Other Applications of Equal Area

- Next we apply the equal area criterion to two different systems of operation:

  (i) sustained line fault, and

  (ii) a line fault cleared after some time by the simultaneous tripping of the breakers at both the ends.
- As shown in Fig. 17.9, it is a parallel feeder fed from one end and at the other end is the infinite bus. The power angle curve corresponding to the healthy condition is given by curve *A in Fig. 17.10.*

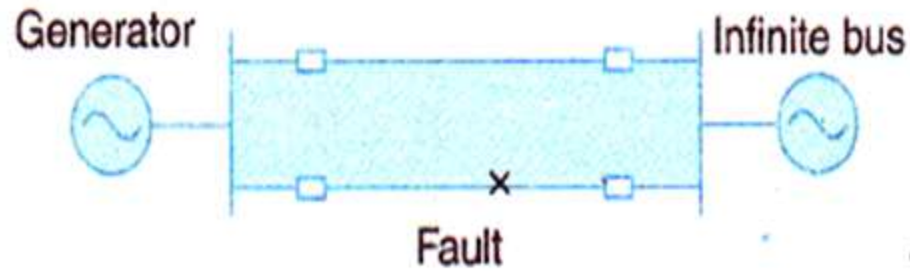


Fig. 17.9 Parallel feeder.

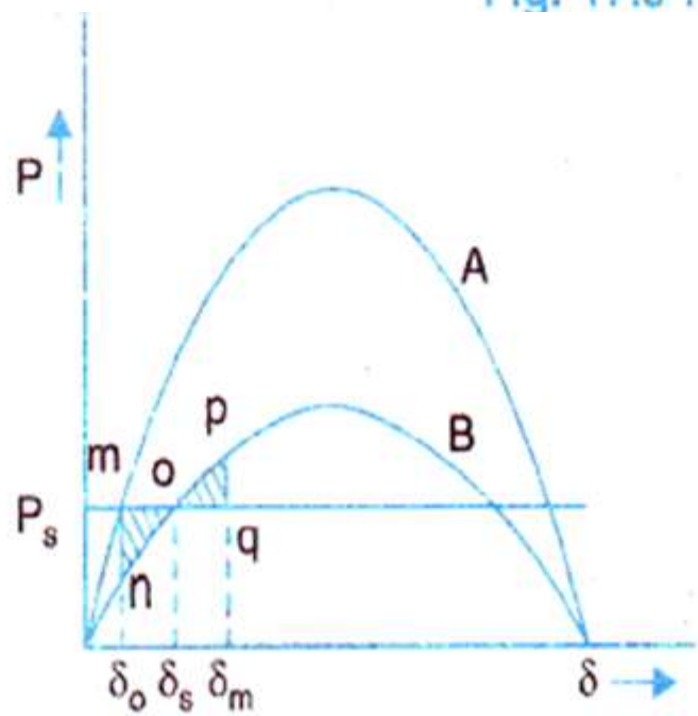
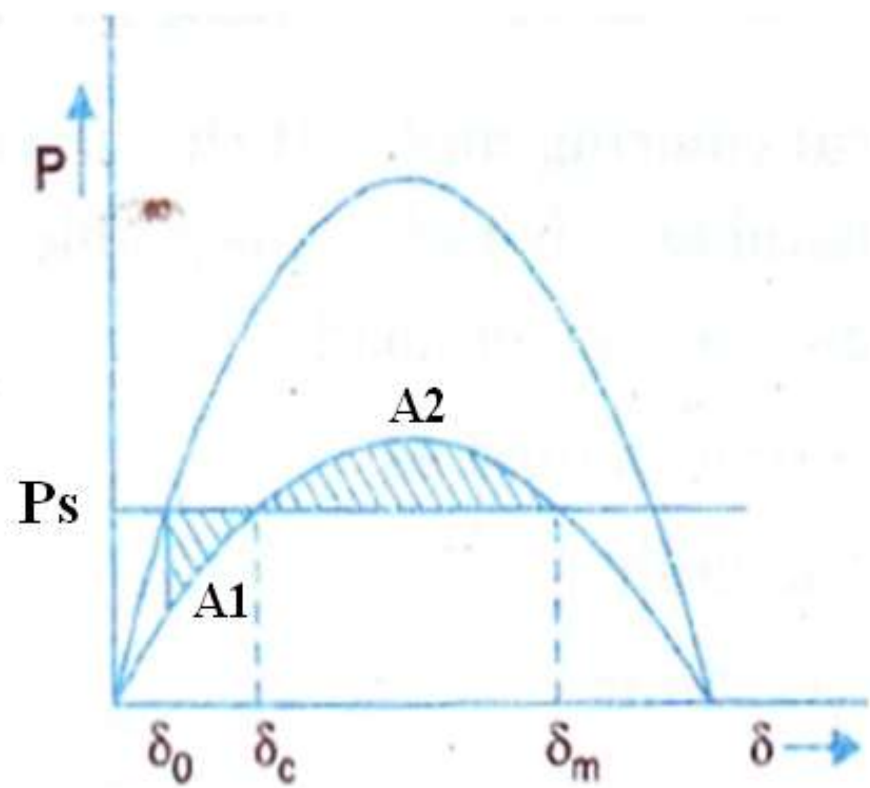


Fig. 17.10 Curve A—Power angle curve under healthy condition; Curve B—Power angle curve under faulted condition.



