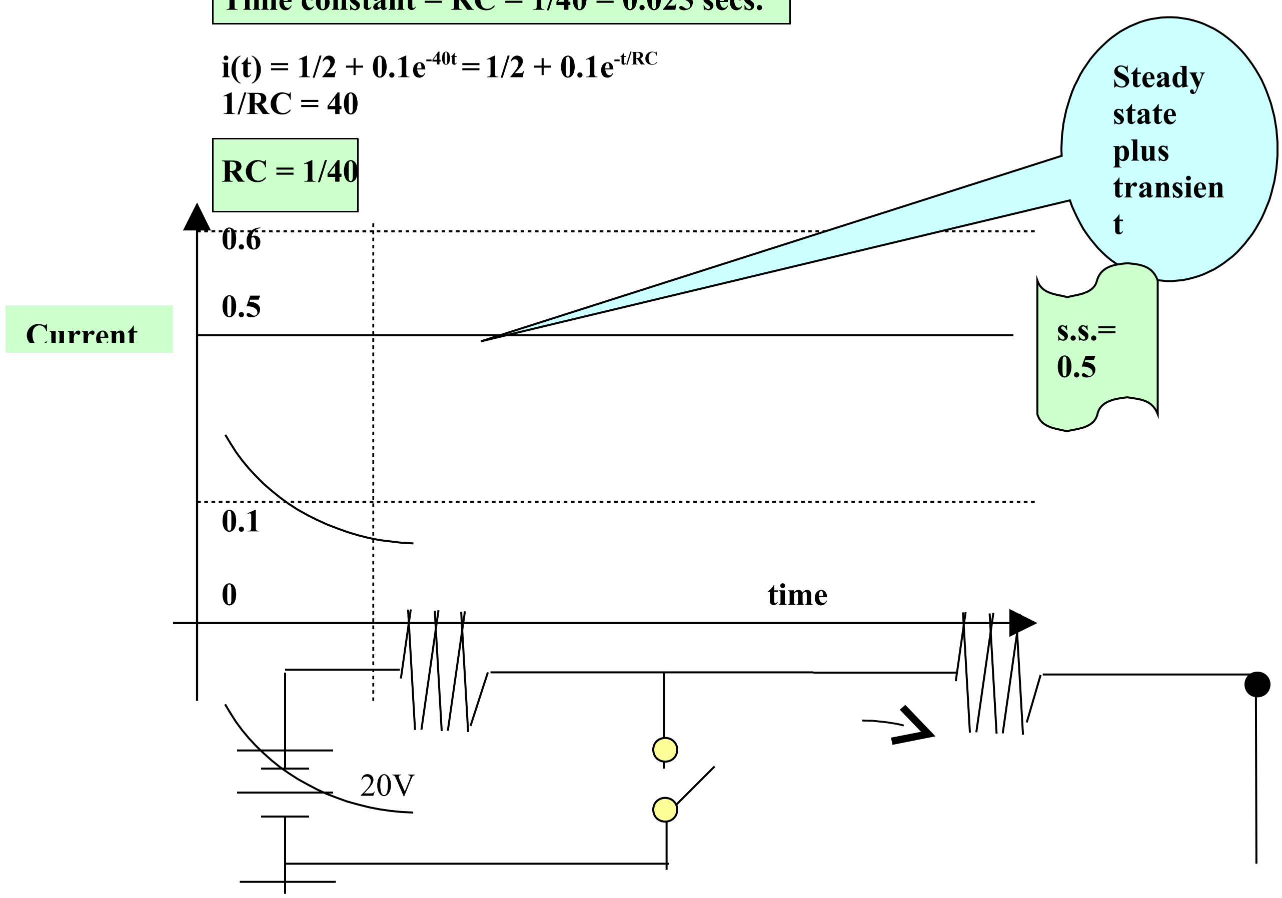
Therefore
$i(t) = 1/2 + ke^{-40t}$

$\boxed{i(t) = 1/2 + 0.1e^{-40t}}$

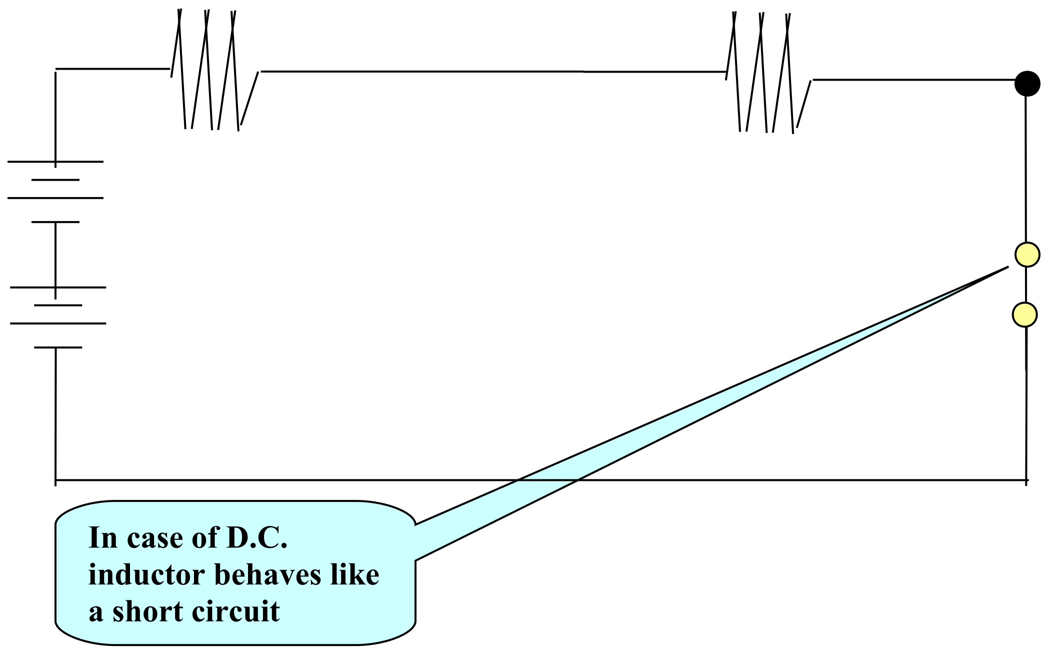
$\boxed{\text{Time constant} = RC = 1/40 = 0.025 \text{ secs.}}$

