Vectors & Scalars

Vectors & Scalars

Vectors are measurements which have both magnitude (size) and a directional component.

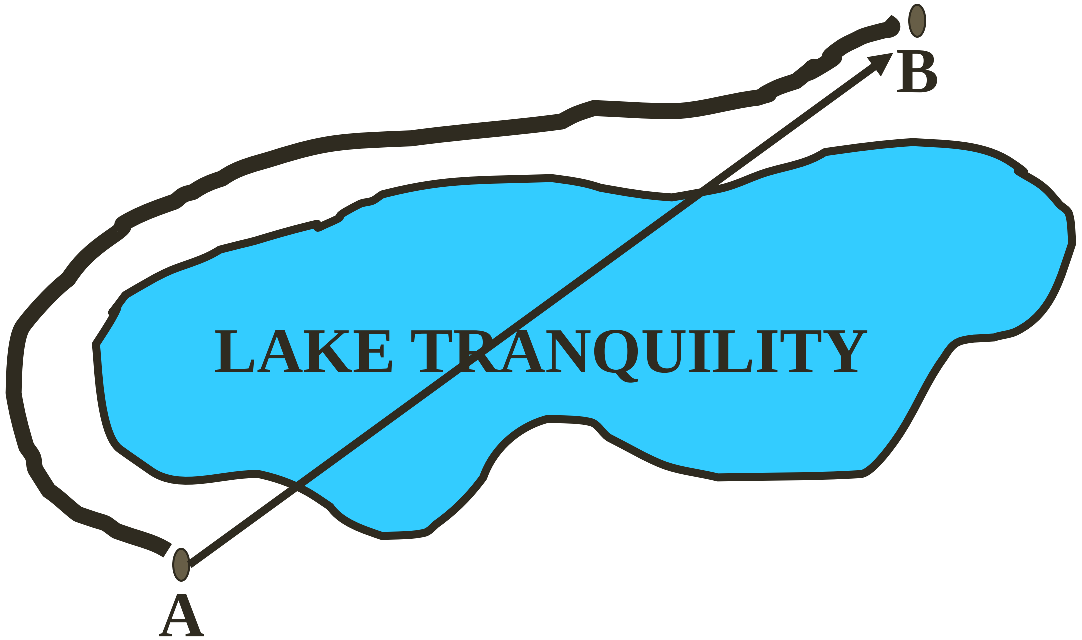
EXAMPLES OF VECTOR VALUES:
Displacement
Velocity
Acceleration
Force
Direction counts in all of these measurements.

Scalars are measurements which have only magnitude (size) and no directional component

EXAMPLES OF SCALAR VALUES:
Distance
Speed
Temperature

Comparing Vector & Scalar Value

Displacement (a vector) versus distance (a scalar)

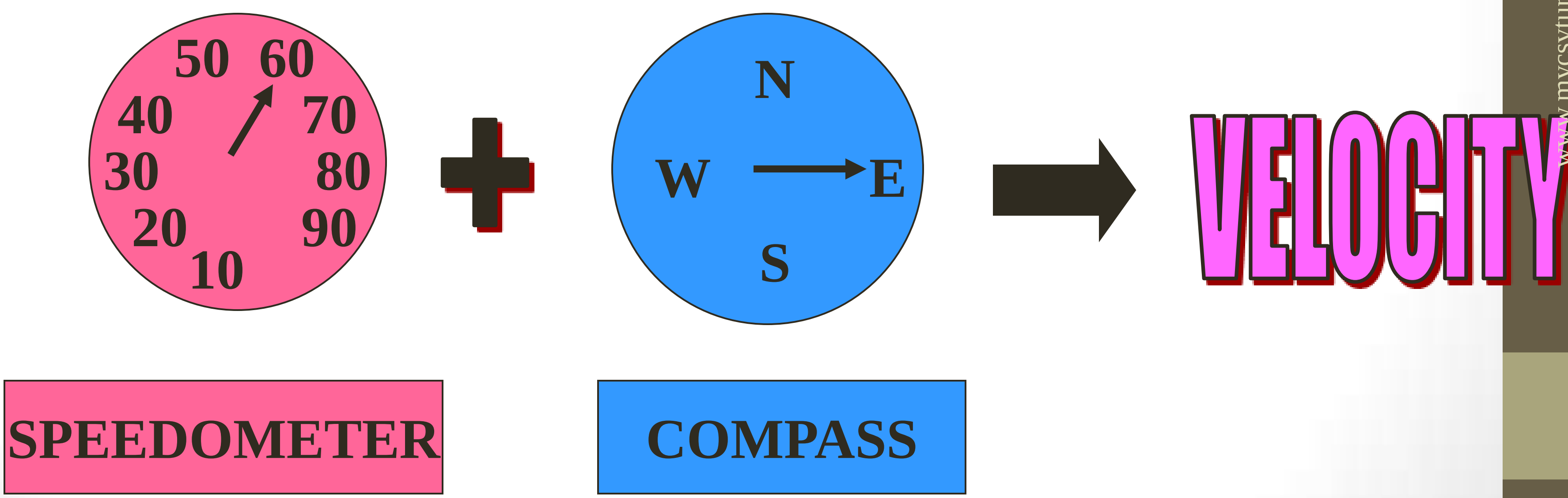


We want to get from point A to point B. If we follow the road around the lake our direction is always changing. There is no specific direction. The distance traveled on the road is a scalar quantity.

A straight line between A and B is the displacement. It has a specific direction and is therefore a vector.

Speed & Velocity

Speed and velocity are not the same.
Velocity requires a directional component and is therefore a vector quantity.
Speed tells us how fast we are going but not which way.
Speed is a scalar (direction doesn't count!)



MEASURING MAGNITUDE

- MAGNITUDE MAY BE MEASURED IN A VARIETY OF DIFFERENT UNITS DEPENDING UPON WHAT IS BEING MEASURED. FOR DISPLACEMENT IT MAY BE METERS, FEET, MILES ETC. FOR VELOCITY IS MIGHT BE METERS PER SECOND OF FEET PER MINUTE, FOR FORCE, IT COULD BE NEWTONS, DYNES OR POUNDS. THE UNITS FOR MAGNITUDE DEPEND UPON WHAT IS MEASURED WHETHER IT IS A VECTOR OR SCALAR QUANTITY.
- WHEN INDICATING THE DIRECTIONAL COMPONENT OF A VECTOR, SEVERAL DIFFERENT METHODS OF CITING THE DIRECTION CAN BE USED. THESE INCLUDE DEGREES, RADIANS OR GEOGRAPHIC INDICATORS SUCH AS NORTH, EAST, NORTH NORTHEAST, ETC.

