Extended Surfaces/Fins

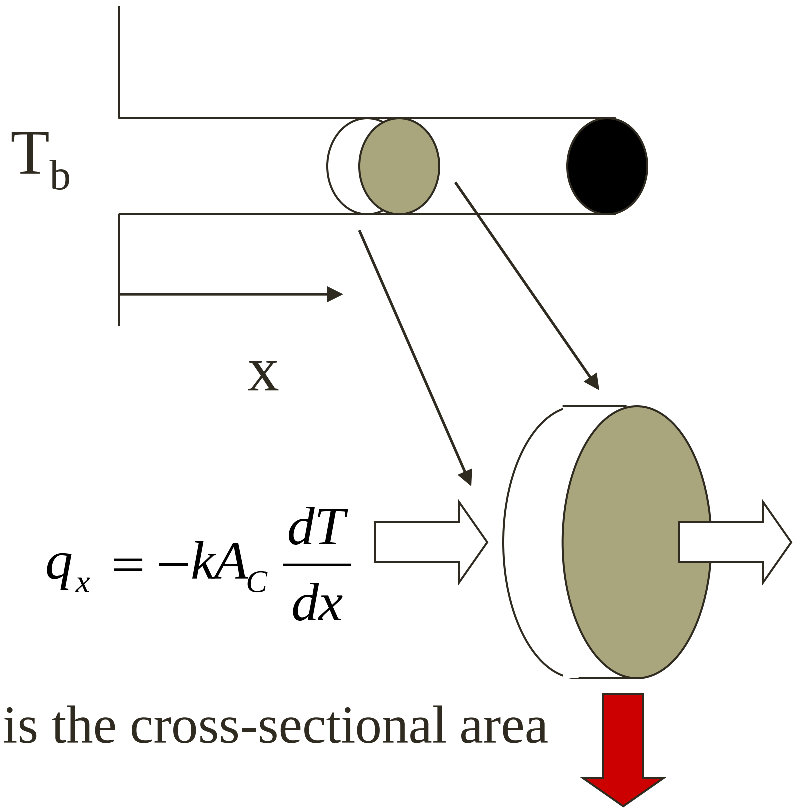
Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(T_s - T_\infty)$. Therefore, to increase the convective heat transfer, one can

- Increase the temperature difference $(T_s - T_\infty)$ between the surface and the fluid.

- Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h. Example: a cooling fan.

- Increase the contact surface area A. Example: a heat sink with fins.

Extended Surface Analysis



P: the fin perimeter

A_c: the fin cross-sectional area

$$q_x = -kA_C \frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx$$

A_C is the cross-sectional area

$dq_{conv} = h(dA_S)(T - T_\infty)$, where dA_S is the surface area of the element

Energy Balance: $q_x = q_{x+dx} + dq_{conv} = q_x + \frac{dq_x}{dx}dx + hdA_S(T - T_\infty)$

$-kA_C \frac{d^2T}{dx^2}dx + hP(T - T_\infty)dx = 0$, if k, A_C are all constants.

Extended Surface Analysis (cont.)

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0, \text{ A second-order, ordinary differential equation}$$

Define a new variable $\theta(x) = T(x) - T_\infty$, so that

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \text{ where } m^2 = \frac{hP}{kA_C}, \ (D^2 - m^2)\theta = 0$$

Characteristics equation with two real roots: +m & -m

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To evaluate the two constants C_1 and C_2, we need to specify two boundary conditions:

The first one is obvious: the base temperature is known as $T(0) = T_b$

The second condition will depend on the end condition of the tip

Extended Surface Analysis (cont.)

For example: assume the tip is insulated and no heat transfer $d\theta/dx(x=L)=0$

The temperature distribution is given by

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

The fin heat transfer rate is

$$q_f = -kA_C \frac{dT}{dx}(x=0) = \sqrt{hPkA_C}\tanh mL = M\tanh mL$$

These results and other solutions using different end conditions are tabulated in the following fins table

Temperature distribution for fins of different configurations

Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\frac{(\theta_L/\theta_b)\sinh m(L-x)+\sinh m(L-x)}{\sinh mL}$	$M\frac{(\cosh mL-\theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	M

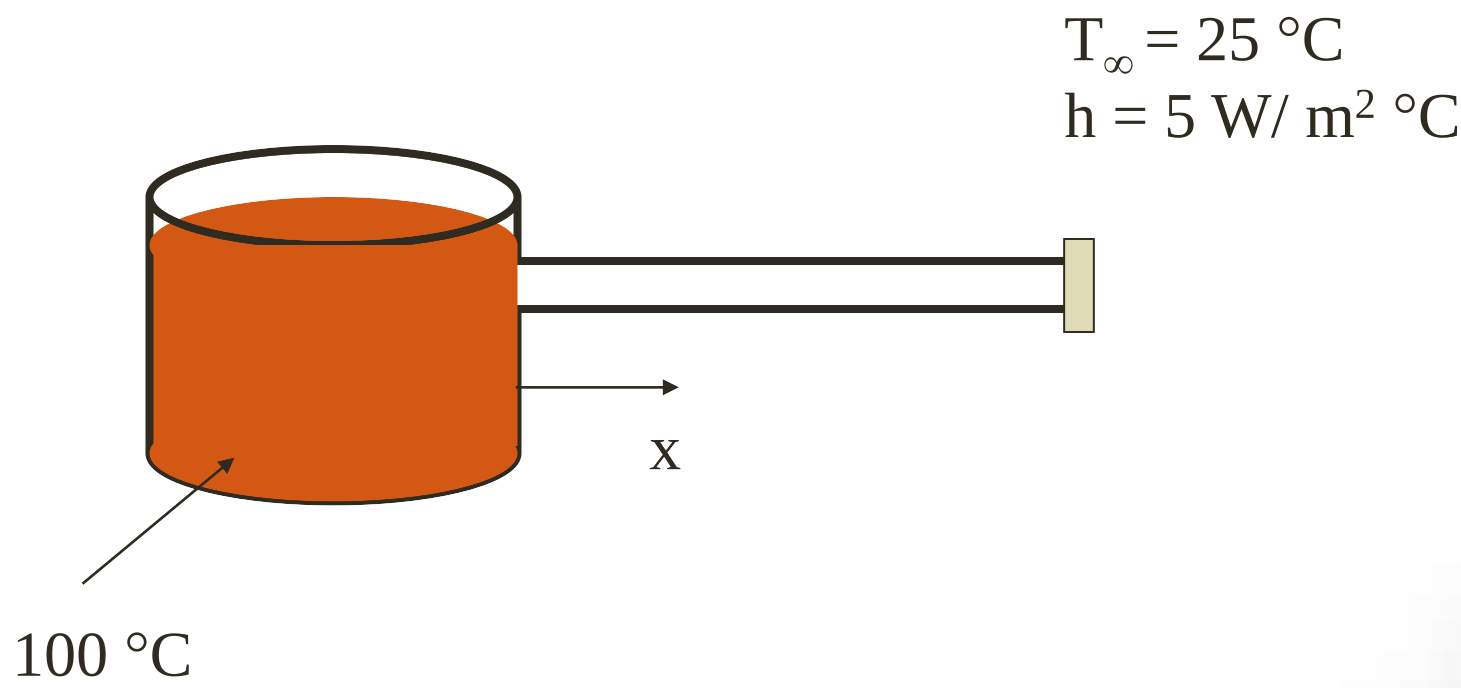
$$\theta \equiv T-T_\infty, \quad m^2 \equiv \frac{hP}{kA_C}$$

$$\theta_b = \theta(0) = T_b - T_\infty, \quad M = \sqrt{hPkA_C}\,\theta_b$$

Note: This table is adopted from *Introduction to Heat Transfer* by Frank Incropera and David DeWitt

Example

An Aluminum pot is used to boil water as shown below. The handle of the pot is 20-cm long, 3-cm wide, and 0.5-cm thick. The pot is exposed to room air at 25°C, and the convection coefficient is 5 W/m^2 °C. Question: can you touch the handle when the water is boiling? (k for aluminum is 237 W/m °C)



Example (cont.)

We can model the pot handle as an extended surface. Assume that there is no heat transfer at the free end of the handle. The condition matches that specified in the fins Table, case B.

h=5 W/ m^2 °C, P=2W+2t=2(0.03+0.005)=0.07(m), k=237 W/m °C, A_C=Wt=0.00015(m^2), L=0.2(m)

Therefore, $m=(hP/kA_C)^{1/2}=3.138$,

$M=\sqrt{(hPkA_C)}(T_b-T_\infty)=0.111\theta_b=0.111(100-25)=8.325(W)$

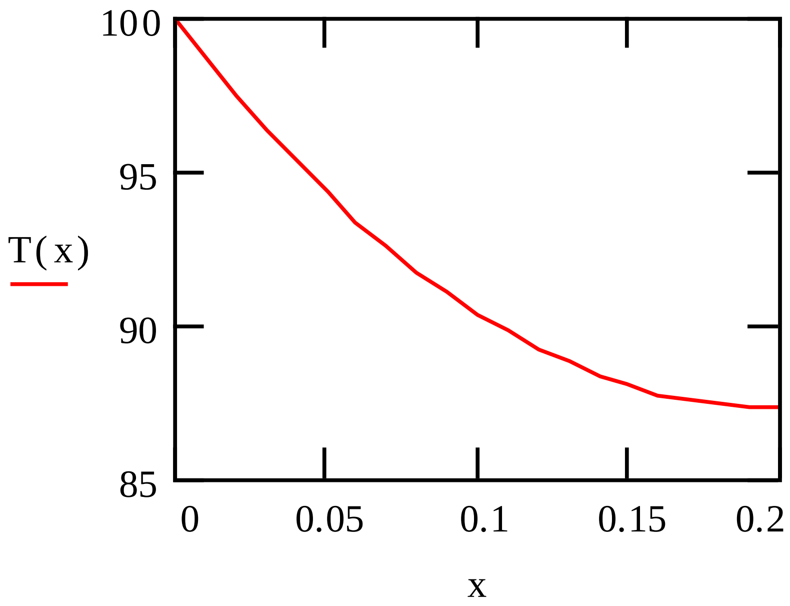
$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$$

$$\frac{T-25}{100-25}=\frac{\cosh[3.138(0.2-x)]}{\cosh(3.138*0.2)},$$

$$T(x)=25+62.32*\cosh[3.138(0.2-x)]$$

Example (cont.)

Plot the temperature distribution along the pot handle



As shown, temperature drops off very quickly. At the midpoint T(0.1)=90.4°C. At the end T(0.2)=87.3°C.
Therefore, it should not be safe to touch the end of the handle

Example (cont.)

The total heat transfer through the handle can be calculated also. q_f=Mtanh(mL)=8.325*tanh(3.138*0.2)=4.632(W) Very small amount: latent heat of evaporation for water: 2257 kJ/kg. Therefore, the amount of heat loss is just enough to vaporize 0.007 kg of water in one hour.