$i(t) = 1/2 + 0.1e^{-40t} = 1/2 + 0.1e^{-t/RC}$
$1/RC = 40$

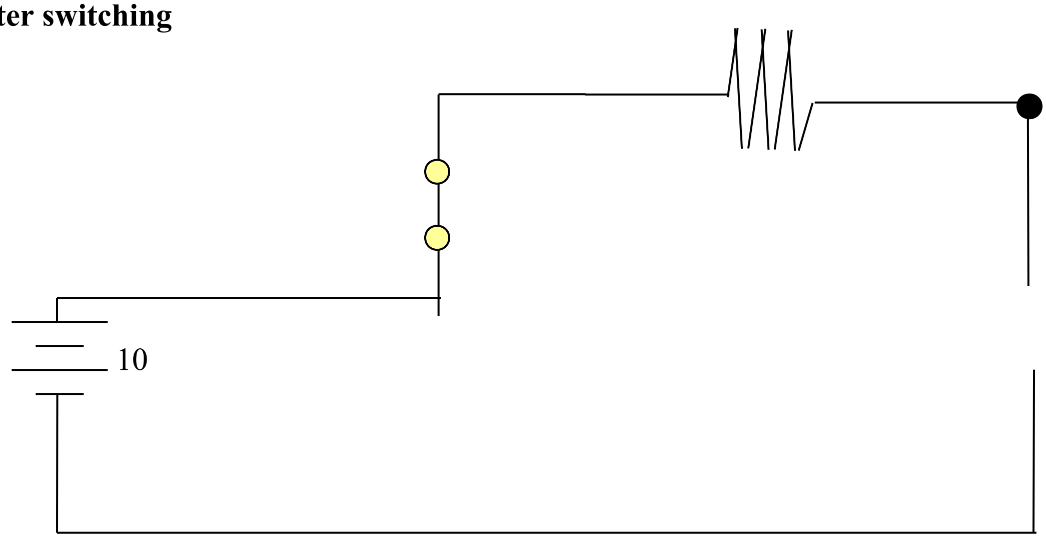
$\boxed{RC = 1/40}$



Before switching



After switching



4-6.
Solution:
Switch is 0pened at t = 0
Initial condition:-
i(0-) = i(0+) = 10/20 = 1/2 A
for t ≥ 0,
According to KVL
Sum of voltage rise = sum of voltage drop
(20 + 30)i + (1/2)di/dt = 30
50i + (1/2)di/dt = 30
Multiplying both the sides by '2'
2(50i) + 2(1/2)di/dt = 30(2)
100i + di/dt = 60
di/dt + 100i = 60
Solving by the method of integrating factor

P = 100	Q = 60

$i(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$i(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$i(t) = e^{-100t}\int e^{100t}.(60)dt + ke^{-100t}$
$i(t) = 60e^{-100t}\int e^{100t}dt + ke^{-100t}$
$i(t) = 60e^{-100t}(e^{100t}/100) + ke^{-100t}$

$\boxed{i(t) = 3/5 + ke^{-100t}}$

Applying initial condition
$i(0+) = 3/5 + ke^{-40(0+)}$
$i(0+) = 3/5 + ke^{0}$
$i(0+) = 3/5 + k(1)$
$1/2 = 3/5 + k$
$1/2 - 3/5 = k$
$0.5 – 0.6 = k$