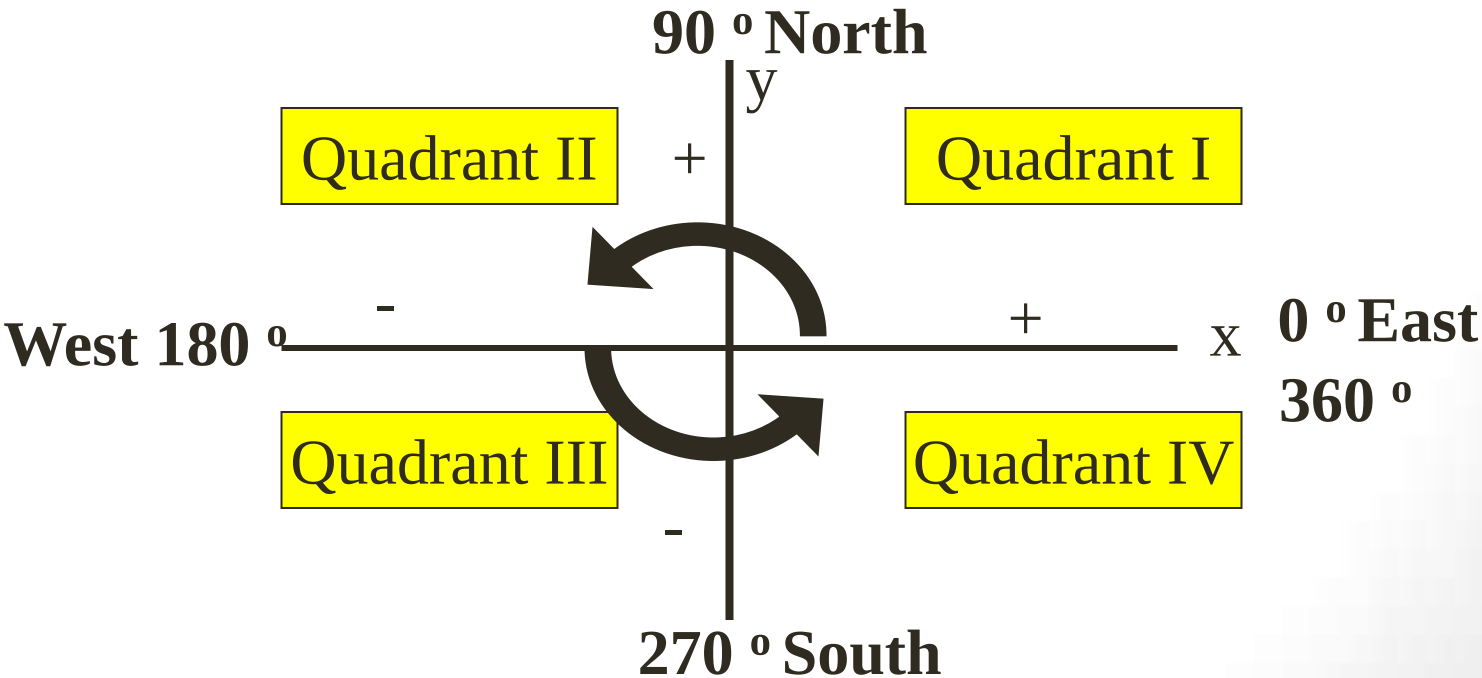
MEASURING DIRECTION

- IN ORDER TO MEASURE DIRECTION CORRECTLY A KNOWLEDGE OF COORDINATE GEOMETRY IS REQUIRED. THIS MEANS THE X-Y PLANES WHICH ARE DIVIDED INTO FOUR SECTIONS OR QUADRANTS DEPENDING ON THE SIGN OF THE X AND Y AXIS IN THAT QUADRANT. THE QUADRANTS ARE NUMBERED IN THE COUNTERCLOCKWISE DIRECTION STARTING FROM THE + X AXIS (OR DUE EAST).
- EACH QUADRANT CONTAINS 90 DEGREES AND, OF COURSE, A FULL CIRCLE REPRESENTS 360 DEGREES.
- ADDITIONALLY, UPWARDS MOTION IS DESIGNATED +, DOWNWARD -, RIGHT MOTION + AND LEFTWARD MOTION -

Physics Mathematics

Up = + | **Down = -** | **Right = +** | **Left = -**

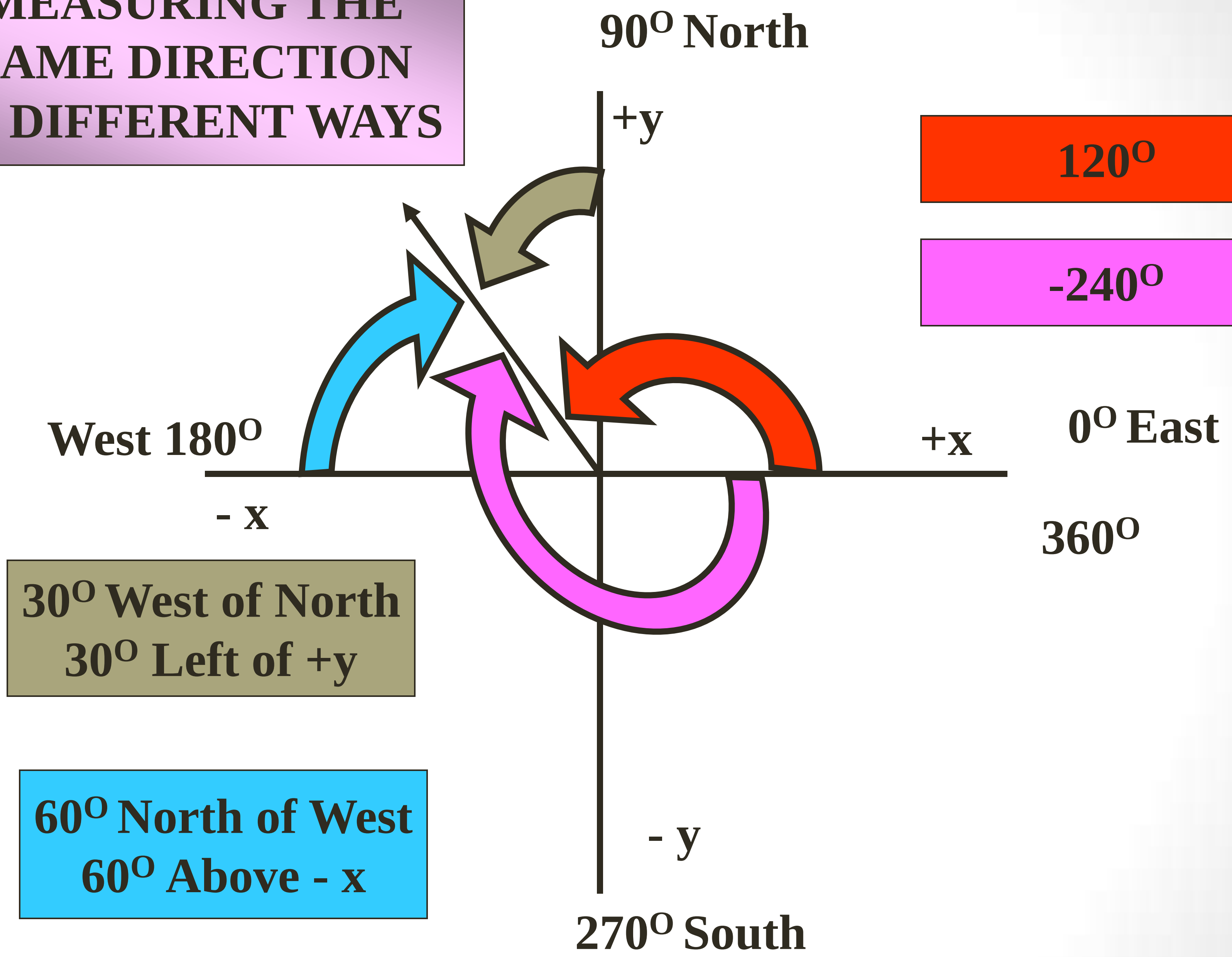
Rectangular Coordinates



MEASURING DIRECTION

- DIRECTION USING THE RECTANGULAR COORDINATE SCALE IS USUALLY REFERENCED FROM THE O DEGREE AXIS BUT ANY REFERENCE MAYBE USED.
- A MEASUREMENT OF 120^O MAYBE RECORDED AS JUST THAT AND WOULD PUT THE VALUE IN QUADRANT II. HOWEVER IT COULD ALSO BE CITED AS A -240^O WHICH MEANS ROTATING CLOCKWISE FROM THE + X AXIS THROUGH 240^O WHICH WOULD PUT US AT THE EXACT SAME LOCATION.
- ADDITIONALLY, A MEASUREMENT OF 30^O WEST OF NORTH (90^O OR VERTICAL) WOULD GIVE THE SAME RESULT. A READING OF 60^O NORTH OF WEST WOULD LIKEWISE GIVE THE SAME READING. USING A READING OF 60^0 ABOVE THE NEGATIVE X AXIS WOULD ALSO GIVE THE SAME RESULT AS WOULD A READING OF 30^O LEFT OF THE POSITIVE Y AXIS. THEY ALL MEAN THE SAME THING!

MEASURING THE SAME DIRECTION IN DIFFERENT WAYS



MEASURING DIRECTION

- **BESIDES THE USE OF DEGREE MEASUREMENTS AND GEOGRAPHIC MEASUREMENTS, DIRECTION CAN ALSO BE MEASURED IN RADIANS. RADIANS ARE DEFINED AS AN ARC LENGTH DIVIDED BY THE RADIUS LENGTH.**
- **A FULL CIRCLE CONTAINS 360^{O} AND ITS CIRCUMFERENCE CAN BER CALCULATED USING CIRCUMFERENCE = ITS DIAMETER TIMES PI (3.14). SINCE THE DIAMETER OF A CIRCLE IS TWICE THE RADIUS, DIVIDING THE ARC LENGTH OR CIRCUMFERENCE (2 x RADIUS x PI) BY THE RADIUS WE FIND THAT ARC DIVIDED BY RADIUS FOR ANY CIRCLE IS ALWAYS 2π**
- **360 DEGREES = 2π RADIANS (6.28 RADIANS)**
- **ONE RADIAN = 57.3 DEGREES**