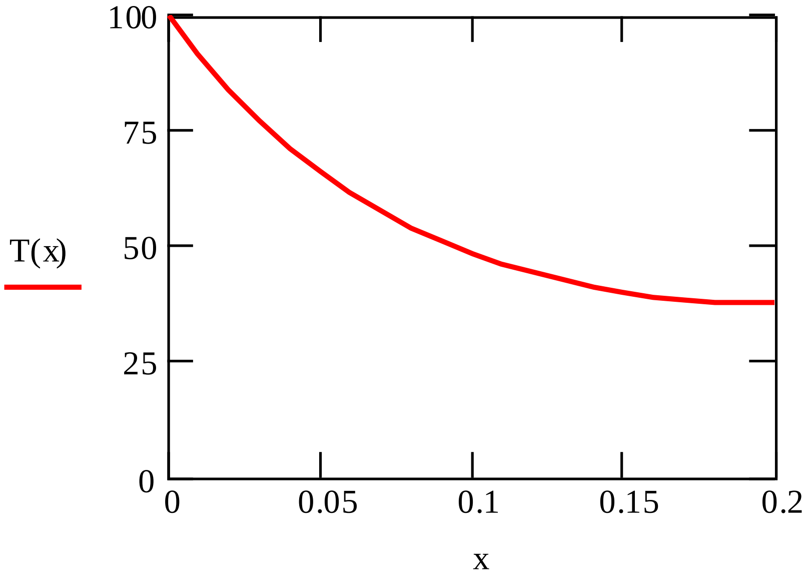
If a stainless steel handle is used instead, what will happen: For a stainless steel, the thermal conductivity k=15 W/m°C. Use the same parameter as before:

$$m = \left(\frac{hP}{kA_C} \right)^{1/2} = 12.47, \quad M = \sqrt{hPkA_C} = 0.0281$$

Example (cont.)

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)}{\cosh mL}$$

$$T(x)=25+12.3\cosh[12.47(L-x)]$$



Temperature at the handle (x=0.2 m) is only 37.3 °C, not hot at all. This example illustrates the important role played by the thermal conductivity of the material in terms of conductive heat transfer.

Transient Conduction

Basic Concepts of Transient Conduction

1. Characteristics and Types of Unsteady-State Conduction

- A heat transfer process for which the **temperature varies with time**, as well as location within a solid.

$$t = f(x, y, z, \tau)$$

- It is initiated whenever a system experiences a **change in operating conditions** and proceeds until a new steady state (**thermal equilibrium**) is achieved.

1. Characteristics and Types of Unsteady-State Conduction

It can be induced by changes in:

—surface convection conditions (h, t_∞),

—surface radiation conditions (h_r, t_{sur}),

—a surface temperature or heat flux, and/or

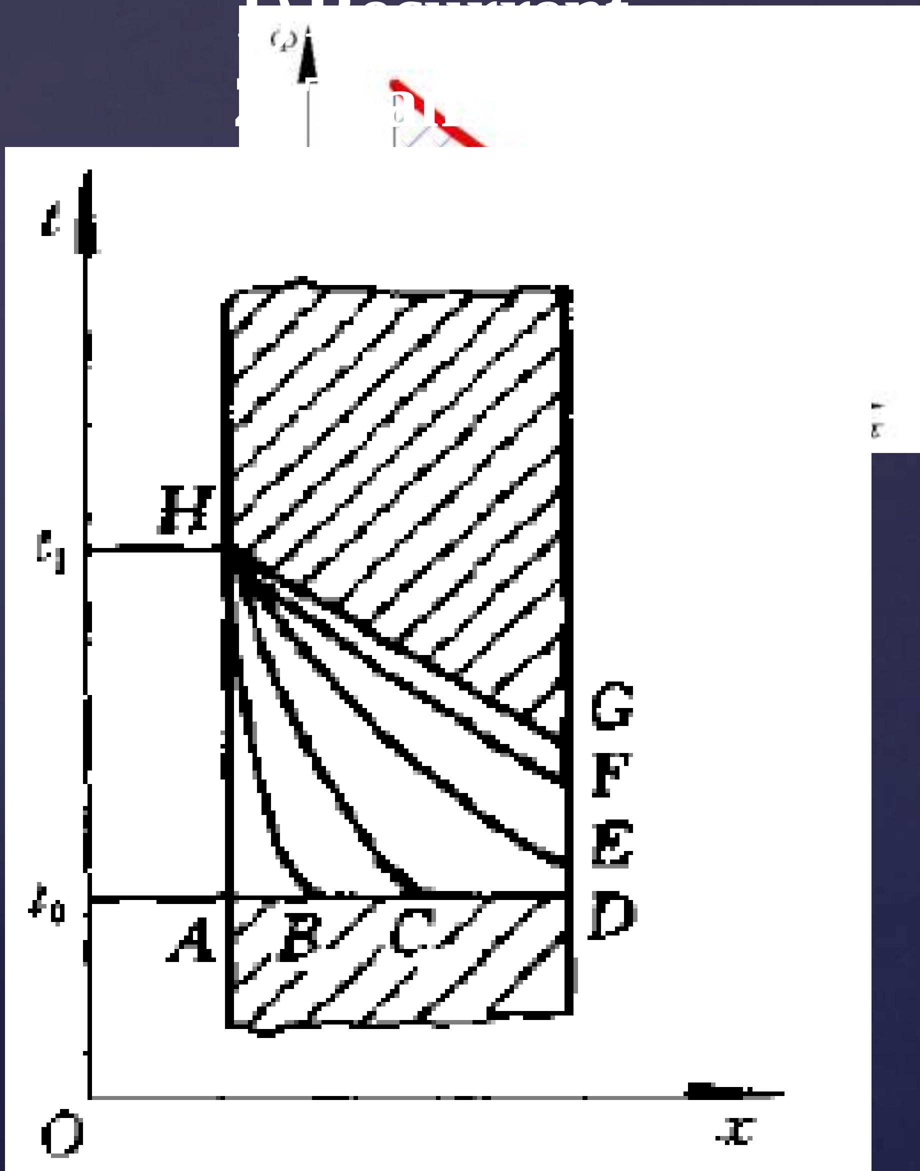
—internal energy generation.

- Solution Techniques

—The **Lumped Capacitance Method**
—**Exact Solutions**
—**The Finite-Difference Method**

- **Types:**

    1) **Recurrent**



- **regime of transient:**

    1) **Non-regular regime**
    2) **Regular regime**

2.The Law of Exclusive Solution of Conductive Differential Equation

- **Based on the assumption of a spatially uniform temperature distribution throughout the transient process.**

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \frac{\dot{\Phi}}{\rho c}$$

$$t(x, y, z, 0) = f(x, y, z)$$

- **The Initial Condition:**

2.The Law of Exclusive Solution of Conductive Differential Equation

- **The Boundary Condition (Convection Condition):**

$$-\lambda\left(\frac{\partial t}{\partial n}\right)_w = h(t_w - t_f)$$

It can be certificated that if a function t (x, y, z, τ) can fit the equation and the conditions above at the same time, it is the exclusive solution of this problem.

3. Influence of Biot Number on Temperature Distribution of Plane Wall Under the 3rd Boundary Condition

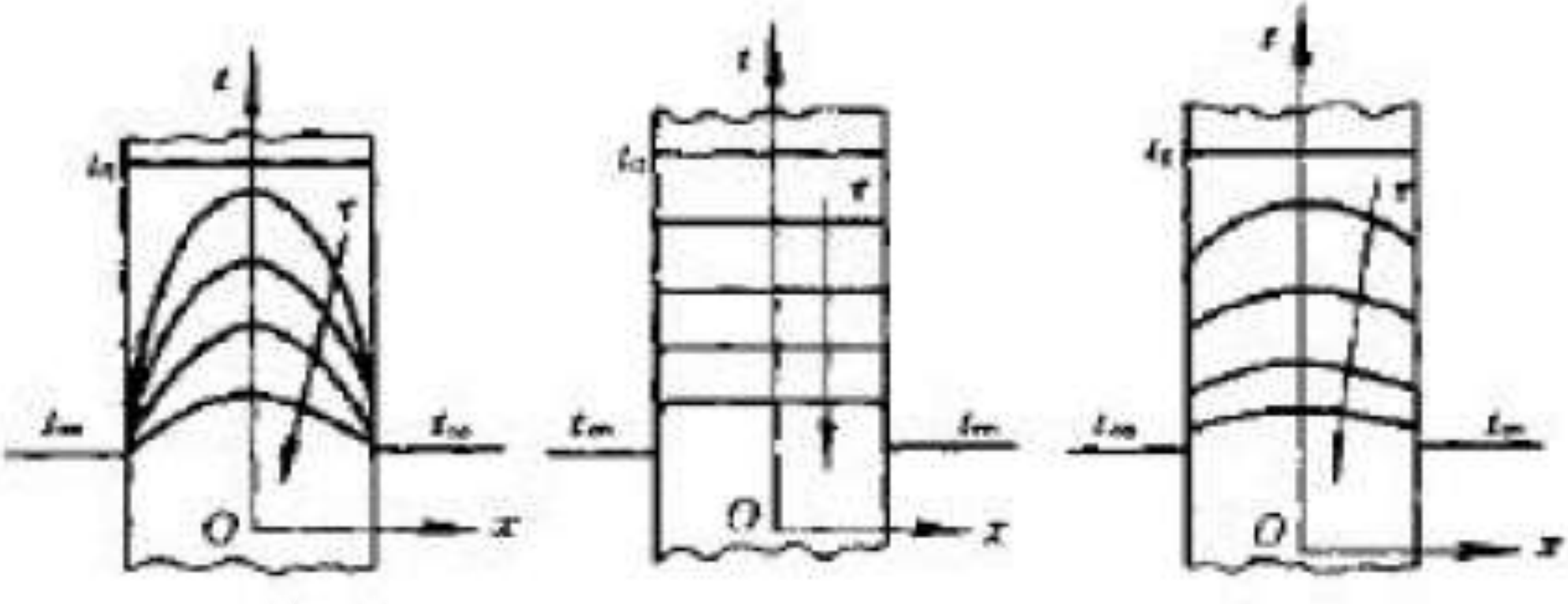
Biot Number, The first of many dimensionless parameters to be considered.

$$Bi \equiv \frac{\delta/\lambda}{1/h} = \frac{\delta h}{\lambda}$$

dimensionless thermal resistance

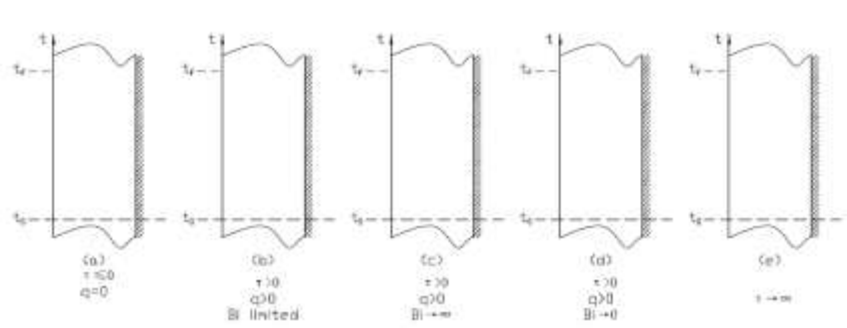
3. Influence of Biot Number on Temperature Distribution of Plane Wall Under the 3rd Boundary Condition

(1) $1/h \ll \delta/\lambda, (Bi \to \infty)$ (2) $1/h \gg \delta/\lambda, (Bi \to 0)$ (3) $1/h \sim \delta/\lambda,$



(a) $Bi \to \infty$ (b) $Bi \to 0$ (c) Bi 为有限大小

Problem: draw out the following temperature distributions according the situations, respectively. λ is constant. The right side of the body is insulated.



