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Arti Verma
lecturer (physics)
RCET Bhilai

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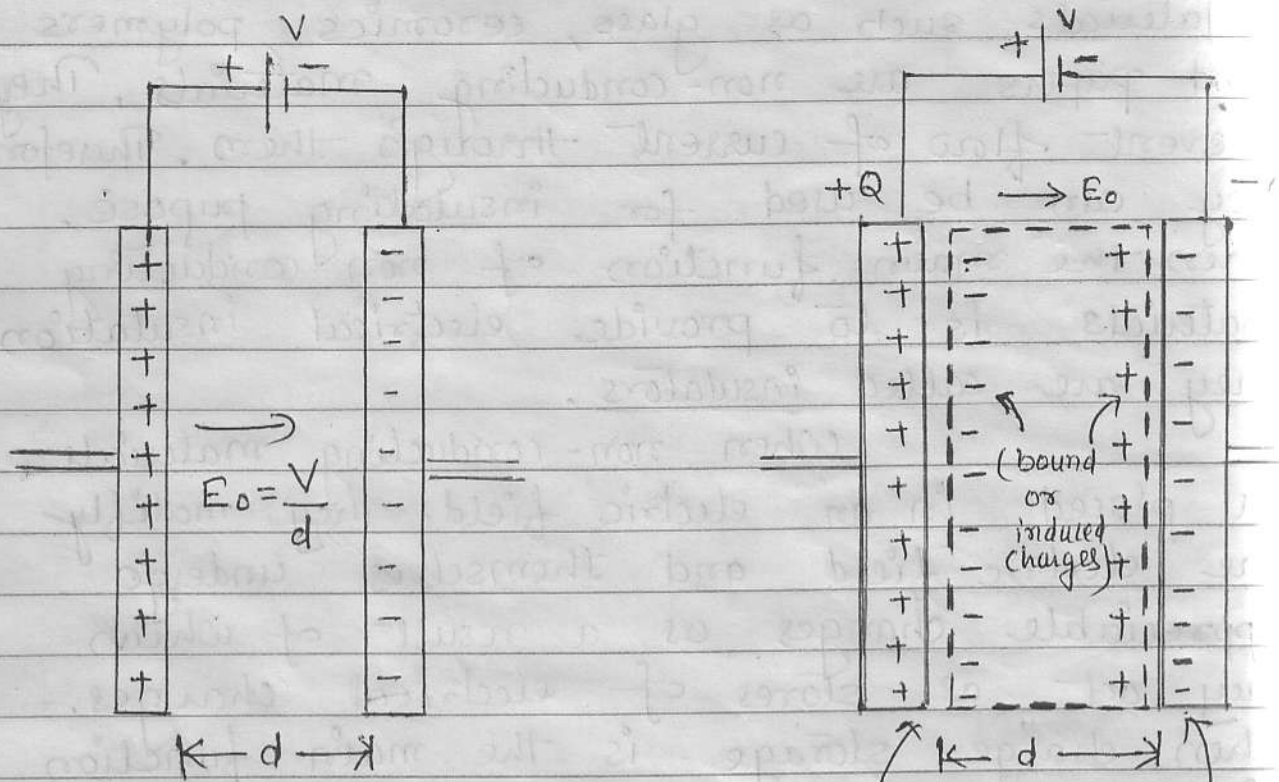
DIELECTRICS & MAGNETIC MATERIALSDIELECTRICS:

Materials such as glass, ceramics, polymers and papers are non-conducting materials. They prevent flow of current through them. Therefore they can be used for insulating purpose. When the main function of non-conducting materials is to provide electrical insulation they are called insulators.

When non-conducting materials are placed in an electric field, they modify the electric field and themselves undergo appreciable changes as a result of which they act as stores of electrical charges. When charge storage is the main function of the materials are called dielectrics. For a material to be good dielectrics, it must be a insulator. Hence any insulator is dielectrics.

A dielectrics may be described in terms of energy band structure. The forbidden gap E_g is very large in dielectrics and excitation of electrons from the normally full valence band to empty conduction band is not possible under ordinary conditions. Therefore conduction cannot occur in dielectrics. Even if a dielectric contains impurity extrinsic conduction is not possible. The resistivity of ideal dielectrics is infinitely high. However in daily practice dielectrics conduct electric current to a negligible extent and their resistivity ranges from 10^{10} to 10^{20} ohm-m.

2.1A DIELECTRIC CONSTANT



(parallel plate capacitor
in presence of vacuum)

(parallel plate capacitor
in presence of
dielectrics)

fig 5.1

Definition: The ratio of the capacitance of a given capacitor with the material filling the entire space between its plates to the capacitance of same capacitor in vacuum is called Dielectric constant.

Dielectrics are mainly characterised by dielectric constant which is also called relative permittivity. When a voltage V is applied across parallel plates of capacitor, let the charges on plate be Q and on other $-Q$. The capacitance is given by

$$C = \frac{Q}{V}$$

Now consider the parallel plate capacitor with vacuum in the region between the plates, the capacitance is

$$C_0 = \frac{\epsilon_0 A}{d}$$

where $A \rightarrow$ area of plate and

$d \rightarrow$ distance between plates;

$\epsilon_0 \rightarrow$ permittivity of vacuum and is equal to $8.85 \times 10^{-12} \text{ F/m}$.

If dielectric is inserted into the region within plates then

$$C = \frac{\epsilon A}{d}$$

where ϵ is permittivity of dielectric medium and $\epsilon > \epsilon_0$. The dielectric constant

$\epsilon_r = \frac{\epsilon}{\epsilon_0} \text{ or } \frac{C}{C_0}$
--

It is a dimensionless quantity and it is independent of size or shape of dielectrics. For dielectrics $\epsilon_r > 1$.

① DIELECTRIC POLARIZATION:

In atoms, because of their spherical symmetry the centre of mass of electrons coincides with nucleus. Therefore atoms do not have permanent electric dipole moments. However when atoms are placed in an electric field they acquire an induced electric dipole moment in the direction of the field. This process is called dielectric polarization, and atoms are said to be polarized.

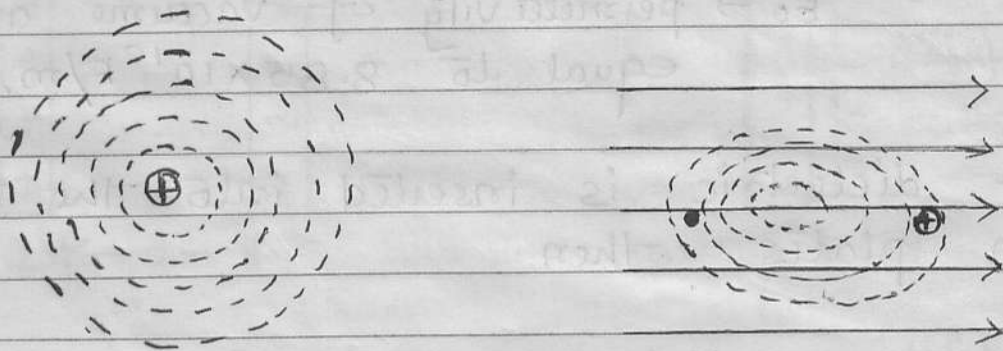


Fig 5.2

The induced dipole moment P_{in} acquired by molecule is

$$\vec{P}_{in} = \alpha \epsilon_0 \vec{E}_0$$

where α is proportionality constant and is called atomic / molecular polarizability and E_0 is an external electric field.

For most of atom, α is of the order of 10^{-29} to 10^{-30} m^3 .

Consider a slab RSTU placed in an external electric field E_0 imposed between two plates. If all the atoms are polarized in the direction of E_0 and x is displacement between

positive charge and negative charge in the atom, the dipole moment of each atom is

$$\vec{P}_{in} = qx$$

If N is number of atom per unit volume then,

$$\vec{P} = N \vec{P}_{in}$$

$$\vec{P} = Nqx$$

This dipole moment \vec{P} is called electric polarization.

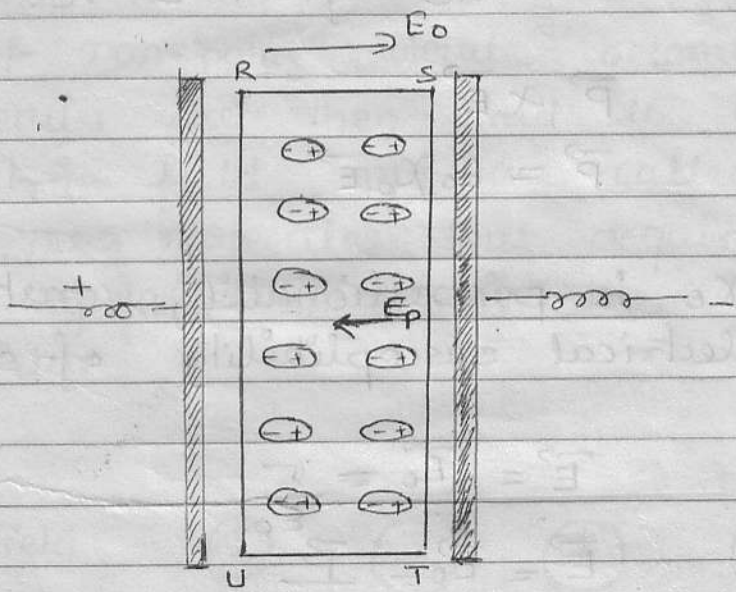


Fig 5.3

Inside the slab because of separation of charge a electric field E_p is set up opposite to E_0 .

$$\text{Effective field} = \vec{E} = \vec{E}_0 - \vec{E}_p$$

$$\text{or } \vec{E} = \vec{E}_0 - \frac{\sigma}{\epsilon_0}$$

If dielectric is placed between plates RU and ST, then surface density of induced charge on S.T plate will be $+\sigma = \vec{P}$ is polarization vector.