Measuring angles in Radians

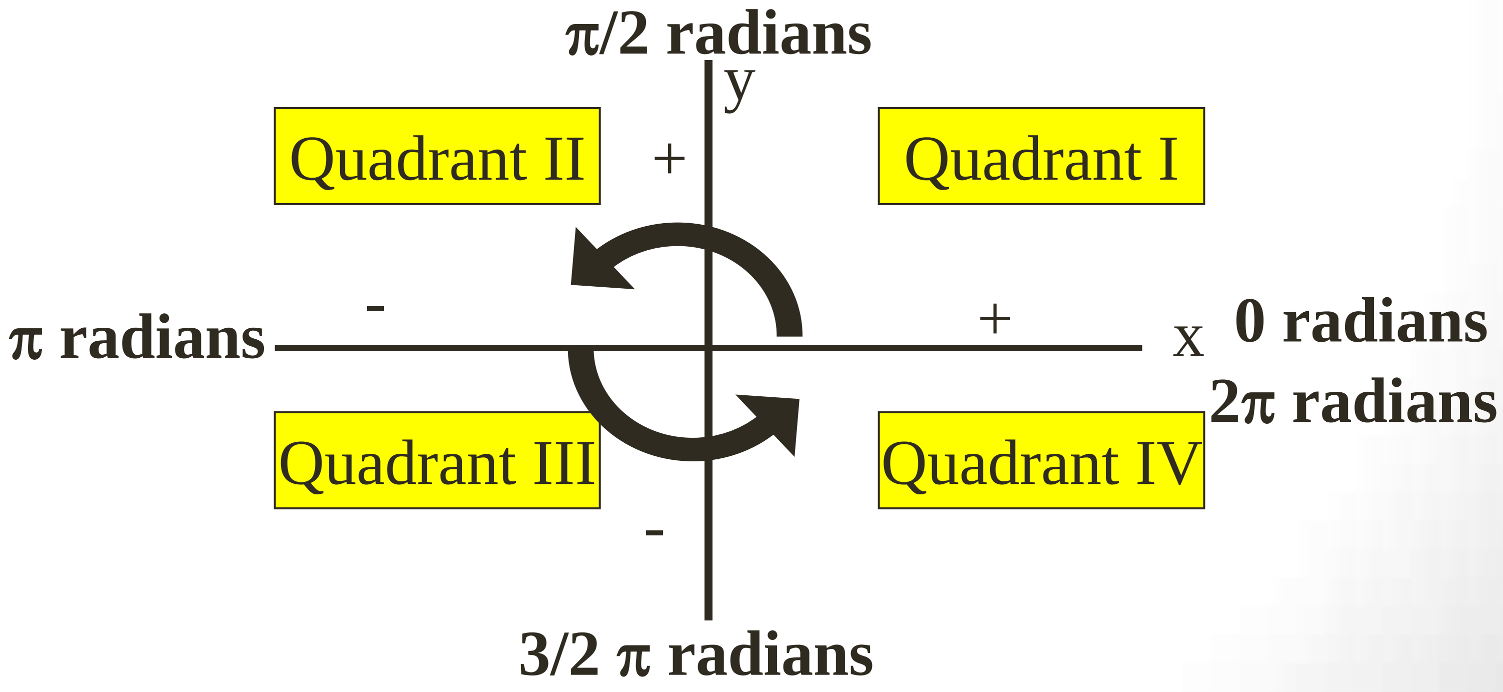
RADIANS = ARC LENGTH / RADIUS LENGTH

CIRCUMFERENCE OF A CIRCLE = 2 π x RADIUS

RADIANS IN A CIRCLE = 2 π R / R

1 CIRCLE = 2 π RADIANS = 360^O

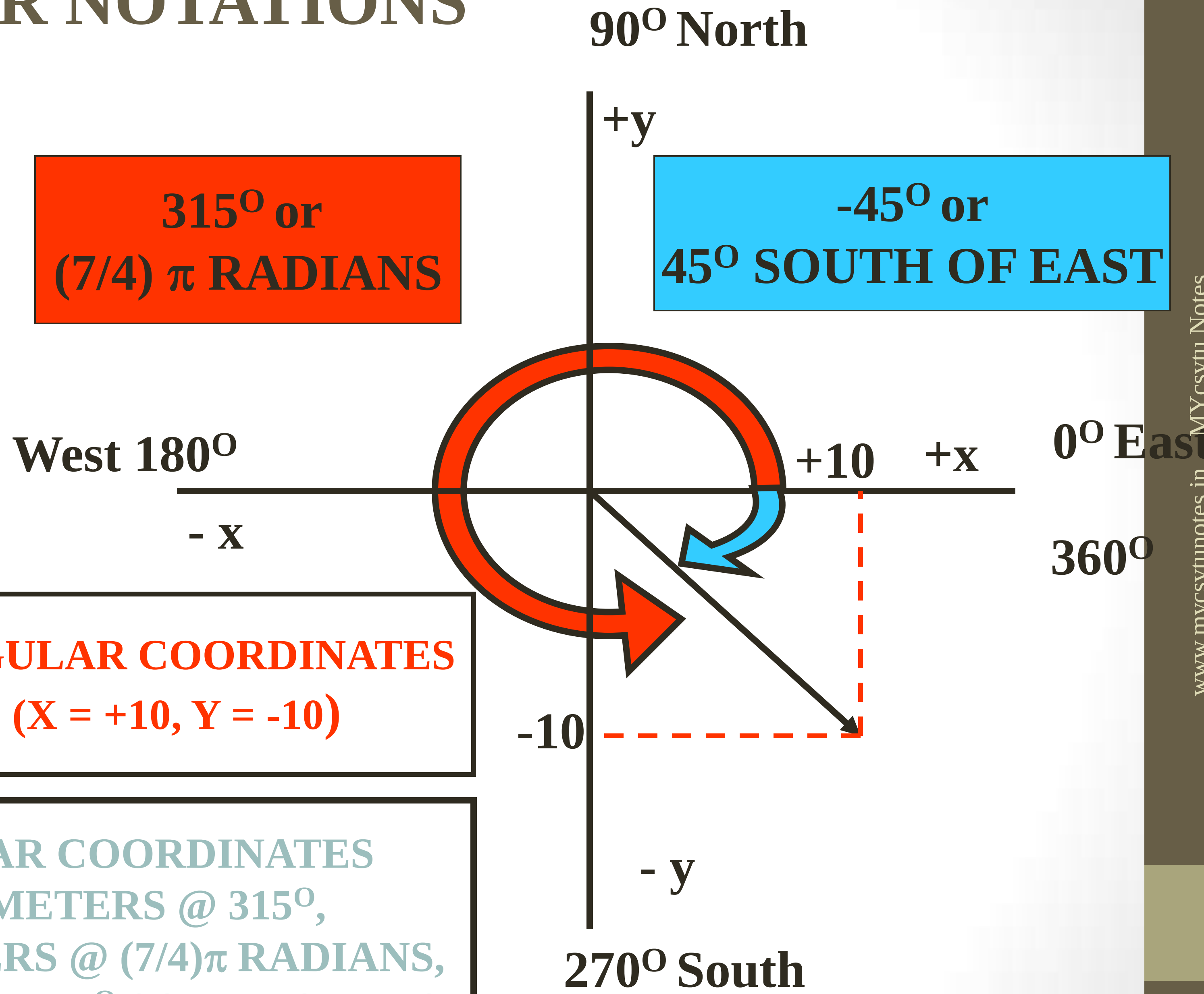
1 RADIAN = 360^O / 2 π = 57.3^O



VECTOR NOTATIONS

- **VECTOR NOTATION MAY TAKE SEVERAL DIFFERENT FORMS:**
- **<u>POLAR FORM</u> INDICATES A MAGNITUDE VALUE AND A DIRECTIONAL VALUE. THE DIRECTION VALUE MAY BE IN DEGREES, RADIANS OR GEOGRAPHIC TERMS.**
- **EXAMPLES: 14.1 METERS @ 315^O, 14.1 METERS @ $(7/4)\pi$ RADIANS, 14.1 FEET AT 45^O SOUTH OF EAST**
- **<u>RECTANGULAR FORM</u> IDENTIFIES THE X-Y COORDINATES OF THE VECTOR. THE VECTOR ITSELF EXTENDS FROM ORIGIN TO THE X-Y POINT.**
- **EXAMPLES: 10, -10 (X = +10, Y = -10) THE MAGNITUDE OF THE VECTOR CAN BE FOUND USING THE PYTHAGOREAN THEOREM $(10^2 + (-10^2))^{1/2} = 14.1$**
- **THE DIRECTION CAN BE FOUND USING AN INVERSE TANGENT FUNCTION TAN^{-1} (10/10) = TAN^{-1} (1.0) = 45^O SINCE X IS POSITIVE AND Y IS NEGATIVE THE ANGLE IS -45^O AND IS IN QUADRANT IV OR 315^O**