Transient Stability limit for system in Fig 17.9

- P_s is the input to the generator which is assumed constant. Curve *B represents the power angle curve when a* fault occurs at one of the two lines and the breakers operate instantaneously and simultaneously at both the ends on that line. As a result the equivalent impedance between the bus bars is increased and hence curve *B will be lower than curve A. Corresponding to input P_s the torque angle of the generator is δ_o initially. Now as soon as there is a fault and instantaneously it is cleared, the output of the generator goes down to point n on curve B and since input remains constant which is higher than the output, the rotor accelerates and hence the torque angle increases and the operating point moves along curve B towards o from n.*

- *As it reaches o, the accelerating power becomes zero and the speed of the generator is more than the infinite bus and the speed continues to increase. From o to p the rotor experiences deceleration but the speed is more than the speed of the infinite bus till it reaches point p where the relative speed is zero and the torque angle ceases to increase. At point p, the output is more than the input to the generator and hence the rotor decelerates and the speed goes down relative to the infinite bus till it reaches point o where the speed is minimum. This torque angle continues to decrease till it reaches point n where again the speed is equal to the speed of infinite bus. The cycle repeats itself if damping is not present. It is found that in practice because of the damping present the rotor operates at point 'o' on curve B and the torque angle is* δ_s*.To determine the transient stability limit for this case we should raise the input line* P_s *such that the area below the line* P_s *and the curve B and above the line* P_s *and the curve B at the intersection of* P_s *and B are equal. This is illustrated in Fig. 17.11. The value of* P_s *corresponding to this situation is known as the transient stability limit. ie, area A1 = A2 for a particular Ps known as transient stability limit.*

Fault cleared after some time

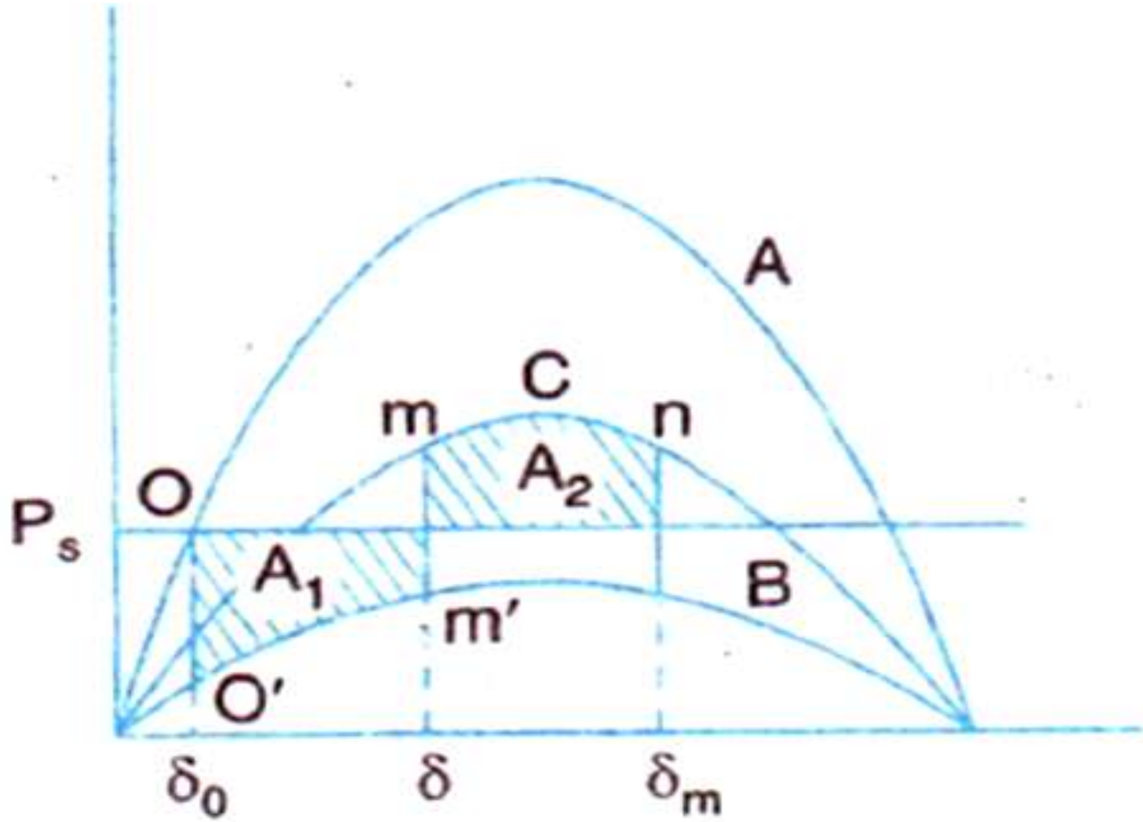


Fig. 17.12 Equal area criterion applied to system of Fig. 17.9 for a fault cleared at an angle $\delta < \delta_c$.

- Curve *A in Fig. 17.12 represents the power angle curve corresponding to healthy condition of system in Fig. 17.9. Curve B represents corresponding to fault on one of the two lines and fault allowed to exist for some time. Curve C corresponds to the situation when the faulted line is removed.*
- Initially for P_s *the torque angle is* δ_o. *At the time of fault, the output of the generator becomes as at O'. Hence, the rotor accelerates and the rotor moves along the curve B up to point m' when the faulted line is removed and the operating point becomes m on curve C where the output is more than the input and the rotor decelerates till the speed becomes equal to the speed of the infinite bus and the torque angle ceases to increase at point n.*

- From Fig. 17.12 it is clear that the transient stability limit not only depends upon the type of disturbance but it also depends upon the clearing time of the breaker. Faster the breaker operation smaller will be the area A_1 and hence larger will be the transient stability limit.

Interesting case

Fig. 17.13 Generator connected to infinite bus through a line.