But dielectric constant of substance

$K = \frac{\text{Electric field } E_0 \text{ in absence of substance}}{\text{Electric field } E \text{ in presence of substance}}$

$$K = \frac{\vec{E}_0}{\vec{E}}$$

$$\vec{E}_0 = K \vec{E}$$

But in homogeneous isotropic dielectric substance, the polarization density P is proportional to electric field E i.e.

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

where χ_e is proportionality constant and is called electrical susceptibility of substance

Since $\vec{E} = \vec{E}_0 - \frac{\sigma}{\epsilon_0}$

$$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\epsilon_0}$$

$$\vec{E} = \vec{E}_0 - \frac{\epsilon_0 \chi_e \vec{E}}{\epsilon_0} = \vec{E}_0 - \chi_e \vec{E}$$

$$\vec{E}_0 = \vec{E} + \chi_e \vec{E} = \vec{E} (1 + \chi_e)$$

$$\frac{\vec{E}_0}{\vec{E}} = (1 + \chi_e)$$

$$K = (1 + \chi_e)$$

TYPES OF DIELECTRICS

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Dielectrics can be classified into two —

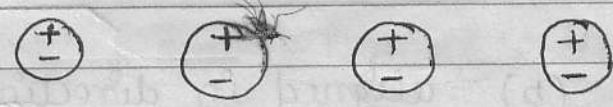
- 1) polar dielectrics
- 2) non-polar dielectrics

A non-polar molecule is one in which the centre of gravity of positive charges coincides with centre of gravity of negative charges. Symmetrical molecules are non-polar. A non-polar molecule has obviously a zero electric dipole moment.

Eg:- H_2 , N_2 and O_2

If the dielectric with non-polar molecules is placed in an electric field, the charge centres of non-polar molecule become displaced. The molecules are then said to be polarized, by electric field and are called induced dipoles. The molecules thus acquires an induced electric dipole moment in the direction of the field.

No field



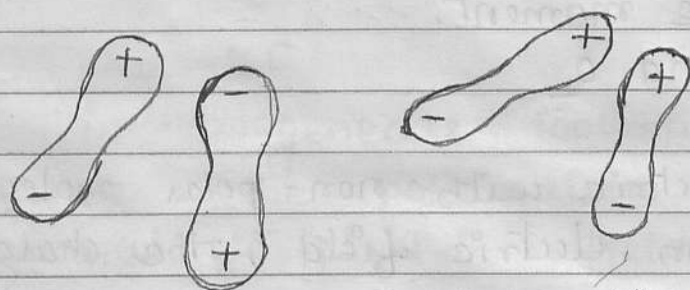
field E



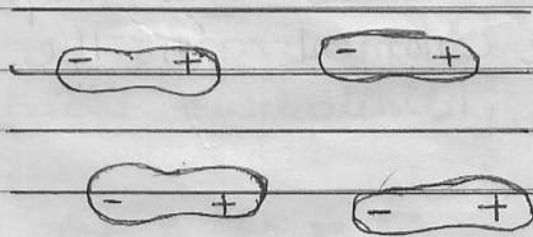
fig 5.4

A polar molecule is one in which centre of gravity of positive charges is separated from centre of gravity of negative charges by finite distance. The polar

molecule is thus an electric dipole and has intrinsic permanent electric dipole moment. In the absence of any external electric field the individual dipoles are oriented at random and no net dipole moment is observed in the dielectric. When an electric field is applied, the force on a dipole give rise to a torque, whose effect is to orient the dipole along the direction of electric field.



(a) Random aligned.



(b) aligned in direction of field.

fig 5.5

3

GIAUSS'S LAW IN DIELECTRICS

According to Gauss's law "the total flux through any closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed in by the closed surface.

$$\text{i.e. } \oint \vec{E}_0 \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Consider a parallel plate capacitor with plate area A and having vacuum between its plates. Let $+q$ and $-q$ be the charges on the plates and E_0 the uniform electric field between the plates. Let PQRS be a Gaussian surface.

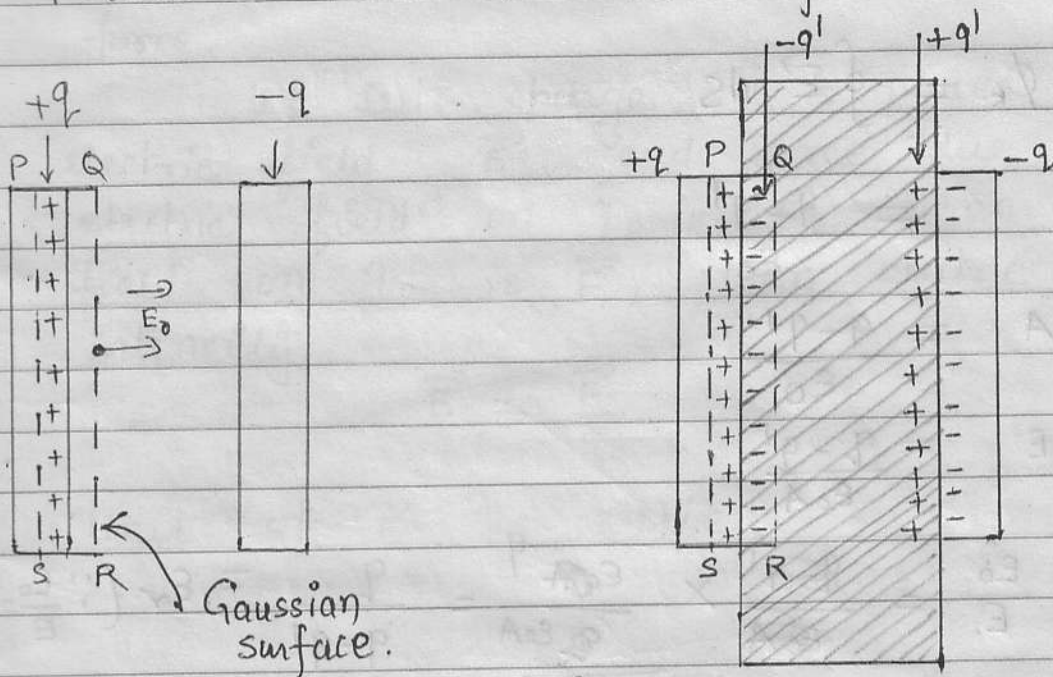


Fig 5.6

By Gauss's law $\oint \vec{E}_0 \cdot d\vec{s} = \frac{q}{\epsilon_0}$

$$\oint \vec{E}_0 \cdot d\vec{s} = \int E_0 ds = E_0 A \quad \left\{ \because \oint ds = A \right\}$$

$$\therefore E_0 A = \frac{q}{\epsilon_0}$$

$$\text{or } E_0 = \frac{q}{\epsilon_0 A}$$

Let dielectric slab of dielectric constant K is placed between two plates so dielectric is polarized. $-q'$ appears on

left surface and an equal positive charge on right surface. These polarization charges produce their own field which opposes the external field \vec{E}_0 .
 All \vec{E} be net electric field within the dielectric.

\therefore Now electric flux through the Gaussian surface PQRS is :-

$$\phi_k = \oint \vec{E} \cdot d\vec{s}$$

$$= \frac{q - q'}{\epsilon_0}$$

$$\Rightarrow EA = \frac{q - q'}{\epsilon_0}$$

$$\Rightarrow E = \frac{q - q'}{\epsilon_0 A}$$

$$\therefore \frac{\epsilon_0}{E} = \frac{q - q'}{\epsilon_0 A} \times \frac{\epsilon_0 A q}{q - q'} = \frac{q}{q - q'} = \epsilon_r \left(\because \frac{E}{E} \right)$$

$$\therefore q - q' = \frac{q}{\epsilon_r}$$

$$\phi_k = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_r \epsilon_0}$$

or $\oint \epsilon_r \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$

Therefore it is clear from above equation that the charge q contained within the Gaussian surface is taken to be free charge carrier only.

Electric intensity (\vec{E}), Electric polarization (\vec{P}), Displacement vector (\vec{D})

Electric intensity (\vec{E}):

In an electric field at any point, the magnitude of electric intensity vector is equal to the force on unit positive charge (test charge) at that point and direction is in direction of force.

If test charge q_0 is placed in an electric field \vec{E} and force due to the electric field on q_0 is \vec{F} , then electric field on q_0 is \vec{F} , then electric field intensity

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Unit of \vec{E} is N/C.