VECTOR NOTATIONS



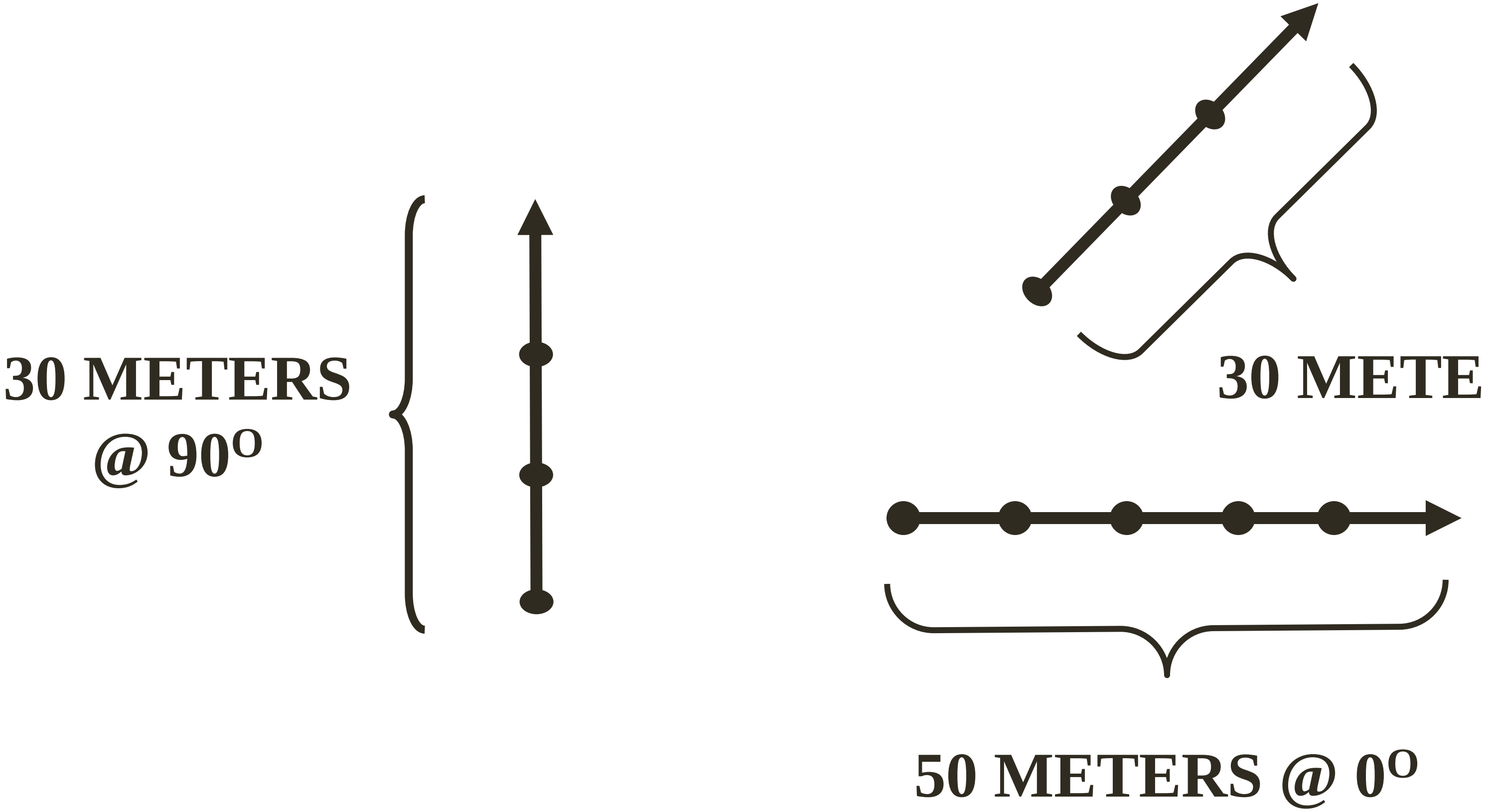
- **RRECTANGULAR COORDINATES**
  10, -10 (X = +10, Y = -10)

- **PPOLAR COORDINATES**
  14.1 METERS @ 315°,
  14.1 METERS @ (7/4)π RADIANS,
  14.1 FEET AT 45° SOUTH OF EAST

WORKING WITH VECTORS

- VECTORS CAN BE ADDED OR SUBTRACTED HOWEVER NOT IN THE USUAL ARITHEMATIC MANNER. THE DIRECTIONAL COMPONENTS AS WELL AS THE MAGNITUDE COMPONENTS MUST EACH BE CONSIDERED.
- THE ADDITION AND SUBTRACTION OF VECTORS CAN BE ACCOMPLISHED USED GRAPHIC METHODS (DRAWING) OR COMPONENT METHODS (MATHEMATICAL).
- GRAPHICAL ADDITION AND SUBTRACTION REQUIRES THAT EACH VECTOR BE REPRESENTED AS AN ARROW WITH A LENGTH PROPORTIONAL TO THE MAGNITUDE VALUE AND POINTED IN THE PROPER DIRECTION ASSIGNED TO THE VECTOR.

GRAPHIC REPRESENTATION OF VECTORS



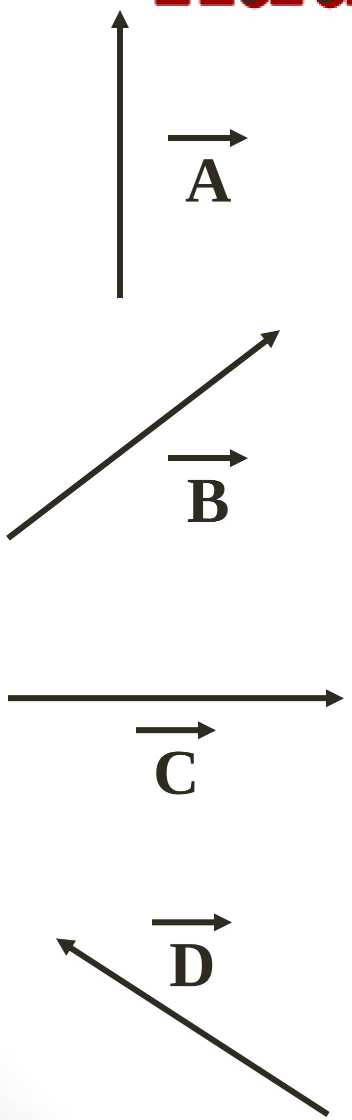
VECTOR ARROWS MAY BE DRAWN ANYWHERE ON THE PAGE AS LONG AS THE PROPER LENGTH AND DIRECTION ARE MAINTAINED

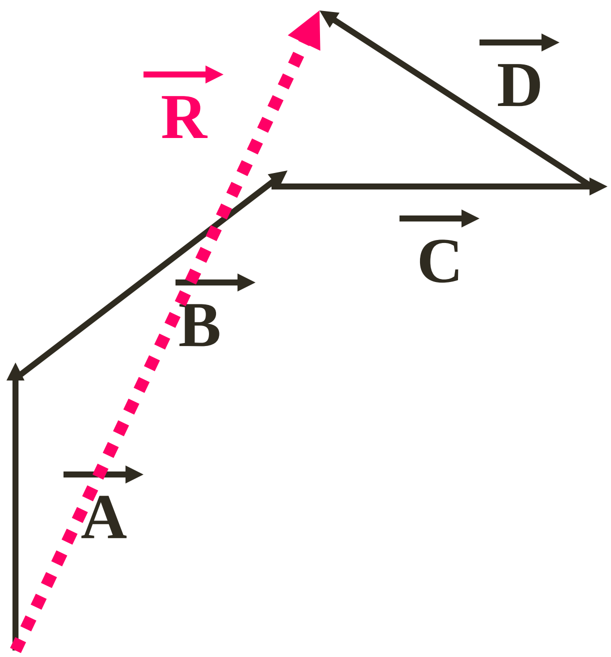
WORKING WITH VECTORS GRAPHIC ADDITION

- **VECTORS ARE ADDED GRAPHICALLY BY DRAWING EACH VECTOR TO SCALE AND ORIENTED IN THE PROPER DIRECTION. THE VECTOR ARROWS ARE PLACED HEAD TO TAIL. THE ORDER OF PLACEMENT DOES NOT AFFECT THE RESULT (VECTOR A + VECTOR B = VECTOR B + VECTOR A)**
- **THE RESULT OF THE VECTOR ADDITION IS CALLED THE RESULTANT. IT IS MEASURED FROM THE TAIL OF THE FIRST VECTOR ARROW TO THE HEAD OF THE LAST ADDED VECTOR ARROW.**
- **THE LENGTH OF THE RESULTANT VECTOR ARROW CAN THEN BE MEASURED AND USING THE SCALE FACTOR CONVERTED TO THE CORRECT MAGNITUDE VALUE. THE DIRECTIONAL COMPONENT CAN BE MEASURED USING A PROTRACTOR.**