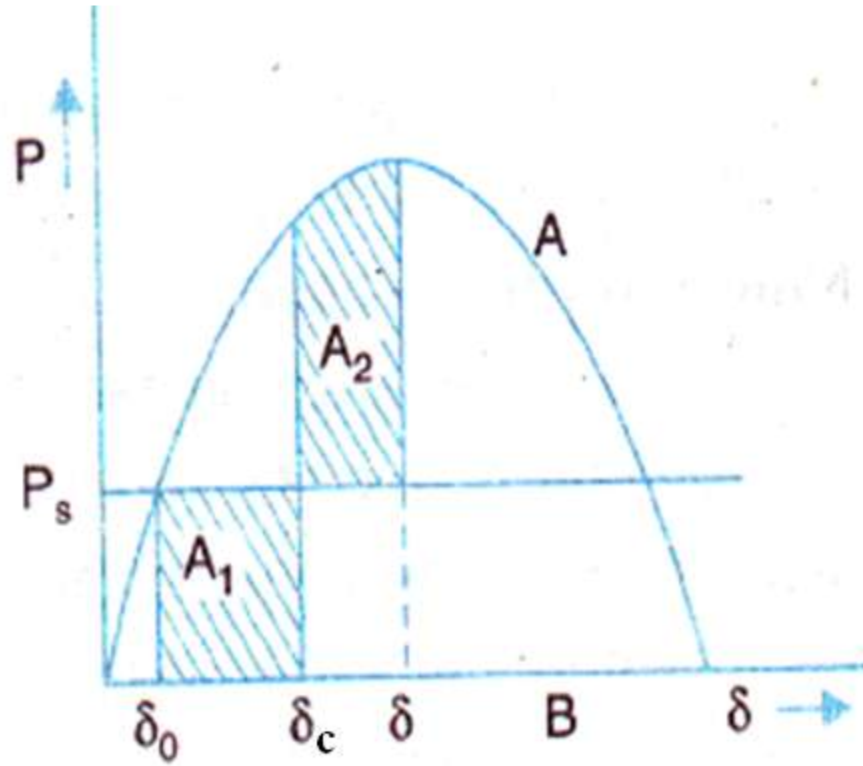


Fig. 17.14 Power angle curves: A—Healthy condition B—3-phase fault.

- Say there occurs a three-phase fault on the line temporarily (Fig. 17.14). The power angle curve will correspond to the horizontal axis because power transferred is zero. If the breaker reclose after some time corresponding to clearing angle δ_c when the fault is vanished, the output will be more than the input and hence the rotor decelerates. Finally, if the clearing angle δ_C is such that $A_1 = A_2$*, the system becomes stable.*

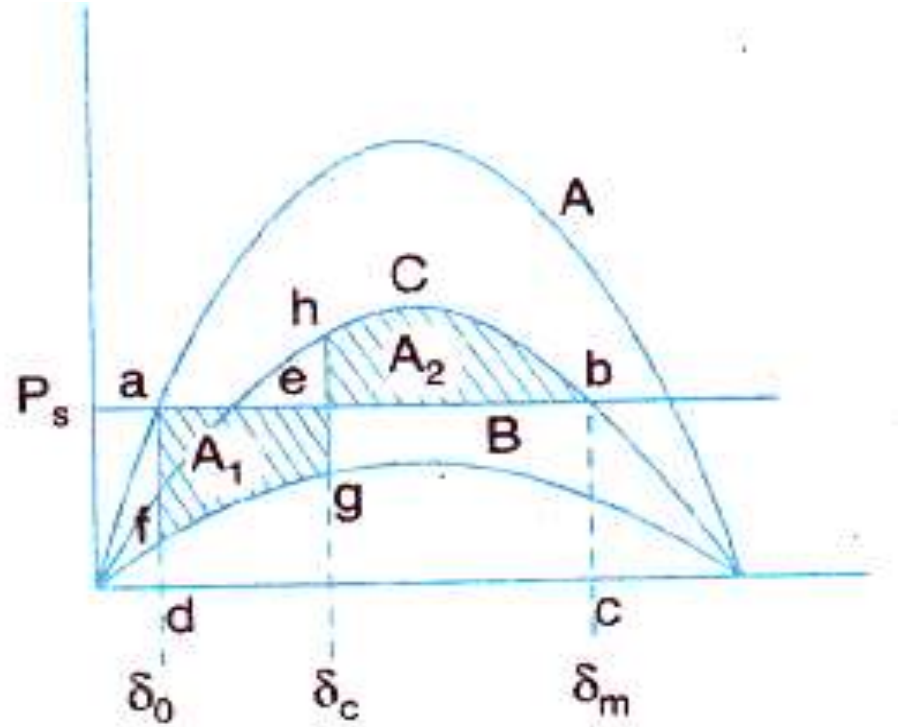
Critical clearing from wadhwa

Let three power angle curves:

$A = P_m \sin\delta$; before the fault

$B = r_1 P_m \sin\delta$; during fault

$C = r_2 P_m \sin\delta$; after the fault



- For transient stability limit, the two areas $A_l = A_2$ *or equivalently the area under the curve abcd (a rectangle) should be equal to the area under the curve dfghbc i.e.*

$$(\delta_m - \delta_o)P_s = \int_{\delta_o}^{\delta_c} r_1 P_m \sin\delta \, d\delta + \int_{\delta_c}^{\delta_m} r_2 P_m \sin\delta \, d\delta$$

$$= r_1 P_m[\cos\delta_o - \cos\delta_c] + r_2 P_m[\cos\delta_c - \cos\delta_m]$$

Now substituting $P_s = P_m \sin\delta_o$

$$(\delta_m - \delta_o)P_m \sin\delta_o = r_1 P_m[\cos\delta_o - \cos\delta_c] + r_2 P_m[\cos\delta_c - \cos\delta_m]$$

$$(\delta_m - \delta_o)\sin\delta_o = (r_2 - r_1)\cos\delta_c + r_1\cos\delta_o - r_2\cos\delta_m$$

$$\therefore \quad \cos\delta_c = \frac{(\delta_m - \delta_o)\sin\delta_o - r_1\cos\delta_o + r_2\cos\delta_m}{r_2 - r_1} \quad (17.25)$$

Now from the curves,

$$P_s = P_m \sin \delta_o = r_2 P_m \sin \delta_m = r_2 P_m \sin (\pi - \delta_m)$$

or $$\sin \delta_o = r_2 \sin (\pi - \delta_m)$$

or $$\delta_m = \pi - \sin^{-1}\left(\frac{\sin \delta_o}{r_2}\right) \tag{17.26}$$

Thus if r_1, r_2 and δ_0 are known, the critical clearing angle δ_c can be obtained.

