UNIT - III

GYROSCOPE

Introduction. 1.When a body moves along a curved path, a force in the direction of centripetal acceleration (centripetal force) has to be applied externally. This external force is known as active force.

2. When a body is moving along a circular path, it is subjected to the centrifugal force radially outwards. This centrifugal force is known as reactive force.

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Note: Whenever the effect of any force or couple is to be considered, it should be with respect to reactive force. or couple.

Precessional Angular Motion

Consider a disc spinning about the axis OX (axis of spin) with an angular velocity ω. After a short interval of time δt, let the disc be spinning about the new axis of spin OX’

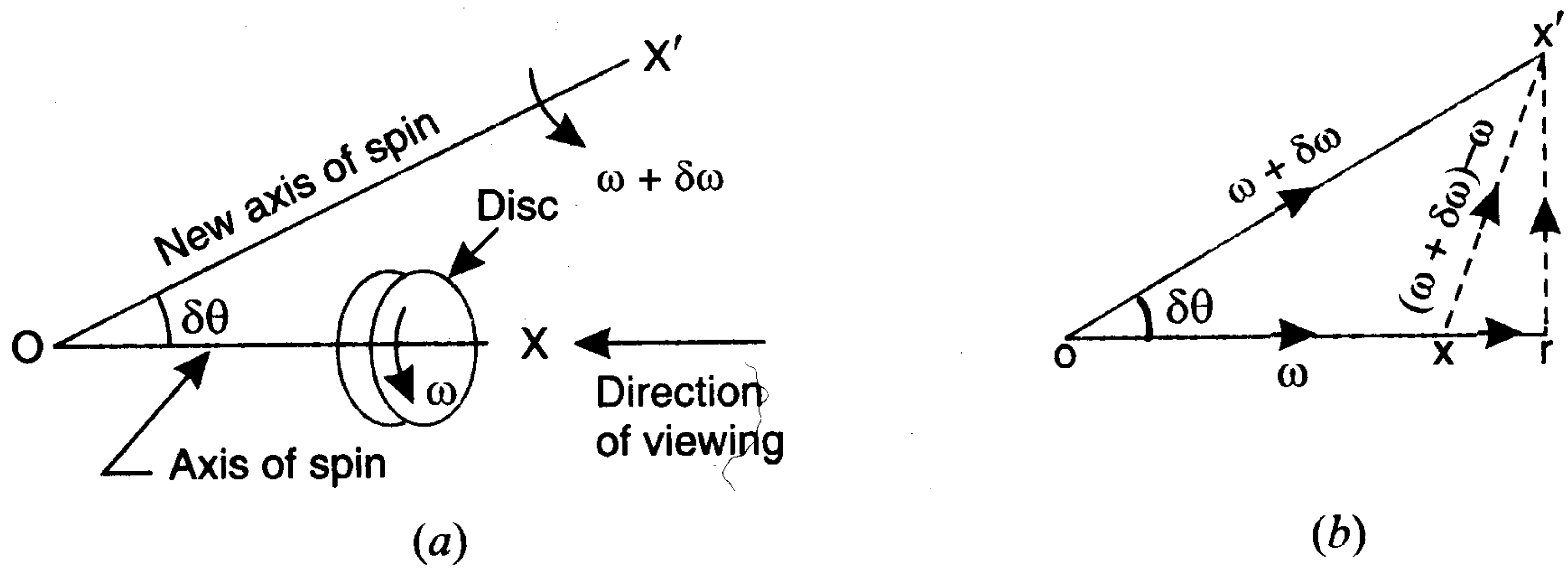

Fig. 14.1. Precessional angular motion.

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Total angular acceleration of the disc

$$= \frac{d\omega}{dt} + \omega.\omega_P$$

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Angular velocity of the axis of spin ($d\Theta/dt$) is known as angular velocity of precession. The axis about which the axis of spin is to turn is known as axis of precession.

If the angular velocity of the disc changes direction but remains constant in magnitude, then angular acceleration of the disc is given by

$\alpha_c = \omega . d\Theta/dt = \omega . \Omega p$

The angular acceleration α_c is known as gyroscopic acceleration.

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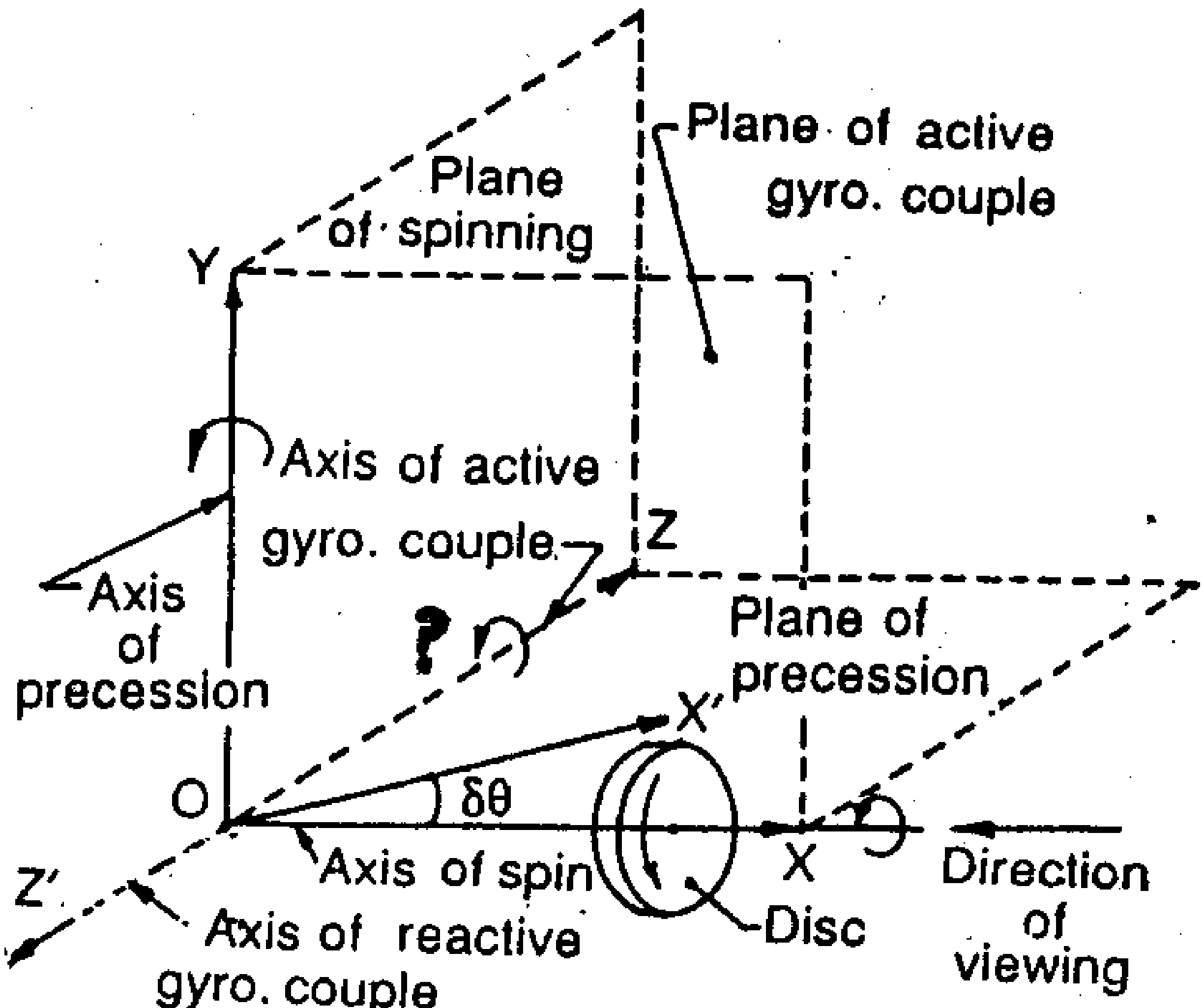
Gyroscopic Couple

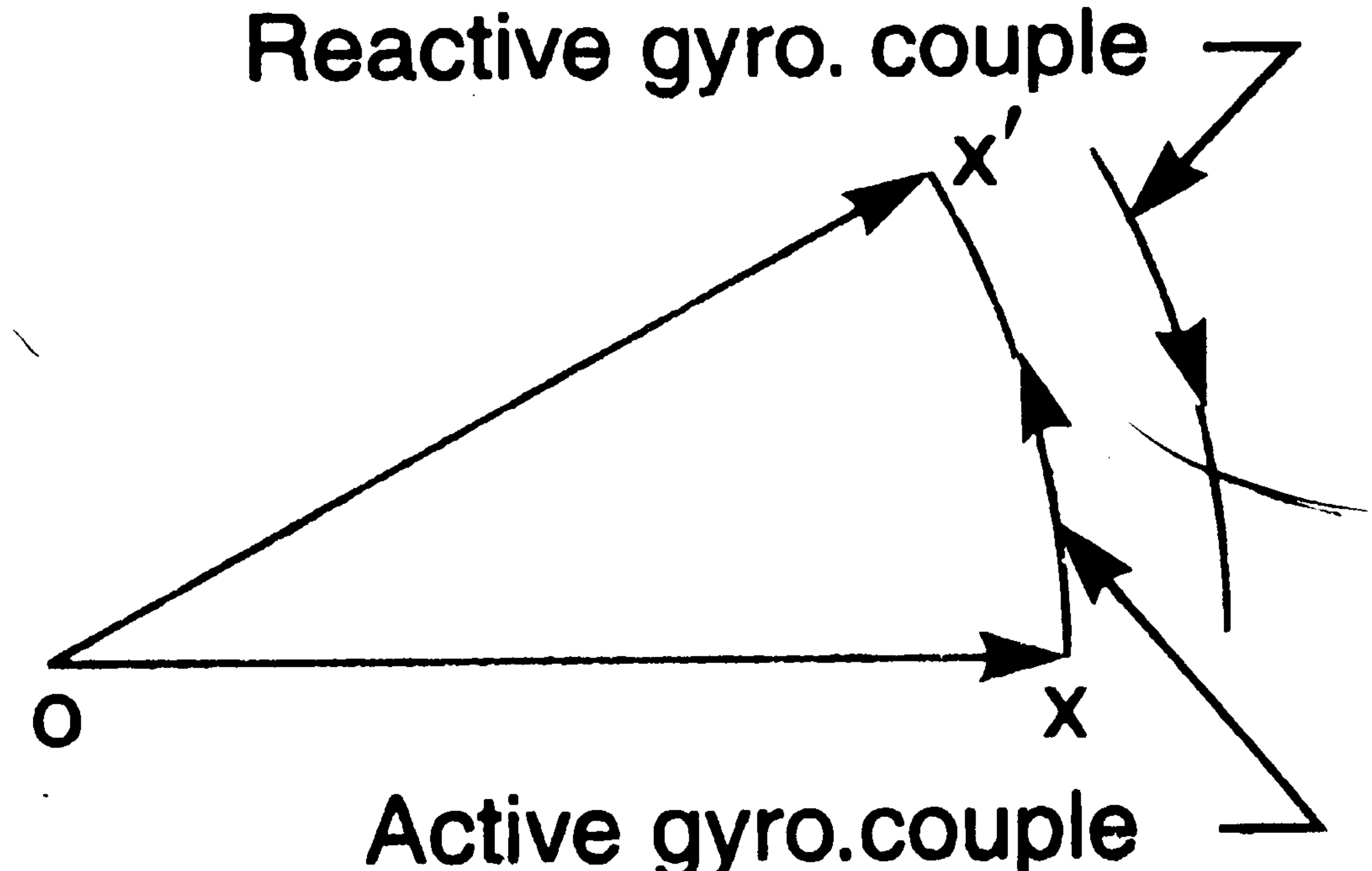
Consider a disc spinning with an angular velocity ω rad/s.

Angular momentum of the disc = $I.\omega$

The couple applied to the disc causing precession

$C = I.\omega.\omega_P$


Plane of spinning
Plane of active gyro. couple
Y
Axis of active gyro. couple
Z
Axis of precession
Plane of precession
X'
O
δθ
Axis of spin
X
Direction of viewing
Z'
Axis of reactive gyro. couple
Disc


Reactive gyro. couple
x′
o
x
Active gyro.couple

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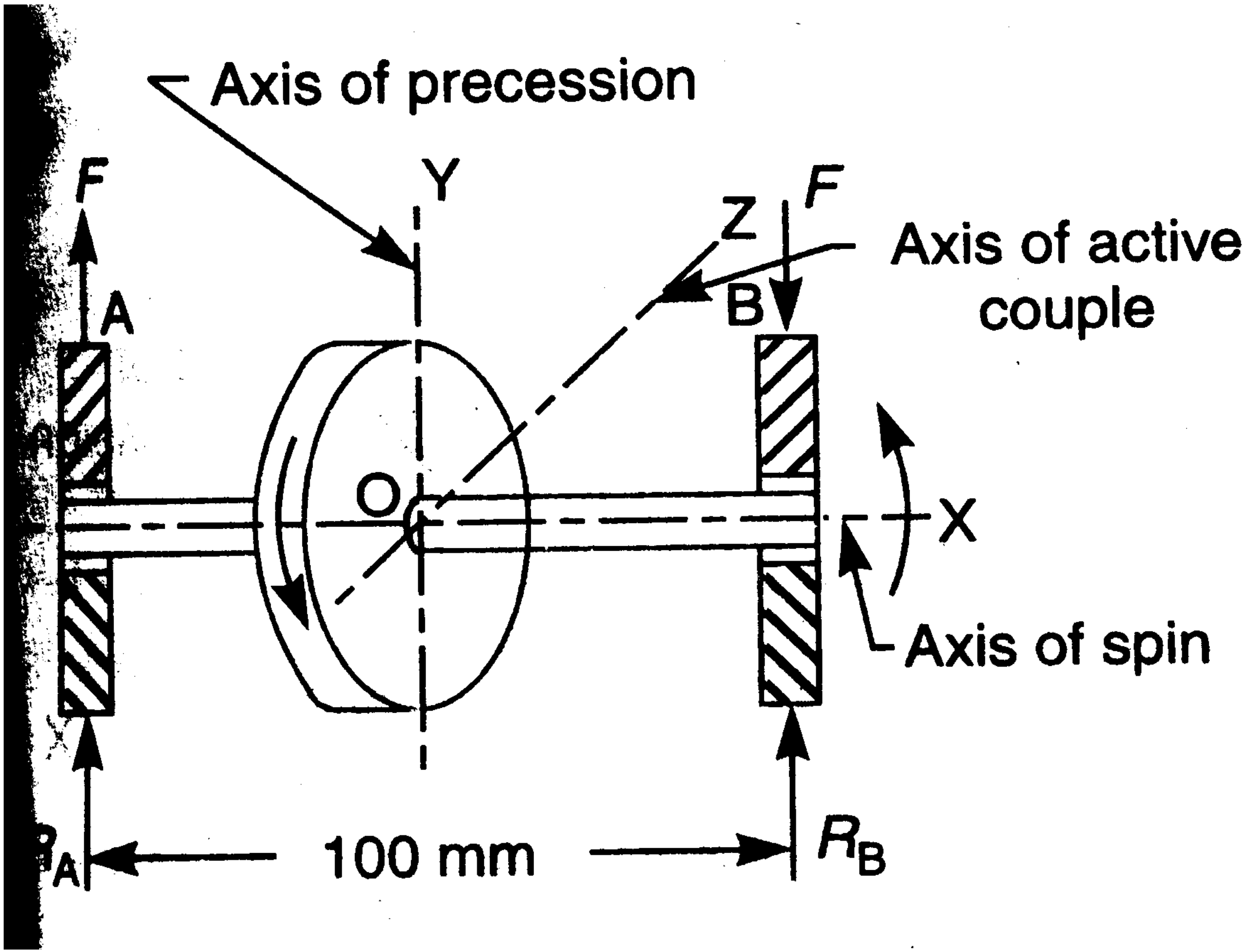
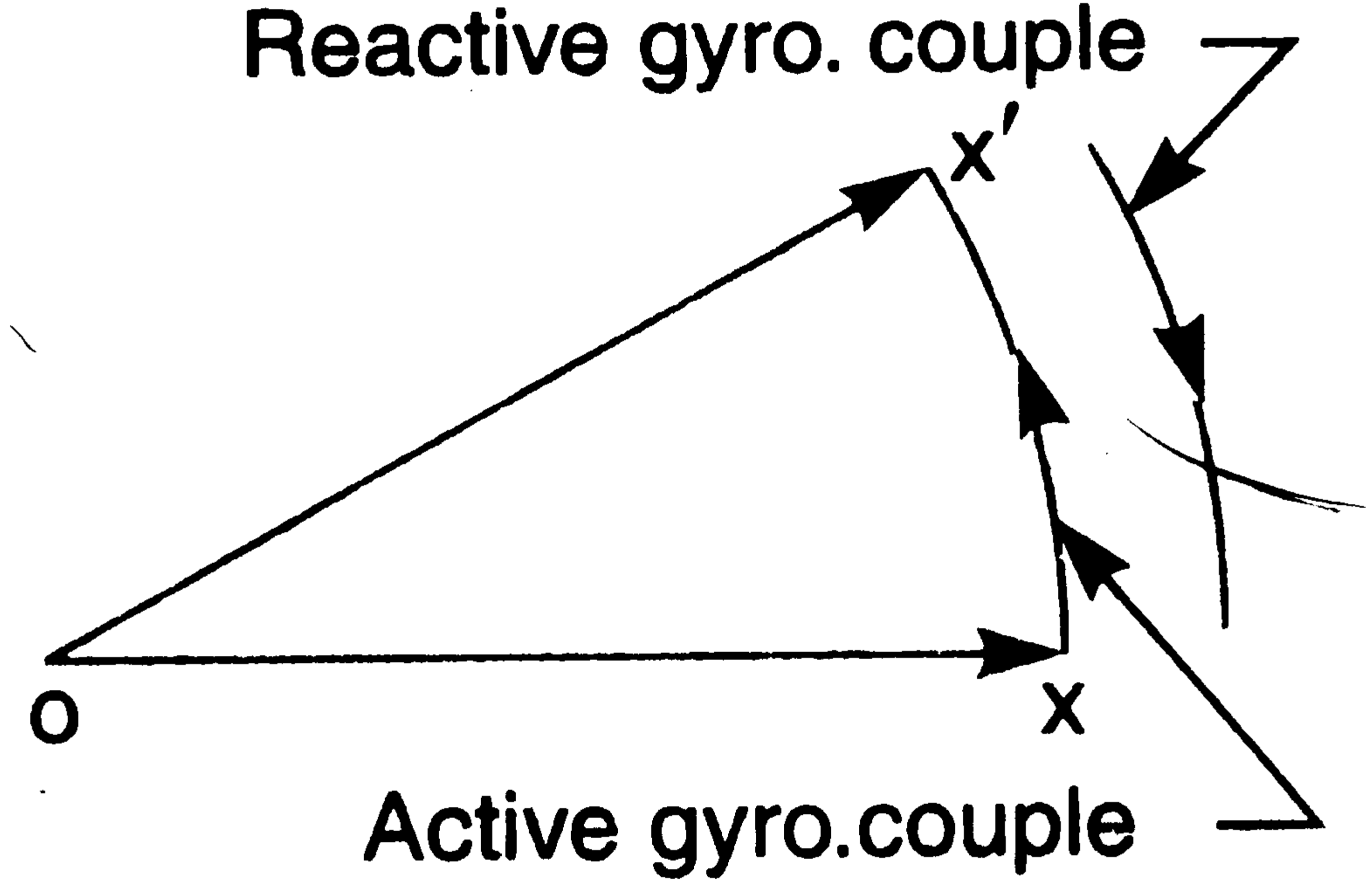
The couple $I.\omega.\omega_P$ in the direction of the vector xx' (representing the change in angular momentum) is the active gyroscopic couple which has to be applied over the disc.

When the axis of spin moves with an angular velocity ω_P, the disc is subjected to reactive gyroscopic couple which is opposite in direction to that of active couple.



Axis of precession
F
Y
Z
F
Axis of active couple
A
B
O
X
Axis of spin
R_A
100 mm
R_B



Reactive gyro. couple
x′
O
X
Active gyro.couple

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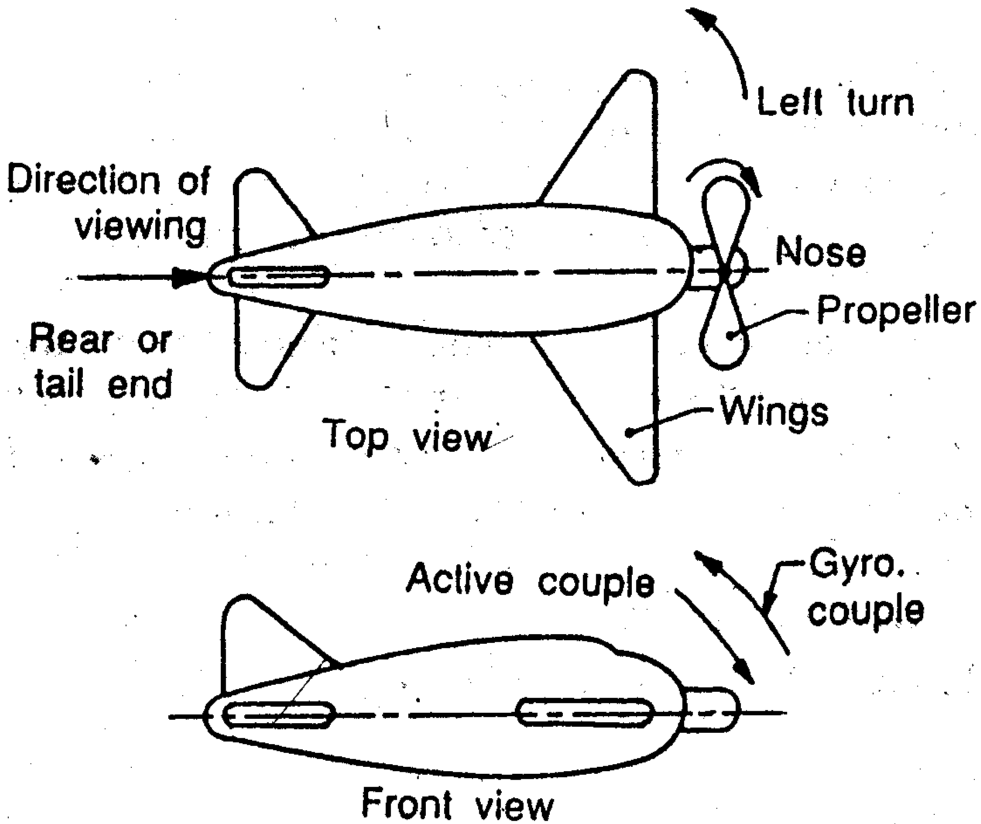
Effect of the Gyroscopic Couple on an Aeroplane

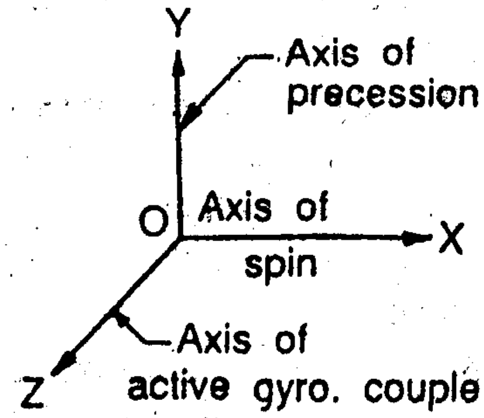
Let ω = Angular velocity of the engine in rad/s,

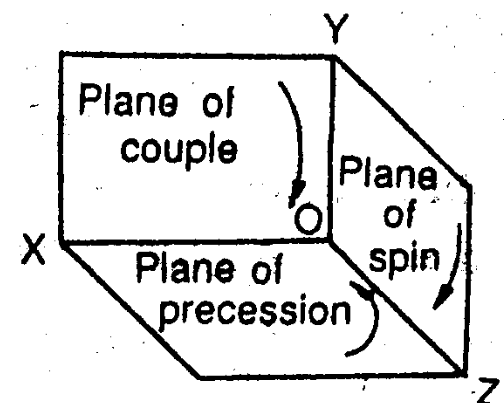
m = mass of the engine and propeller in kg,

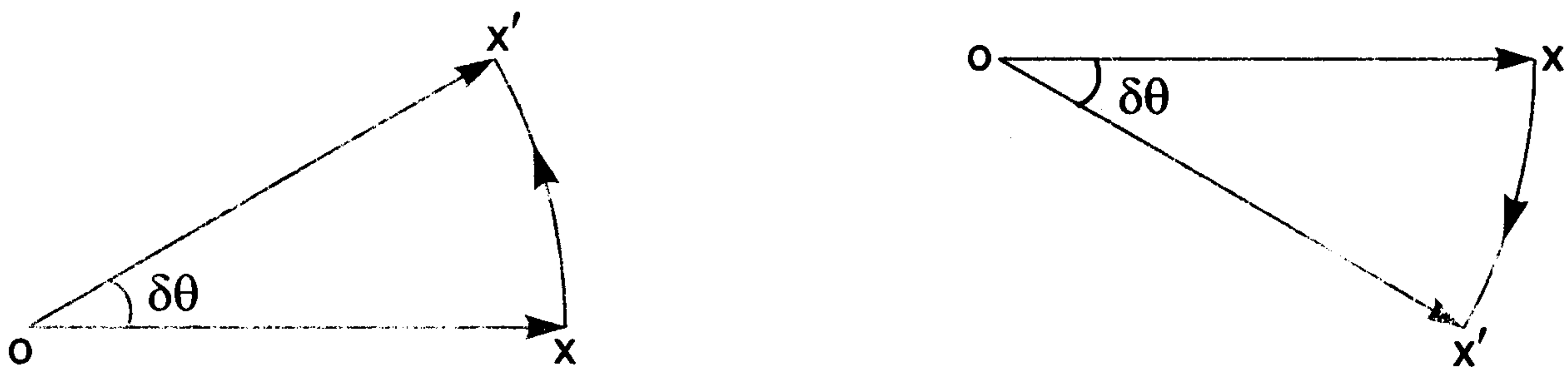
k = Its radius of gyration in metres,

I = Mass moment of inertia of the engine and propeller in kg-m^2 = m.k^2


Left turn
Direction of viewing
Nose
Propeller
Rear or tail end
Wings
Top view
Active couple
Gyro. couple
Front view


Y
Axis of precession
O
Axis of spin
X
Z
Axis of active gyro. couple


Y
Plane of couple
Plane of spin
O
X
Plane of precession
Z



(*a*) Aeroplane taking left turn. (*b*) Aeroplane taking right turn.

Fig. 14.6. Effect of gyroscopic couple on an aeroplane.

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v = Linear velocity of the aeroplane in m/s

R = radius of curvature in metres, and

ω_P = Angular velocity of precession = rad/s = $\dfrac{v}{R}$

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Notes:

1. .When the aeroplane takes a left turn, the effect of the reactive gyroscopic couple will be to raise the nose and dip the tail.
2. .When the aeroplane takes a right turn, the effect will be to dip the nose and raise the tail.
3. .When the engine rotates in anticlockwise direction when viewed from the front and the aeroplane takes a left turn, the effect will be to raise the tail and dip the nose.

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4. When the aeroplane takes a right turn and the engine rotates in anticlockwise direction, the effect will be to raise the nose and dip the tail.
5. When the engine rotates in clockwise direction and the aeroplane takes a left turn, the effect will be to raise the tail and dip the nose.
6. When the aeroplane takes a right turn and the engine rotates in clockwise direction, the effect will be to raise the nose and dip the tail.

Terms Used in a Naval Ship

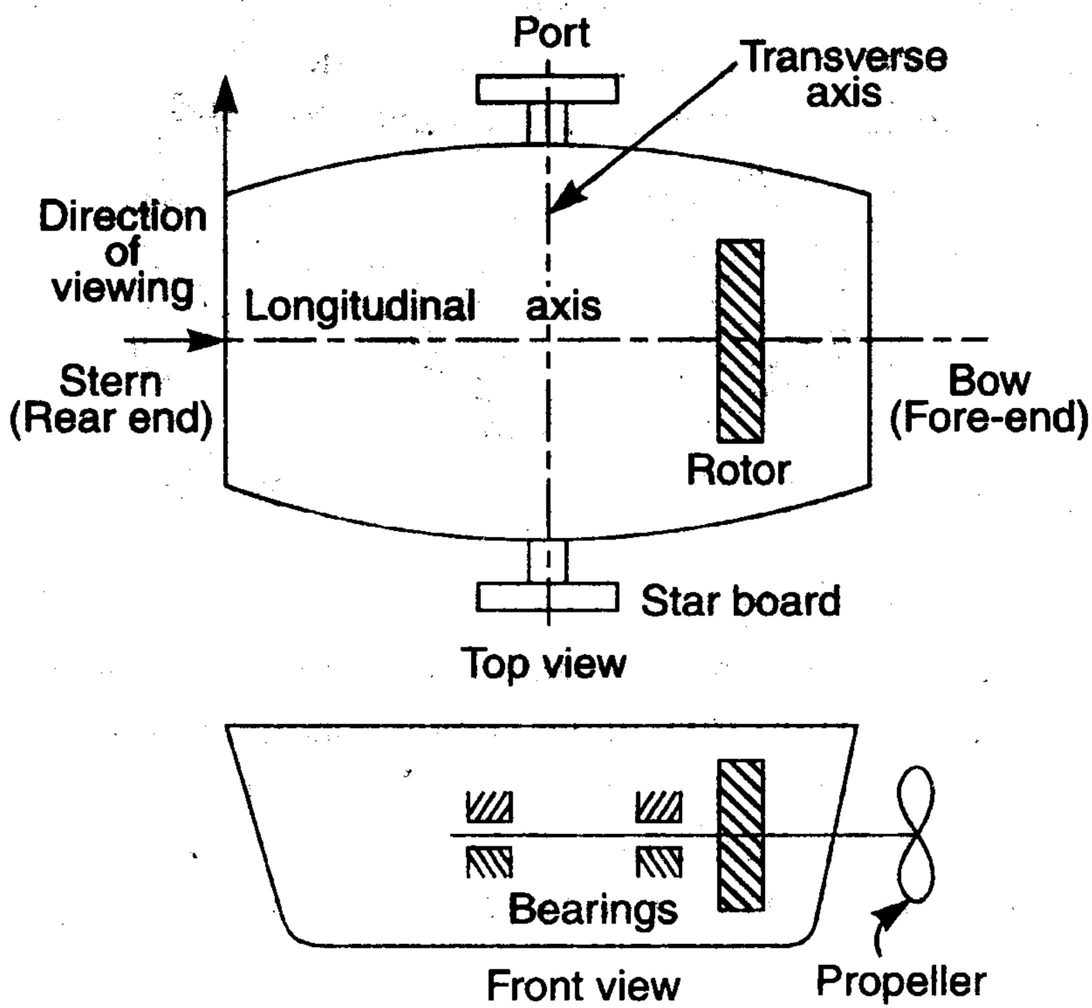

Fig. 14.7. Terms used in a naval ship.

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Effect of Gyroscopic Couple on a Naval Ship during Steering

1. When the rotor of the ship rotates in the clockwise direction when viewed from the stern, and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

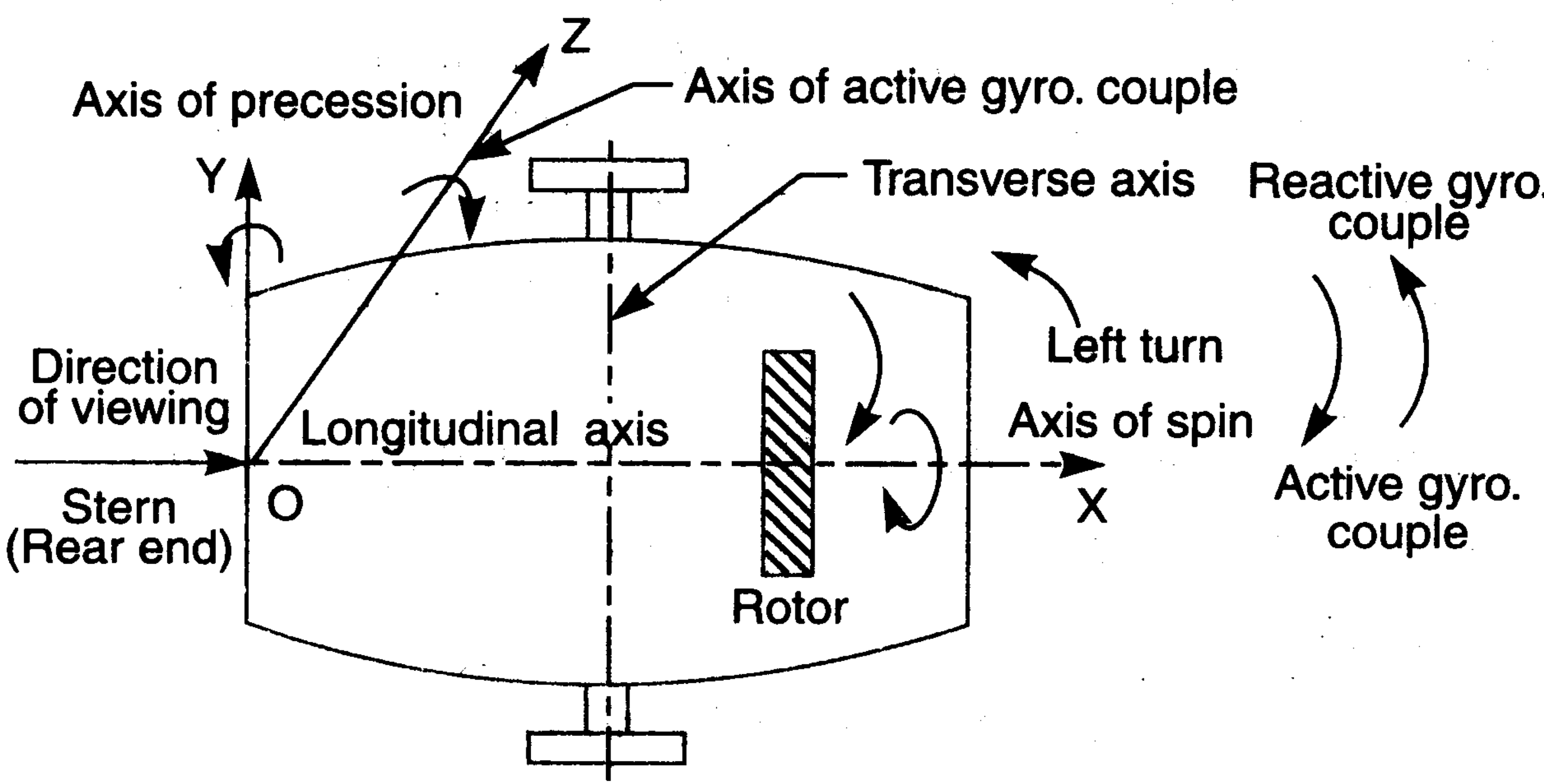

Fig. 14.8. Naval ship taking a left turn.

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2. When the ship steers to the right and the rotor rotates in the clockwise direction, the effect will be to raise the stern and lower the bow.
3. When the rotor rotates in the anticlockwise direction and the ship steers to the left and the effect will be to lower the bow and raise the stern

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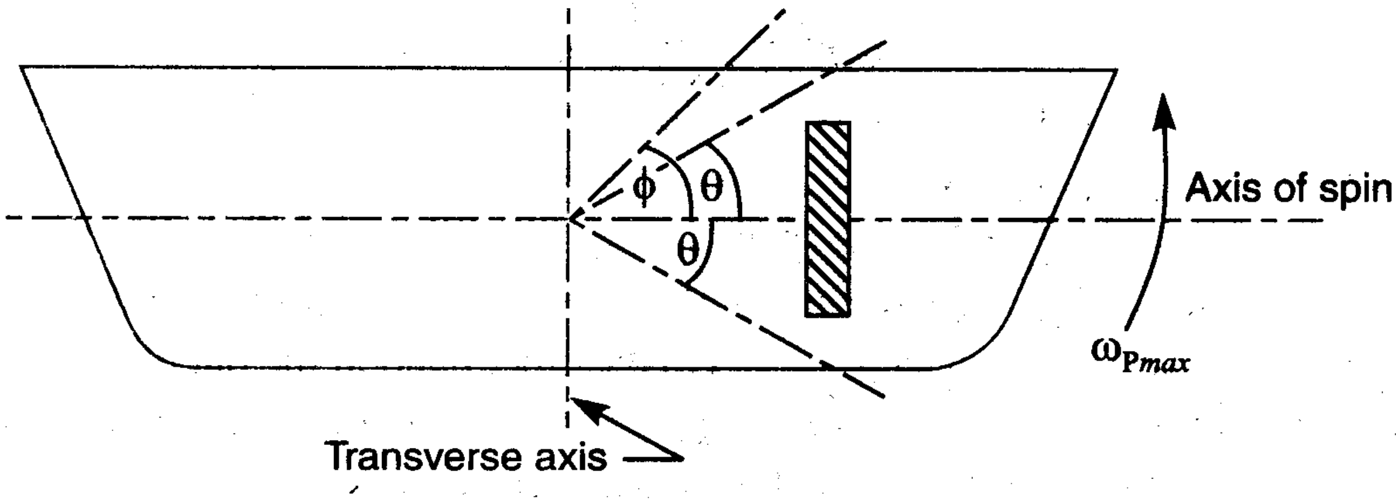
Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis. The pitching of the ship is assumed to takes place with simple harmonic motion.



(*a*) Pitching of a naval ship

(*b*) Pitching upward

(*c*) Pitching downward

Fig. 14.10. Effect of gyroscopic couple on a naval ship during pitching.

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Angular displacement of the axis of spin from mean position after time t seconds

$\Theta = \phi \sin \omega_1 . t$

where ϕ = Amplitude of swing, t_p

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{Time\ period\ of\ S.H.M.\ in\ seconds} = \frac{2\pi}{t_P}$$

.

Maximum angular velocity of precession

$\omega_{Pmax} = \phi.\omega_1 = \phi \times 2\pi/t_P$

Maximum gyroscopic couple

$C_{max} = I.\ \omega.\ \omega_{Pmax}$

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When the pitching is upward, the reactive gyroscopic couple will try to move the ship toward star-board. If the pitching is downward, the effect is to turn the ship towards port side.

Note: There is no effect of the gyroscopic couple acing on the body of the ship during rolling.

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Stability of a Four Wheel Drive Moving in a Curved Path

Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons =m.g,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres,

h = Distance of c.g. vertical above the road surface in metres,

x = Width of track in metres,

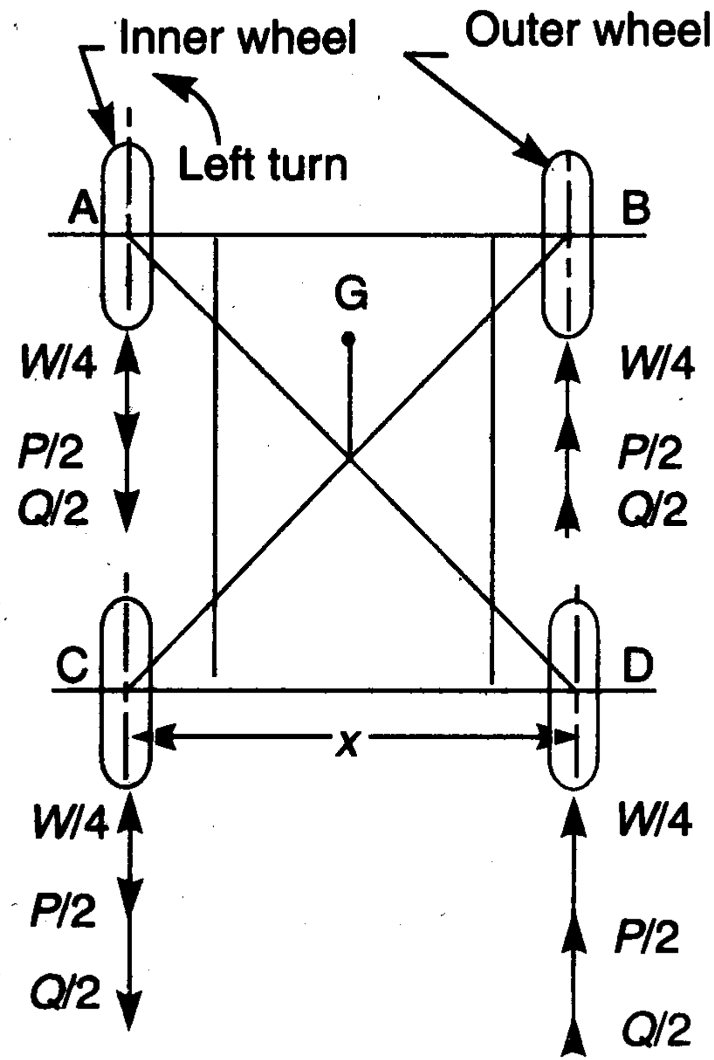

Fig. 14.11. Four wheel drive moving in a curved path.

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I_W = Mass moment of inertia of one of the wheels

in kg-m^2,

ω_W = Angular velocity of the wheels,

I_E = Mass moment of inertia of the rotating parts of the engine in rad/s,

G = gear ratio = ω_E/ω_W

v = Linear velocity of the vehicle in m/s = $\omega_W.r_W$

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1. Effect of the gyroscopic couple

Velocity of precession

$\omega_P = v/R$

Gyroscopic couple due to four wheels,

$C_W = 4I_{W.}\omega_W.\omega_P$

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Gyroscopic couple due to rotating parts of the engine

$C_E = I_{E.}\omega_E.\omega_P = I_{E.}G.\omega_W.\omega_P$

Net gyroscopic couple = $C_W \pm C_E$

$= \omega_W.\omega_P\ (4I_W \pm G.I_E)$

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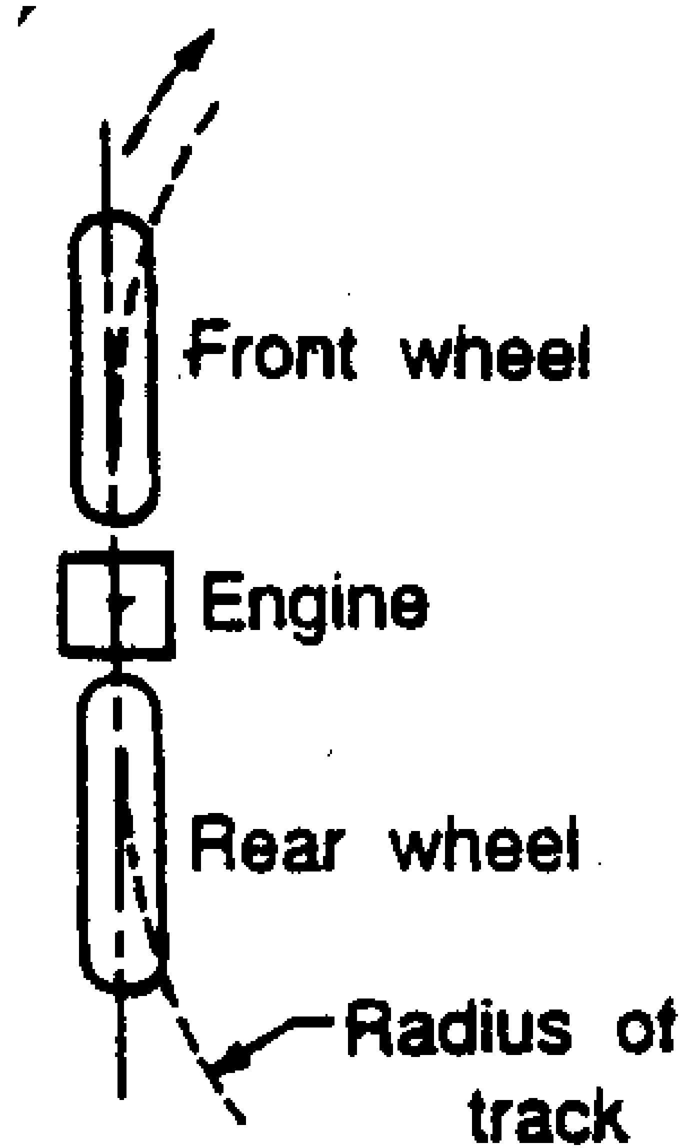
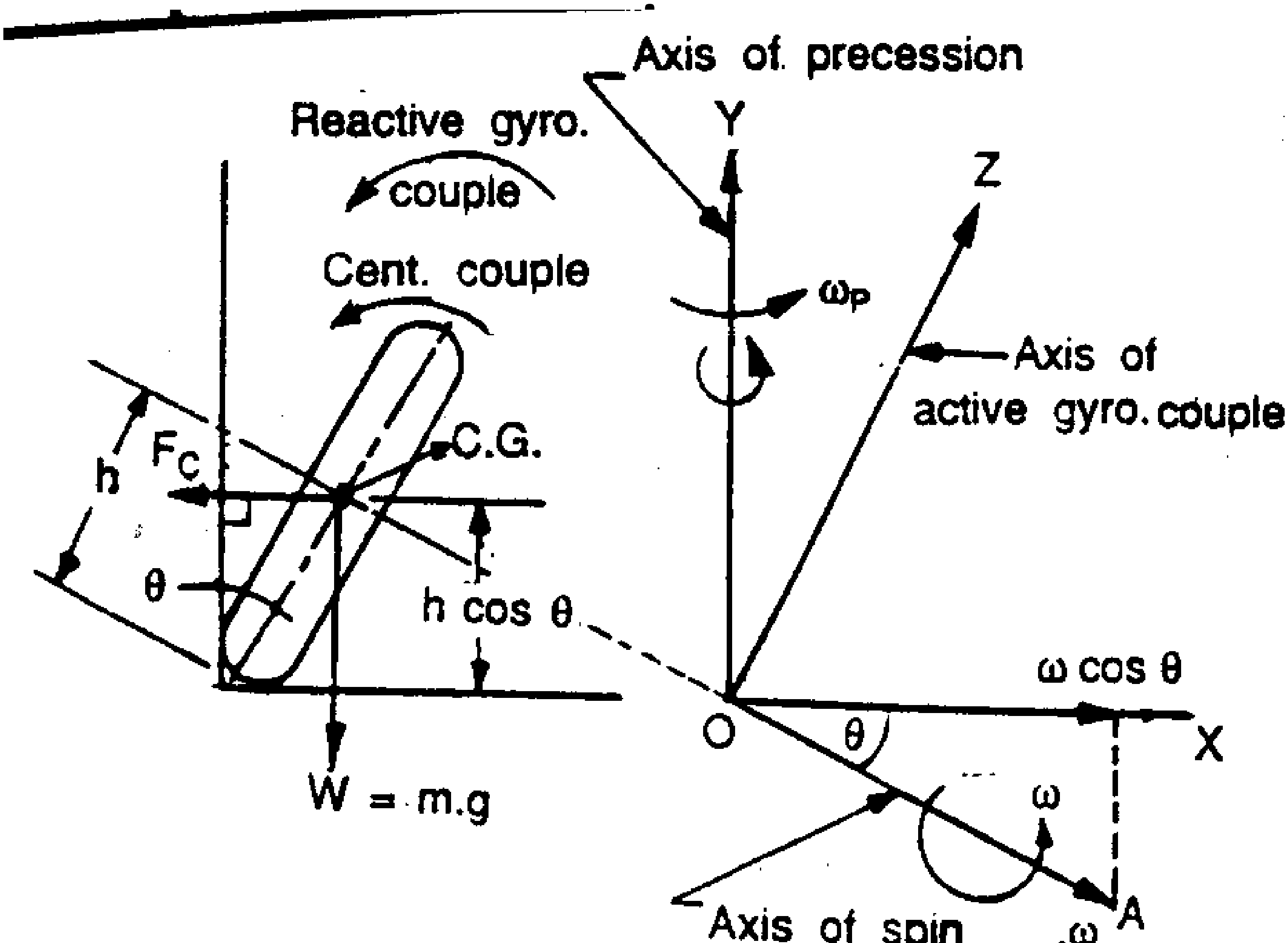
Stability of a Two Wheel Vehicle taking a Turn

Total angular momentum

$$(I \times \omega) = \frac{v}{r_W}\left(2I_W \pm G.I_E\right)$$



Front wheel
Engine
Rear wheel
Radius of
track



Axis of precession
Reactive gyro. couple
Y
Z
Cent. couple
ωp
Axis of active gyro. couple
C.G.
Fc
h
θ
h cos θ
ω cos θ
O
θ
X
W = m.g
ω
Axis of spin
.ω
A

.

Gyroscopic couple,

C1 = I.ω cos ϴ × ω_P

$$= \frac{v}{r_W}\left(2I_W \pm G.I_E\right)\cos\theta \times \frac{v}{R}$$

$$= \frac{v^2}{R.r_W}\left(2I_W \pm G.I_E\right)\cos\theta$$

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Centrifugal couple

$C_2 = F_C \times h \cos \Theta = \left(\frac{m.v^2}{R} \right) h \cos\theta$

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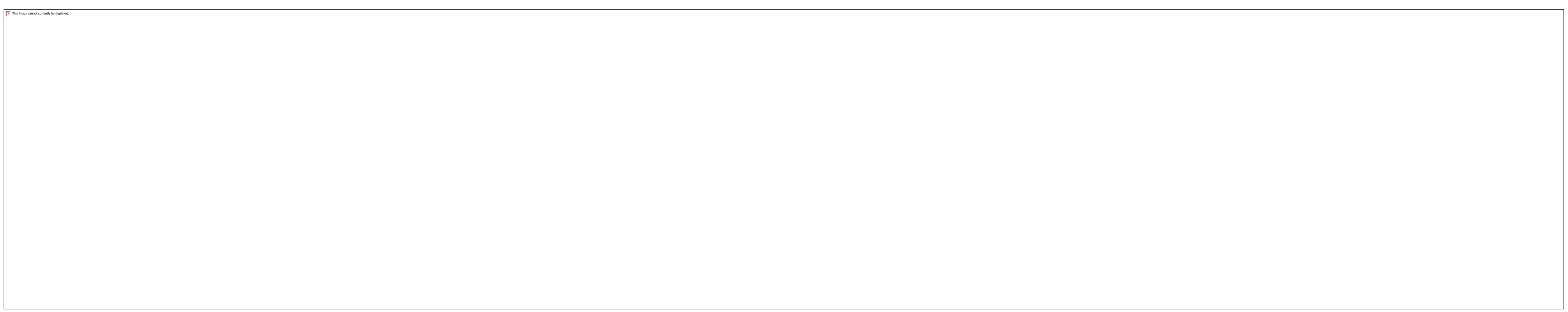
Total overturning couple

C_O = Gyroscopic couple + Centrifugal couple

$$= \frac{v^2}{R}\left[\frac{2I_W + G.I_E}{r_W} + m.h\right]\cos\theta$$

Balancing couple = m.g.h sin ϴ

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple i.e.



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Example 1: A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing . If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis.

K/

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Solution: $r = 0.15$ m; $m = 5$ kg; $l = 0.6$ m; $\omega = 31.42$ rad/s

Mass moment of inertia of the disc

$$I = m.r^2 / 2 = 0.056 \text{ kg-m}^2$$

Couple due to mass of disc,

$$C = m.g.l = 29.43 \text{ N-m}$$

Let ω_P = speed of precession,

$$C = I.\omega.\omega_P$$

$$\omega_P = 16.7 \text{ rad/s}$$

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Example 2: A uniform disc of 150 mm diameter has a mass of 15 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about its axle with a constant speed of 100 rpm while axle precesses uniformly about the vertical at 60 rpm. The directions of rotation are as shown in the figure. If the distance between the bearing is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effect. **K/ /(S08)**

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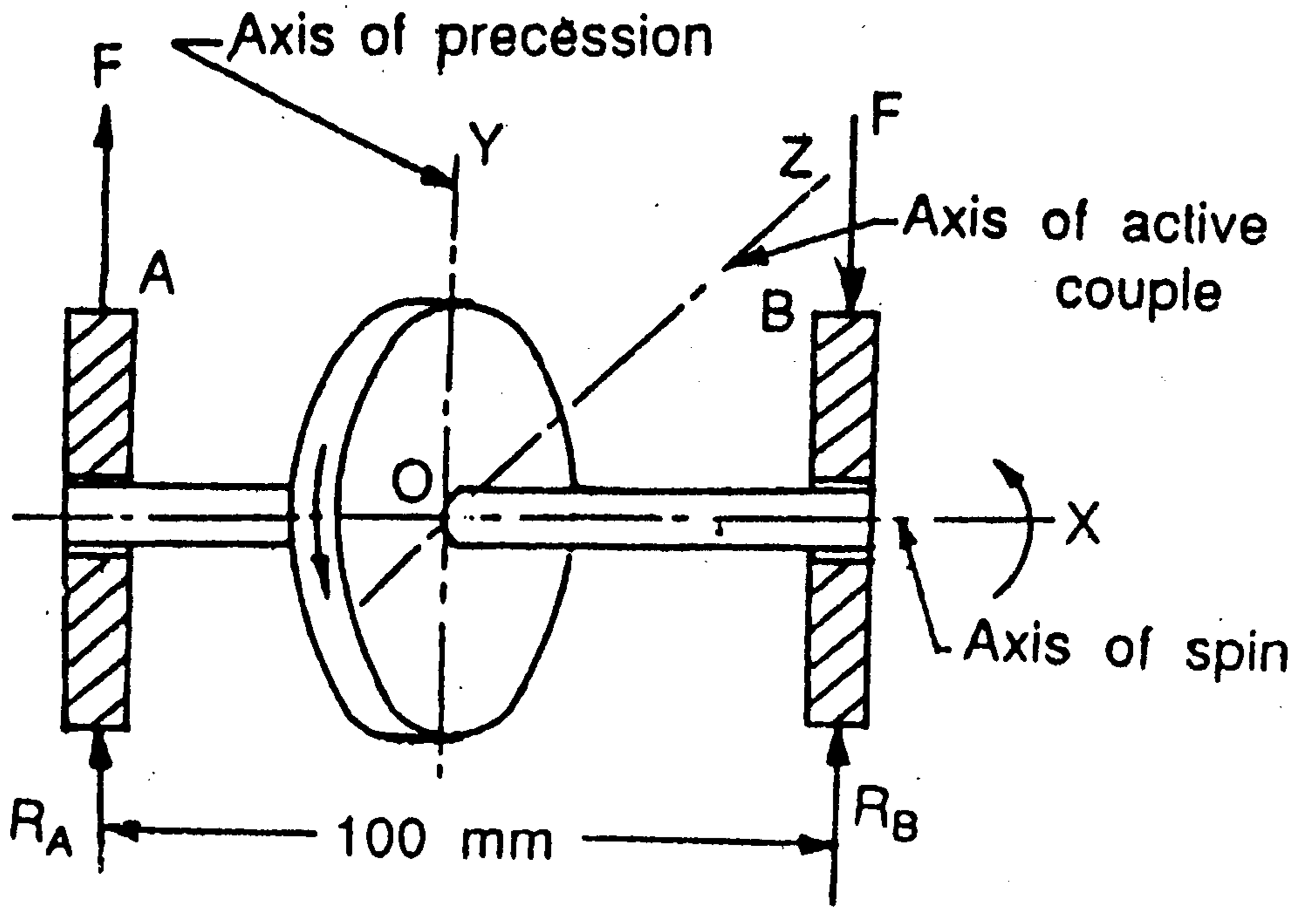
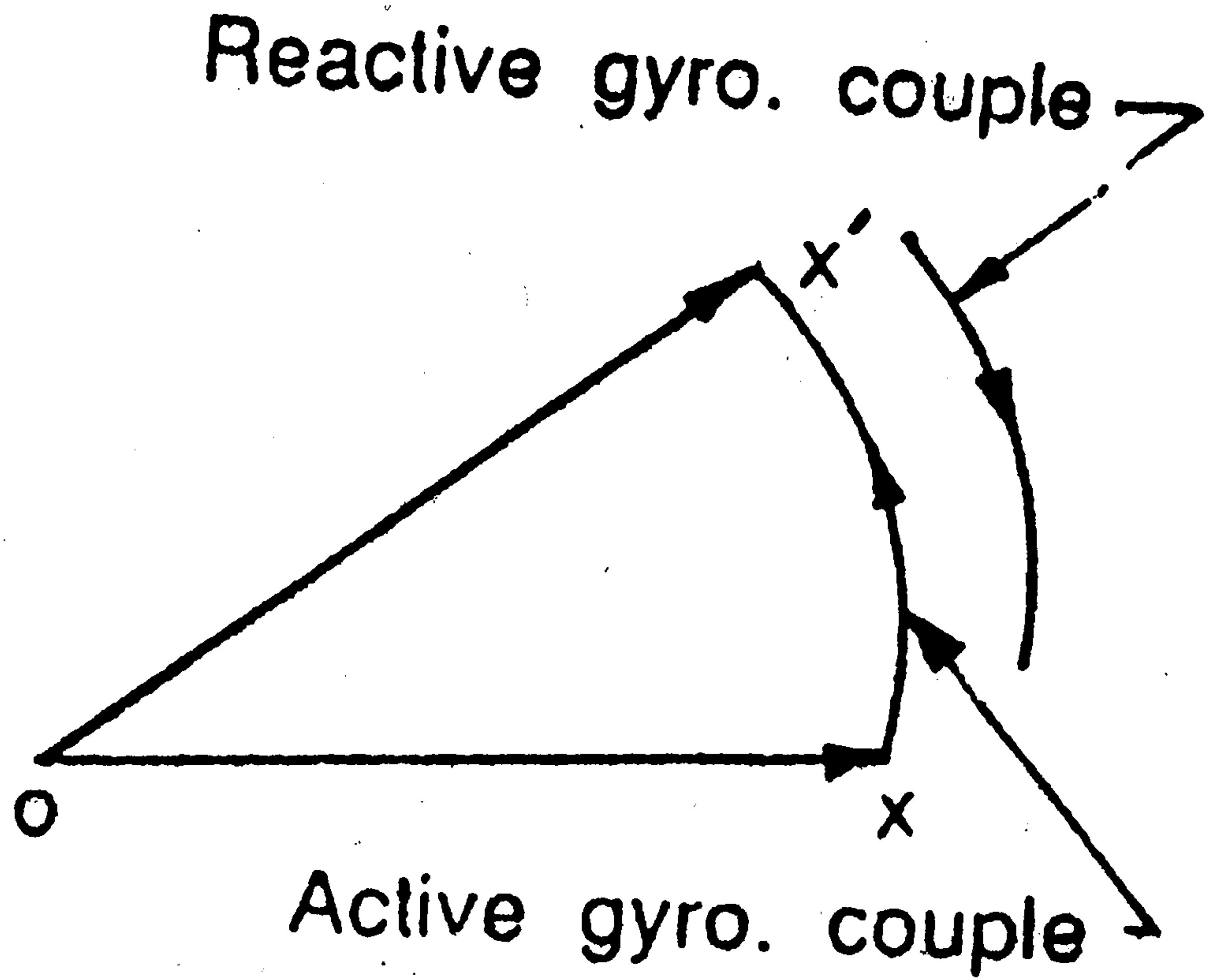
Solution:



Axis of precession
F
Y
Z
F
Axis of active couple
A
B
O
X
Axis of spin
R_A
100 mm
R_B

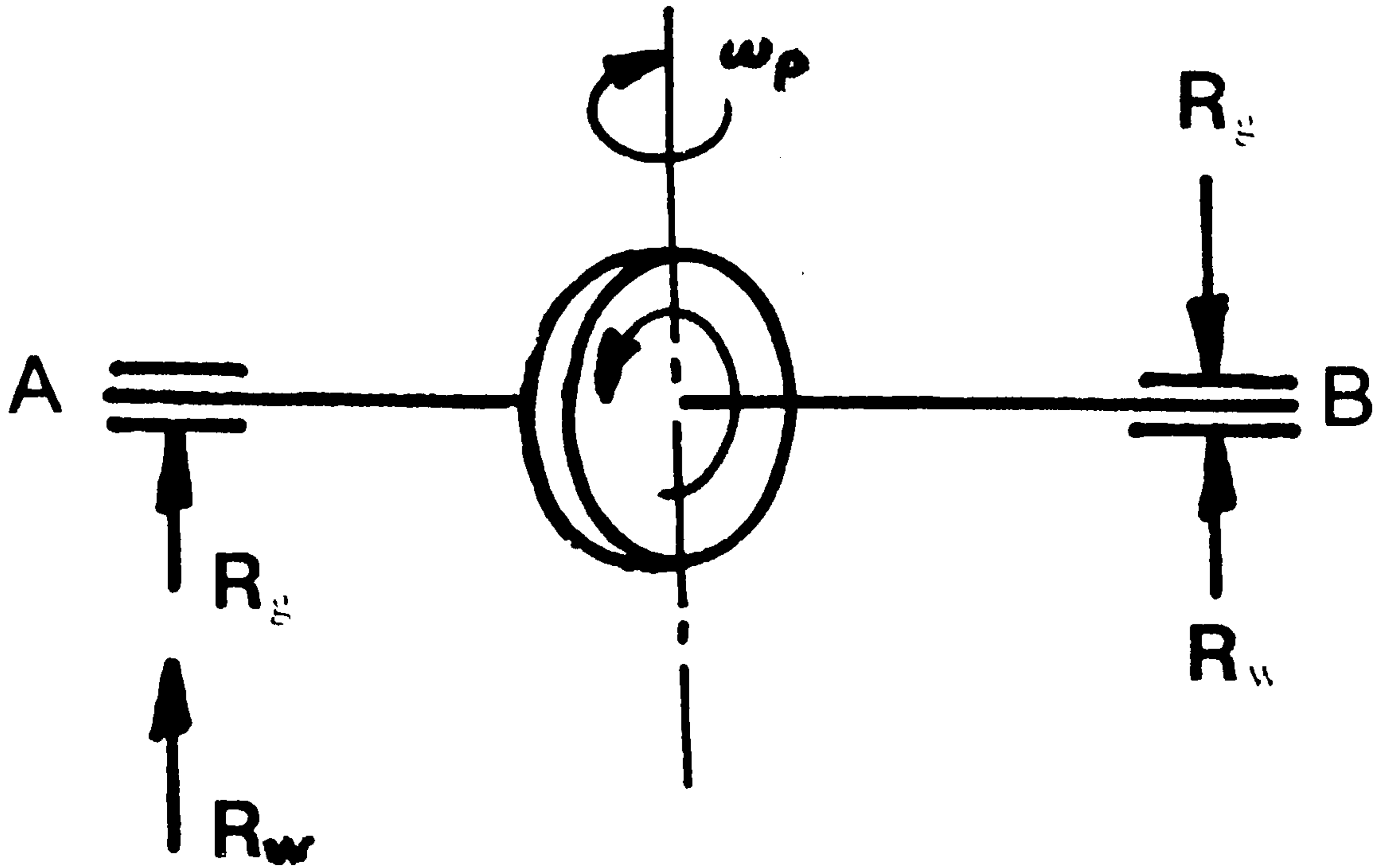


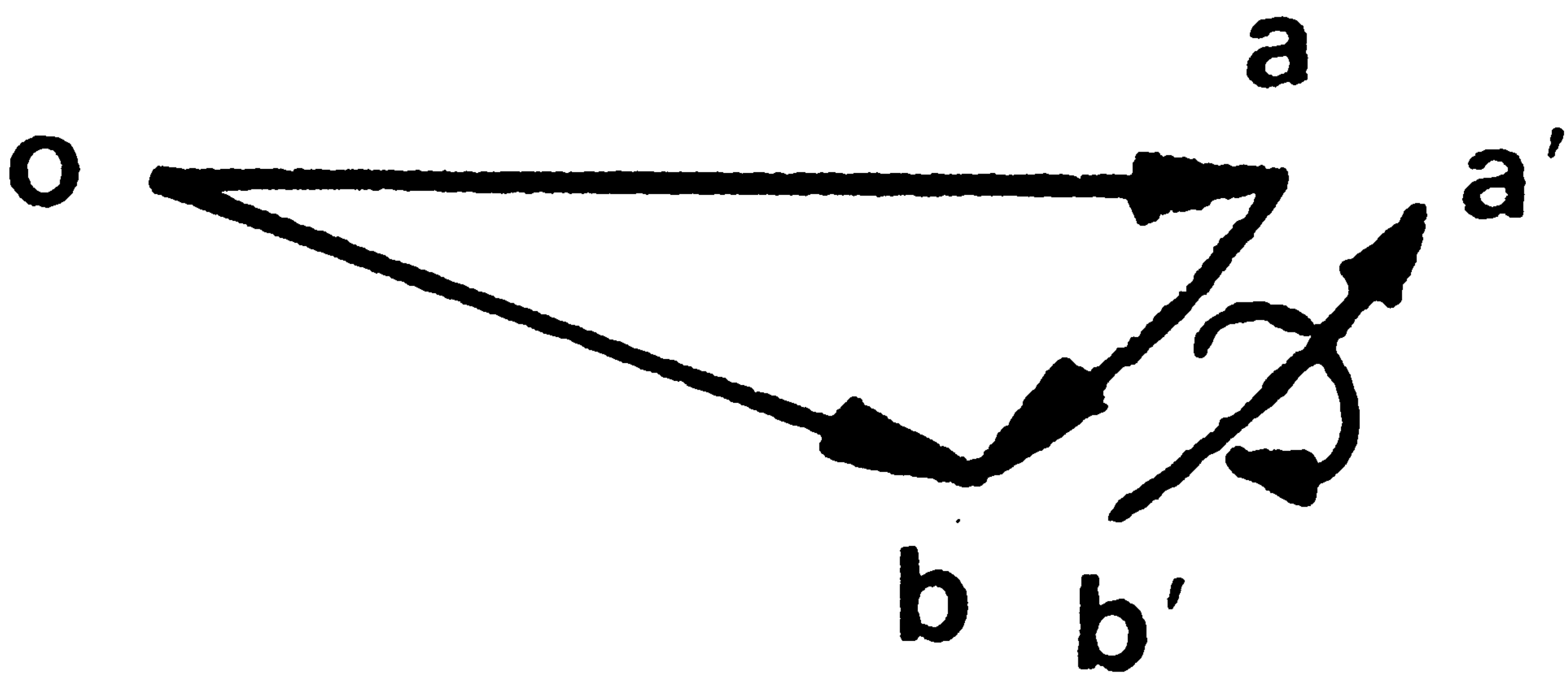
Reactive gyro. couple
X'
O
X
Active gyro. couple

Example 3: A disc with radius of gyration of 60 mm and a mass of 4 kg is mounted centrally on a horizontal axle of 80 mm length between the bearings. It spins about the axle at 800 rpm counter clockwise when viewed from the right hand side bearing. The axle precesses about a vertical axis at 50 rpm in the clockwise direction when viewed from the above. Determine the resultant reaction at each bearing due to the mass and gyroscopic effect.

R/600

Solution:





o
a
a′
b
b′

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Example 4: A flywheel having a mass of 20 kg and a radius of gyration of 300 m is given a spin of 500 rpm about its axis which is horizontal. The flywheel is suspended at a point that is 250 mm from the plane of rotation of the flywheel. Find the rate of precession of the wheel. **R/614/(W08)**

Solution:

$\omega_P = 0.52$ rad/s

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Example 5: An aircraft consists of propeller and engine. The mass moment of inertia of propeller and engine is 100 kg-m2. The engine rotates at 2500 rpm in the clockwise direction if viewed from front. The aircraft completes half circle of radius of 1000 m towards right while flying at 50 km/h. Determine the gyroscopic couple on the aircraft and state its effect. **(W07)**

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Example 6: An aeroplane makes a complete half circle of 50 metres radius, towards left, When flying at 200 km/hr. The rotary engine and propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

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Example 7: The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/h and steer to the left in a curve of 75 m radius. **K/**

Solution:

Gyroscopic couple = $I.\omega.\omega_P$

$$= 8 \times 1000 \times (0.6)^2 \times \frac{2\pi \times 1800}{60} \times \frac{100 \times 1000}{60 \times 60 \times 75}$$

$$= 201062\, N-m = 201 kN-m$$

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left the effect of gyroscopic couple is to raise the bow and lower the stern.

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Example 8: The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 rpm clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.
2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching

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Is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

K/ /(S08)

Solution:

1. When the ship is steering to the left

Gyroscopic couple $C = I.\omega.\omega_P$

$$= 3500 \times (0.45)^2 \times \frac{2\pi \times 3000}{60} \times \frac{36 \times 1000}{60 \times 60 \times 100}$$

$$= 22270\, N - m = 22.27\, kN - m$$

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left the effect of gyroscopic couple is to raise the bow and lower the stern.

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2. When the ship is pitching with the bow falling

$t_p = 40$ s

Amplitude of swing $\Phi = 12/2 = 6^\circ$

$= 6 \times \pi/180 = 0.105$ rad/s

Angular velocity of S.H.M. ω_1

$= 2\,\pi/tp = 0.157$ rad.s

Maximum angular velocity of precession

$\omega_P = \Phi.\omega_1 = 0.0165$ rad/s

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Gyroscopic couple = $I.\omega.\omega_P$ = 3675 N-m = 3.675 k-N

When the bow is falling, the effect of the reactive gyroscopic couple is to move the ship towards port side.

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Example 9: A ship is propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 rpm. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions:

1. The ship sails at a speed of 30 km/hr. and steers to the left in a curve having 60 m radius.

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2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds.
3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern.

Determine also the maximum angular acceleration during pitching. Explain

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How the direction of motion due to gyroscopic effect is determined in each case.

K/ /(W08)

Solution:

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Example 10: The rotor of the turbine of a ship has a mass of 2500 kg and rotates at a speed of 3200 rpm counter clockwise when viewed from stern. The rotor has radius of gyration of 0.4 m. determine the gyroscopic couple and its effects when

(i) the ship steers to the left in a curve of 80 m radius at a speed of 15 knots (1 knot = 1860 m/h)

(ii) the ship pitches 5 degrees above and 5 degrees below th normal position and the

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bow is descending with its maximum velocity – the pitching motion is simple harmonic with a periodic time of

(iii) the ships rolls and at the instant its angular velocity is 0.4 rad/s clockwise when viewed from stern.

Also find the maximum angular acceleration during pitching. **R/604/(S09)**

Solution:

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Example 11: Each wheel of a four-wheeled rear engine automobile has a moment of inertia of 2.4 kg-m^2 and an effective diameter 660 mm. The rotating parts of the engine has moment of inertia of 1.2 kg-m^2. The gear ratio of the engine to the back wheel is 3 to 1. The engine axis is parallel to the rear axle and the crank shaft rotates in the same sense as the road wheels. The mass of the vehicle is 2200 kg and the centre of the mass is 550 mm above the road level. The track width of

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the vehicle is 1.5 m. Determine the limiting speed of the vehicle around a curve with 80 m radius so that all the four wheels maintain contact with the road surface. **R/607/(W07)**

Solution:

$R_W = 5395.5$ N (upward)

$C_W = 0.364\ v^2$

$C_E = 0.136\ v^2$

$C_G = C_W + C^E = 0.5\ v^2$

.

$R_{go} = 0.167\ v^2$

$R_{gi} = 0.167\ v^2$

$C_C = 15.125\ v^2$

$R_{co} = 5.042\ v^2$

$R_{ci} = 5.042\ v^2$

For maximum safe speed, the condition is

$R_W = R_{Gi} + R_{ci}$

$V = 115.9$ km/h

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Example 12: A car is of total mass 2000 kg. It has wheel base equal to 2.4 m and track width 1.4 m. The centre of gravity lies at 500 mm above ground level. The effective diameter of each wheel is 800 mm and mass moment of inertia of each wheel is 1 kg-m^2. The rear axle ratio is 4. The mass moment of inertia of engine rotating parts is 3 kg-m^2. The engine is rotating in the same sense as the wheels.

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Determine the critical speed of the car when it takes a right turn of 100 m radius. (Critical speed is a speed up to which wheels remains in contact with road.

(W08)

Solution:

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Example 13: A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/hr. The track is banked at 8˚ The wheels have an external diameter of 0.7 m and each pair with axle has mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m.

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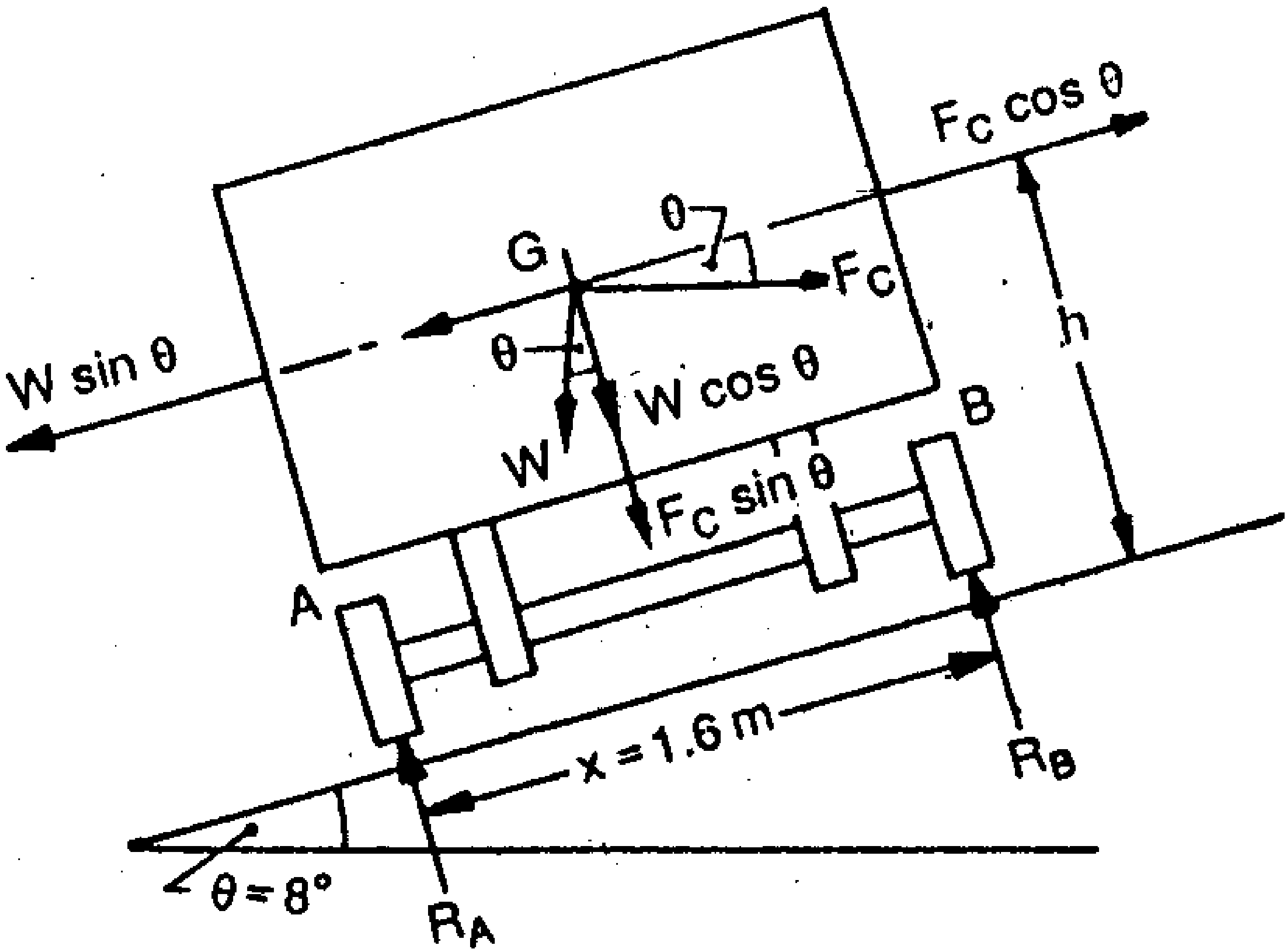
Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail. **K/502/(S09)**

Solution:



$F_C \cos\theta$
θ
G
F_C
h
W sin θ
θ
W cos θ
B
W
$F_C \sin\theta$
A
x = 1.6 m
R_B
θ = 8°
R_A

.

Resolving the forces perpendicular to the track

$R_A + R_B = W \cos \Theta + F_C \sin \Theta$

$$= m.g \cos\theta + \frac{m.v^2}{R} \sin\theta = 21518\, N$$

.

Taking moments about B,

$$R_A \times x = \left(W\cos\theta + F_C\sin\theta\right)\frac{x}{2}$$

$$+W\sin\theta \times h - F_C\cos\theta \times h$$

$$R_A = \left(m.g\cos\theta + \frac{m.v^2}{R}\sin\theta\right)\frac{1}{2}$$

$$= \left(m.g\sin\theta - \frac{m.v^2}{R}\cos\theta\right)\frac{h}{x} = 3182N$$

$R_B = (R_A + R_B) – R_A = 18336$ N

$$\omega_W = \frac{v}{r_W} = 42.86\, rad/s$$

$$\omega_P = \frac{v}{R} = 0.5\, rad/s$$

Gyroscopic couple,

$C = I\, \omega_W \cos \Theta \times \omega_P = 382$ N-m

.

Due to this gyroscopic couple, the car will overturn about the outer wheel. Let P will be the force at each pair of wheels due to the gyroscopic couple

$P = C/x = 382/1.6$

Pressure on the inner rail,

$P_1 = R_A – P = 2943.25$

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Example 14: A rear engine automobile is travelling along a track of 100 metres mean radius. Each of the four road wheels has a moment of inertia of 2.5 kg-m^2 and an effective diameter of 0.6 m. The rotating parts of the engine have a moment of inertia of 1.2 kg-m^2. The engine axle is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The ratio of the engine speed to back axle speed is 3:1. The automobile has a mass of 1600 kg and has its centre of gravity 0.5 m above road level.

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The width of the track of the vehicle is 1.5 m.

Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface. Assume that the road surface is not cambered and centre of gravity of the automobile lies centrally with respect to the four wheels. **K/ /(W09)**

Solution:

Example 15: A motor cycle with a rider has a mass of 250 kg. The centre of gravity of the motor cycle and the rider falls 60 cm above the ground when running straight in vertical position. Each road wheel diameter is 60 cm with polar mass moment of inertia of 1 kg-cm^2. The engine rotates 6 times faster than the wheels in the same direction and the rotating parts of the engine have a mass moment of inertia of 0.175 kg-m^2. Determine the angle of inclination of the motorcycle or heel required if it is speeding at 80 km/hr. and rounding a curve of radius 50 m.

(S08)

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Example 16: Find the angle of inclination with respect to the vertical of a two wheeler negotiating a turn. Given: combined mass of the vehicle with its rider 250 kg; moment of inertia of engine flywheel 0.3 kg-m2; moment of inertia of each road wheel 1 kg-m2; speed of engine flywheel 5 times that of road wheel and in the same direction; height of centre of gravity of rider with vehicle 0.6 m; two wheeler speed 90 km/h. wheel radius 300 mm; radius of turn 50 m.

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Gyroscopic couple

$$C_1 = \frac{v^2}{R \times r_W}\left(2I_W + G.I_E\right)\cos\theta = 146\cos\theta\ N-m$$

Centrifugal couple,

$$C_2 = \frac{m.v^2}{R} \times h\cos\theta = 1875\cos\theta\ N-m$$

Total overturning couple,

$=C_1 + C_2 = 2021 \cos \Theta$ N-m

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Balancing couple

$= m.g.h \sin \theta = 1471.5 \sin \theta$ N-m

Since the overturning couple must be equal to the balancing couple, therefore

$2021 \cos \theta = 1471.5 \sin \theta$

$\tan \theta = 1.3734$ or $\theta = 53.94^\circ$