Electric polarization (\vec{P}):

When dielectric is placed in external uniform electric field E_0 , the centre of gravity of positive and negative charge of each molecule aligned in one direction while centre of gravity of negative charge in the direction opposite to electric field. As a result a dipole moment is induced in each molecule which is directly proportional to intensity of external electric field. If there are n molecules/volume of dielectric and \vec{P}_m is the induced dipole moment in each molecule, electric polarization

(or) polarization density \vec{P} is equal to induced dipole moment per unit volume of material i.e.

$$\vec{P} = n \vec{P}_{in}$$

since $\vec{P}_{in} \propto \vec{E}_0$ (external electric field)

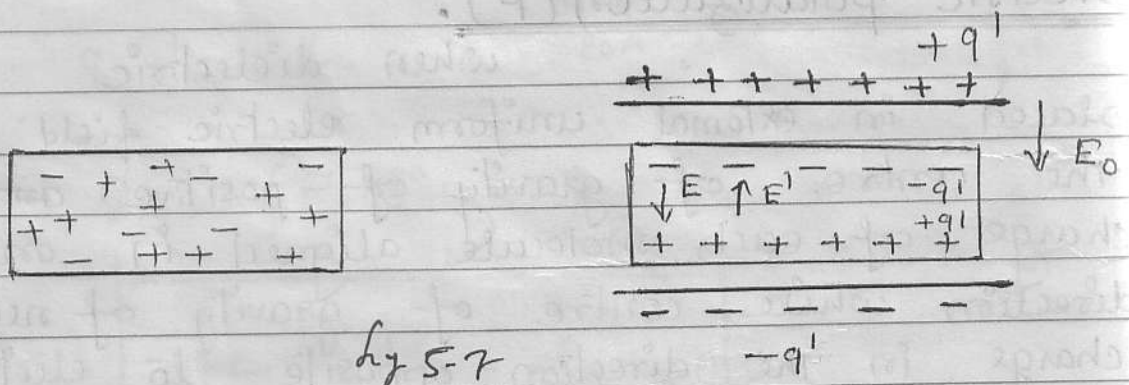
$$\vec{P}_{in} = \epsilon_0 \alpha_{in} \vec{E}_0$$

$\alpha_{in} \rightarrow$ molecular polarizability

Hence density of polarization

$$\vec{P} = \epsilon_0 \alpha_{in} n \vec{E}_0$$

Thus electric polarization vector \vec{P} is in the direction of external electric field \vec{E}_0 and its unit is coulomb/meter² i.e. (C/m²)



(a) Dielectric substance
in absence of E.F.

(b) Dielectric in presence
of E.F.

Suppose ' l ' be the length of each plate
' t ' is distance between them and ' b '
is breadth.

If V is potential difference between plates
The electric field produced between plates

$$\vec{E}_0 = \frac{V}{t}$$

When dielectric substance is placed between the plates and if $-q'$ is the negative charge induced towards +ve plate, $+q'$ is positive charge induced towards the negative plate then induced dipole moment within the substance is

$$\vec{P}_{in} = q' \times t$$

$$\text{Volume of substance} = l \times b \times t$$

Therefore electric polarization or dipole moment induced in unit volume of substance

$$P = \frac{q't}{lbt} = \frac{q'}{lb} = \frac{q'}{a} = \sigma'$$

The density of polarization or electric polarization P is measured by surface density of charge induced on the surface of dielectric material normal to the direction of electric field.

Electric displacement vector (\vec{D}):

The electric field due to a point charge q coulomb at a distance x in air (or vacume) is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \hat{x}$$

Where $\epsilon_0 \rightarrow$ permittivity of vacuum.

But when dielectric medium is placed in electric field we get electric field for dielectrics

$$\vec{E} = \frac{1}{4\pi\epsilon} \cdot \frac{q}{x^2} \cdot \hat{x}$$

$$= \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q}{x^2} \cdot \hat{x}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q}{x^2} \cdot \hat{x}$$

$\epsilon \rightarrow$ absolute permittivity of medium

$\epsilon_r \rightarrow$ relative permittivity of medium

$K \rightarrow$ dielectric constant.

It is obvious that electric field depends on nature of medium, so to study the properties of the dielectrics we consider a electric vector \vec{D} which does not depend upon nature of medium i.e. does not depend upon charge induced on the surface of medium but depends only on real charges and their distribution. Such electric vector is called displacement field or displacement vector \vec{D} .

The only difference between \vec{D} and \vec{E} is that the electric field \vec{E} at a point is defined as the forces on a unit positive charge kept at that point, while the displacement vector \vec{D} is defined by the displacement

flux linked with unit area at that point.

For a point charge 'q' displacement flux associated with da surface is equal to charge 'q' i.e

$$\iint_S \vec{D} \cdot d\vec{a} = q$$

or $\vec{D} = q/a = \sigma$ (surface density of real or free charge)

The unit of \vec{D} is coulomb/meter². The direction of displacement vector \vec{D} within homogeneous and isotropic dielectrics is same as that of electric field \vec{E} .

(By Gauss's theorem of Dielectrics) $\iint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_r \epsilon_0} q$

$$\Rightarrow \vec{E} = \frac{1}{\epsilon} \frac{q}{a}$$

$$\Rightarrow \vec{E} = \frac{1}{\epsilon_0 k} \frac{q}{a} \quad (\because \epsilon = \epsilon_0 k)$$

$$\Rightarrow \frac{q}{a} = \epsilon_0 k \vec{E}$$

but $\vec{D} = \epsilon_0 k \vec{E}$ (since $q/a = \vec{D}$)

$$\boxed{\vec{D} = \epsilon_0 k \vec{E}}$$

Relation between \vec{D} , \vec{E} and \vec{P} :

A parallel plate capacitor with surface charge density of positive charge $+\sigma$ on upper plate and surface density of charge $-\sigma$ on lower plate, with no dielectric slab, the electric field between plates is

$$E_0 = \frac{\sigma}{\epsilon_0} \text{ (downwards)}$$

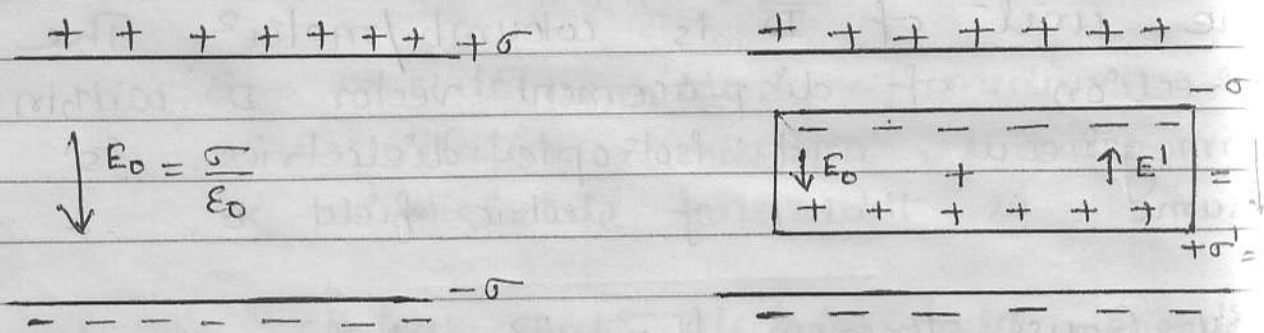


Fig. 5.8

If a dielectric substance is placed between the plates of capacitor, the substance gets polarized due to external field E_0 . As a result a negative charge is induced on the upper surface of the dielectric substance and positive charge is induced on lower surface of dielectric substance.

In this condition surface density of induced charge on upper surface of dielectric substance $-\sigma'$ is equal to polarization vector $-\vec{P}$ i.e. $-\sigma' = -P$ and surface density of induced charge on

\vec{P} is the polarization vector of dielectric substance which is in direction of external electric field E_0 .

Due to induced charge on two surface of dielectric substance, an electric field E' is produced which is in opposite direction to external field E_0 and is given by

$$\vec{E}' = \frac{\sigma'}{\epsilon_0} = -\frac{\vec{P}}{\epsilon_0} \text{ (upwards)} \quad \text{--- (2)}$$

Hence resultant electric field \vec{E} at a point within dielectric substance is

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

$$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\epsilon_0}$$

$$\vec{E}_0 = \vec{E} + \frac{\vec{P}}{\epsilon_0} \quad \text{--- (3)}$$

but dielectric constant $K = \frac{\vec{E}_0}{E}$

$$\text{or } \vec{E}_0 = K E$$

$$\begin{aligned} \text{(3)} \Rightarrow K E &= \vec{E} + \frac{\vec{P}}{\epsilon_0} \\ \vec{P} &= \epsilon_0 (K-1) \vec{E} \end{aligned}$$

$$\text{ie } \boxed{\vec{P} = \epsilon_0 (K-1) \vec{E}} \quad \text{--- (4)}$$

This is the relation between \vec{P} and \vec{E} .
But in homogeneous and isotropic dielectric substance the polarization density P is proportional to the electric field E i.e.

$$P \propto E$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} \quad \text{--- (5)}$$

comparing (4) and (5)

$$\chi_e = (K-1)$$

This is the relation between dielectric constant and susceptibility of the substance.

From equation (4)

$$\vec{P} = \epsilon_0 (K-1) \vec{E}$$

$$\Rightarrow P + \epsilon_0 \vec{E} = \epsilon_0 K \vec{E}$$

but we know that $\vec{D} = \epsilon_0 K \vec{E}$

$$\therefore \vec{D} = \epsilon_0 \vec{E} + P$$

which is the desired relation between \vec{E} , \vec{D} and \vec{P} .

In vacuum or air $K=1$

$$\therefore \chi_e = 0 \quad \text{and} \quad \vec{D} = \epsilon_0 \vec{E}$$

(4)

DIFFERENT TYPES OF POLARIZATION:

polarization occurs due to following mechanism-

- (1) electronic polarization (P_e)
- (2) Ionic polarization (P_i)
- (3) Orientational polarization (P_o)
- (4) space charge polarization (P_{sc})

Electronic polarization :-

This is a polarization that results from the displacement of electron clouds of atoms, molecules or ions with respect to heavy fixed nucleus to the distance that is less than the dimension of atom or molecule.