- AM09 14 MARK
- A motor is receiving 25% of the power that it is capable of receiving from an infinite bus. If the load on the motor is doubled, calculate the maximum value of δ during the swinging

Solution:

$\sin \delta_o = 0.25 \qquad \therefore \quad \delta_o = 14.48°$

$\sin \delta_c = 0.5 \qquad \delta_c = 30°.$

$\delta_m = ?$

$$(\delta_m - \delta_o)\, 0.5 = \int_{\delta_o}^{\delta_m} \sin \delta \, d\delta$$

$$0.5(\delta_m - 0.2527) = (\cos \delta_o - \cos \delta_m)$$

$$0.5\delta_m = \cos \delta_o - \cos \delta_m + 0.1263$$

$$0.5\delta_m + \cos \delta_m = 0.96823 + 0.1263 = 1.094535$$

For solution of this equation we make guess for δ_m such that δ_m should be greater than 30° (as $\delta_c = 30°$). After some trials δ_m is found to be 45°.

$\delta_m =$ 14.45, but since it should be greater than 30 therefore 30 + 14.45 = 44.45 (45 approx.)

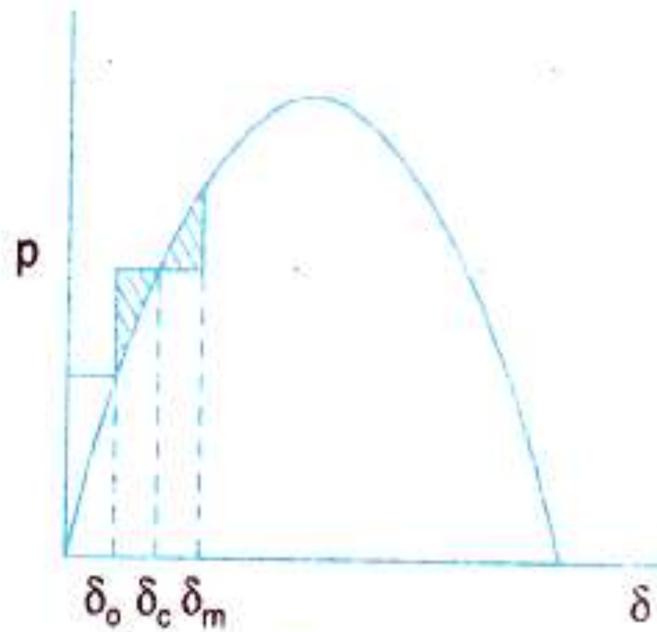


Fig. E.17.3

- A 50 Hz generator is delivering 50% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and the infinite bus to 500% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 75% of the original maximum value. Determine the critical clearing angle for the condition described.

$$0.5P_m = 0.75P_m \sin \delta_m$$

$\therefore$ $$\delta_m = 41.8^\circ \text{ or } \delta_m = 180 - 41.8 = 138.2^\circ$$

or $$\delta_m = 2.412 \text{ radians}$$

Substituting these values in the above expression, we get

$$= \cos^{-1} \frac{0.5(2.412 - 0.5236) - 0.75 \times 0.74547 - 0.2 \times 0.866}{0.55}$$

$$= \cos^{-1} 0.3836$$

$$= 67.44^\circ. \quad \textbf{Ans.}$$

Example 19.5 A loss-free generator supplies 50 MW to an infinite bus, the steady-state limit of the system being 100 MW. Determine whether the generator will remain in synchronism if the prime mover input is abruptly increased by 30 MW.

Solution

The power-angle curve is shown in Fig. 19.16. The equation of the power-angle curve is

$$P_e = P_{\max} \sin \delta \tag{E 19.5.1}$$

The initial operating point is at a where $P_e = 50$ MW. The prime mover input is abruptly increased by 30 MW. The desired operating point is at b so that $ag = 30$ MW. The maximum swing of the rotor can be upto point c. The system will be stable if the area agb is less than area bmc. The load angles corresponding to points a and b can be found from Eq. (E 19.5.1).

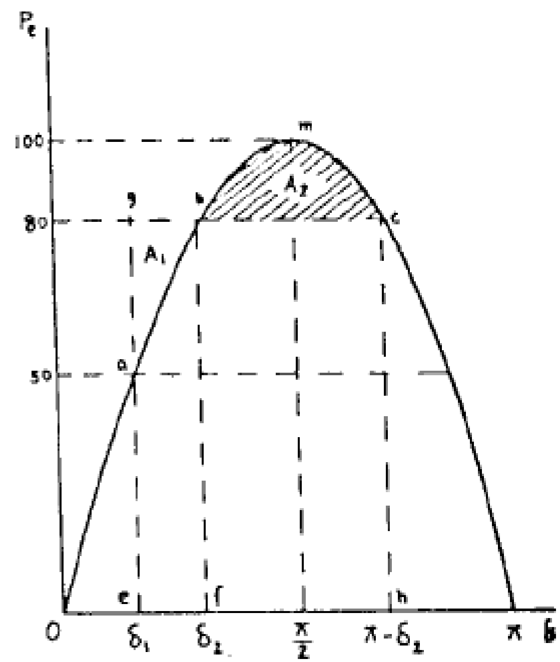


Fig. 19.16.