$\boxed{- 0.1 = k}$

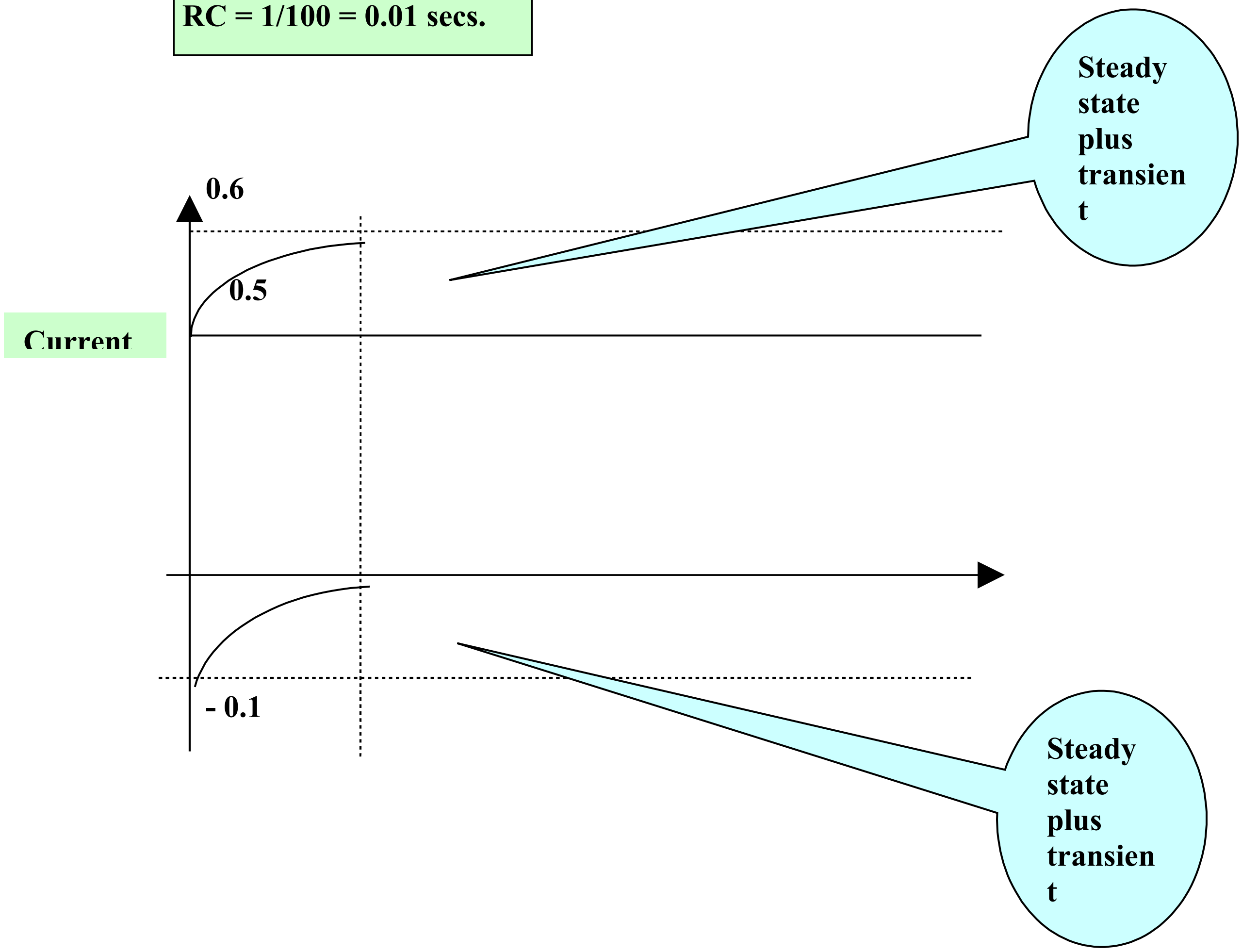
Therefore
$i(t) = 3/5 + ke^{-100t}$

$\boxed{i(t) = 3/5 - 0.1e^{-100t}}$

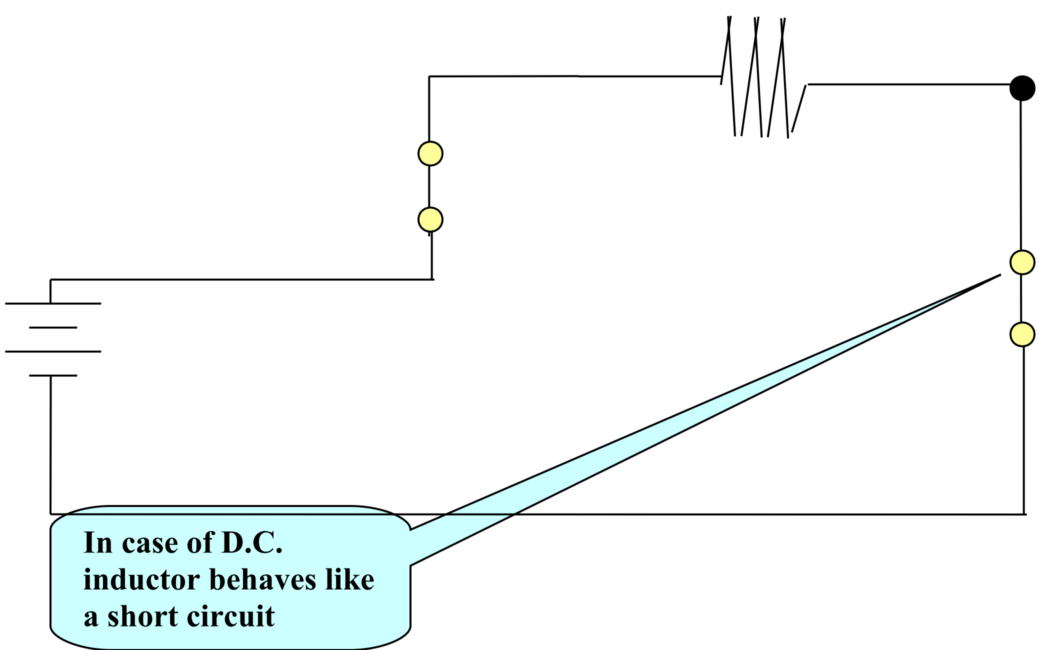
$\boxed{\text{Time constant} = RC = 1/40 = 0.025 \text{ secs.}}$