3.2 The Lumped Capacitance Method

1. The Lumped Parameter Method

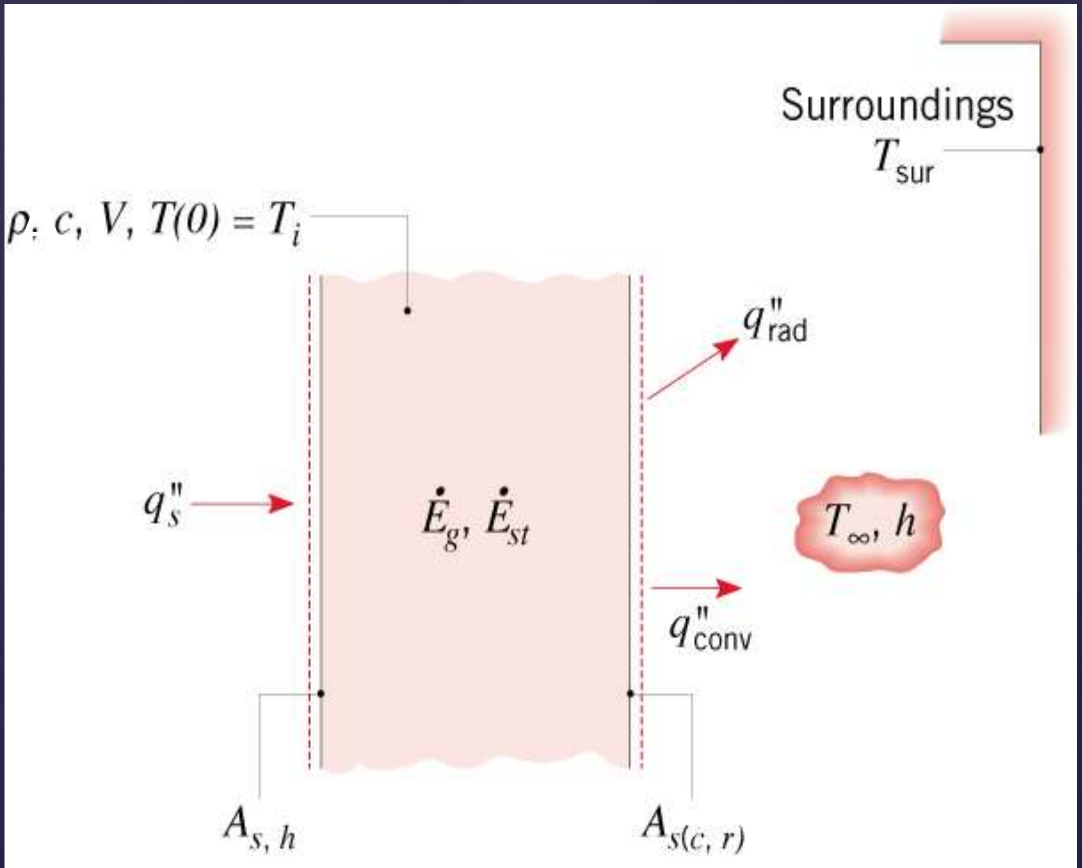
- **Based on the assumption of a spatially uniform temperature distribution throughout the transient process. $t(r, \tau) \approx t(\tau)$.**

- **Why is the assumption never fully realized in practice?**

1. The Lumped Parameter Method

- **General Lumped Capacitance Analysis:**

Consider a general case, which includes convection, radiation and/or an applied heat flux at specified surfaces ($A_{s,c}$, $A_{s,h}$) as well as internal energy generation.



➢ **The Differential Equation of Transient Conduction with Thermal Energy Generation:**

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \frac{\dot{\Phi}}{\rho c} \qquad \Rightarrow \frac{dt}{d\tau} = \frac{\dot{\Phi}}{\rho c}$$

Assuming energy outflow due to combined convection and radiation.

$$-\dot{\Phi} V = Ah(t - t_\infty)$$

$$\rho c V \frac{dt}{d\tau} = -hA(t - t_\infty)$$

$$t(0) = t_0$$

➢ The Differential Equation of Transient Conduction with Thermal Energy Generation:

- Is this expression applicable in situations for which convection and/or radiation provide for energy inflow?

- May h and h_r be assumed to be constant throughout the transient process?

- How must such an equation be solved?

The Excess Temperature:

$$\theta = t - t_{\infty}$$

The Differential Equation:

$$\Rightarrow \frac{d\theta}{\theta} = -\frac{hA}{\rho c V} d\tau$$

Using the Method of Separation of Variables and then integral:

$$\int_{\theta_0}^{\theta} \frac{d\theta}{\theta} = -\int_0^{\tau} \frac{hA}{\rho c V} d\tau$$

$$\frac{\theta}{\theta_0} = \frac{t - t_\infty}{t_0 - t_\infty} = \exp\left(-\frac{hA}{\rho c V}\tau\right)$$

Note the power:

$$\frac{hA}{\rho c V}\tau = \frac{hl_c}{\lambda}\frac{\lambda}{\rho c}\frac{\tau}{l_c^2} = \frac{hl_c}{\lambda}\frac{a\tau}{l_c^2} = Bi \cdot Fo$$

$$\frac{\theta}{\theta_0} = \exp(-Bi \cdot Fo)$$

2. Heat Rate, Time Constant, and the Fourier Number

Heat Rate

$$\Phi = -\rho c V \frac{dt}{d\tau} = -\rho c V (t_0 - t_\infty)\left(-\frac{hA}{\rho c V}\right)\exp\left(-\frac{hA}{\rho c V}\tau\right)$$
$$= (t_0 - t_\infty) hA \exp\left(-\frac{hA}{\rho c V}\tau\right)$$

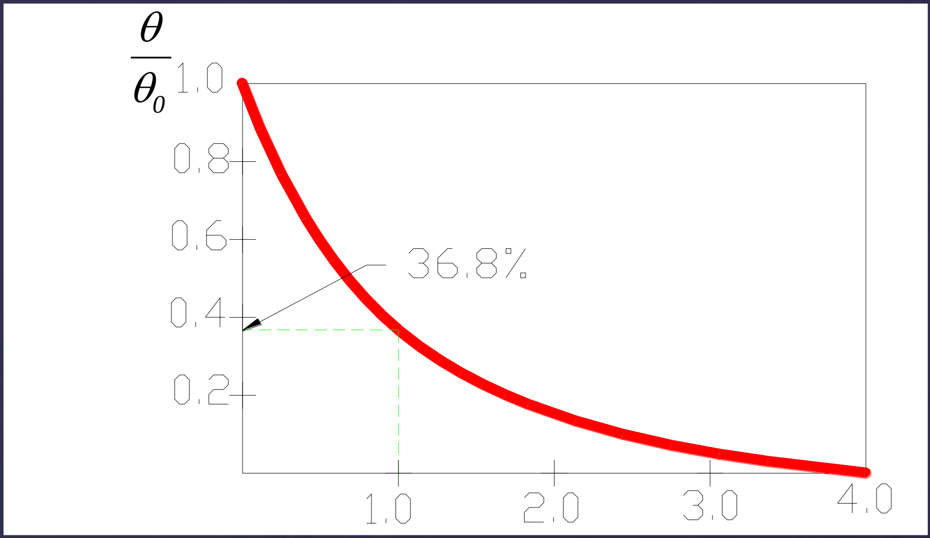
$$Q = \int_0^\tau \Phi d\tau = (t_0 - t_\infty)\int_0^\tau hA \exp\left(-\frac{hA}{\rho c V}\tau\right) d\tau$$
$$= (t_0 - t_\infty)\rho c V\left[1 - \exp\left(-\frac{hA}{\rho c V}\tau\right)\right]$$

Total heat from τ = 0 to τ

Time Constant

$$\tau_c = \frac{\rho c V}{hA}$$

$$\frac{\theta}{\theta_0} = \frac{(t - t_\infty)}{(t_0 - t_\infty)} = \exp(-1) = 0.368 = 36.8\%$$



The Fourier Number

dimensionless time

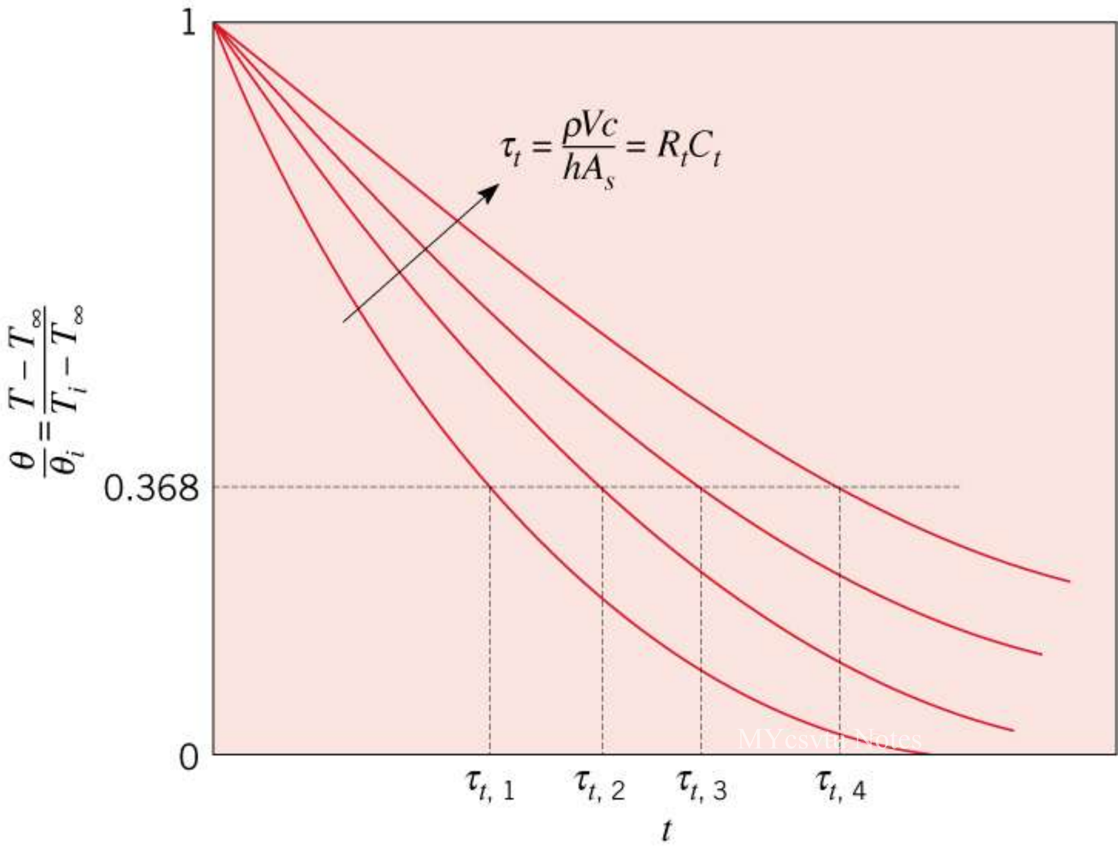
$$Fo = \frac{a\tau}{l_c^2} = \frac{\tau}{l_c^2 / a}$$