For point a, $P_e = 50$, $\delta = \delta_1$

Substituting these values in Eq. (E 19.5.1) we get

$$50 = 100 \sin \delta_1$$

$$\sin \delta_1 = \frac{50}{100}, \ \delta_1 = 30° = 0.523 \text{ rad}$$

For point b, $P_e = 80$, $\delta = \delta_2$

$$80 = 100 \sin \delta_2$$

$$\sin \delta_2 = \frac{80}{100}, \ \delta_2 = 53.2° = 0.927 \text{ rad}$$

A_1 = area agb = area of rectangle $gbfe$ – area $abfe$ under the power angle sine curve

$$= (eg \times ef) - \int_{\delta_1}^{\delta_2} P_e \, d\delta = 80\,(\delta_2 - \delta_1) - \int_{\delta_1}^{\delta_2} 100 \sin\delta \, d\delta$$

$$= 80\,(0.927 - 0.523) + 100\,(\cos\delta_2 - \cos\delta_1) = 32.32 + 100\,(0.600 - 0.866) = 5.72 \text{ MW rad}$$

A_2 = area *bmc* = area *bmchf* under the sine curve – area of the rectangle *bchf*

$$= \int_{\delta_2}^{\pi - \delta_2} P_e \, d\delta - 80\,(\pi - \delta_2 - \delta_2) = \int_{\delta_2}^{\pi - \delta_2} 100 \sin\delta \, d\delta - 80\,(\pi - 2\delta_2)$$

$$= -100\,[\cos(\pi - \delta_2) - \cos\delta_2] - 80\,(\pi - 2 \times 0.927)$$

$$= 200 \cos\delta_2 - 103 = 200 \times 0.6 - 103 = 117 \text{ MW rad.}$$

Since area *agb* (A_1) is less than area *bmc* (A_2) the system is stable.

Example 19.6 A generator with constant excitation supplies 30 MW through a step-up transformer and a high voltage line to an infinite busbar. If the steady-state stability limit of the system is 60 MW, estimate the maximum permissible sudden increase of generator output (resulting from a sudden increase in prime mover input) if the stability is to be maintained. The resistances of the generator, transformer and line may be neglected.

Solution

The power-angle curve is shown in Fig. 19.17. The initial operating point is at a where $P_e = 30$ MW. With the sudden increase of load to P_1 the rotor advances corresponding to the point b. The maximum swing of the rotor can be upto point c. For the system to remain stable the accelerating area A_1 = area agb should be less than or equal to the retardation area A_2 = area bmc.

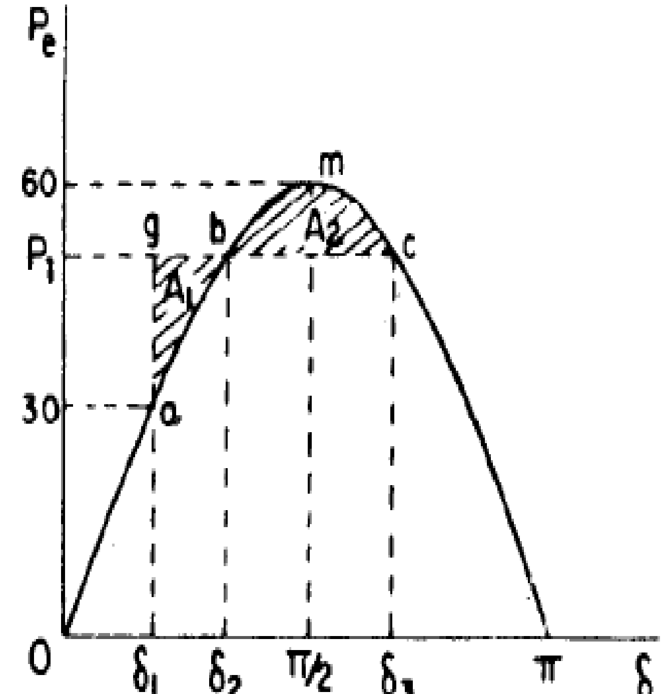


Fig. 19.17.

The equation of the power-angle curve is

$$P_e = P_{max} \sin \delta$$

or

$$P_e = 60 \sin \delta \tag{E 19.6.1}$$

$$A_1 = \text{area } agb = \int_{\delta_1}^{\delta_2} (P_1 - 60 \sin \delta)\, d\delta$$

$$A_2 = \text{area } bmc = \int_{\delta_2}^{\delta_3} (60 \sin \delta - P_1)\, d\delta$$

For stability, $A_1 = A_2$ or $A_1 - A_2 = 0$

$$\int_{\delta_1}^{\delta_2} (P_1 - 60 \sin \delta)\, d\delta - \int_{\delta_2}^{\delta_3} (60 \sin \delta - P_1)\, d\delta = 0$$

$$\int_{\delta_1}^{\delta_2} (P_1 - 60 \sin \delta)\, d\delta + \int_{\delta_2}^{\delta_3} (P_1 - 60 \sin \delta)\, d\delta = 0$$

$$\int_{\delta_1}^{\delta_3} (P_1 - 60 \sin \delta)\, d\delta = 0$$

$$\left[P_1\, \delta + 60 \cos \delta\right]_{\delta_1}^{\delta_3} = 0$$

$$P_1 (\delta_3 - \delta_1) + 60 (\cos \delta_3 - \cos \delta_1) = 0$$

At point a, $P_e = 30$, $\delta = \delta_1$

Substituting these values in Eq. (E 19.6.1) we get

$$30 = 60 \sin \delta_1$$

$$\therefore \quad \sin \delta_1 = 0.5, \quad \delta_1 = \frac{\pi}{6} \text{ radians}, \quad \cos \delta_1 = 0.866$$