$i(t) = 3/5 - 0.1e^{-100t} = 3/5 - 0.1e^{-t/RC}$
$1/RC = 100$

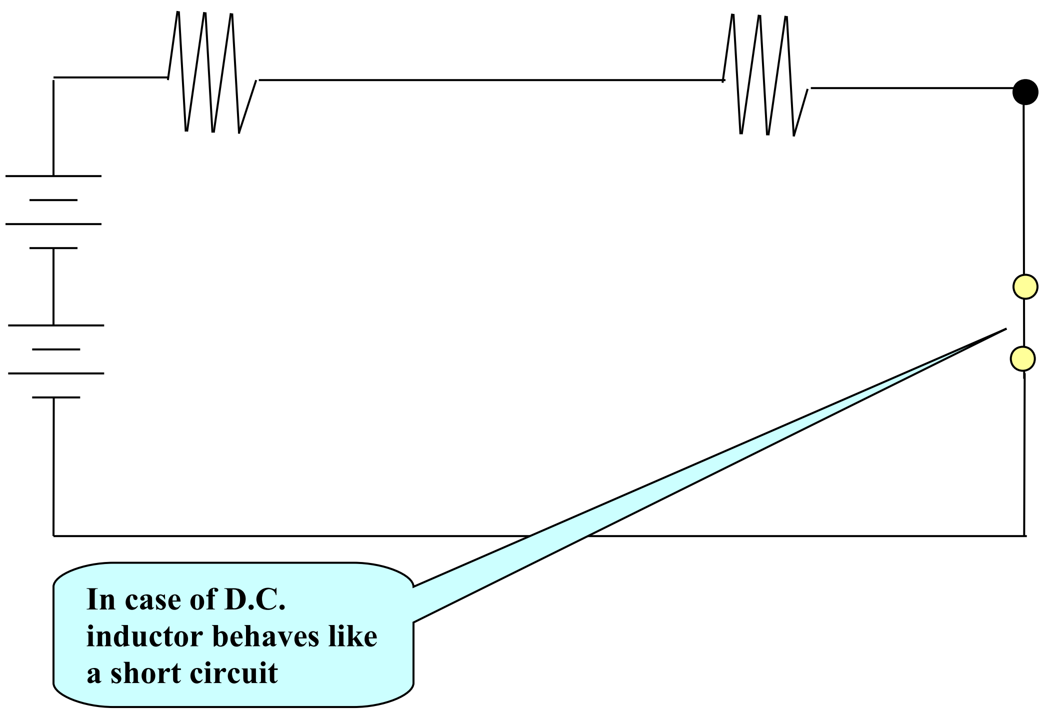
RC = 1/100 = 0.01 secs.



Before switching:



After switching:



4-7.

Solution:

Initial condition $v_c(0-) = v_c(0+) = v_2(0+) = 0$

for $t \geq 0$

$(v_2 - v_1)/R_1 + Cdv_2/dt + v_2/R_2 = 0$

$v_2/R_1 - v_1/R_1 + Cdv_2/dt + v_2/R_2 = 0$

$v_2/R_1 + Cdv_2/dt + v_2/R_2 = v_1/R_1$

$v_2/R_1 + v_2/R_2 + Cdv_2/dt = v_1/R_1$

$v_2(1/R_1 + 1/R_2) + Cdv_2/dt = v_1/R_1$

Dividing both the sides by 'C'

$v_2(1/R_1 + 1/R_2)/C + Cdv_2/Cdt = v_1/CR_1$

$v_2(1/R_1 + 1/R_2)/C + dv_2/dt = v_1/CR_1$

C = (1/20) F	R_1 = 10-ohm	R_2 = 20-ohm

$v_2(1/10 + 1/20)/(1/20) + dv_2/dt = e^{-t}/\{(1/20)(10)\}$
$v_2(0.1 + 0.05)/(0.05) + dv_2/dt = e^{-t}/\{0.5\}$
$v_2(0.15)/(0.05) + dv_2/dt = e^{-t}/\{0.5\}$
$3v_2 + dv_2/dt = 2e^{-t}$
Here

P = 3	Q = $2e^{-t}$

Solving by the method of integrating factor
$v_2(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$v_2(t) = e^{-3t}\int e^{3t}.(2e^{-t})dt + ke^{-3t}$
$v_2(t) = 2e^{-3t}\int e^{3t}e^{-t}dt + ke^{-3t}$
$v_2(t) = 2e^{-3t}\int e^{2t}dt + ke^{-3t}$
$v_2(t) = 2e^{-3t}(e^{2t})/2 + ke^{-3t}$
$v_2(t) = e^{-t} + ke^{-3t}$
Applying initial condition
$v_2(t) = e^{-t} + ke^{-3t}$
$v_2(0+) = e^{-0} + ke^{-(0)t}$
$0 = 1 + k(1)$
$0 = 1 + k$