$$l_c = \frac{V}{A}$$ —— **Characteristic Length of the solid**

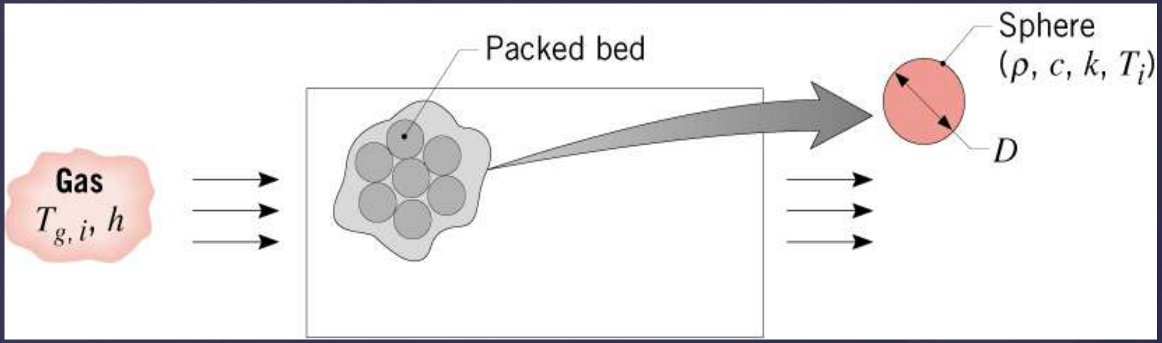
3. The Application Condition of Lumped Method

$$\text{For}\begin{cases} l=\delta, & \textbf{Plane wall with thickness } 2\delta \\ l=R, & \textbf{Cylinder with radius R} \\ l=R, & \textbf{Sphere with radius R} \end{cases}$$

$$Bi=\frac{hl}{\lambda}\leq 0.1$$



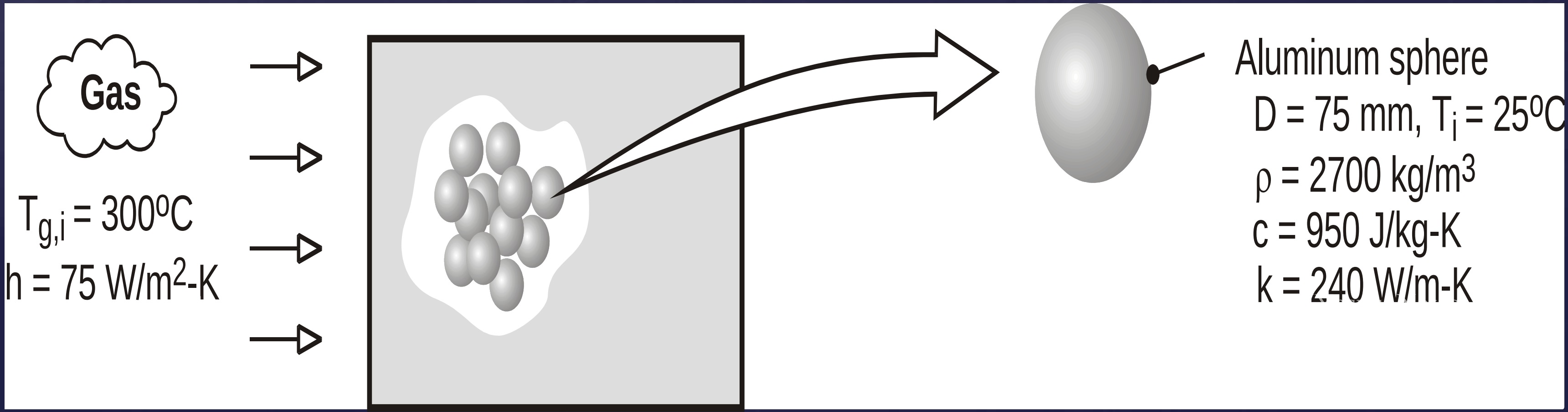
Problem 1: Charging a thermal energy storage system consisting of a packed bed of aluminum spheres.



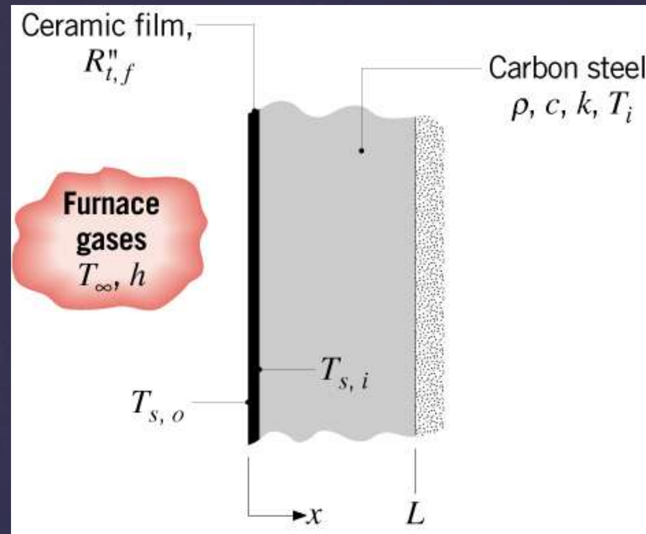
KNOWN: Diameter, density, specific heat and thermal conductivity of aluminum spheres used in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere at inlet to acquire 90% of maximum possible thermal energy and the corresponding center temperature.

Schematic:



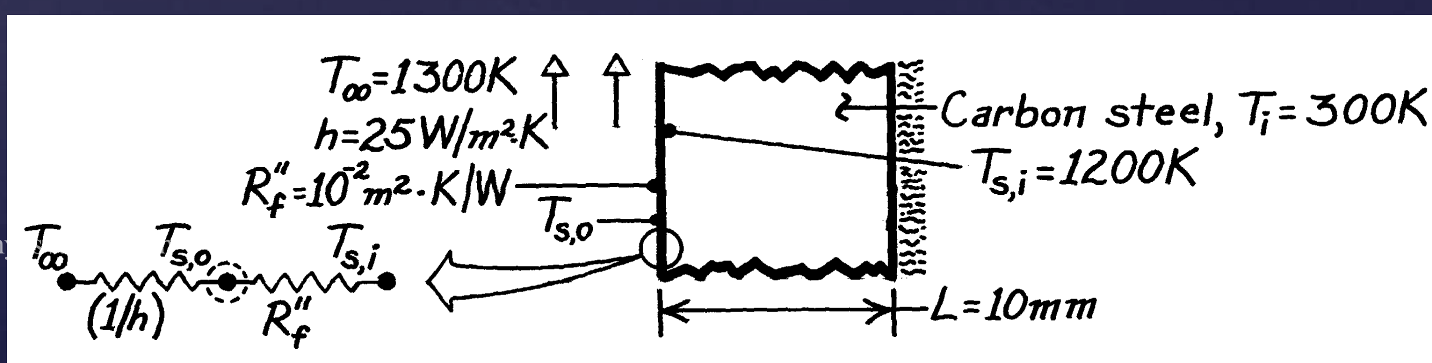
Problem 2: Heating of coated furnace wall during start-up.



KNOWN: Thickness and properties of furnace wall. Thermal resistance of ceramic coating on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of coating surface temperature.

Schematic:

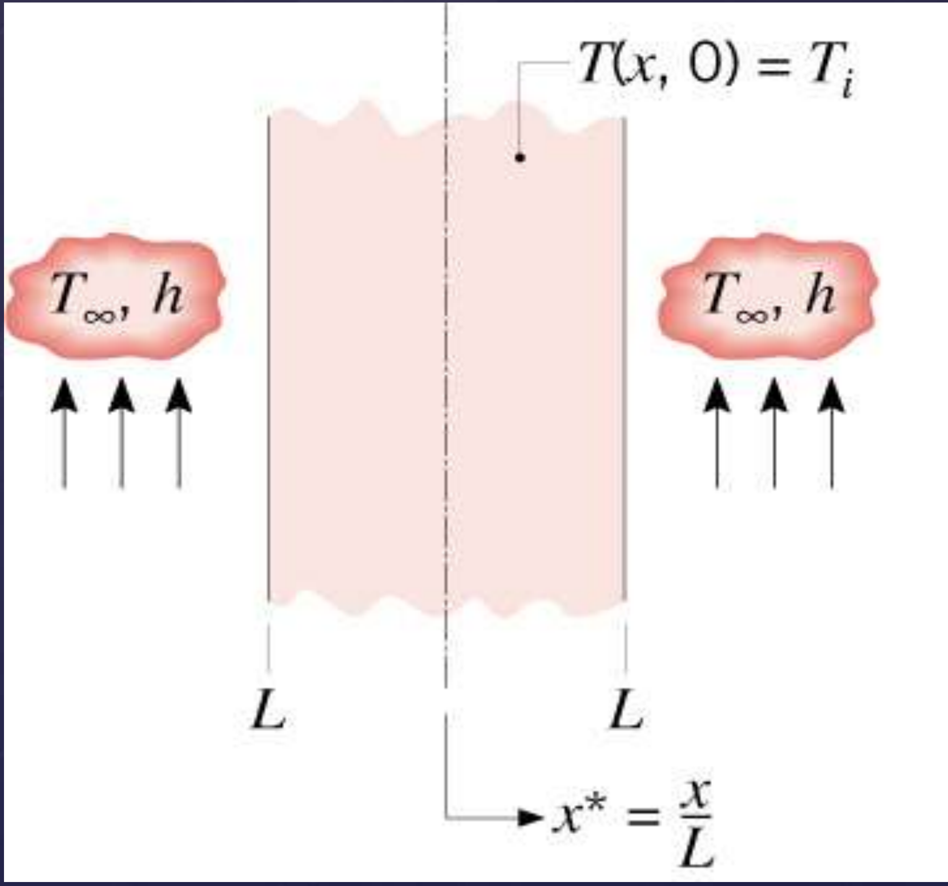


3.3 Analytical Solutions of Typical One-Dimensional Transient Conduction

1. Analytical Solutions of Temperature Field of Three Kinds of Solids

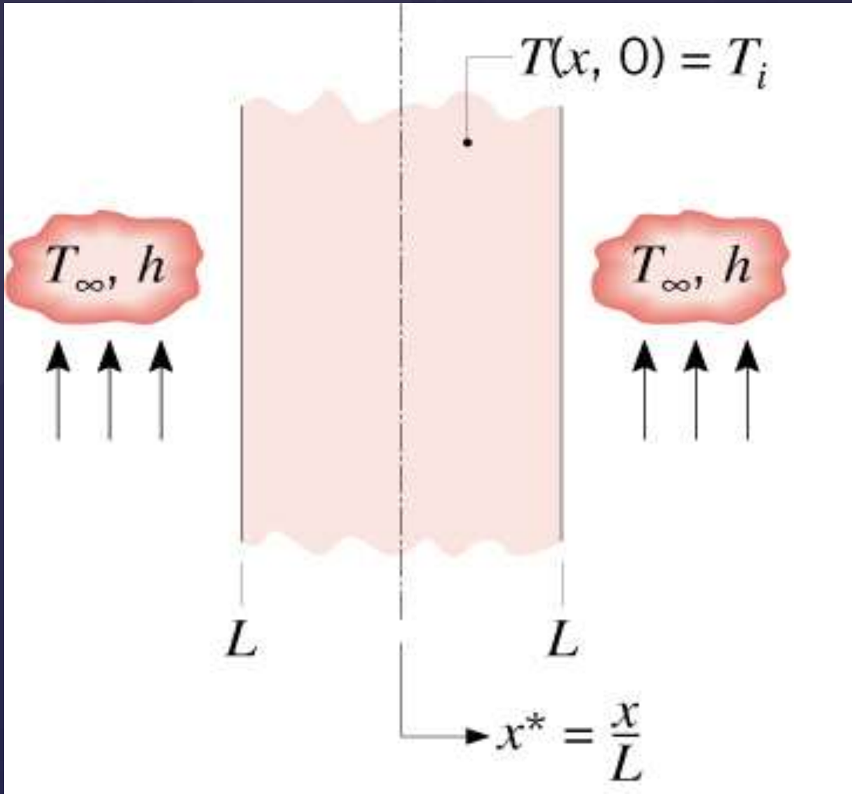
(1) Plane Wall

- **If the lumped capacitance approximation can not be made, consideration must be given to spatial, as well as temporal, variations in temperature during the transient process.**