Adding Vectors

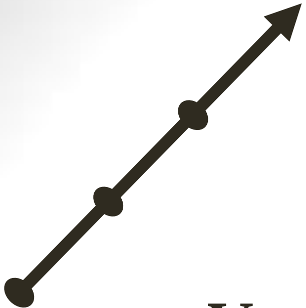
ALL VECTORS MUST BE DRAWN TO SCALE & POINTED IN THE PROPER DIRECTION





$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$$

Drawing Vectors to Scale



Vector A
30 METERS @ 45°

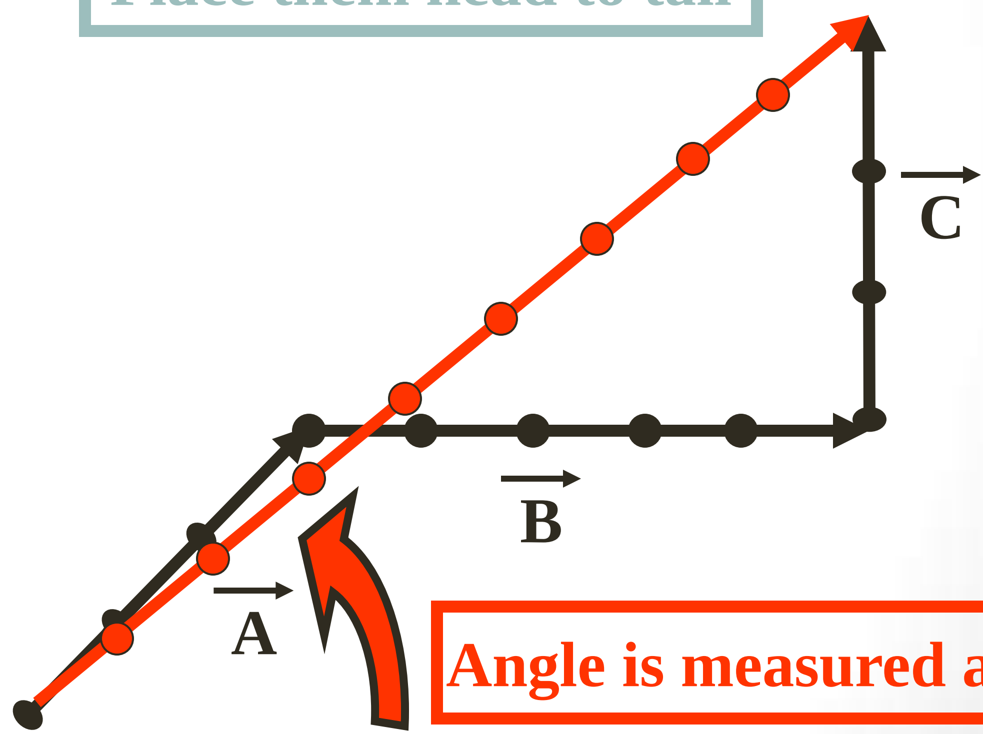
Vector B
50 METERS @ 0°

Vector C
30 METERS
@ 90°


= 10 METERS

SCALE

To add the vectors
Place them head to tail

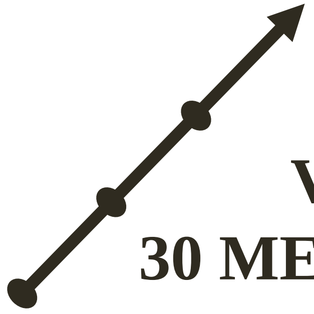


Angle is measured at 40°

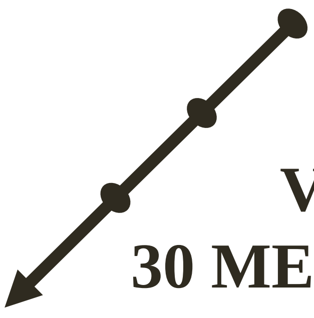
Resultant = 9 x 10 = 90 meters

WORKING WITH VECTORS GRAPHIC SUBTRACTION

- **IN ALGEBRA, A – B = A + (-B) OR IN OTHER WORDS, ADDING A NEGATIVE VALUE IS ACTUALLY SUBTRACTION. THIS IS ALSO TRUE IN VECTOR SUBTRACTION. IF WE ADD A NEGATIVE VECTOR B TO VECTOR A THIS IS REALLY SUBTRACTING VECTOR B FROM VECTOR A.**
- **VECTOR VALUES CAN BE MADE NEGATIVE BY REVERSING THE VECTOR'S DIRECTION BY 180 DEGREES. IF VECTOR A IS 30 METERS DIRECTED AT 45 DEGREES (QUADRANT I), NEGATIVE VECTOR A IS 30 METERS AT 225 DEGREES (QUADRANT II).**

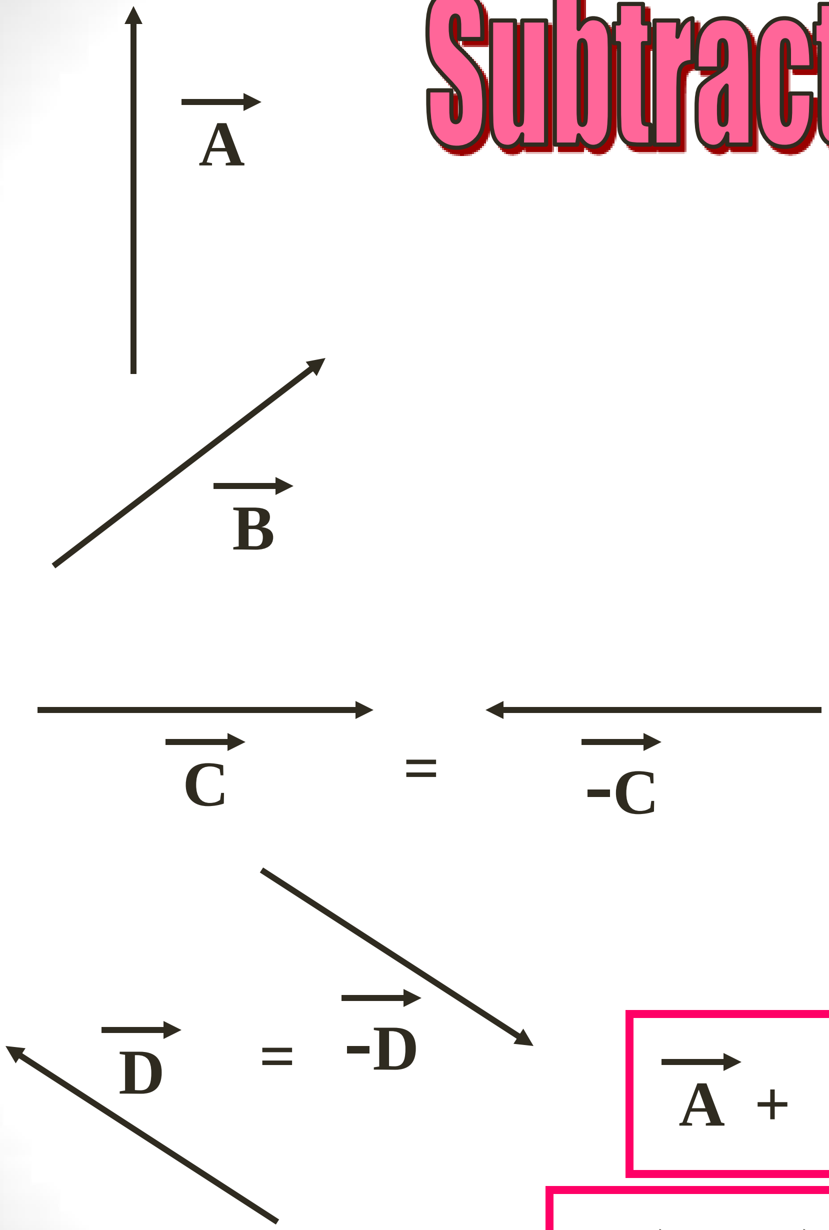


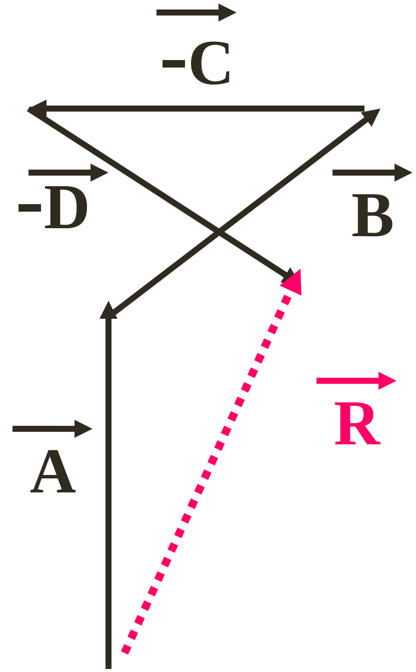
Vector A
30 METERS @ 45^{O}



Vector - A
30 METERS @ 225^{O}

Subtracting Vectors





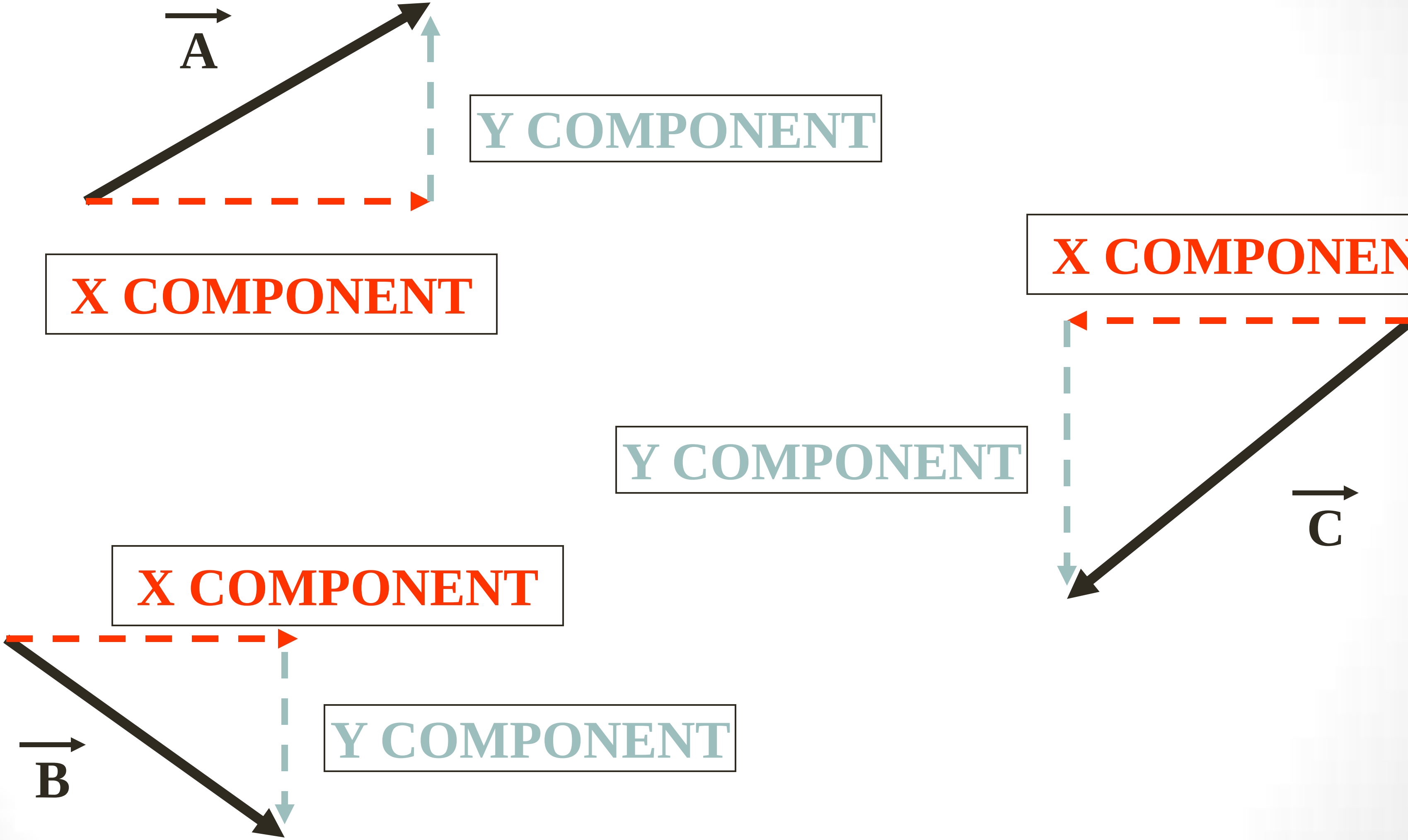
$$\vec{A} + \vec{B} - \vec{C} - \vec{D} = \vec{R}$$

$$\vec{A} + \vec{B} + (-\vec{C}) + (-\vec{D}) = \vec{R}$$

VECTOR COMPONENTS

- **AS WE HAVE SEEN TWO OR MORE VECTORS CAN BE ADDED TOGETHER TO GIVE A NEW VECTOR. THEREFORE, ANY VECTOR CAN CONSIDERED TO BE THE SUM OF TWO OR MORE OTHER VECTORS.**
- **WHEN A VECTOR IS RESOLVED (MADE) INTO COMPONENTS TWO COMPONENT VECTORS ARE CONSIDERED, ONE LYING IN THE X AXIS PLANE AND THE OTHER LYING IN THE Y AXIS PLANE. THE COMPONENT VECTORS ARE THUS AT RIGHT ANGLES TO EACHOTHER.**
- **THE X-Y AXIS COMPONENTS ARE CHOSEN SO THAT RIGHT TRIANGLE TRIGONOMETRY AND THE PYTHAGOREAN THEOREM CAN BE USED IN THEIR CALCULATION.**

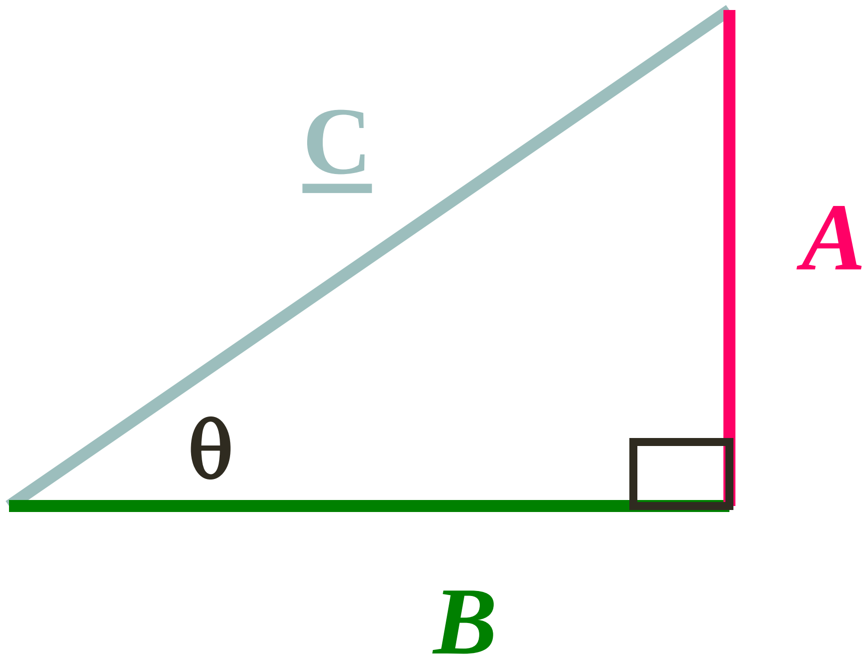
Vector Components



VECTOR COMPONENTS

- **VECTOR COMPONENTS CAN BE FOUND MATHEMATICALLY USING SINE AND COSINE FUNCTIONS. RECALL SINE OF AN ANGLE FOR A RIGHT TRIANGLE IS THE SIDE OPPOSITE THE ANGLE DIVIDED BY THE HYPOTENUSE OF THE TRIANGLE AND THE COSINE IS THE SIDE ADJACENT TO THE ANGLE DIVIDED BY THE HYPOTENUSE.**
- **USING THESE FACTS, THE X COMPONENT OF THE VECTOR IS CALCULATED BY MULTIPLYING THE COSINE OF THE ANGLE BY THE VECTOR VALUE AND THE Y COMPONENT IS CALCULATED BY MULTIPLYING THE SINE OF THE ANGLE BY THE VECTOR VALUE. ANGULAR VALUES ARE MEASURED FROM 0 DEGREES (DUE EAST OR POSITIVE X) ON THE CARTISIAN COORDINATE SYSTEM.**

Fundamental Trigonometry



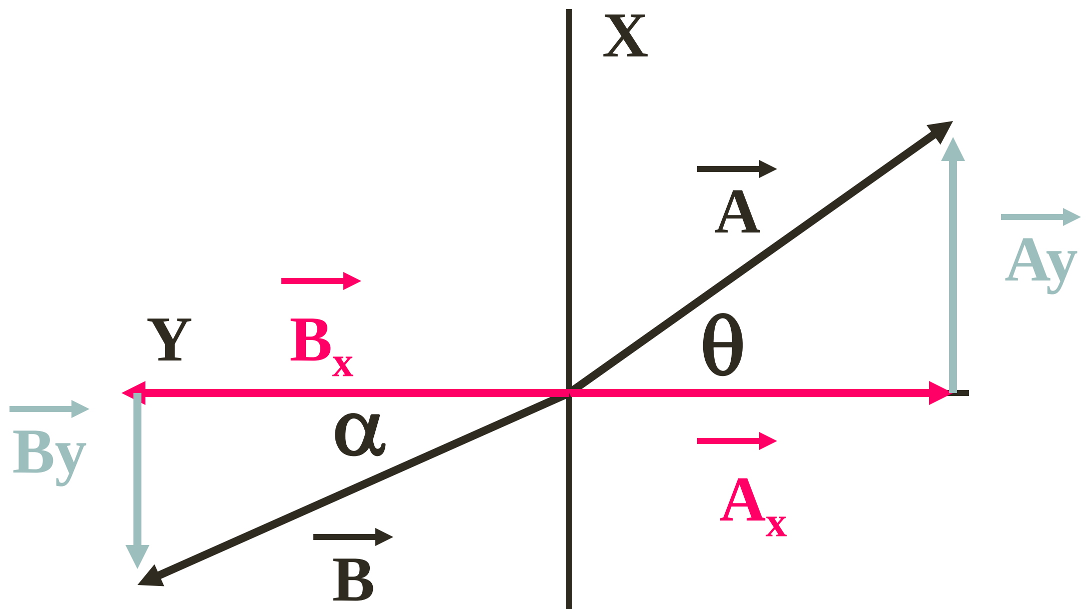
A RIGHT TRIANGLE

Sin θ = A / C

Cos θ = B / C

Tan θ = A / B

Vector Components



$$\vec{B}_x = \vec{B}\ \cos\ \alpha$$

$$\vec{B}_y = \vec{B}\ \sin\ \alpha$$

$$\vec{A}_x = \vec{A}\ \cos\ \theta$$

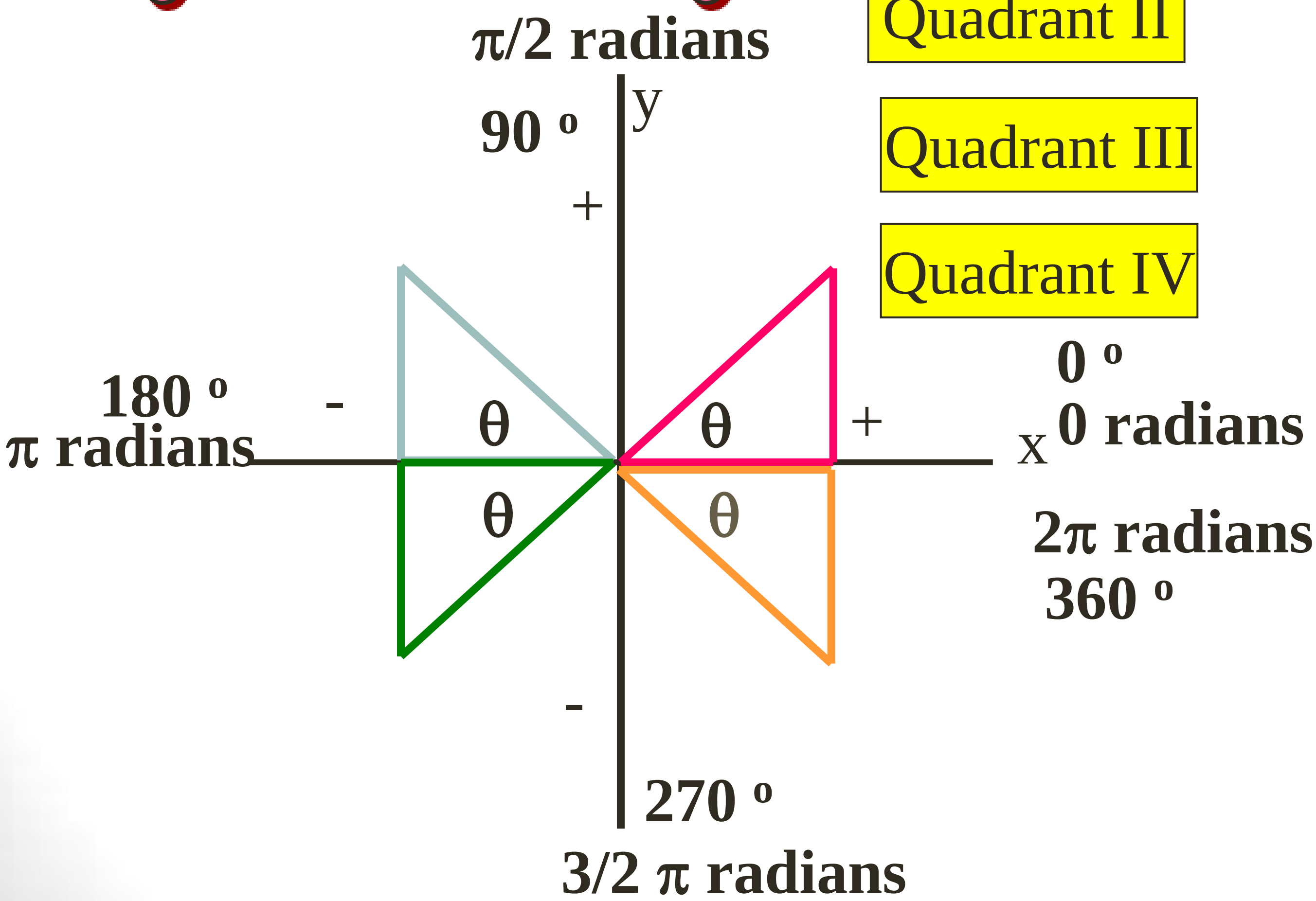
$$\vec{A}_y = \vec{A}\ \sin\ \theta$$

VECTOR COMPONENTS

- THE SIGNS OF THE X AND Y COMPONENTS DEPEND ON WHICH QUADRANT THE VECTOR LIES.
- VECTORS IN QUADRANT I (0 TO 90 DEGREES) HAVE POSITIVE X AND POSITIVE Y VALUES
- VECTORS IN QUADRANT II (90 TO 180 DEGREES) HAVE NEGATIVE X VALUES AND POSITIVE Y VALUES.
- VECTORS IN QUADRANT III (180 TO 270 DEGREES) HAVE NEGATIVE X VALUES AND NEGATIVE Y VALUES.
- VECTORS IN QUADRANT IV (270 TO 360 DEGREES) HAVE POSITIVE X VALUES AND NEGATIVE Y VALUES.

Trig Function Signs

	Sin θ	Cos θ	Tan θ
Quadrant I	+	+	+
Quadrant II	+	-	-
Quadrant III	-	-	+
Quadrant IV	-	+	-



VECTOR COMPONENTS

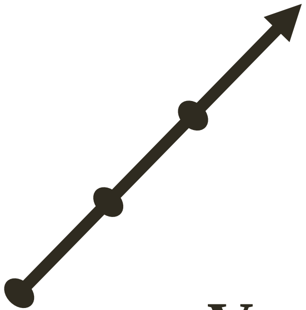
$\vec{A}_x = \vec{A} \cos\theta$

$\vec{A}_y = \vec{A} \sin\theta$

- WHAT ARE THE X AND Y COMPONENTS OF A VECTOR 40 METERS @ 60^O ?
- A_X = 40 METERS x COS 60^0 = 20 METERS
- A_Y = 40 METERS x SIN 60^0 = 34.6 METERS

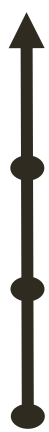
- WHAT ARE THE X AND Y COMPONENTS OF A VECTOR 60 METERS PER SECOND @ 245^0 ?
- B_X = 60 M/SEC x COS 245 ^{0} = - 25.4 M/SEC
- B_Y = 60 M/SEC x SIN 245 ^{0} = - 54.4 M/SEC

ADDING & SUBTRACTING VECTORS USING COMPONEN



Vector A
30 METERS @ 45^O

Vector B
50 METERS @ 0^O



Vector C
30 METERS
@ 90^O

ADD THE FOLLOWING THREE VECTORS USING COMPONENTS

(1) RESOLVE EACH INTO X AND Y COMPONENTS

$$\vec{V_y} = \vec{V}\ \text{SIN}\ \theta$$

$$\vec{V_x} = \vec{V}\ \text{COS}\ \theta$$

ADDING & SUBTRACTING VECTORS USING COMPONENTS

- A_X = 30 METERS x COS 45^0 = 21.2 METERS
- A_Y = 30 METERS x SIN 45^0 = 21.2 METERS

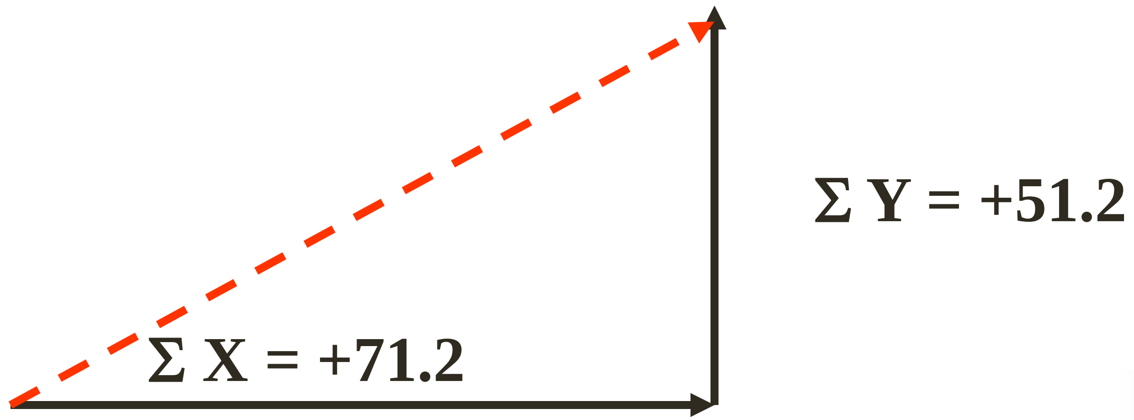
- B_X = 50 METERS x COS 0^0 = 50 METERS
- B_Y = 50 METERS x SIN 0^0 = 0 METERS

- C_X = 30 METERS x COS 90^0 = 0 METERS
- C_Y = 30 METERS x SIN 90^0 = 30 METERS

(2) ADD THE X COMPONENTS OF EACH VECTOR
ADD THE Y COMPONENTS OF EACH VECTOR

Σ X = SUM OF THE Xs = 21.2 + 50 + 0 = +71.2
Σ Y =SUM OF THE Ys = 21.2 + 0 + 30 = +51.2

(3) CONSTUCT A NEW RIGHT TRIANGLE USING THE Σ X AS THE BASE AND Σ Y AS THE OPPOSITE SIDE



THE HYPOTENUSE IS THE RESULTANT VECTOR

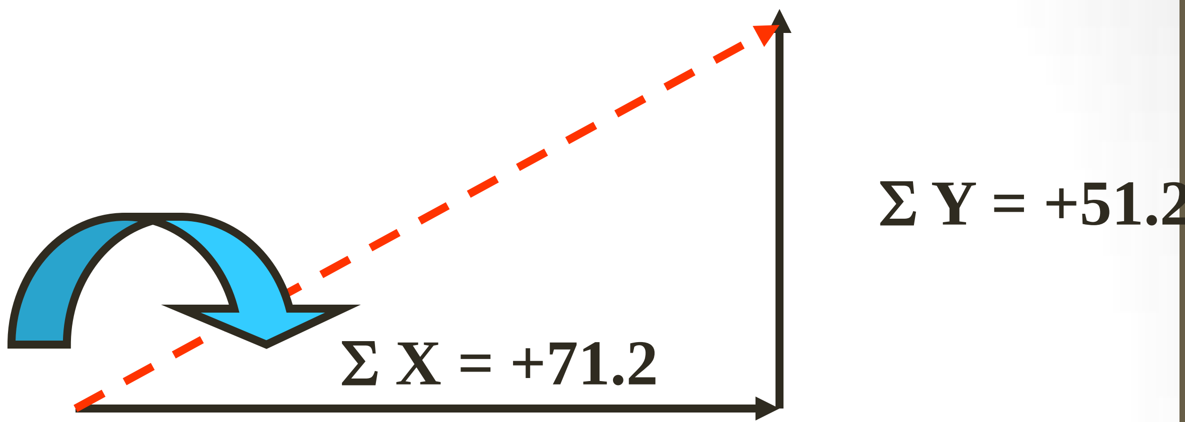
(4) USE THE PYTHAGOREAN THEOREM TO THE LENGTH (MAGNITUDE) OF THE RESULTANT VECTOR

ANGLE

TAN^{-1} (51.2/71.2)

ANGLE = 35.7 ^{O}

QUADRANT I



$$\sqrt{(+71.2)^2 + (+51.2)^2} = 87.7$$

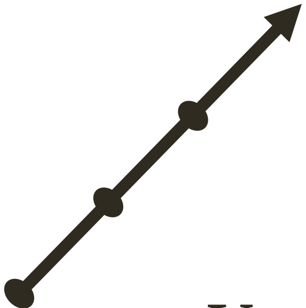
(5) FIND THE ANGLE (DIRECTION) USING INVERSE TANGENT OF THE OPPOSITE SIDE OVER THE ADJACENT SIDE

RESULTANT = 87.7 METERS @ 35.7 ^{O}

SUBTRACTING VECTORS USING COMPONENTS

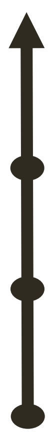
$$\vec{A} - \vec{B} + \vec{C} = \vec{R}$$

$$\vec{A} + (-\vec{B}) + \vec{C} = \vec{R}$$



Vector A
30 METERS @ 45^{O}

Vector B
50 METERS @ 0^{O}



Vector C
30 METERS
@ 90^{O}

Vector A **30 METERS @ 45^{O}**

- Vector B **50 METERS @ 180^{O}**

Vector C **30 METERS @ 90^{O}**

(1) RESOLVE EACH INTO X AND Y COMPONENTS

- A_X = 30 METERS x COS 45^0 = 21.2 METERS
- A_Y = 30 METERS x SIN 45^0 = 21.2 METERS

•B_X = 50 METERS x COS 180^0 = - 50 METERS

•B_Y = 50 METERS x SIN 180^0 = 0 METERS

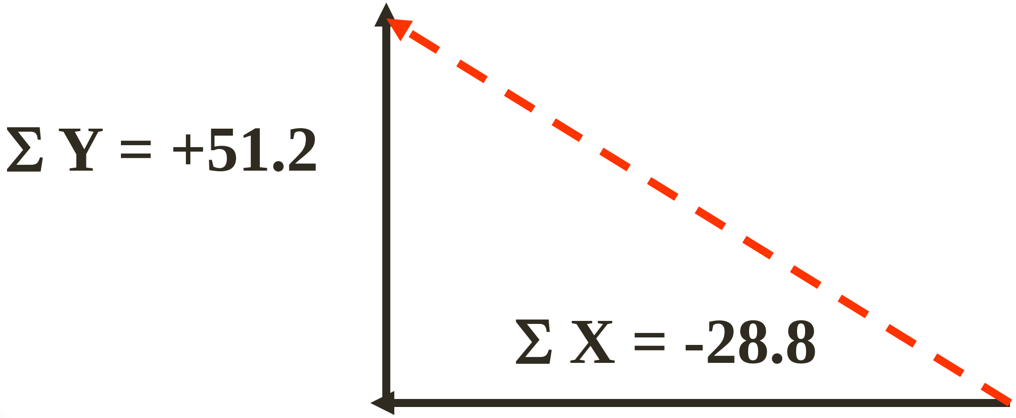
•C_X = 30 METERS x COS 90^0 = 0 METERS

•C_Y = 30 METERS x SIN 90^0 = 30 METERS

(2) ADD THE X COMPONENTS OF EACH VECTOR
ADD THE Y COMPONENTS OF EACH VECTOR

ΣX = SUM OF THE Xs = 21.2 + (-50) + 0 = -28.8
ΣY =SUM OF THE Ys = 21.2 + 0 + 30 = +51.2

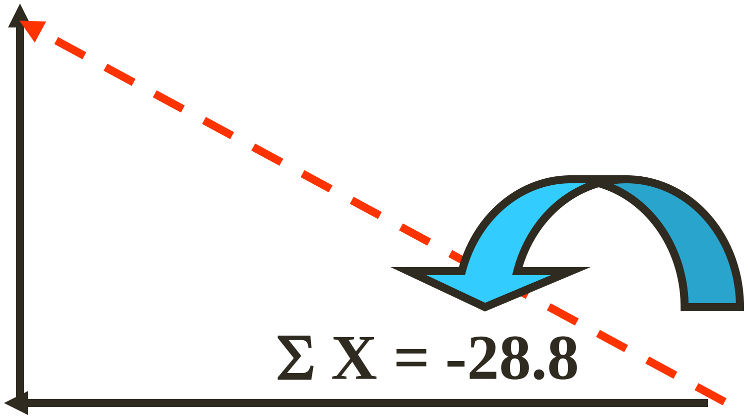
(3) CONSTUCT A NEW RIGHT TRIANGLE USING THE ΣX AS THE BASE AND ΣY AS THE OPPOSITE SIDE



THE HYPOTENUSE IS THE RESULTANT VECTOR

(4) USE THE PYTHAGOREAN THEOREM TO THE LENGTH (MAGNITUDE) OF THE RESULTANT VECTOR

$\Sigma Y = +51.2$



ANGLE

TAN^{-1} **(51.2/-28.8)**

ANGLE = -60.6 ^{0}

$(180^{0} - 60.6^{0}) = 119.4^{0}$

QUADRANT II

$$\sqrt{(-28.8)^2 + (+51.2)^2} = 58.7$$

(5) FIND THE ANGLE (DIRECTION) USING INVERSE TANGENT OF THE OPPOSITE SIDE OVER THE ADJACENT SIDE

RESULTANT = 58.7 METERS @ 119.4^{O}

the end