At point b, $P_1 = 60 \sin \delta_2$

Also, $\delta_3 = \pi - \delta_2, \quad \cos\delta_3 = -\cos\delta_2$

Substituting these values in Eq. (E 19.6.2) we get

$$(60\sin\delta_2)\left(\pi - \delta_2 - \frac{\pi}{6}\right) + 60(-\cos\delta_2 - 0.866) = 0$$

$$\left(\frac{5\pi}{6} - \delta_2\right)\sin\delta_2 = \cos\delta_2 + 0.866 \qquad \text{(E 19.6.3)}$$

Equation (E 19.6.3) can be solved by trial and error method. Put $\delta_2 = \pi/3$ radians = 60°

$$\text{L.H.S.} = \left(\frac{5\pi}{6} - \frac{\pi}{3}\right)\sin\frac{\pi}{3} = 1.36035$$

$$\text{R.H.S.} = \cos\frac{\pi}{3} + 0.866 = 1.366$$

Let us put $\delta_2 = 60.4° = 1.0541789$ radians

$$\text{L.H.S.} = \left(\frac{5\pi}{6} - \frac{60.4\times\pi}{180}\right)\sin 60.4° = 1.35973$$

$$\text{R.H.S.} = (\cos 60.4°) + 0.866 = 1.35994$$

$\therefore$ L.H.S. = R.H.S. (approx.)

If $\delta_2 = 60.4°$

$$P_1 = 60\sin\delta_2 = 60\sin 60.4° = 52.17 \text{ MW}$$

The maximum permissible sudden increase of load $= P_1 - 30 = 52.17 - 30 = 22.17$ MW

Equation (E 19.6.3) can also be solved by graphical method. The left-hand side and right-hand side of this equation are plotted for arbitrary values of δ_2 which lie between δ_1 and 90°. The intersection of the two curves gives the value of δ_2.

Question

- Q5. b). Explain the "Equal Area criterian " of stability with reference to a power system. What is the significance of "critical clearing angle"? A generator having H = 6.0 MJ/MVA is delivering power of 1.0 p.u to an infinite bus through a purely reactive network when the occurrence of a fault reduces the power output to zero. The maximum power that can be transmitted is 2.5p.u when the fault is cleared original network conditions are restored. Determine critical clearing angle & critical clearing time.

Question

- b). Explain "Equal Area Criterian" for stability of a 2 machine power system.
- A 60 Hz generator is supplying 60% of Pmax to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and infinite bus by 400%. When the fault is cleared, the maximum power that can be delivered is 80% of the original maximum. Determine the critical clearing angle for the condition. Derive the formula used, if any.

Question

- 8 Explain the 'equal area ' criterion of stability as applied to a power system .Derive an expression for critical clearing angle and use the result to determine the same for the following :
- A synchronous generator is delivering 50% of maximum power to an infinite bus through a transmission line. A fault occurs such that the new maximum power that can be delivered is 30% of original ,When the fault is cleared, the maximum power that can be delivered is 80% of original maximum. Determine the critical clearing angle. If the fault is cleared as $\delta = 75^0$ find the maximum value of δ for which machine swings around before reaching steady state .

Example 19.7 A double-circuit three-phase feeder connects a single generator to a large network. The power corresponding to the limit of steady-state stability for each circuit is 100 MW. The line is transmitting 80 MW when one of the circuits is suddenly switched out. Determine with reference to appropriate diagram whether the generator is likely to remain in synchronism.

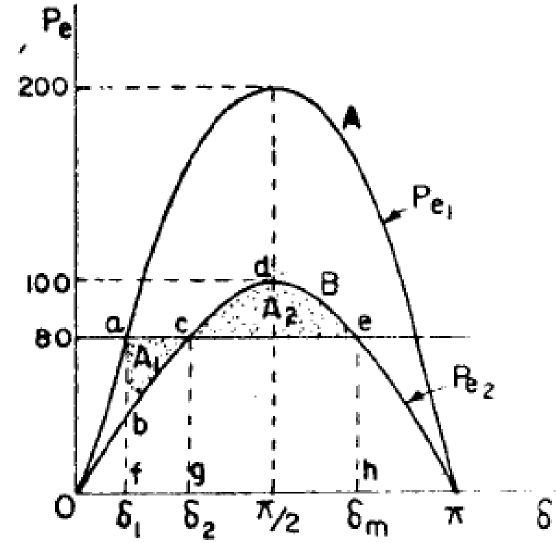


Fig. 19.21.

Solution

The equation of the power-angle curve is

$$P_e = P_{\max} \sin \delta$$

For curve A in Fig. 19.21,

$$80 = 200 \sin \delta_1,\ \delta_1 = \sin^{-1} \frac{80}{200}$$

$$\delta_1 = 23.578° = 0.4115 \text{ rad}$$

For curve B,

$$80 = 100 \sin \delta_2,\ \delta_2 = \sin^{-1} \frac{80}{100}$$

$$\delta_2 = 53.13° = 0.9273 \text{ rad}$$

Area A_1 = area abc = area of rectangle $acgf$ – area $bcgf$ under the power–angle sine curve B

$$= 80 (\delta_2 - \delta_1) - \int_{\delta_1}^{\delta_2} 100 \sin \delta \, d\delta = 80 (0.9273 - 0.4115) + [100 \cos \delta]_{\delta_1}^{\delta_2}$$

$$= 41.26 + 100 (\cos \delta_2 - \cos \delta_1) = 41.26 + 100 (0.6 - 0.9165) = 9.6 \text{ MW radians}$$

Area A_2 = area cde = area $cdehg$ under the sine curve B – area of rectangle $cehg$

$$= \int_{\delta_2}^{\delta_m} 100 \sin \delta \, d\delta - 80 (\delta_m - \delta_2) = [-100 \cos \delta]_{\delta_2}^{\delta_m} - 80 (\delta_m - \delta_2)$$

$$\delta_m = \pi - \delta_2$$

$\therefore$
$$A_2 = -100 \{\cos (\pi - \delta_2) - \cos (\delta_2)\} - 80 (\pi - \delta_2 - \delta_2)$$

$$= 100 \times 2 \cos 53.13° - 80 (\pi - 2 \times 0.9273) = 120 - 102.96 = 17.04 \text{ MW radians}$$

Since area A_1 is less than area A_2, the system is stable.

Point by Point method (B.R.Gupta)

- Why
- 1). The power system engineer is not interested in knowing the clearing angle but interested in corresponding critical clearing time so that he can design the operating times of the relay and circuit breaker so that the total time taken by them should be less than the critical clearing time for stable operation of the system, the point-by-point method is used for the solution of critical clearing time associated with critical clearing angle.

Point by Point method (B.R.Gupta)

Swing equation is given by

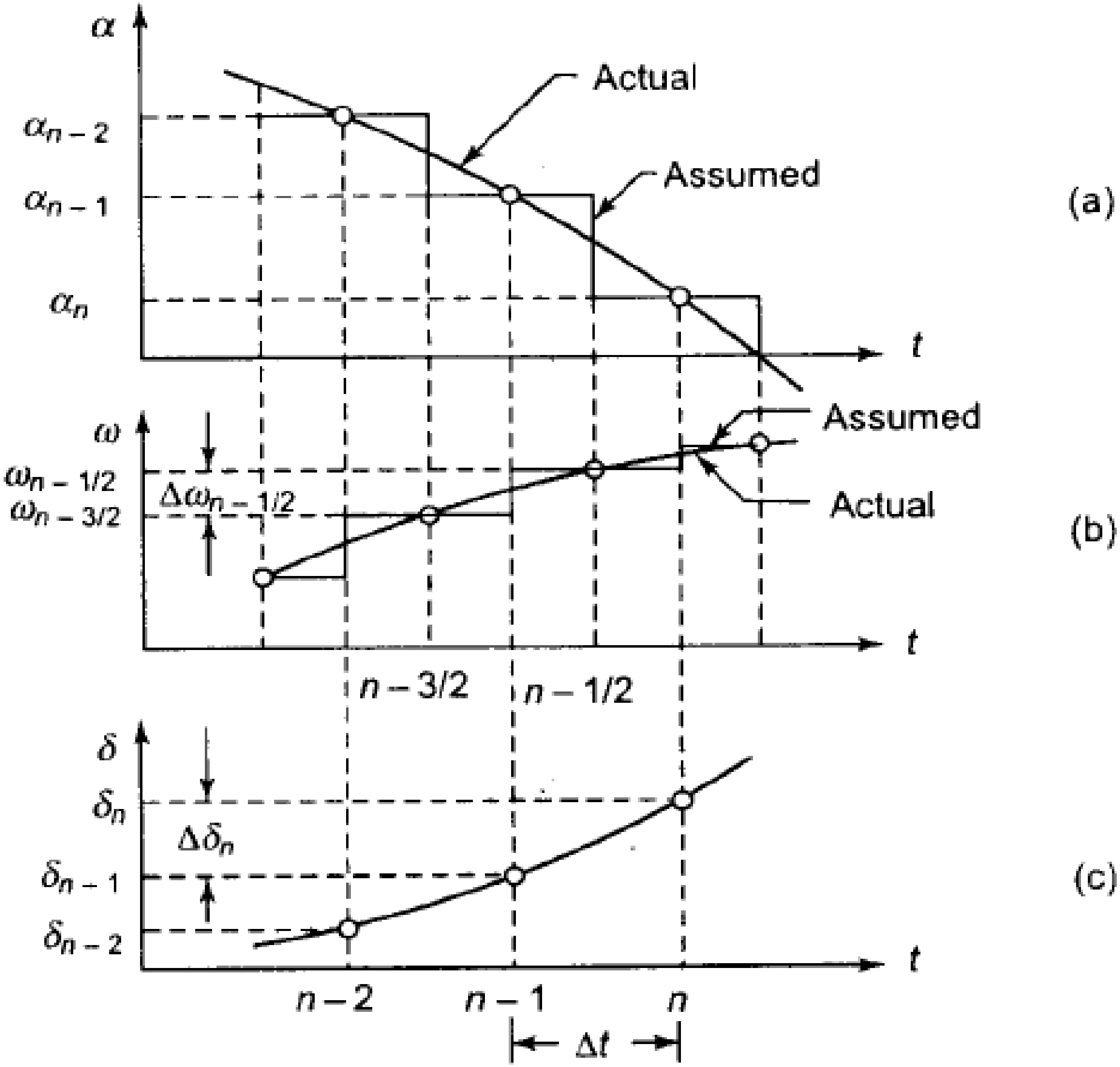
$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{1}{M}(P_i - P_m \sin\delta)$$

- Its solution gives a plot of δ vs t known as swing curve. If the swing curve indicates that δ starts decreasing after reaching a maximum value, the system can be assumed to be stable. The swing equation is a non-linear equation and a formal solution is not feasible. Point by point method is very simple for solving this equation.

- The point-by-point method calculates the change in the angular position (δ) of the rotor during a short interval of time (say 0.05sec). The following assumptions are made during the computational procedure:

  (i) The accelerating power P_a computed at the beginning of an interval is assumed to be constant from the middle of the preceding interval to the middle of the interval under consideration.

  (ii) The angular velocity, computed at the middle of an interval, remains constant over the interval. Refer to Fig. 17.16 for better understanding of the assumptions.



α
Actual
αn − 2
Assumed
αn − 1
(a)
αn
t
ω
Assumed
ωn − 1/2
Δωn − 1/2
Actual
ωn − 3/2
(b)
t
n − 3/2
n − 1/2
δ
δn
Δδn
δn − 1
(c)
δn − 2
t
n − 2
n − 1
n
Δt

- Let us consider the nth time interval which begins at $t = (n-1)\Delta t$. The angular position of the rotor at this instant is δ_{n-1} (fig c). The accelerating power $P_{a(n-1)}$ and hence, acceleration at this instant is assumed to be constant from

$$t = \left(n - \frac{3}{2}\right)\Delta t \text{ to } \left(n - \frac{1}{2}\right)\Delta t.$$

- The procedure for the first iteration is outlined below:

i). Evaluate the accelerating power P_a

$$Pa(0+) = Ps - Pe(0+)$$

ii). From the swing equation $$\frac{d^2\delta}{dt^2} = \alpha_{(0+)} = \frac{P_{a(0+)}}{M}$$

where α is the acceleration. Evaluate α.

(iii). The change in angular velocity for the first interval

$$\Delta\omega_1 = \alpha_{(0+)} \cdot \Delta t$$

$$\omega_1 = \omega_0 + \Delta\omega_1 = \omega_0 + \alpha_{(0+)}\Delta t$$

Here ωo is the relative angular velocity and is zero at $t = 0$.

iv) The change in rotor angle for the first interval,

$$\Delta\delta_1 = \Delta\omega_1 . \Delta t$$

$$\delta_1 = \delta_0 + \Delta\delta_1 = \delta_o + \Delta\omega_1 . \Delta t = \delta_o + \alpha_{(0+)}(\Delta t)^2$$

Question

- (c) Enlist and explains in brief the computational Algorithm for obtaining swing curves using numerical integration (step by step Method) state the assumption made very clearly.

Factors affecting transient stability

- From the swing equation $d^2\delta/dt^2 = Pa/M$ the acceleration of rotor $d^2\delta/dt^2$ is inversely proportional to the moment of inertia of the machine when accelerating power is constant which means higher the moment of inertia the slower will be the change in the rotor angle of the machine and thus allows a longer time for breaker operation to isolate the fault before the machine passes through the critical clearing angle. Higher moment of inertia means the heavier rotor which requires higher SCR. Since the p.u. synchronous impedance is the reciprocal of the SCR ratio, the p.u. impedance becomes smaller with higher values of SCR and, therefore, the short circuit currents in the system increase. Therefore, higher moment of inertia method of improving the transient stability is uneconomical and is normally not used.

- The methods normally used are
- *(i)* Higher system voltage
- *(ii)* Use of parallel lines to reduce the series reactance.
- *(iii)* Use of high speed circuit breakers and auto-reclosing breakers.

- From equation (17.5) it is clear that an increase in system voltage results in higher value of power P_m that can be transferred between nodes. Since shaft power Ps = *Pm* sin δo with higher value of P_m, δ_o is reduced and, therefore, the difference between the critical clearing angle and the initial angle δ_0 is increased. Therefore, increasing P_m allows the machine to rotate through large angle before it reaches the *critical clearing angle which results in greater critical clearing time and the probability of maintaining stability.*

- *Reducing the series reactance by using series capacitor is normally economical for lines of length more than 320 kms. For lines of length less than 320 kms, the objectvie is achieved by running parallel lines.* When parallel lines are used, instead of a single line, some power can *be transferred over the healthy line even during a three-phase fault* on one of the lines, unless of course when a fault takes place at the paralleling bus when no power can be transferred out the parallelled lines. For other types of faults on one line more power can be transferred during the fault if there are two lines in parallel than can be transferred over a single faulted line. The effect of reducing the series reactance is to increase P_m which, therefore, increases the transient stability limit of a system.

- The quicker a breaker operates, the faster the fault is removed from the system and the better is the tendency of the system to restore to normal operating conditions. The use of high speed breakers has materially improved the transient stability of the power systems and does not require any other methods for the purpose.

Questions

- (a) When a sudden Disturbance on the power system occurs, mechanical power input to the generator is taken constant throughout the analysis , explain why?
- (b) The application of equal area criterion gives maximum value of power angle during swing, explain,
- (a) A generator is connected to an infinite bus ,a fault takes place in between ,Discuss with reasons, What will happen to the rotor of generator, if system remains stable.
- (b)high speed circuit breaking improve transient stability diagram.
- (c) A motor is receiving 25% of P_{max} from on infinite bus if the load on the motor is doubled, calculate the maximum value of δ while the motor swings around its new equilibrium position.

Question

- (a) A power system having several synchronous machines is stable. What does this statement mean?
- (b) When a single line to ground fault occurs on a power system of two machines : power transferred from generator increases or decrees .Explain
- (a) derive the value of power angle for maximum power delivered by a generator to motor considering generalized circuit constants.
- (b) A 60 Hz four pole turbo generator rated 20 MVA 13.2kV has inertia constant of H =9.0 KW s/ KVA .Find the kinetic energy stored in the rotor at synchronous speed .Find the acceleration if the input is the rotation if the input less the rotational losses is 26800 H.P. and the electric developed is 16 MW.
- b). Derive swing equation for a synchronous generator. Show swing curve for synchronous generator attaining stability after sudden disturbance. What mathematical property of the curve differentiates between a stable system and unstable system.