- **For a plane wall with symmetrical convection conditions and constant properties, the heat equation and initial/boundary conditions are:**

$$
\begin{cases}
\dfrac{\partial t}{\partial \tau} = a\dfrac{\partial^2 t}{\partial x^2} (0 < x < \delta, \tau > 0) \\
t(x,0) = t_0 (0 \le x \le \delta) \\
\left.\dfrac{\partial t(x,\tau)}{\partial x}\right|_{x=0} = 0 \\
h[t(\delta,\tau) - t_\infty] = -\lambda \left.\dfrac{\partial t(x,\tau)}{\partial x}\right|_{x=\delta}
\end{cases}
$$



- **Excess temperature difference:**

$$\theta = t - t_\infty$$

- **The Heat Equation and Initial/Boundary Conditions described using θ:**

$$\begin{cases} \dfrac{\partial \theta}{\partial \tau} = a \dfrac{\partial^2 \theta}{\partial x^2} (0 < x < \delta, \tau > 0) \\ \theta(x,0) = \theta_0 (0 \le x \le \delta) \\ \left. \dfrac{\partial \theta(x,\tau)}{\partial x} \right|_{x=0} = 0 \\ h\theta(\delta,\tau) = -\lambda \left. \dfrac{\partial \theta(x,\tau)}{\partial x} \right|_{x=\delta} \end{cases}$$

- **Exact Solution (with the separation of variables) :**

$$\frac{\theta(\eta,\tau)}{\theta_0} = \sum_{n=1}^{\infty} C_n \exp(-\mu_n^2 Fo)\cos(\mu_n \eta)$$

$$Fo = \frac{a\tau}{\delta^2}, \eta = \frac{x}{\delta}, Bi = \frac{h\delta}{\lambda}, C_n = \frac{2\sin\mu_n}{\mu_n + \cos\mu_n \sin\mu_n}, \tan\mu_n = \frac{Bi}{\mu_n}, n = 1,2,\cdots\cdots$$

The eigenvalue μ_n is positive roots of the transcendental equation.

(2) Cylinder

- **Exact Solution:**

$$\frac{\theta(\eta,\tau)}{\theta_0} = \sum_{n=1}^{\infty} C_n \exp(-\mu_n^2 Fo) J_0(\mu_n \eta)$$

$$Fo = \frac{a\tau}{R^2}, \eta = \frac{r}{R}, Bi = \frac{hR}{\lambda}, C_n = \frac{2}{\mu_n} \frac{J_1(\mu_n)}{J_0^2(\mu_n) + J_1^2(\mu_n)},$$

$$\mu_n \frac{J_1(\mu_n)}{J_0(\mu_n)} = Bi, n = 1, 2, \cdots\cdots$$

The eigenvalue μ_n is positive roots of the transcendental equation.

(3) Sphere

- **Exact Solution:**

$$\frac{\theta(\eta,\tau)}{\theta_0}=\sum_{n=1}^{\infty}C_n\exp(-\mu_n^2 Fo)\frac{\sin(\mu_n\eta)}{\mu_n\eta}$$

$$Fo=\frac{a\tau}{R^2},\eta=\frac{r}{R},Bi=\frac{hR}{\lambda},C_n=2\frac{\sin(\mu_n)-\mu_n\cos(\mu_n)}{\mu_n-\sin(\mu_n)\cos(\mu_n)},$$

$$1-\mu_n\cos(\mu_n)=Bi,n=1,2,\cdots\cdots$$

The eigenvalue μ_n is positive roots of the transcendental equation.

- **Conclusion: The distribution θ/θ_0 is a function of *Fo*, *Bi*, and η.**

$$\frac{\theta}{\theta_0} = \frac{t(\eta,\tau) - t_\infty}{t_0 - t_\infty} = f(Fo, Bi, \eta)$$

2. The One-Term Approximation of Analytical Solution of Regular Regime of Transient Conduction

When *Fo*>0.2

For plane wall

$$\frac{\theta}{\theta_0}=\frac{2\sin\mu_1}{\mu_1+\sin\mu_1\cos\mu_1}\exp(-\mu_1^2 Fo)\cos(\mu_1\eta)$$

For cylinder

$$\frac{\theta}{\theta_0}=\frac{2}{\mu_1}\frac{J_1(\mu_1)}{J_0^2(\mu_1)+J_1^2(\mu_1)}\exp(-\mu_1^2 Fo)J_0(\mu_1\eta)$$

For sphere

$$\frac{\theta}{\theta_0}=\frac{2(\sin\mu_1-\mu_1\cos\mu_1)}{\mu_1-\sin\mu_1}\exp(-\mu_1^2 Fo)\frac{\sin(\mu_1\eta)}{\mu_1\eta}$$

The total heat transferred from initial to τ and the maximum heat:

$$Q = \rho c \int_V \left[t_0 - t(x,\tau)\right] dV \qquad Q_0 = \rho c V (t_0 - t_\infty)$$

$$\frac{Q}{Q_0} = \frac{\rho c \int_V \left[t_0 - t(x,\tau)\right] dV}{\rho c V (t_0 - t_\infty)} = \frac{1}{V}\int_V \frac{(t_0 - t_\infty) - (t - t_\infty)}{(t_0 - t_\infty)} dV = 1 - \frac{1}{V}\int_V \frac{(t - t_\infty)}{(t_0 - t_\infty)} dV = 1 - \frac{\bar{\theta}}{\theta_0}$$

For plane wall

$$\frac{Q}{Q_0} = 1 - \frac{\sin \mu_1}{\mu_1} \frac{2 \sin \mu_1}{\mu_1 + \sin \mu_1 \cos \mu_1} \exp(-\mu_1^2 Fo)$$

For cylinder

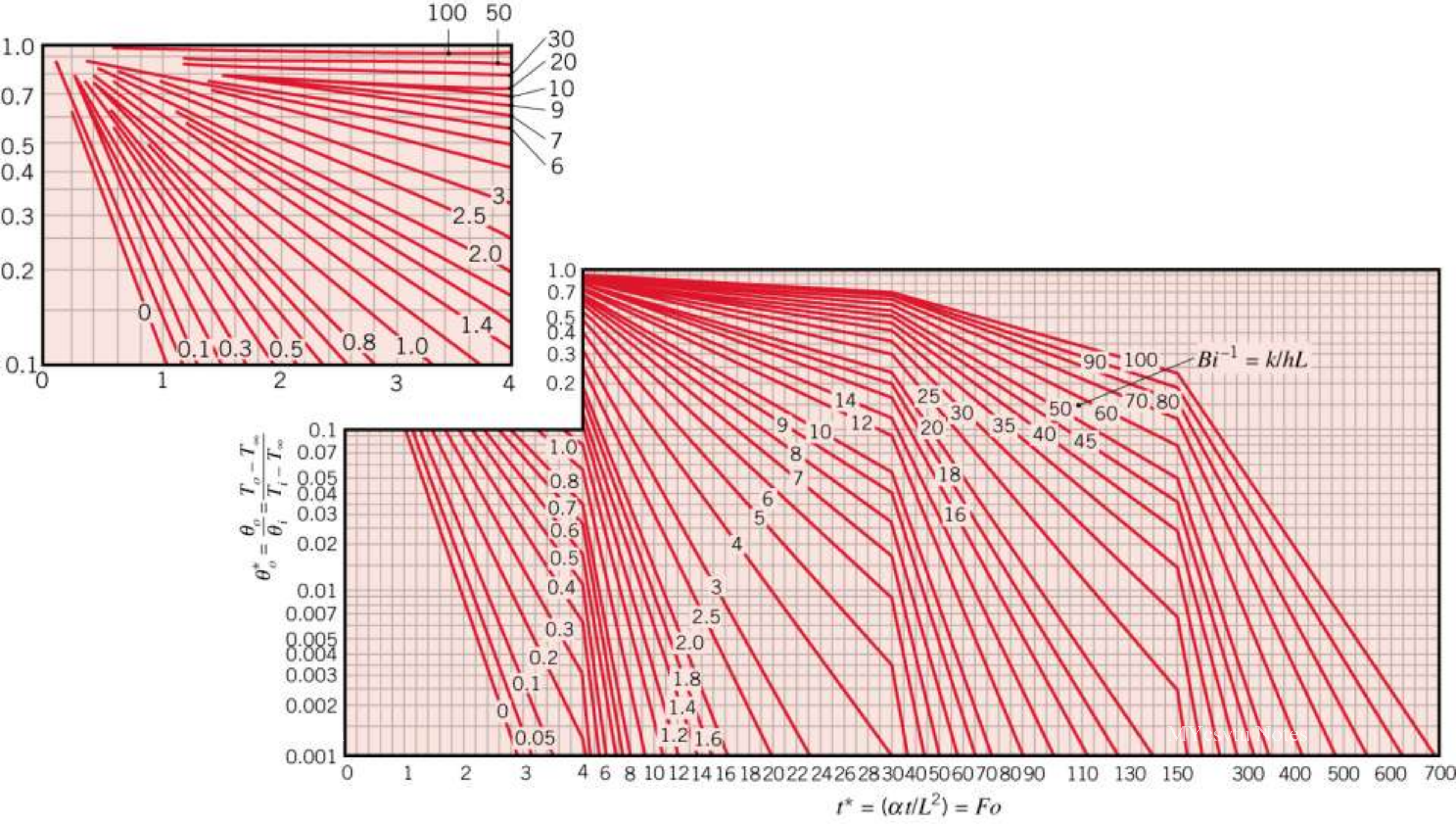
$$\frac{Q}{Q_0} = 1 - \frac{2 J_1(\mu_1)}{\mu_1} \frac{2}{\mu_1} \frac{J_1(\mu_1)}{J_0^2(\mu_1) + J_1^2(\mu_1)} \exp(-\mu_1^2 Fo)$$