The electronic polarization is set in over a very short period of time of the order of 10^{-14} to 10^{-15} sec. It is independent of temperature.

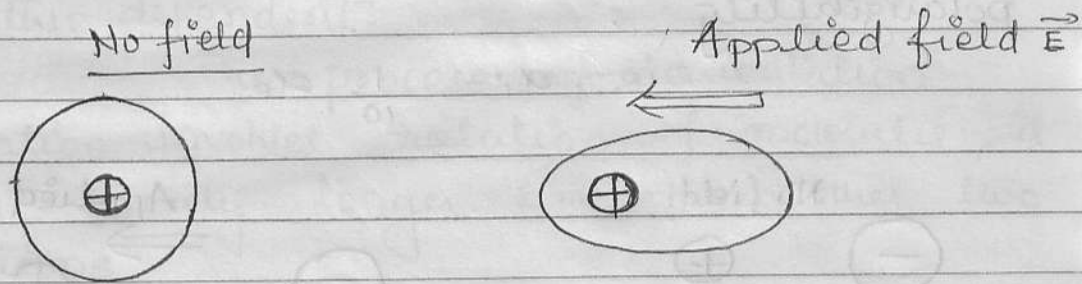


Fig 5-9

The polarization is given by

$$P_e = N \alpha_e E$$

where α_e is electronic polarizability.

The contribution of P_e to dielectric constant may be obtained as follows -

$$\epsilon_r = 1 + \chi = 1 + \frac{P_e}{\epsilon_0 E} = 1 + \frac{N \alpha_e E}{\epsilon_0 E}$$

$$\epsilon_r = 1 + \frac{N \alpha_e}{\epsilon_0}$$

Electronic polarizability of monoatomic gas is

$$\alpha_e = 4\pi \epsilon_0 R^3$$

Ionic polarization:-

Ionic polarization occurs in ionic crystal. When electric field is applied to an ionic material, cation and anion gets displaced in opposite directions which give rise to net dipole moment. The ionic polarization is given by

$$\vec{P}_i = N \alpha_i \vec{E}$$

The ionic polarizability for most material is less than electronic polarizability.

$$\text{ie } \alpha_i = \frac{1}{10} \alpha_e$$

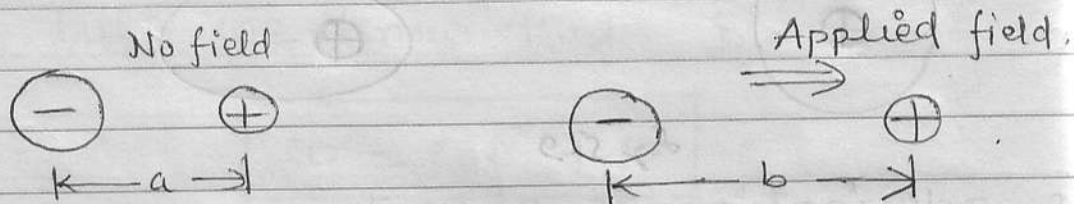


Fig 5-10

Orientation polarization:-

The orientation polarizability is found only in materials that possess a permanent dipole moments.

Eg CH3Cl.

In absence of external electric field the orientations of dipoles is random resulting in complete cancellations of each others effect. When electric field is impressed molecular dipoles rotate about their axis of symmetry to align with applied field.

In case of electronic and ionic polarizations the force due to

external field is balanced by a restoring force due to Coulomb attraction but orientational polarization restoring force do not exist. However dipole alignment is counteracted by thermal agitation. Higher the temperature greater is the thermal agitation, dipole may turn through small angles, even in case of liquids and gases where the molecules are free to rotate a complete alignment cannot take place due to randomizing effect of temperature. Thus orientational polarization is strongly temperature dependent.

The process of orientation polarization involves rotation of molecules, it takes relatively longer time than other two polarizations.

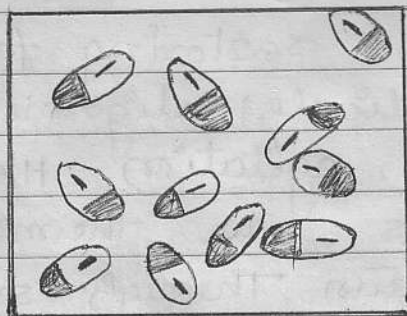
The orientation polarization α_0 is calculated to be

$$\alpha_0 = \frac{\mu^2}{3kT}$$

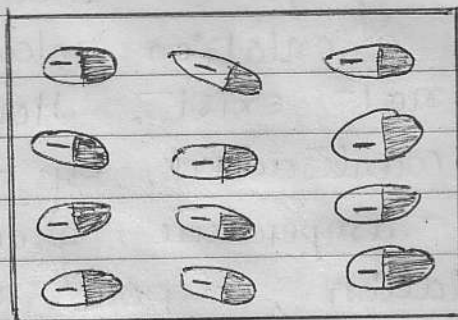
and orientational polarization is given by Debye's law

$$P_0 = \frac{N\mu^2 E}{3kT}$$

which states that the orientation polarization is inversely proportional to temperature and proportional to square of permanent dipole moment.



(randomly arranged dipole when $E=0$)



(when field is applied dipoles are partially aligned)

Fig 5-11

Total polarization:

In a material which can experience all forms of polarization, the total polarizability is equal to sum of the electronic, ionic and orientation polarizabilities-

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

$$\alpha = \alpha_e + \alpha_i + \frac{\mu^2}{3kT}$$

Total polarization may be written as

$$P = P_e + P_i + P_o$$

$$P = N \left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT} \right) E$$

The dielectric constant of polar gaseous dielectrics may be expressed as

$$\epsilon_0 (\epsilon_r - 1) = N \left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT} \right)$$

Let us consider a simple case of an elemental solid dielectric which exhibits only electronic polarizability. Solids such as diamond, silicon and germanium crystals are made up of single type of atoms. If α_e is an electronic polarizability per atom, it is equal to induced dipole moment for unit polar field of molecule.

$$\vec{P}_{in} = \alpha_e \vec{E}_0$$

If 'n' is a number of molecules per unit volume then

$$\vec{P} = n \vec{P}_{in}$$

$$\vec{P} = n \alpha_e \vec{E}_0$$

From Lorentz field equation, polar field working on molecule of an isotropic dielectric in macroscopic electric field is

$$\vec{E}_0 = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$$

$$\vec{P} = n \alpha_e \left(\vec{E} + \frac{\vec{P}}{3\epsilon_0} \right) \quad \text{--- (1)}$$

But we know that $\vec{P} = \chi_e \vec{E}$ --- (2)

and $K = 1 + \frac{\chi_e}{\epsilon_0}$

$$\chi_e = (K-1) \epsilon_0 \quad \text{--- (3)}$$

Using (1) (2) and (3)

$$(k-1)\epsilon_0 \vec{E} = n\alpha \left[E + \frac{(k-1)\epsilon_0 E}{3\epsilon_0} \right]$$

$$\Rightarrow (k-1)\epsilon_0 = n\alpha \left[1 + \frac{k-1}{3} \right]$$

$$\Rightarrow (k-1)\epsilon_0 = n\alpha \left(\frac{k+2}{3} \right)$$

$$\Rightarrow \frac{k-1}{k+2} = \frac{n\alpha}{3\epsilon_0}$$

$$\Rightarrow \alpha = \frac{3\epsilon_0(k-1)}{n(k+2)}$$

$$\Rightarrow \boxed{\alpha = \frac{3\epsilon_0(\epsilon_r - 1)}{n(\epsilon_r + 2)}}$$

This is Clausius-Mossotti equation

Limitations:

Clausius-Mossotti equation is derived based on following assumptions —

- (1) polarization is proportional to the field.
- (2) polarizable molecules are isotropic
- (3) Absence of short-range interaction

These conditions are satisfied with neutral molecule having a constant-dipoles i.e. non-polar. Thus the equation is applicable to neutral liquids and specially to gases. This is not applicable to strong solutions and solids.

Specific polarization of dielectrics:

$$\text{As } \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0}$$

Also dielectric constant $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$$\therefore \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} = \frac{n\alpha}{3\epsilon_0}$$

So $\frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0}$ is known as specific polarization of a dielectrics

5 LANGEVIN THEORY OF DIPOLAR ORIENTATION:

In absence of an electric field, a microscopic piece of polar dielectrics is not polarised because the individual molecules (dipoles) are randomly oriented so that the net electric dipole moment of the piece is zero. When an electric field is applied the individual molecule experiences torques which leads to align them in direction of the field. But because of thermal motion of the molecules, the alignment is incomplete. This favours random orientations. The average alignment produced gives rise to a net dipole moment per unit volume.

(5) As the temperature of the specimen is increased, the polarization becomes even smaller due to the increase in thermal agitation of

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the molecules. Thus for polar dielectrics, the orientational polarizability and hence dielectric constant and the electric susceptibility depends upon temperature.

To calculate net dipole moment per unit volume produced due to alignment of the molecule at a temperature T , we suppose that the number of molecules per unit volume of specimen be ' n ' and ' θ ' is angle at which permanent dipole moment \vec{P} of a molecule makes with the polarizing field \vec{E} . The potential energy in the field is

$$U = -PE \cos \theta \quad \text{--- (1)}$$

According to statistical mechanics the number of molecules having energy U at a temperature ' T ' is proportional to the quantity $e^{-U/K_B T}$

where K_B is Boltzmann's constant.

Now the number of molecules (dipoles) per unit volume (dn) having energy U and oriented at angles between θ and $\theta + d\theta$ with respect to direction of \vec{E} is

$$dn = C e^{-U/K_B T} d\Omega \quad \text{--- (2)}$$

where $C \rightarrow$ constant and $d\Omega$ - solid angle contained between the directions θ and

$\theta + d\theta$.

The solid angle is given by

$$d\Omega = \frac{\text{Area of ring between } \theta \text{ \& } \theta + d\theta}{r^2}$$

$$= \frac{2\pi (r \sin \theta) (r d\theta)}{r^2}$$

$$d\Omega = 2\pi \sin \theta d\theta$$

P

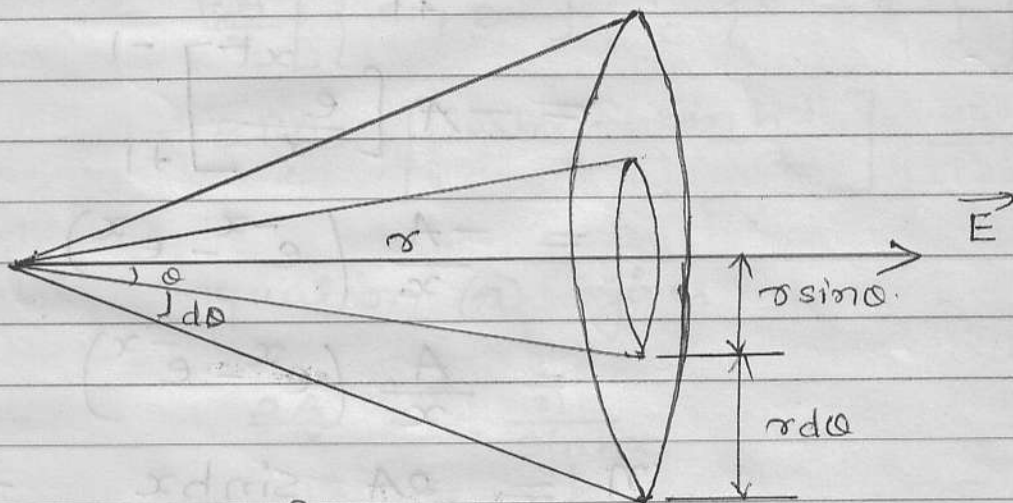


Fig. 5.12

putting the value of $d\Omega$ and value of U in eq. (2)

$$dn = c e^{\frac{\vec{P} \cdot \vec{E} \cos \theta}{k_B T}} \cdot 2\pi \sin \theta d\theta$$

$$\text{Let } \frac{\vec{P} \cdot \vec{E}}{k_B T} = \alpha$$

$$dn = 2\pi c e^{\alpha \cos \theta} \sin \theta d\theta$$

$$dn = A e^{\alpha \cos \theta} \sin \theta d\theta \quad (\text{where } A = 2\pi c)$$

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Now total number of molecules per unit volume is given by

$$n = \int_0^{\pi} dn$$

$$n = A \int_0^{\pi} e^{x \cos \theta} \sin \theta d\theta$$

$$\text{Let } \cos \theta = t$$

$$\therefore \sin \theta d\theta = -dt$$

Thus
$$n = -A \int_{+1}^{-1} e^{xt} dt$$

$$= -A \left[\frac{e^{xt}}{x} \right]_{+1}^{-1}$$

$$= -\frac{A}{x} (e^{-x} - e^x)$$

$$= \frac{A}{x} (e^x - e^{-x})$$

$$n = \frac{2A}{x} \sinh x \quad \text{--- (3)}$$

The total dipole moment along the field direction due to all the molecules per unit volume is given by

$$\vec{P} = \int_0^{\pi} \vec{P}_m \cos \theta dn$$

$$= \vec{P}_m A \int_0^{\pi} e^{x \cos \theta} \cos \theta \sin \theta d\theta$$

Let $\cos \theta = t$ and $\sin \theta d\theta = -dt$
Thus

$$\begin{aligned} \vec{P} &= -P_{in} \int_{-1}^{+1} t e^{xt} dt \\ &= -P_{in} \left[\frac{t \cdot e^{xt}}{x} - \int \frac{e^{xt}}{x} dt \right]_{-1}^{+1} \\ &= -P_{in} \left[\frac{t e^{xt}}{x} - \frac{e^{xt}}{x^2} \right]_{-1}^{+1} \\ &= -P_{in} \left[\left\{ -\frac{e^{-x}}{x} - \frac{e^{-x}}{x^2} \right\} - \left\{ \frac{e^x}{x} - \frac{e^x}{x^2} \right\} \right] \\ &= \frac{P_{in}}{x} \left[e^{-x} + \frac{e^{-x}}{x} + e^x - \frac{e^x}{x} \right] \\ &= \frac{P_{in}}{x} \left[\left\{ e^x + e^{-x} \right\} - \frac{1}{x} \left\{ e^x - e^{-x} \right\} \right] \\ &= \frac{2P_{in}}{x} \left[\cosh x - \frac{\sinh x}{x} \right] \quad \text{--- (4)} \end{aligned}$$

But equation (3) gives

$$\frac{2A}{x} = \frac{\eta}{\sinh x}$$

$$\therefore P = \eta P_{in} \left[\coth x - \frac{1}{x} \right]$$

$\left(\coth x - \frac{1}{x} \right)$ is known as "Langevin's function" and is denoted by $L(x)$. Thus.

$$\vec{P} = \eta P_{in} L(x)$$

where $\boxed{x = \frac{PF}{k_B T}} \quad \text{--- (5)}$

This is known as "Langevin's equation".

At ordinary temperatures for fields even upto the dielectric strength the value α is of the order of 10^{-3} for most polar materials. For such small values of α the series expansion of $L(\alpha)$ may be taken as :

$$L(\alpha) = \left(\coth \alpha - \frac{1}{\alpha} \right) \approx \frac{\alpha}{3}$$

∴ Equation (5) becomes

$$P \approx n P_m \frac{\alpha}{3} \approx \frac{n P_m^2}{3 k_B T} E$$

Thus, the polarization P becomes a linear function of polarizing field E . The polarizability α_0 due to the alignment of molecular dipoles of the polar dielectrics is given by —

$$\alpha_0 = \frac{P}{nE} = \frac{P_m^2}{3k_B T}$$

This equation represents the "orientational polarizability". As the induced dipole moments are also present in polar dielectrics which gives rise to "deformation polarizability" α_d . Thus total polarizability of molecule of a polar dielectrics is given by

$$\alpha = \alpha_d + \alpha_0$$

$$\alpha = \alpha_d + \frac{p_m^2}{3k_B T}$$

This equation is known as "Langevin - Debye's equation".

⑤ INTERNAL FIELDS IN SOLIDS (Molecular field)

The electric field which is responsible for the polarizing of dielectric is called molecular field \vec{E}_m . It is produced by all external sources and by polarizing molecules in dielectrics with the exception of one molecule at point under consideration. The internal field \vec{E}_i which is defined as electric field acting the location of given atom is given by sum of electric fields created by neighbouring atoms and applied field.

In evaluation of the bulk polarization, the additional effects of surrounding polarized atom is to be taken into account. The effective field intensity is given by.

$$\vec{E}_i \text{ or } \vec{E}_m = \vec{E}_0 + E'$$

The value of E' is evaluated by summation of all effects of surrounding atoms.

Figure shows a dielectrics placed in field \vec{E}_0 between plates

A and B. The electric field is uniform and dielectric is polarized.

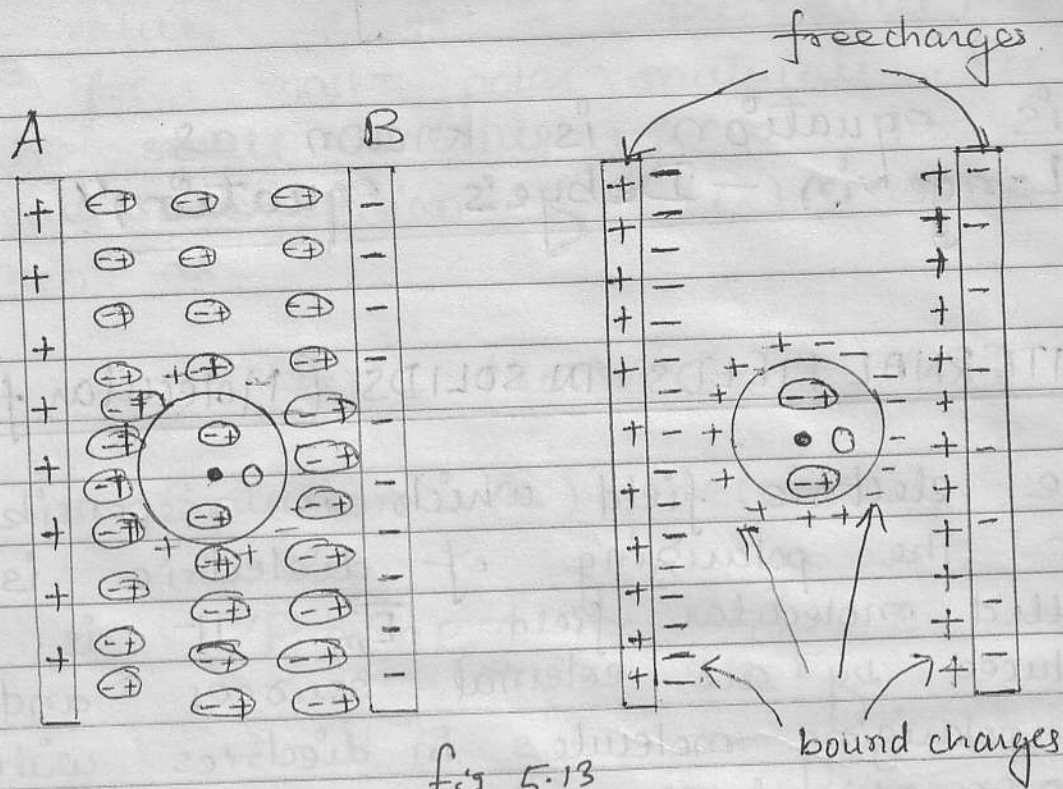


fig 5.13

To calculate the field at position O of a molecule, we assume that this molecule is not present at all.

We draw a sphere of radius r around O. Let us assume that

the dielectric consists of two parts:-

(a) The dielectric outside the sphere is treated as continuum of dipoles.

The part of dielectric outside the sphere can be replaced by a system of polarization charges.

(b) The molecules inside the sphere are treated as individual dipoles.

The molecular field at O consists of four components -

$$\vec{E}_m \text{ or } \vec{E}_i = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \vec{E}_3.$$

where

(1) \vec{E}_0 is external field due charged plates of capacitor

$$\vec{E}_0 = \frac{\sigma}{\epsilon_0} = \frac{D}{\epsilon_0}$$

where ' σ ' surface charge density.

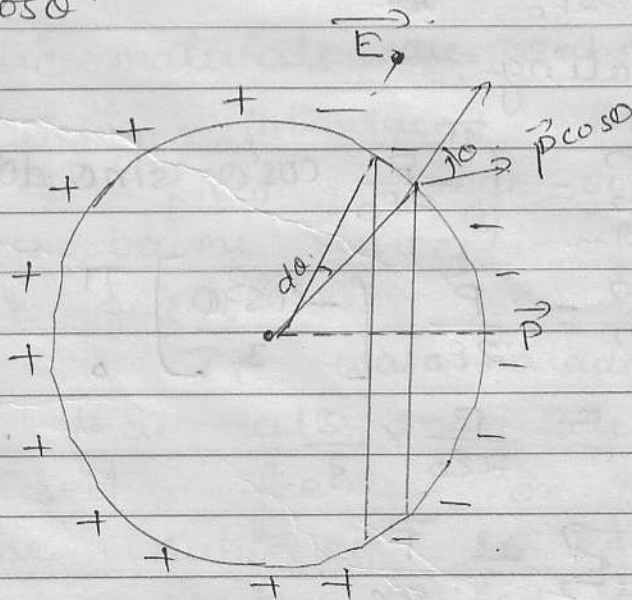
(2) \vec{E}_1 is depolarizing field bound charges on the outer faces of the dielectrics.

$$\vec{E}_1 = -\frac{\sigma'}{\epsilon_0} = -\frac{P}{\epsilon_0}$$

(3) To find \vec{E}_2 , the field at O due to the bound charges on surface of the sphere.

Let ds be the surface area element with polar coordinates (r, θ) .

The component of \vec{P} normal to ds is $P \cos \theta$.



Therefore charges on surface element da lying between θ and $\theta + d\theta = \rho \cos\theta \cdot ds$

$$\therefore E.F \text{ at } O = \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \cos\theta \cdot ds}{r^2}$$

The component of field in the direction of

$$= \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \cos\theta \cdot ds}{r^2} \right) \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \cos\theta}{r^2} \cdot (ds)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \cdot \cos^2\theta}{r^2} \left(2\pi r \cdot d\theta \right) \cdot (r \sin\theta)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho \cos^2\theta}{r^2} \left(2\pi r^2 \sin\theta \cdot d\theta \right)$$

$$= \frac{\rho}{2\epsilon_0} \cos^2\theta \cdot \sin\theta \cdot d\theta$$

Therefore component of field along the direction of \vec{E}

$$dE_2 = \frac{\rho}{2\epsilon_0} \cos^2\theta \sin\theta \cdot d\theta$$

Integrating.

$$\vec{E}_2 = \int_0^\pi \frac{\rho}{2\epsilon_0} \cos^2\theta \cdot \sin\theta \cdot d\theta$$

$$\vec{E}_2 = \frac{\rho}{2\epsilon_0} \left[\frac{-\cos^3\theta}{3} \right]_0^\pi$$

$$= \frac{\rho}{2\epsilon_0} \times \frac{2}{3}$$

$$\vec{E}_2 = \frac{\rho}{3\epsilon_0}$$

(4) \vec{E}_3 is field due to polarized

molecules within the sphere
practically $\vec{E}_3 = 0$

$$\vec{E}_m = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_m = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} + \frac{\vec{P}}{3\epsilon_0}$$

$$\vec{E}_m = \frac{\vec{D} - \vec{P}}{\epsilon_0} + \frac{\vec{P}}{3\epsilon_0}$$

$$\vec{E}_m = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$$

This gives the expression for internal fields or molecular field in solids. Above equation is called Lorentz field equation.

5 MAGNETIC MATERIALS

Magnetic materials are substances, which upon being introduced into an external magnetic field, change so that they themselves become sources of additional magnetic field.

It was Faraday who showed that all materials can be magnetised, feebly or strongly, when they are subjected to external field. Materials can be divided broadly into the following types -

- (1) Diamagnetic substances.
- (2) paramagnetic materials
- (3) Ferromagnetic materials.

DIAMAGNETIC MATERIALS:

The materials which when placed a magnetic field acquire feeble magnetism in a direction opposite to that of field are known as diamagnetic materials.

The main properties of diamagnetic materials are as follows:—

- (1) Diamagnetic materials exhibit negative magnetic susceptibility. The magnetization M is directed opposite to the direction of applied magnetic field. The absolute value of susceptibility is 10^{-6} .
- (2) As diamagnetic permeability is negative, the relative permeability μ_r is less than unity and substance repelled by magnetic field.
- (3) The magnetic susceptibility of diamagnetic materials is practically independent of temperature.
- (4) The magnetization M is linear function of magnetic field H when the field is not too strong.
- (5) When a rod is suspended freely

in uniform magnetic field, the rod align itself in a direction perpendicular to the direction of magnetic field.

(6) Diamagnetic substance loses its magnetization as soon as magnetization field is removed.

(7) For diamagnetic materials

$$\chi = \frac{M}{H} = -1$$

Eg:- metals, hydrogen, air, gold, silver etc.

PARAMAGNETIC MATERIALS:

The substance which when placed in a magnetic field acquire feeble magnetism in the direction of magnetic field.

Eg:- Oxygen, copper chloride, chromium and platinum etc.

properties paramagnetic materials:

(1) paramagnetic materials exhibit positive magnetic susceptibility as magnetization coincides in direction of magnetic field H .
The susceptibility is of the order of 10^6 .

(2) The relative permeability (μ_r) is slightly more than unity ($\mu_r > 1$) for paramagnetic materials.

(3) When a rod of paramagnetic material is freely suspended in magnetic field,

it aligns itself along the direction of lines of induction.

(4) The magnetization is linear function of magnetic field H when field is not too strong.

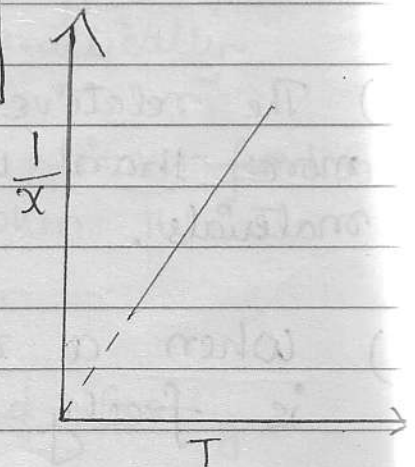
(5) The paramagnetic susceptibility is strongly dependent on temperature. It was discovered by Pierre Curie in 1895 that the susceptibility of paramagnetic substance varies inversely with the temperature. Thus

$$\boxed{\chi_{\text{para}} = \frac{C}{T}} \quad (\text{Curie law})$$

$C \rightarrow$ Curie constant.

A departure from above equation is observed in case of solid and liquid paramagnetic substances at low temperature. The behaviour of χ_{para} in these cases can be described by the Curie-Weiss law

$$\boxed{\chi_{\text{para}} = \frac{C}{T - \theta_p}}$$



FERROMAGNETIC MATERIALS

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Ferromagnetic materials are metallic crystals when placed in magnetic field becomes strongly magnetized in the direction of field.

Eg: Iron, nickel and some steels.

properties of ferromagnetic materials :

(1) When ferromagnetic substances is placed in a non-uniform magnetic field the magnetic lines of force tends to crowd into the specimen.

(2) Ferromagnetic materials exhibits very high values of magnetic susceptibility and relative permeability. Susceptibility is of the order of 10^6 .

(3) Magnetization of ferromagnetic materials does not vary linearly with applied field. ^{as shown in fig 5-16} It is very complex non-linear function of H.

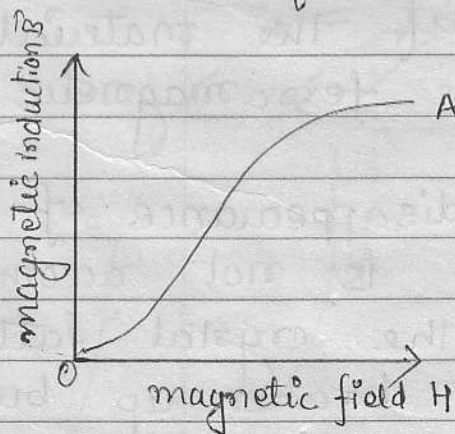


Fig 5-16

(5) The specific properties of ferromagnetic materials are found in crystalline state only. In the liquids and gaseous states, ferromagnetic materials behaves like ordinary paramagnetic substances.

(6) The ferromagnetic properties of crystals are found to be dependent on the direction of magnetization. The magnetization is more readily obtained along certain crystallographic axes than along others. The direction in which the magnetization is strongest for given value of field is called direction of easy magnetization while the direction corresponding to the lowest magnetization for a given field is called the direction of hard magnetization.

(7) Ferromagnetic materials are characterized by definite temperature T_c called Curie temperature, above which ferromagnetic behaviour is lost. At T_c material transform into paramagnetic. It is a reversible transition i.e. if the material cooled back through T_c , the ferromagnetic character reappear.

The disappearance of ferromagnetic character at T_c is not accompanied by changes in the crystal lattice. The lattice is not broken up but changes its type of symmetry. Such changes are known as second order phase transitions.

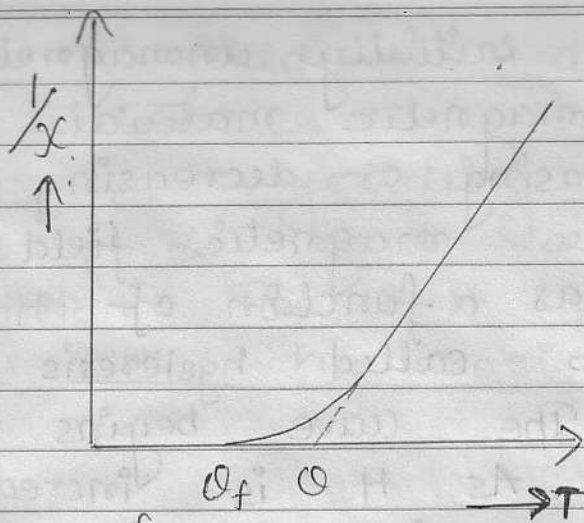


Fig 5.17

(8) Hysteresis loop:

A typical property of ferromagnetic material is hysteresis. Hysteresis may be defined as lag in changes of magnetization behind variations of the magnetic field. Because of hysteresis, the magnetization of ferromagnetic material depends not only strength of magnetic field at the given instant - but also on the magnetization history of material.

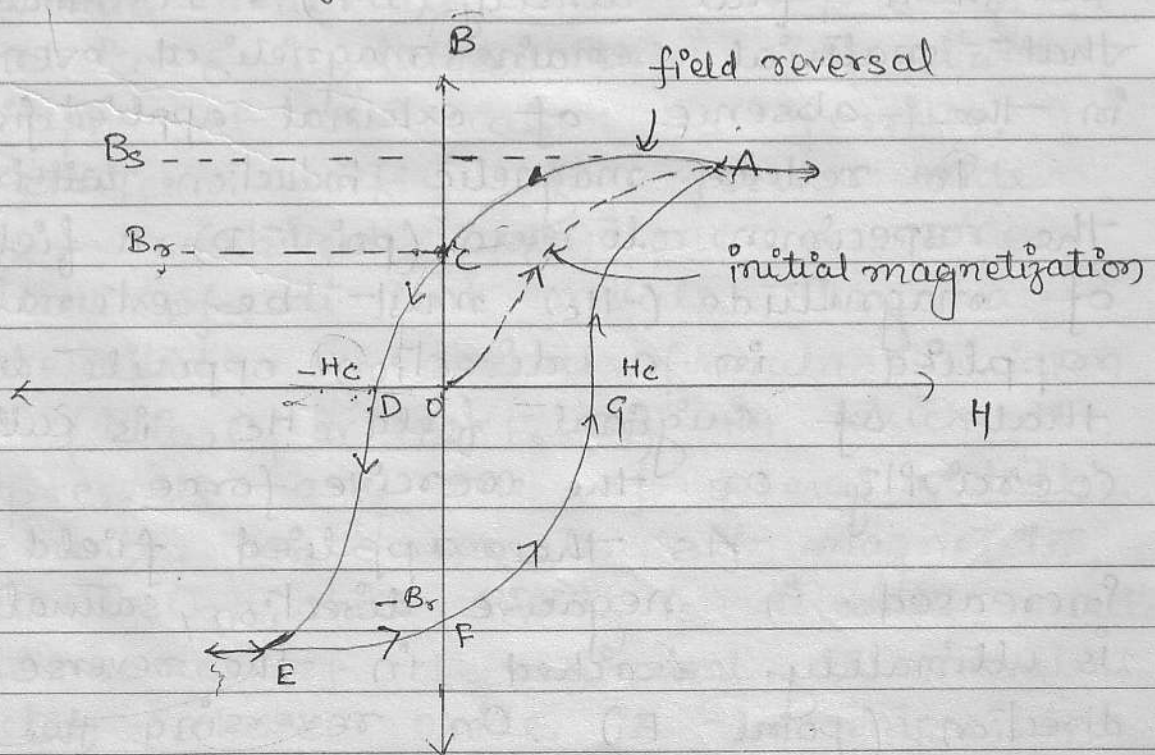


Fig 5.18

If an initially unmagnetized specimen of ferromagnetic material is subjected to increasing or decreasing magnetic field the magnetic field induction B varies as a function of H along a closed loop, called hysteresis loop.

The curve begins at the origin O . As H is increased the field B begins to increase slowly, then more rapidly and finally attaining a saturation value and becoming independent of H . The maximum value of B is the saturation flux density B_s and the corresponding magnetization is the saturation magnetization M_s .

From saturation (point A) the curve does not retrace its original path as the field H is reduced. At zero field (point C) there exists a residual field which is called remanence or remanent flux density (B_r). It indicates that material remains magnetized even in the absence of external applied field H .

To reduce magnetic induction within the specimen to zero (point D) a field of magnitude ($-H_c$) must be externally applied in a direction opposite to that of original field H_c is called coercivity or the coercive force.

As the applied field is increased in negative direction, saturation is ultimately reached in the reverse direction (point E). On reversing the

variation of field H , a curve similar to ACDE is traced through points EFGA, yielding a negative remanence ($-B_r$) and a positive coercivity $+H_c$. The B-H curve is known as the hysteresis loop.

FERRIMAGNETISM (FERRITES)

Ferrimagnetic materials are v. much similar to ferromagnetic materials in their macroscopic magnetic characteristics. The saturation magnetization in these materials is not as high as ferromagnetic materials. The saturation magnetization decreases with increasing temperature until it vanishes at a curie temperature T_c . Above (T_c) they exhibit paramagnetic behaviour.

Ferrimagnetic materials are ceramic materials and therefore good electrical insulators. This important group of materials is ferrites.

Ferrites chiefly of ferric oxide Fe_2O_3 combined with one or more oxides of divalent metals. They are represented by general formula written as $(MFe_2O_4$ or $MO \cdot Fe_2O_3$) in which M represents any one of several metallic elements. An example of magnetite, $FeO \cdot Fe_2O_3$ where Fe^{++} is divalent metal ion. The name of the divalent or monovalent metal, whose oxide is in composition

of ferrite, is given to ferrite.

Composite ferrites, which are solid solutions of one simple ferrite shows the best magnetic properties.

Ni-Zn ferrite is an example of composite ferrite.

Garnets constitute another important group of ferrimagnetic materials. The garnets have a very complicated crystal structure which may be represented by the general formula (M_3FeO_{12}) where M represents rare earth ion such as samarium, europium, gadolinium or yttrium. Yttrium iron garnet ($Y_3Fe_5O_{12}$) denoted by YIG is the most common material in this type.

9) ANTIFERROMAGNETISM

Antiferromagnetic materials are crystalline materials, which exhibit a small positive susceptibility of the order of 10^{-3} to 10^{-5} . The variation of susceptibility with temperature follows a peculiar pattern in these materials.

The susceptibility increases with temperature and reaches a maximum at certain temperature called Neel temperature (T_N). With a further increase in temperature, the material goes into paramagnetic state.

The material is antiferromagnetic if below T_N . The transition temperature lies far below room temperature for

most of the materials.

The variation of susceptibility with temperature obeys modified Curie - Weiss law

$$\chi_{af} = \frac{C}{T - (-\theta)} = \frac{C}{T + \theta}$$

$\theta \rightarrow$ paramagnetic curi

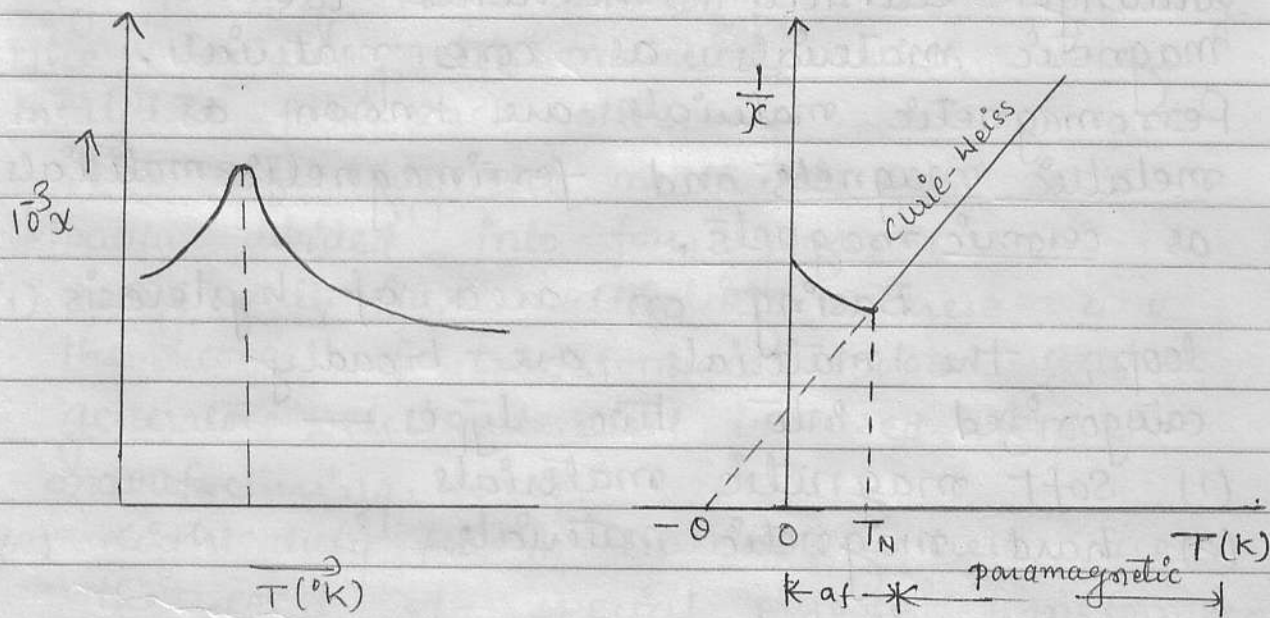


fig 5.19

The elements, manganese and chromium exhibit antiferromagnetism at room temp. Most of the antiferromagnetic materials are ionic compound. MnO , MnS , Cr_2O_3 and NiCr are some of the compound which exhibit antiferromagnetism.

TYPES OF MAGNETIC MATERIALS

The magnetic flux produced by an electric is proportional to relative permeability of the material through which the flux passes. As ferromagnetic and ferrimagnetic material possess high permeability they produce required flux densities with relatively small currents. Therefore both these materials are widely used as flux path and flux multipliers in electrical machines. Transformers, motors, and generators etc. rotating electrical machines use the magnetic materials as core materials. Ferromagnetic materials are known as metallic magnets and ferrimagnetic materials as ceramic magnets.

Basing on area of hysteresis loop, the materials are broadly categorized into two types —

- (1) Soft magnetic materials.
- (2) hard magnetic materials.

Soft magnetic materials :

The materials having narrow hysteresis loop are known as soft magnetic materials. These magnetic materials can be easily magnetized and demagnetized. These materials have a high initial permeability and low coercivity.

Because of high initial permeability material reaches its saturation magnetization with low applied field and exhibits low hysteresis loss energy. Since the area of hysteresis loop represents energy lost-

as heat during a cycle. The smaller the area, the lower the power losses and greater is the possibility of using the material at high frequency.

When magnetic flux through conducting material changes, voltage are induced which cause the flow of eddy currents. Eddy current losses can be reduced by increasing the electrical resistivity of the materials. It is accompanied by forming solid solutions rather than using the ferromagnetic material, which are insulated from each other.

The soft magnetic materials can be broadly divided into four groups:

- (1) heavy duty flux multipliers: these are the cores of transformers, motors and generators, electrical steel is used for manufacturing cores.
- (2) light-duty flux multipliers: these are the cores of special purpose transformers, inductors etc used in communication equipments. Ni-Fe alloys and soft ferrites are used in these applications.
- (3) Square loop materials: They are used in magnetic amplifiers, saturable core devices, computers etc. For these applications Ni-Fe alloy and soft ferrites are used.
- (4) Microwave system components: soft ferrites and garnets are used in this applications.

④ Hard magnetic materials:

The materials with large area of hysteresis loop are called hard materials. permanent magnets, magnetic tapes and disks used in entertainment and computer industries are made from hard magnetic materials.

Hard magnetic materials are characterised by high remanence, high permeability, a high coercive field and large hysteresis loop. A desirable requirement is immunity of the material to loss of magnetization by ac fields, mechanical ill-treatment and changes in temperature.

The hard magnetic materials are magnetized in magnetic field strong enough to orient the magnetic moments of their domains in the direction of applied field. Part of energy of applied field is converted into potential energy which is stored in the permanent magnet produced. A permanent magnet in the fully magnetized condition is thus in relatively high energy state as compared to demagnetized state. The power or external energy of a hard magnetic material is directly related to the size of hysteresis loop. The magnetic potential energy of a hard material is measured by its maximum energy product $(BH)_{max}$.

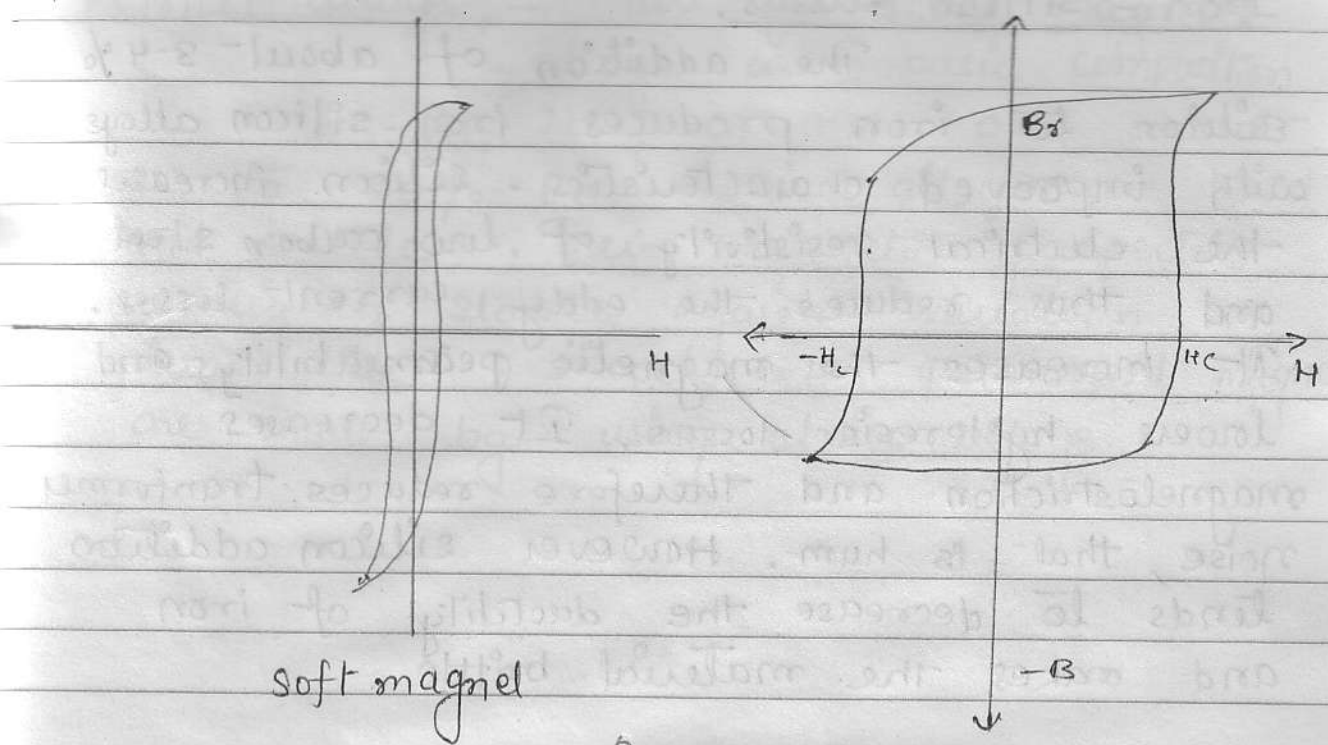


Fig 8.20

Application of magnetic materials

Soft magnetic materials are used in wide variety of electrical machines in daily uses, such as power transformers, motors, generators, electromagnets etc. Electrical steels are used as core materials of these machines. Hard magnetic materials are used in fabrication of permanent magnets, which are required to retain their magnetic field indefinitely.

Low carbon steel:

Pure iron has higher permeability but its higher electrical conductivity causes more eddy current losses. Low carbon steel (Fe - 0.05%) has relatively small permeability and a higher resistivity. It is least expensive core material and is used where low cost is more important than other factors.

Iron - silicon alloys:

The addition of about 3-4% silicon to iron produces iron-silicon alloys with improved characteristics. Silicon increases the electrical resistivity of low carbon steel and thus reduces the eddy current losses. It increases the magnetic permeability and lowers hysteresis losses. It decreases magnetostriction and therefore reduces transform noise, that is hum. However silicon addition tends to decrease the ductility of iron and makes the material brittle.

Grain Orientation:

The permeability can be substantially increased and hysteresis losses can be decreased by making use of favourable grain orientation in the material. Often, aluminium and manganese of less than 1% are added in iron-silicon alloys to improve the grain orientation and reduce the hysteresis losses.

Nickel - iron alloys:-

Nickel-iron alloys are used for communication application such as pulse transformer, relays, recording heads, small motors etc.

Soft ferrites:- has high electrical resistance 10^5 to 10^{15} times the metallic ferromagnets. They are used in memory core, audiovisual and recording applications.

Alnico alloys :- are permanent magnets alloys with basic composition of Al, Ni, Co and about 25 to 50% Fe. Alnico alloys are hard and brittle. Their magnetic properties are highly stable against variation in temperature, shock etc. Permanent magnets are made by using alnico alloys.