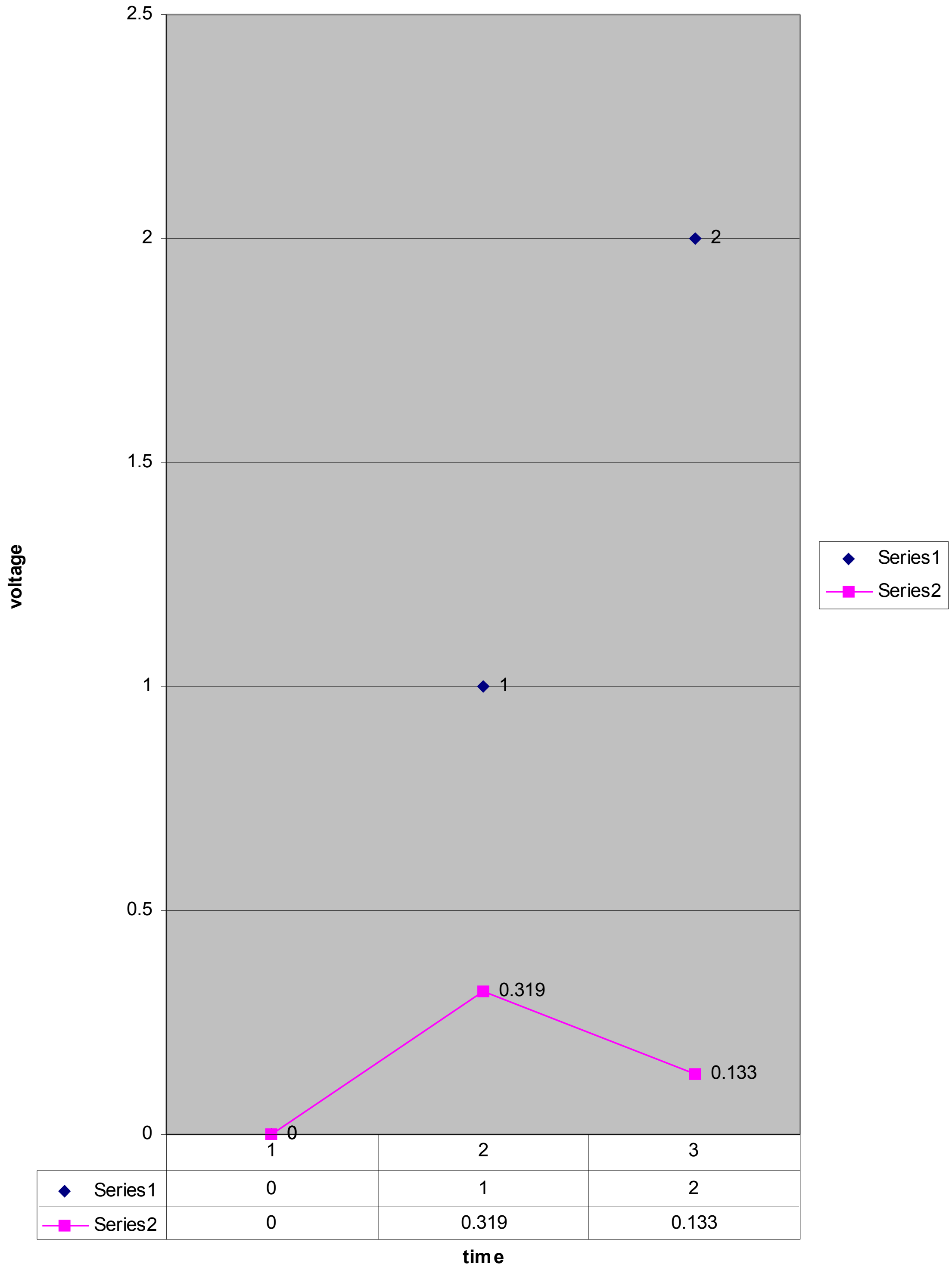
$k = -1$

$v_2(t) = e^{-t} + ke^{-3t}$

$v_2(t) = e^{-t} - e^{-3t}$

Time constant of e^{-t} = 1sec.
Time constant of e^{-3t} = 0.33 secs.

$v_2(t) = e^{-t} - e^{-3t}$
Sketch $v_2(t)$



You should implement a program using JAVA for the solution of the equation $v_2(t) = e^{-t} - e^{-3t}$.

```
import java.io.*;
public class Addition {
   public static void main (String args []) throws
IOException {

     BufferedReader stdin = new BufferedReader
            (new InputStreamReader(System.in));

   double e = 2.718;
   double a, b;
   String string2, string1;
   int num1, num2;

   System.out.println("enter the value of X:");
   string2 = stdin.readLine();
   num2 = Integer.parseInt (string2);

   for(int c = 0; c <= num2; c++){
   System.out.println("enter the value of t:");
   string1 = stdin.readLine();
   num1 = Integer.parseInt (string1);
   a =(double)(1/Math.pow(e, num1));
   b =(double)(1/Math.pow(e, 3*num1));
   System.out.println("The solution is:" + (a - b));
   }//for loop
}//method main
}//class Addition
```

4-9.
Solution:
Network attains a steady state
Therefore
$i_{R2}(0-) = V_0/R_1 + R_2$
$i_{R2}(0-) = 3/10 + 5 = 3/15 = 1/5$ Amp.
$v_a(0+) = i_{R2}(0+)(R_2)$

$v_a(0+) = (1/5)(5) = 1$ Volt

for $t \geq 0$
According to kirchhoffs current law:

$(v_a - V_0)/R_1 + v_a/R_2 + (1/L)\int v_a dt = 0$

By putting $R_1 = 10$, $R_2 = 5$, $V_0 = 3$ & $L = ½$

$(v_a - 3)/10 + v_a/5 + (1/(1/2))\int v_a dt = 0$

$(v_a - 3)/10 + v_a/5 + 2\int v_a dt = 0$

$v_a/10 - 3/10 + v_a/5 + 2\int v_a dt = 0$

$3v_a/10 + 2\int v_a dt = 3/10$

Differentiating with respect to 't'

$d/dt\{3v_a/10 + 2\int v_a dt\} = d/dt\{3/10\}$

$d/dt\{3v_a/10\} + d/dt\{2\int v_a dt\} = d/dt\{3/10\}$

$(3/10)d/dt\{v_a\} + 2v_a = 0$

$(3/10)d/dt\{v_a\} = - 2v_a$

$d/dt\{v_a\} = - 2v_a/(3/10)$

$d/dt\{v_a\} = - 20v_a/3$

$dv_a/v_a = - 20dt/3$

Integrating both the sides

$\int dv_a/v_a = \int - 20dt/3$

$\ln v_a = - 20t/3 + C$

$v_a = e^{-20t/3 + C}$

$v_a = e^{-20t/3} e^C$

$$\boxed{v_a = ke^{-20t/3}}$$

Applying initial condition

$v_a(0+) = ke^{-20(0+)/3}$

$v_a(0+) = ke^0$

$v_a(0+) = k(1)$

$$\boxed{1 = k}$$

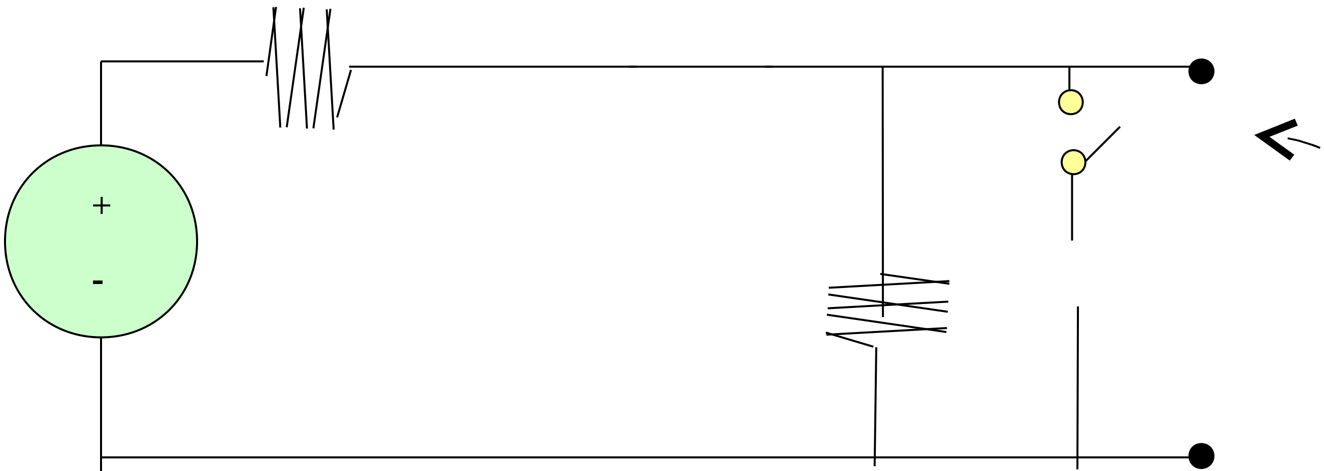
Therefore

$v_a = ke^{-20t/3}$

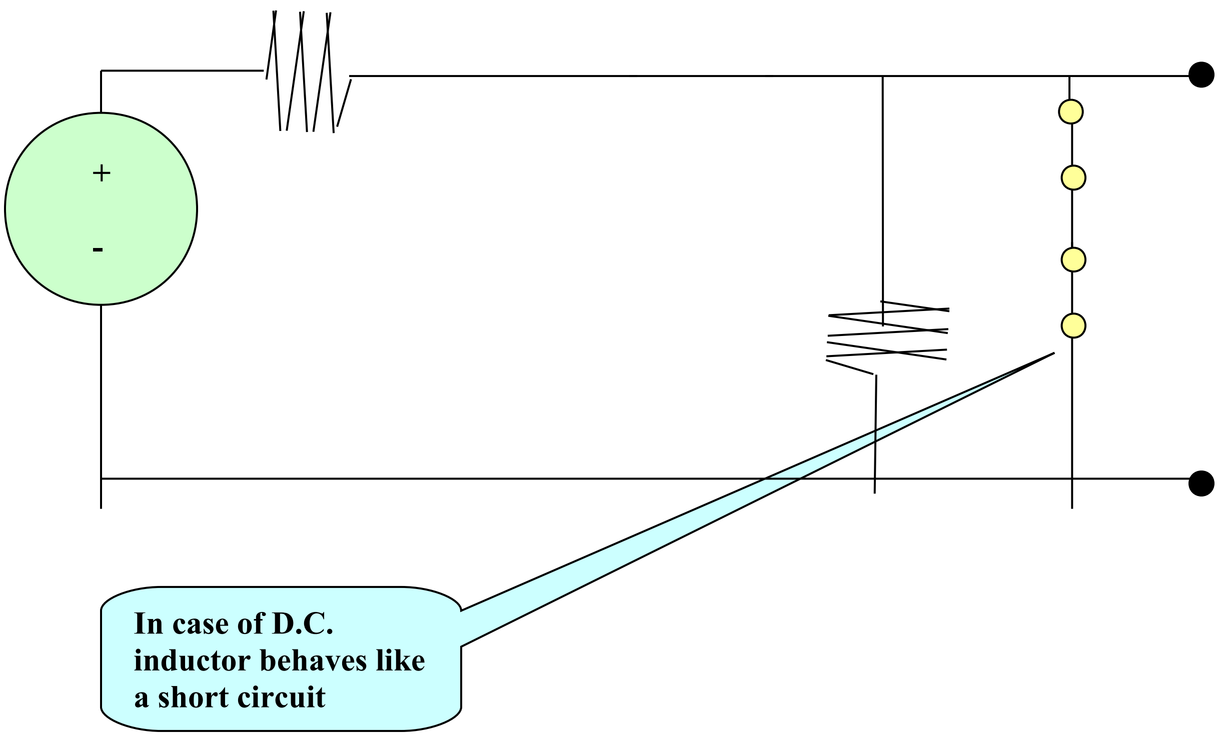
$v_a = (1)e^{-20t/3}$

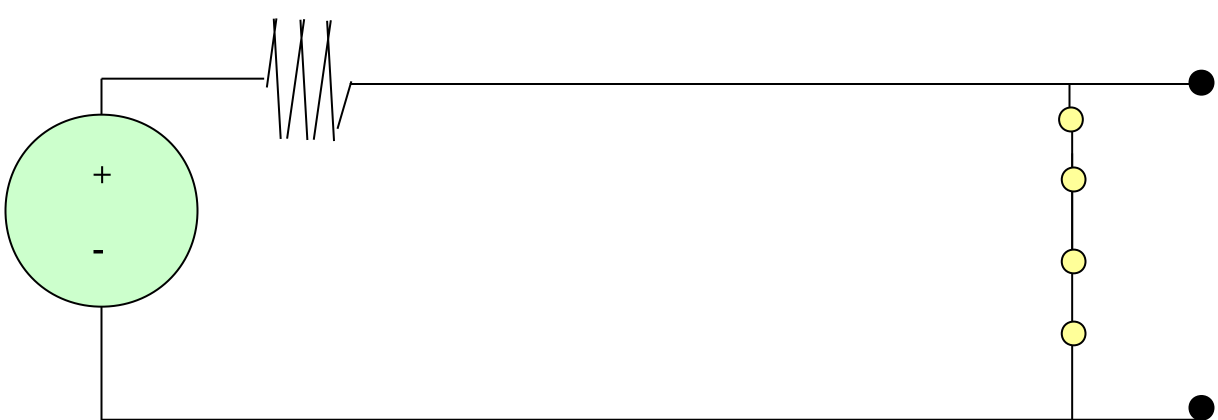
$$\boxed{v_a = e^{-20t/3}}$$

Before switching



After switching





4-10.
Solution:
K is opened at $t = 0$
But $v_2(0-) = v_2(0+) = (1/3)I_0$
for $t \geq 0$, KCL
$v_2/1 + (1/2)dv_2/dt = I_0$
$v_2/1 + (1/2)dv_2/dt = I_0$
Multiplying both the sides by '2'
$2v_2/1 + 2(1/2)dv_2/dt = 2I_0$
$2v_2 + dv_2/dt = 2I_0$

$\boxed{dv_2/dt + 2v_2 = 2I_0}$
Solving by integrating factor method

$P = 2$	$Q = 2I_0$

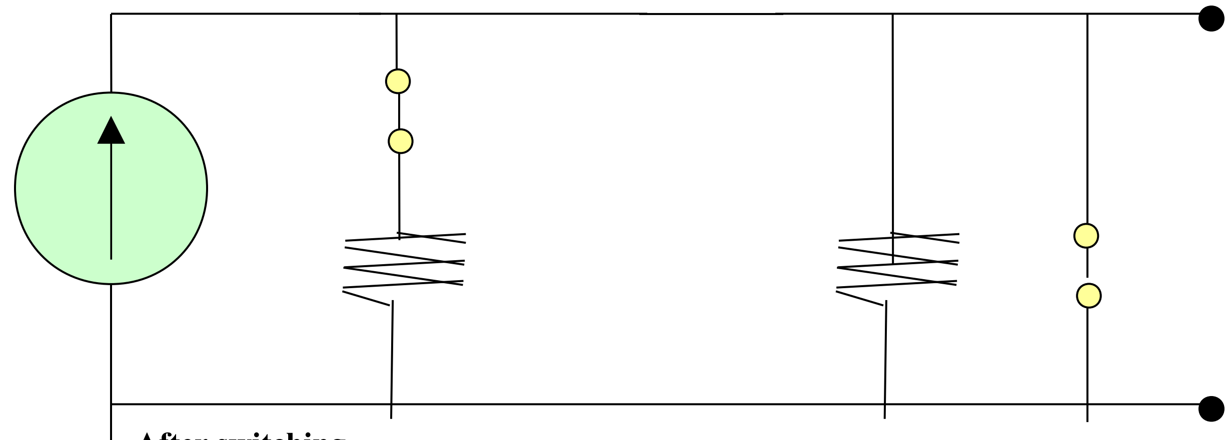
$v_2(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$v_2(t) = e^{-2t}\int e^{2t}.(2I_0)dt + ke^{-2t}$
$v_2(t) = 2I_0e^{-2t}\int e^{2t}dt + ke^{-2t}$
$v_2(t) = 2I_0e^{-2t}(e^{2t})/2 + ke^{-2t}$
$v_2(t) = I_0e^0 + ke^{-2t}$
$v_2(t) = I_0(1) + ke^{-2t}$

$\boxed{v_2(t) = I_0 + ke^{-2t}}$

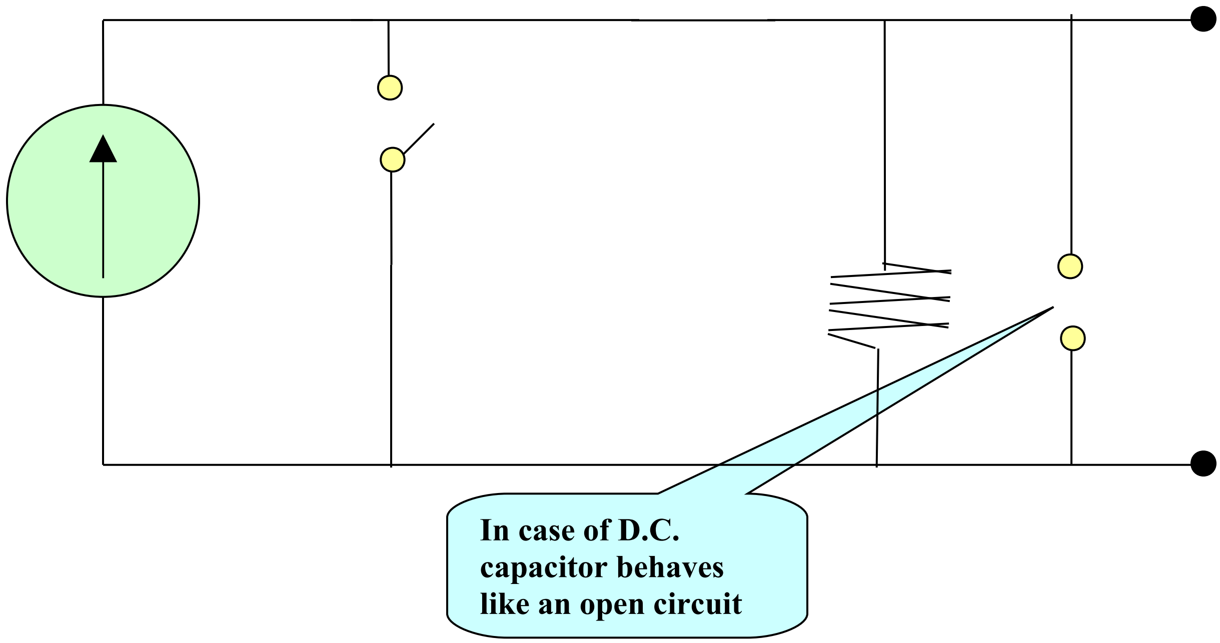
Applying initial condition
$v_2(t) = I_0 + ke^{-2t}$
$v_2(0+) = I_0 + ke^{-2(0+)}$
$v_2(0+) = I_0 + ke^0$
$v_2(0+) = I_0 + k(1)$
$(1/3)I_0 = I_0 + k$
$(1/3)I_0 - I_0 = k$

$\boxed{-(2/3)I_0 = k}$
$v_2(t) = I_0 + ke^{-2t}$
$v_2(t) = I_0 + (-(2/3)I_0)e^{-2t}$

$\boxed{v_2(t) = I_0(1 - (2/3)e^{-2t})}$
Before switching



After switching



4-12.
Solution:
Switch closed at $t = 0$
Initial condition:-
$i_L(0-) = V/(R_1 + R_2)$
$i_L(0-) = i_L(0+) = V/(R_1 + R_2)$
for $t \geq 0$, KVL $\Rightarrow$
$R_1 i + L di/dt = V$
Dividing both the sides by 'L'
$R_1 i/L + di/dt = V/L$
Solving by integrating factor method

$P = R_1/L$	$Q = V/L$

$i(t) = e^{-Pt}\int e^{Pt}.Qdt + ke^{-Pt}$
$i(t) = e^{-(R1/L)t}\int e^{(R1/L)t}(V/L)dt + ke^{-(R1/L)t}$
$i(t) = e^{-(R1/L)t}\{e^{(R1/L)t}/(R_1/L)\}(V/L) + ke^{-(R1/L)t}$

$$i(t) = \{e^0/(R_1/L)\}(V/L) + ke^{-(R1/L)t}$$
$$i(t) = \{1/(R_1/L)\}(V/L) + ke^{-(R1/L)t}$$
$$i(t) = (L/R_1)(V/L) + ke^{-(R1/L)t}$$

$$\boxed{i(t) = V/R_1 + ke^{-(R1/L)t}}$$

Applying initial condition

$$i(t) = V/R_1 + ke^{-(R1/L)t}$$
$$i(0+) = V/R_1 + ke^{-(R1/L)(0+)}$$
$$i(0+) = V/R_1 + ke^0$$
$$i(0+) = V/R_1 + k(1)$$
$$V/(R_1 + R_2) = V/R_1 + k$$

$$\boxed{V/(R_1 + R_2) - V/R_1 = k}$$

$$i(t) = V/R_1 + ke^{-(R1/L)t}$$

$$\boxed{i(t) = V/R_1 + \{V/(R_1 + R_2) - V/R_1\}e^{-(R1/L)t}}$$