For sphere

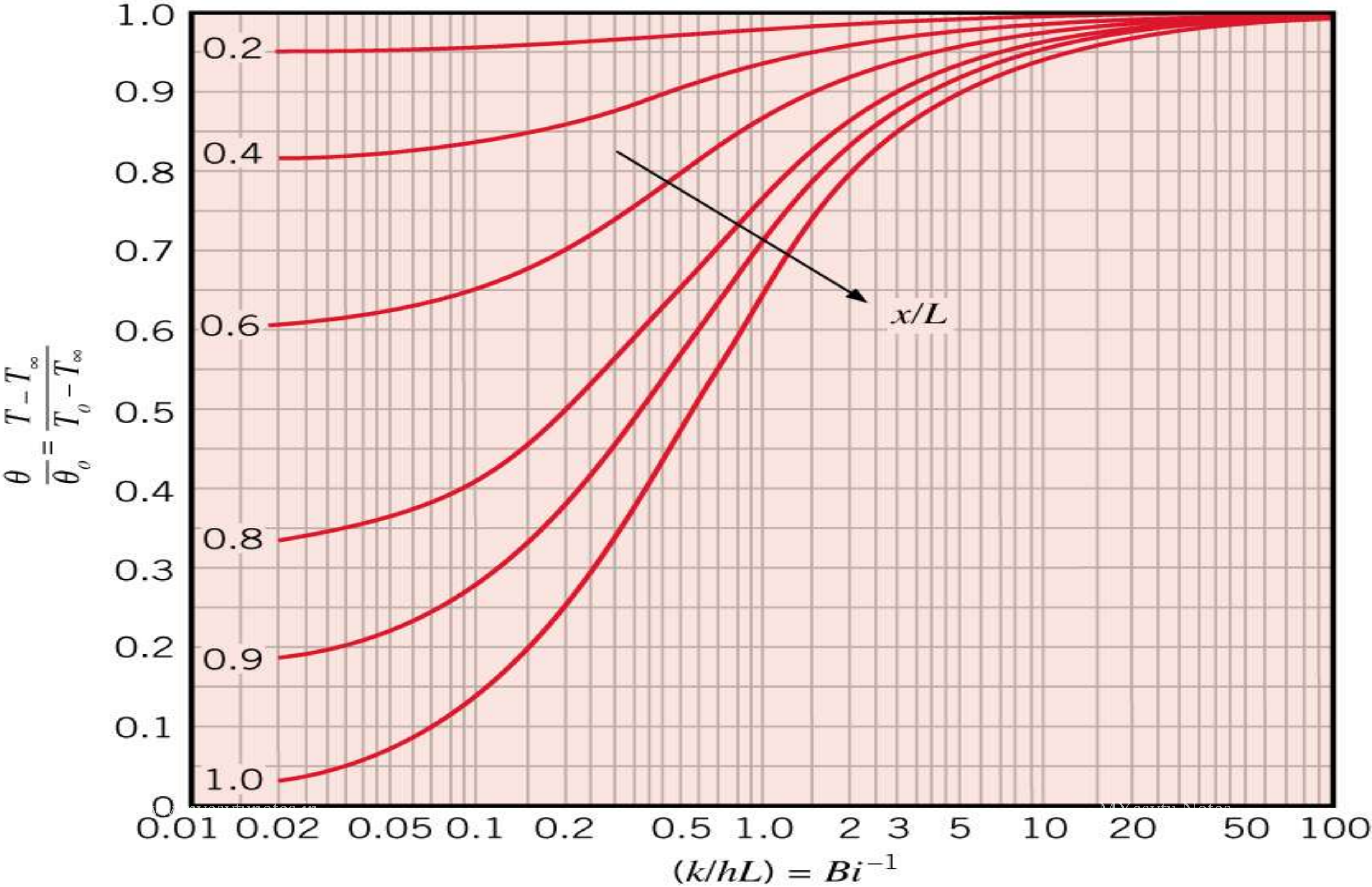
$$\frac{Q}{Q_0} = 1 - \frac{3(\sin \mu_1 - \mu_1 \cos \mu_1)}{\mu_1^3} \frac{2(\sin \mu_1 - \mu_1 \cos \mu_1)}{\mu_1 - \sin \mu_1} \exp(-\mu_1^2 Fo)$$

3. Graphical Representation of the One-Term Approximation

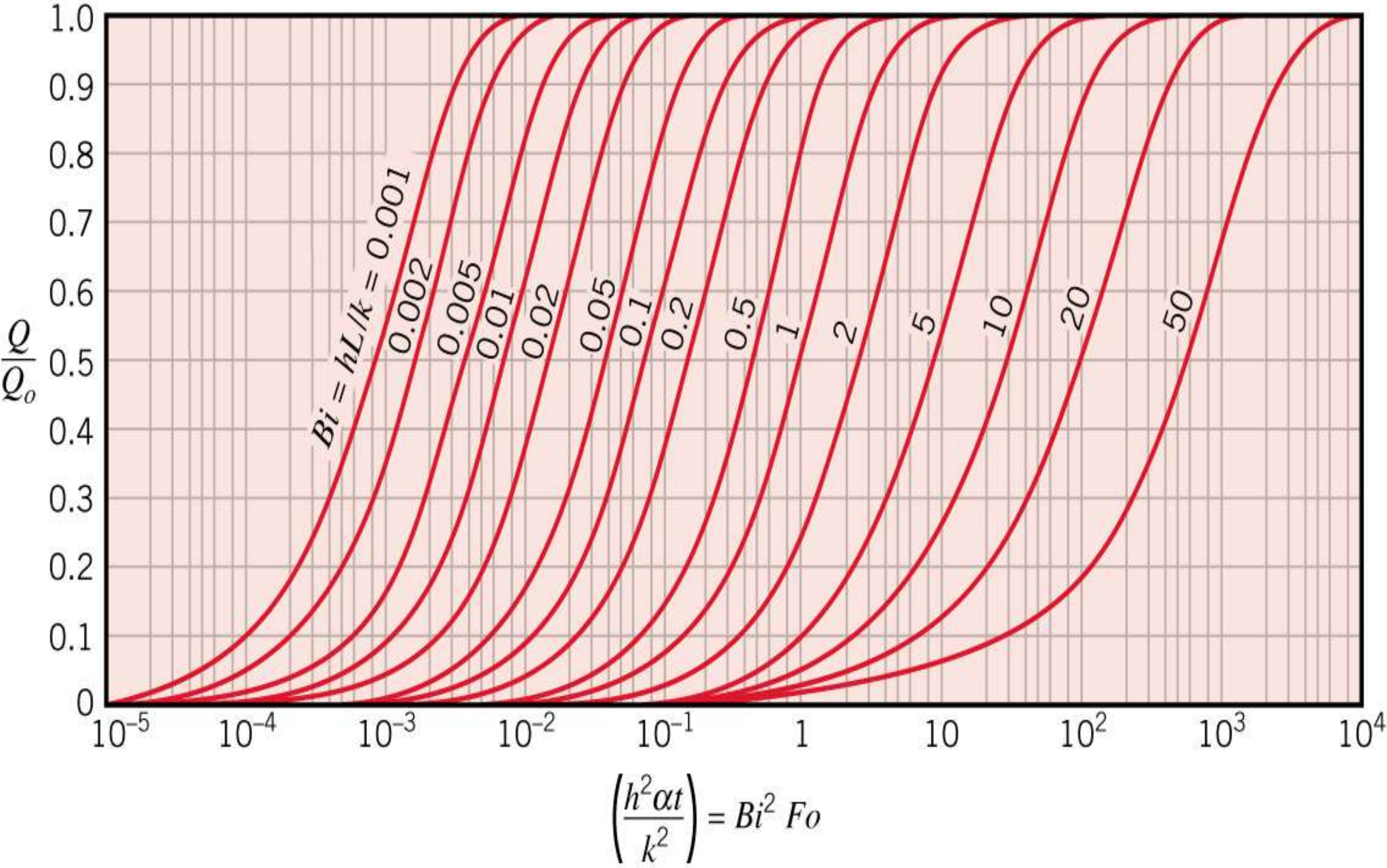
- Midplane Temperature for Plane Wall with thickness 2δ(Heisler Charts):



- **Temperature Distribution:**



• Change in Thermal Energy Storage:



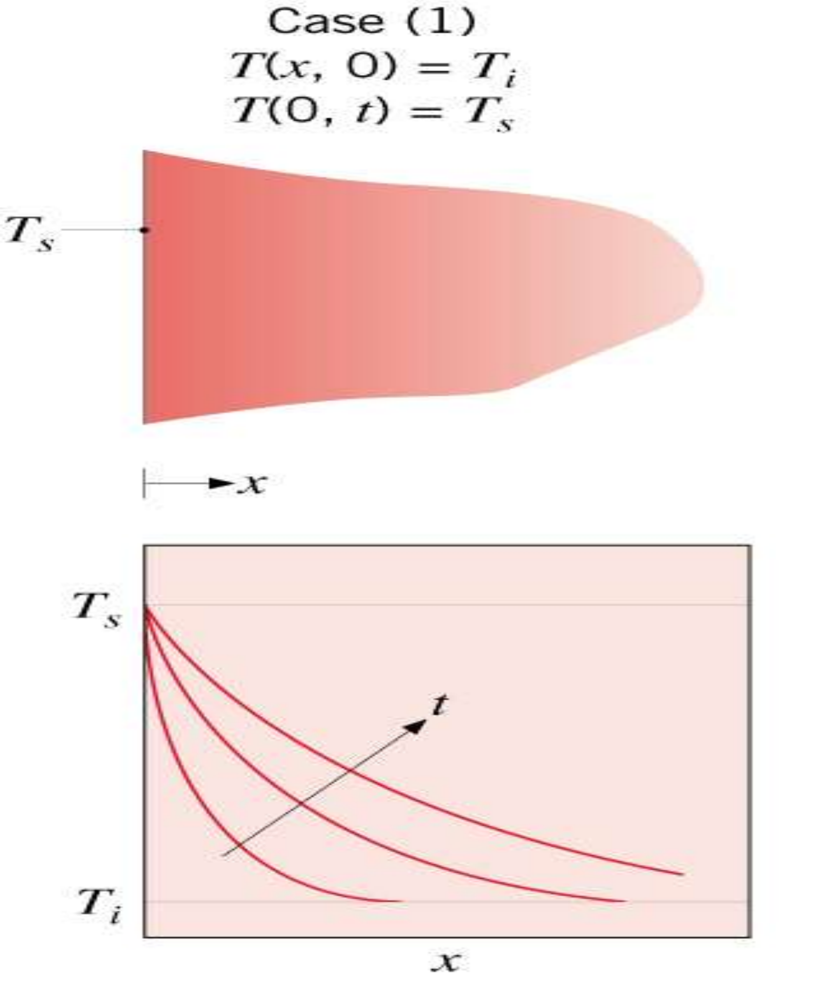
3.4 Transient Conduction of The Semi-Infinite Solid

1. Analytical Solutions under Three Boundary Conditions

- **A solid that is initially of uniform temperature t_0 and is assumed to extend to infinity from a surface at which thermal conditions are altered.**

$$\begin{cases} \dfrac{\partial t}{\partial \tau} = a \dfrac{\partial^2 t}{\partial x^2} (0 < x < \infty) \\ \tau = 0, t(x, \tau) = t_0 \\ x = 0, \end{cases}$$

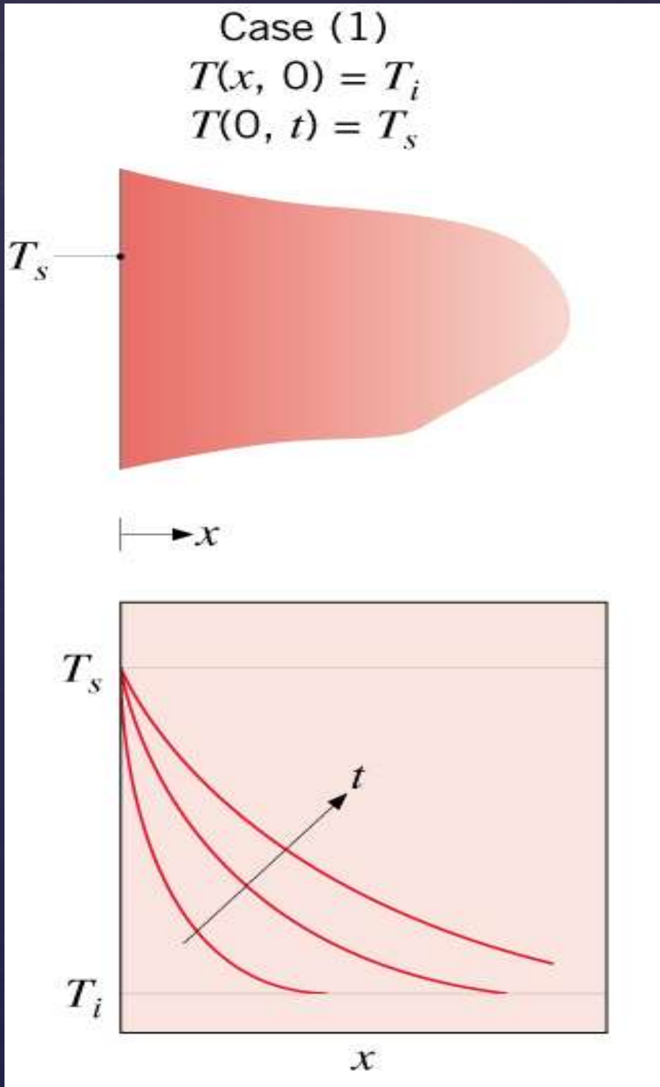
One of Three Kinds Cases



- **Special Cases**:

Case 1: Change in Surface Temperature (t_w)

$$\frac{t(x,\tau)-t_w}{t_0-t_w}=erf\left(\frac{x}{2\sqrt{a\tau}}\right)$$



Heat Flux :

$$q_x=-\lambda\frac{\partial t}{\partial x}=-\lambda(t_0-t_w)\frac{\partial erf\,\eta}{\partial x}$$

$$=\lambda(t_w-t_0)\exp\left[-\frac{x^2}{4a\tau}\right]$$

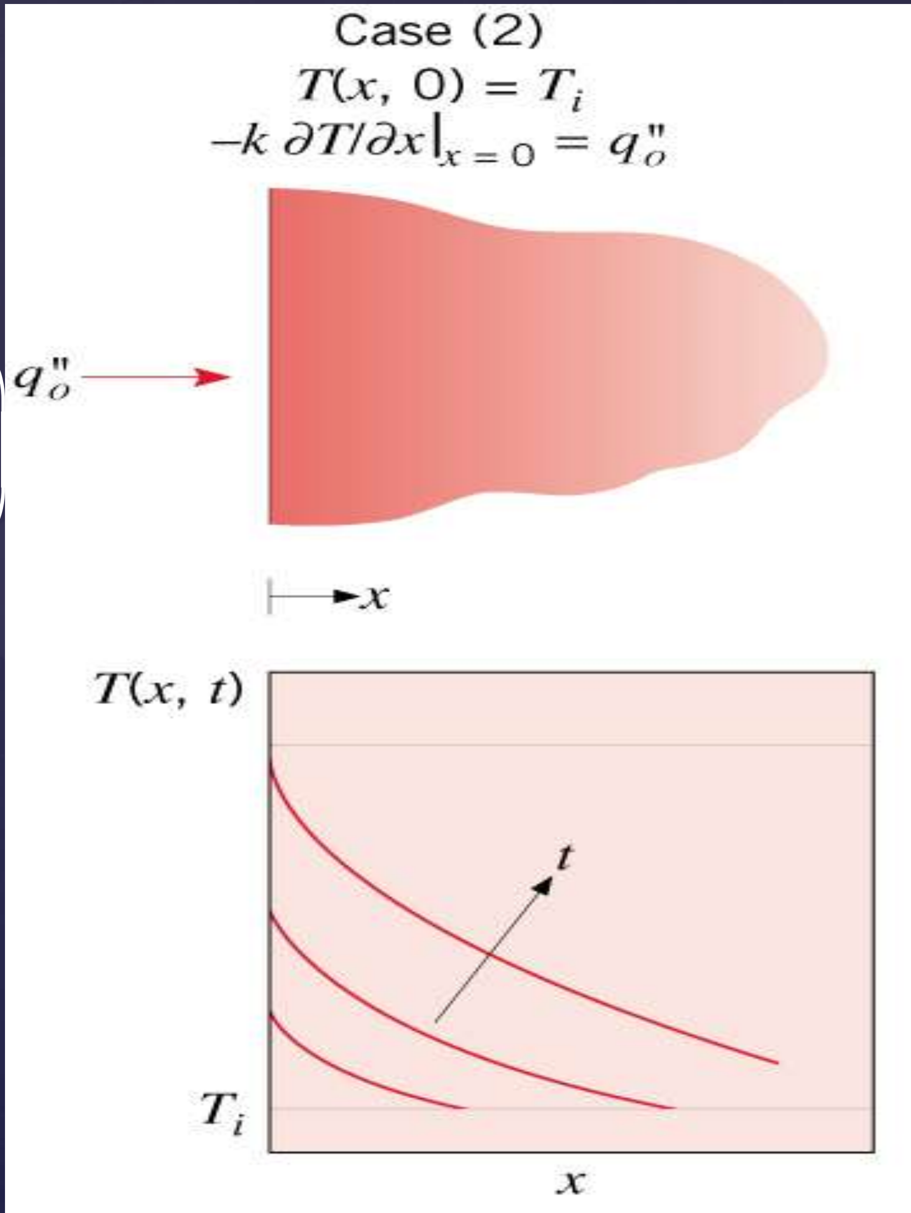
Heat rate:

$$Q=A\int_0^\tau q_w d\tau=A\int_0^\tau\frac{\lambda(t_w-t_0)}{\sqrt{\pi a\tau}}d\tau$$

$$=2A\sqrt{\frac{\tau}{\pi}}\sqrt{\rho c\lambda}(t_w-t_0)$$

Case 2: Uniform Heat Flux q_0

$$t(x,\tau) - t_0 = \frac{2q_0\sqrt{\frac{a\tau}{\pi}}}{\lambda}\exp\left(-\frac{x^2}{4a\tau}\right) - \frac{q_0 x}{\lambda} erfc\left(\frac{x}{2\sqrt{a\tau}}\right)$$



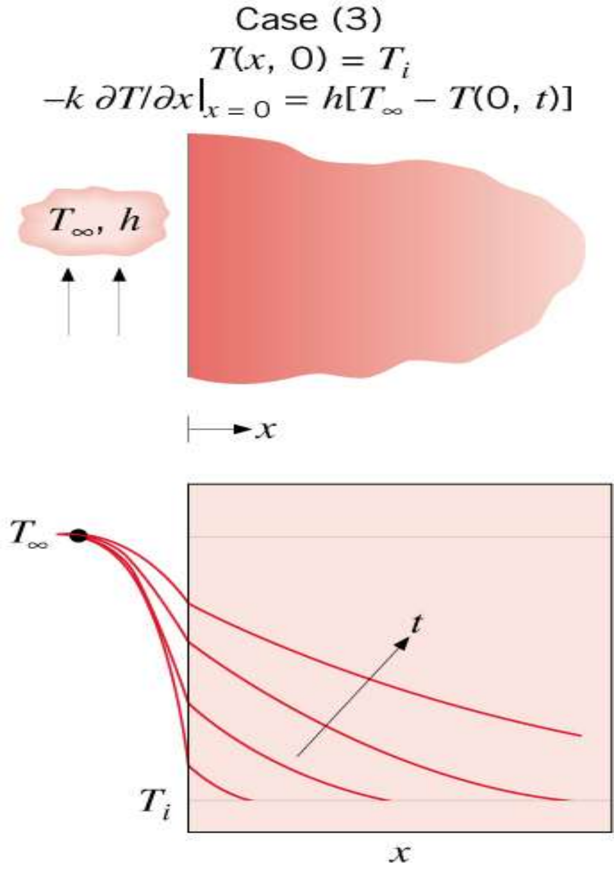
Case 3: Convection Heat Transfer (h, t_∞)

$$\frac{t(x,\tau)-t_0}{t_\infty - t_0} = erf\left(\frac{x}{2\sqrt{a\tau}}\right) - \exp\left(\frac{hx}{\lambda} + \frac{h^2 a\tau}{\lambda^2}\right) erfc\left(\frac{x}{2\sqrt{a\tau}} + \frac{h\sqrt{a\tau}}{\lambda}\right)$$

$$erf\left(\frac{x}{2\sqrt{a\tau}}\right)$$ ——Error function

$$erfc\left(\frac{x}{2\sqrt{a\tau}}\right) = 1 - erf\left(\frac{x}{2\sqrt{a\tau}}\right)$$

——Rest error function

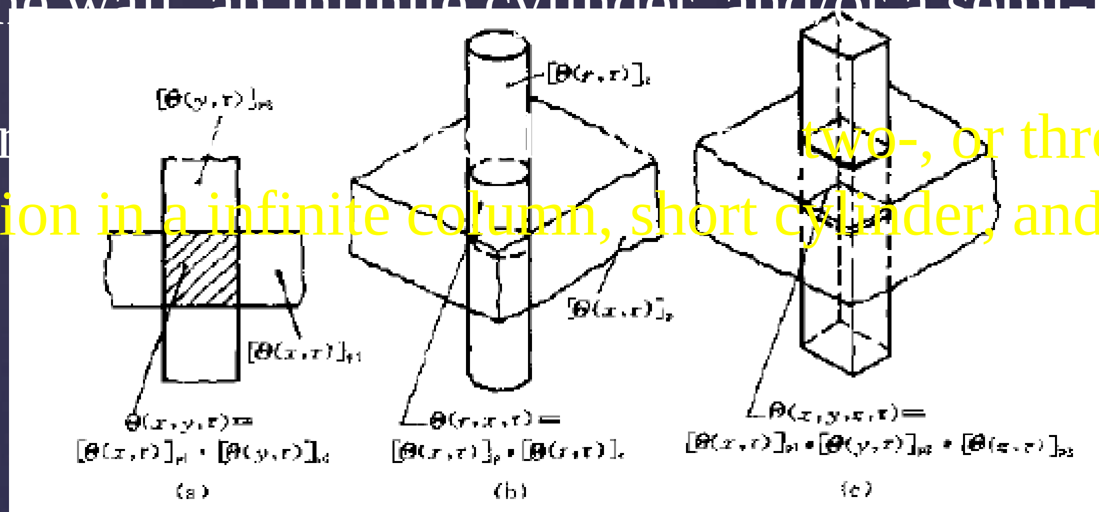


3.5 Analytical Solutions of Multidimensional Transient Conduction

1. Production Solution Method

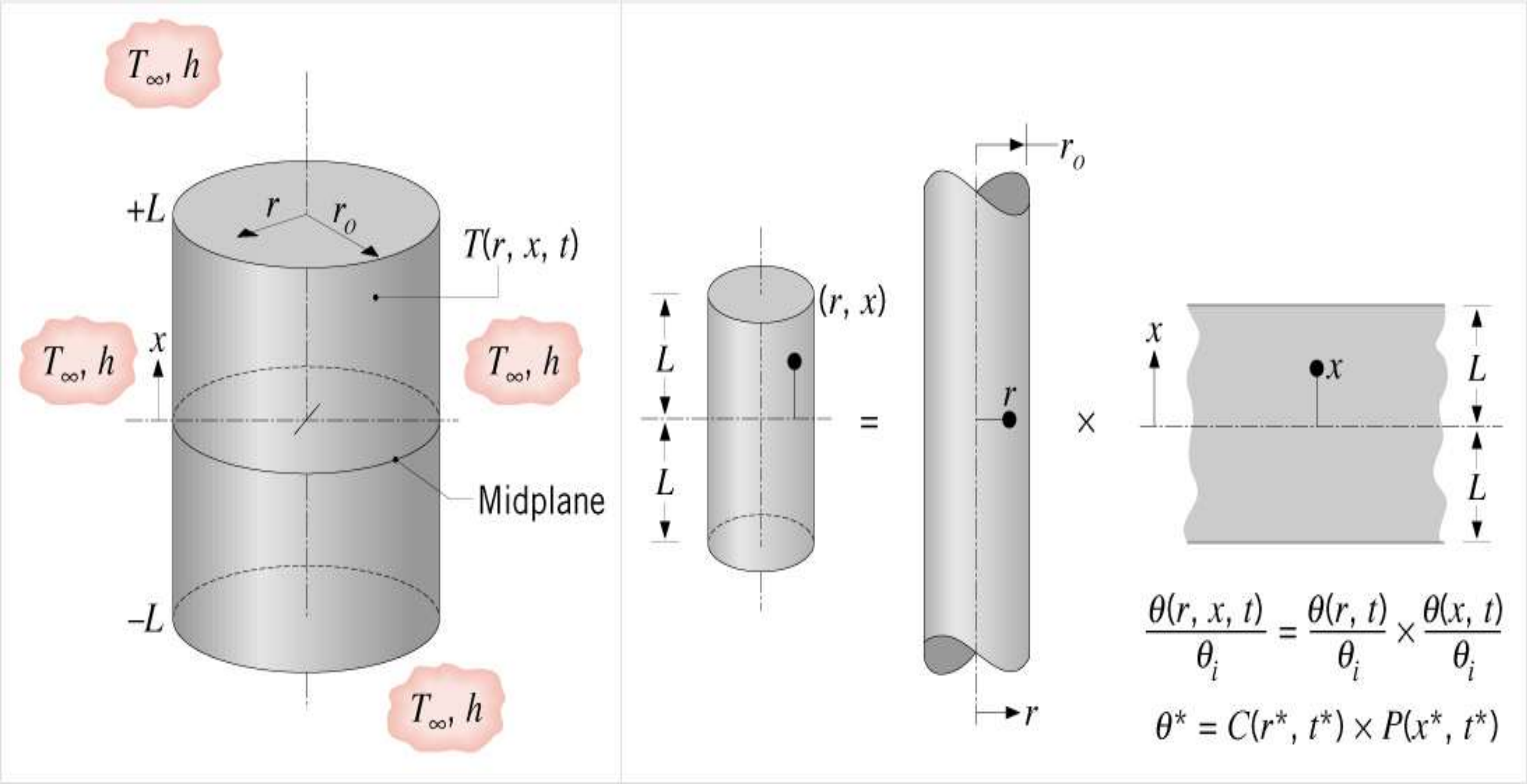
- Solutions for multidimensional transient conduction can often be expressed as a product of related one-dimensional solutions for a plane wall, an infinite cylinder, and/or a semi-infinite solid.

- Consider two-, or three-dimensional conduction in a infinite column, short cylinder, and short column:



for a two-dimensional infinite column:

$$\Theta = \frac{\theta(x, y, \tau)}{\theta_0} = \Theta_{p1}(x, \tau) \cdot \Theta_{p2}(y, \tau)$$



for a short cylinder:

$$\Theta = \frac{\theta(x,r,\tau)}{\theta_0} = \Theta_p(x,\tau)\cdot\Theta_c(r,\tau)$$

for a three-dimensional short column:

$$\Theta = \frac{\theta(x,y,z,\tau)}{\theta_0} = \Theta_{p1}(x,\tau)\cdot\Theta_{p2}(y,\tau)\cdot\Theta_{p3}(z,\tau)$$

Θ_p,Θ_c are the dimensionless temperature solution of plane wall and infinite cylinder under the 3rd boundary condition, respectively.

2. Heat Quantity in the Transient Conduction Process

- For the two-dimensional Transient Conduction:

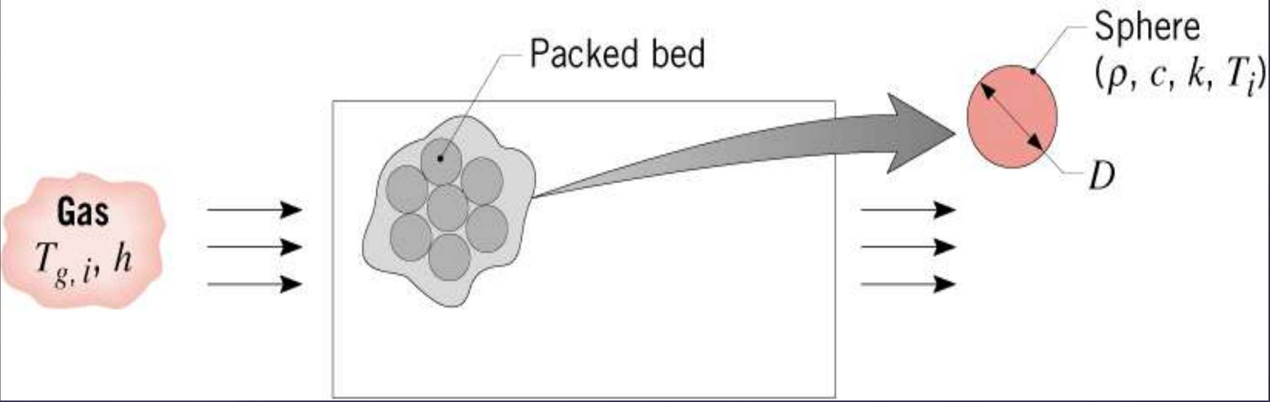
$$\frac{Q}{Q_0} = \left(\frac{Q}{Q_0}\right)_1 + \left(\frac{Q}{Q_0}\right)_2 \left[1 - \left(\frac{Q}{Q_0}\right)_1\right]$$

- For the three-dimensional Transient Conduction:

$$\frac{Q}{Q_0} = \left(\frac{Q}{Q_0}\right)_1 + \left(\frac{Q}{Q_0}\right)_2 \left[1 - \left(\frac{Q}{Q_0}\right)_1\right] + \left(\frac{Q}{Q_0}\right)_3 \left[1 - \left(\frac{Q}{Q_0}\right)_1 - \left(\frac{Q}{Q_0}\right)_2\right]$$

$\left(\frac{Q}{Q_0}\right)_1$ 、 $\left(\frac{Q}{Q_0}\right)_2$ 、 $\left(\frac{Q}{Q_0}\right)_3$ **are three one-dimensional Transient Conduction Heat Quantities.**

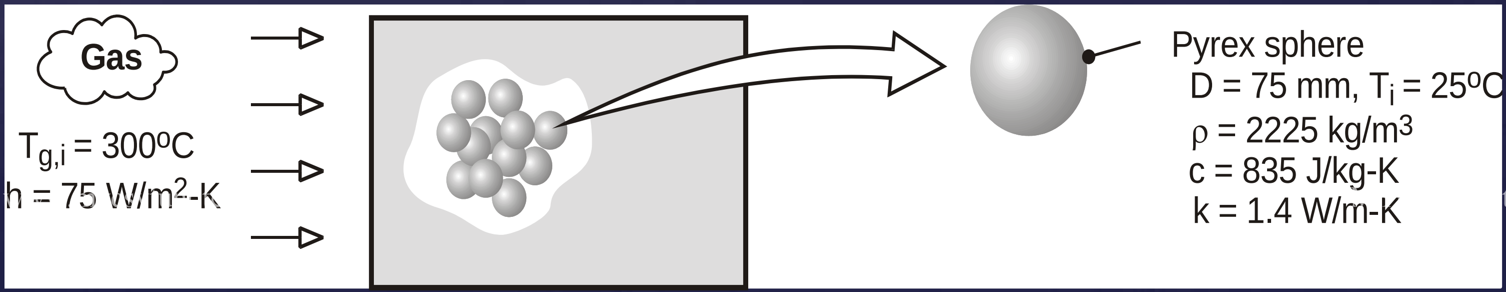
Problem 3: Charging a thermal energy storage system consisting of a packed bed of Pyrex spheres.



KNOWN: Diameter, density, specific heat and thermal conductivity of Pyrex spheres in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere to acquire 90% of maximum possible thermal energy and the corresponding center and surface temperatures.

SCHEMATIC:

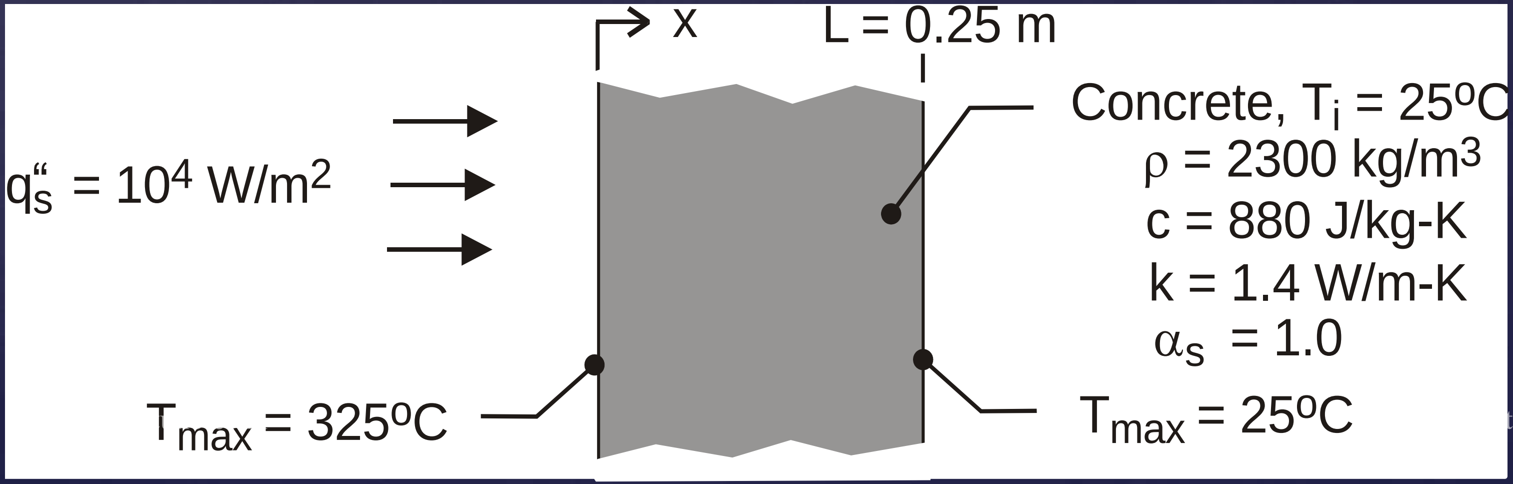


Problem: 4: Use of radiation heat transfer from high intensity lamps $\left(q''_s = 10^4 \text{ W/m}^2\right)$ for a prescribed duration (t=30 min) to assess ability of firewall to meet safety standards corresponding to maximum allowable temperatures at the heated (front) and unheated (back) surfaces.

KNOWN: Thickness, initial temperature and thermophysical properties of concrete firewall. Incident radiant flux and duration of radiant heating. Maximum allowable surface temperatures at the end of heating.

FIND: If maximum allowable temperatures are exceeded.

SCHEMATIC:



Problem: 5: Microwave heating of a spherical piece of frozen ground beef using microwave-absorbing packaging material.

KNOWN: Mass and initial temperature of frozen ground beef. Rate of microwave power absorbed in packaging material.

FIND: Time for beef adjacent to packaging to reach 0°C.

SCHEMATIC: