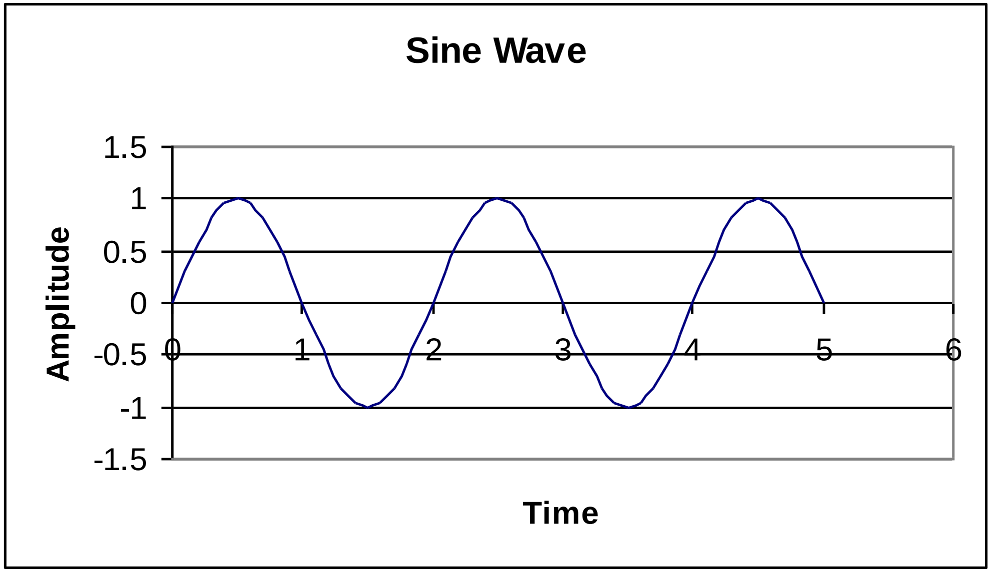
UNIT-3

DIGITAL COMMUNICATION

Sampling Theory

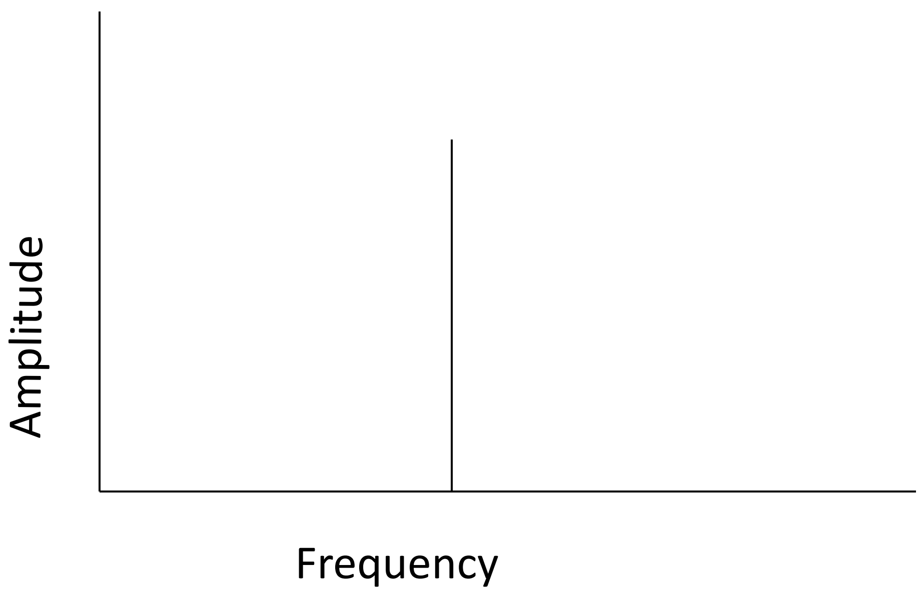
Time domain

- Present a recurring phenomena as amplitude vs. time
  - Sine Wave

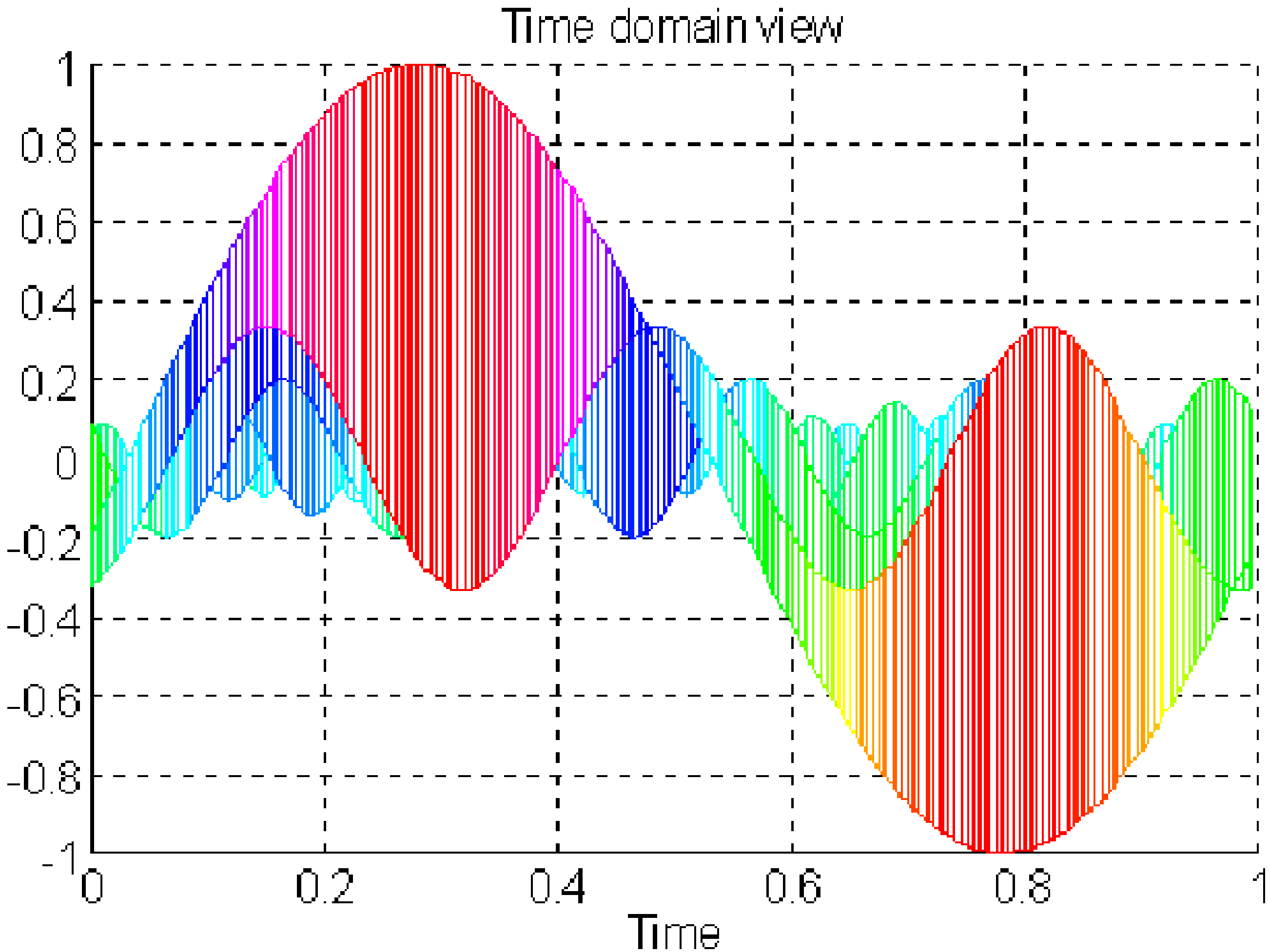


Frequency domain

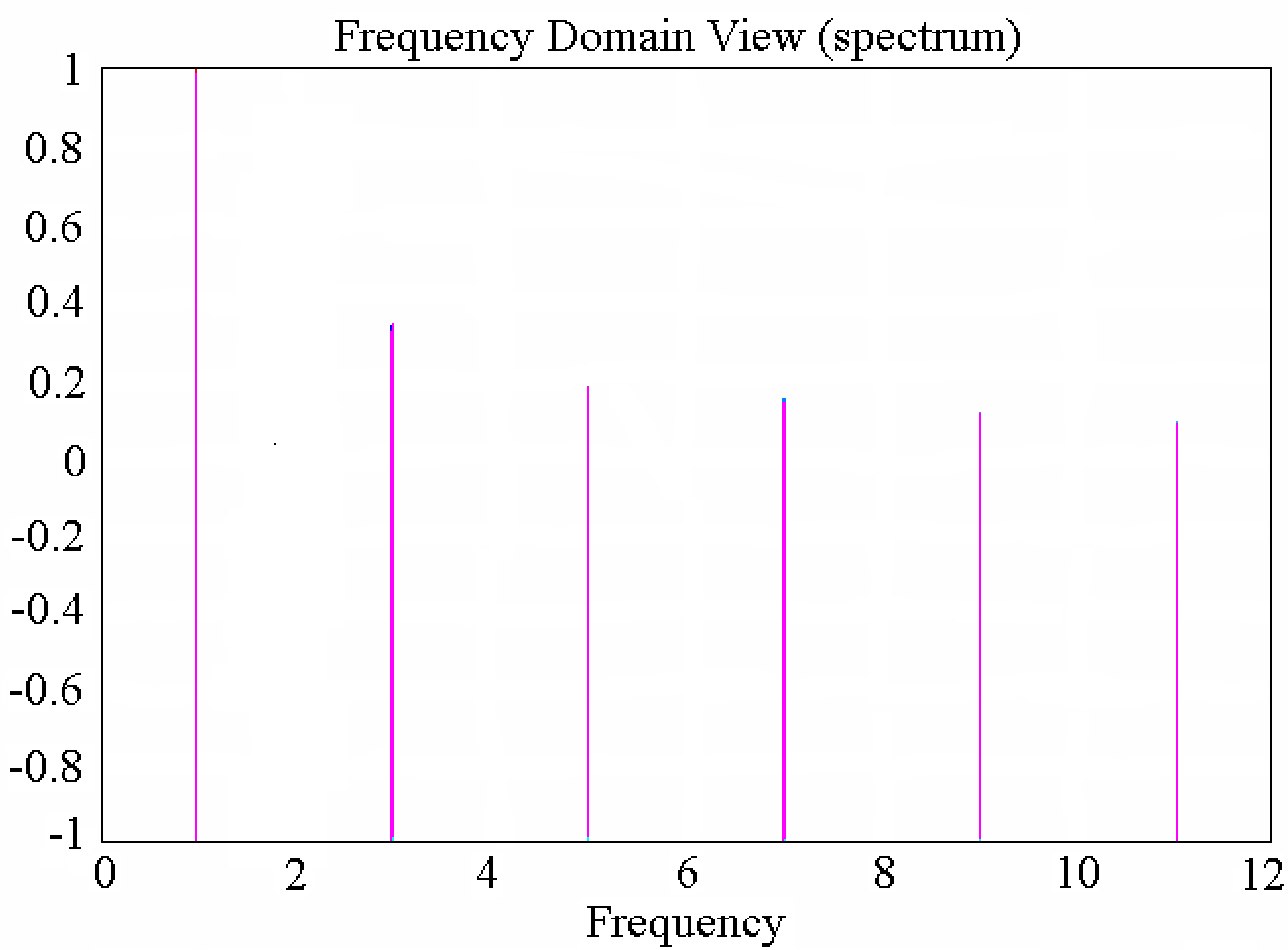
- Present recurring phenomena as amplitude vs. frequency
- Same sine wave looks like –



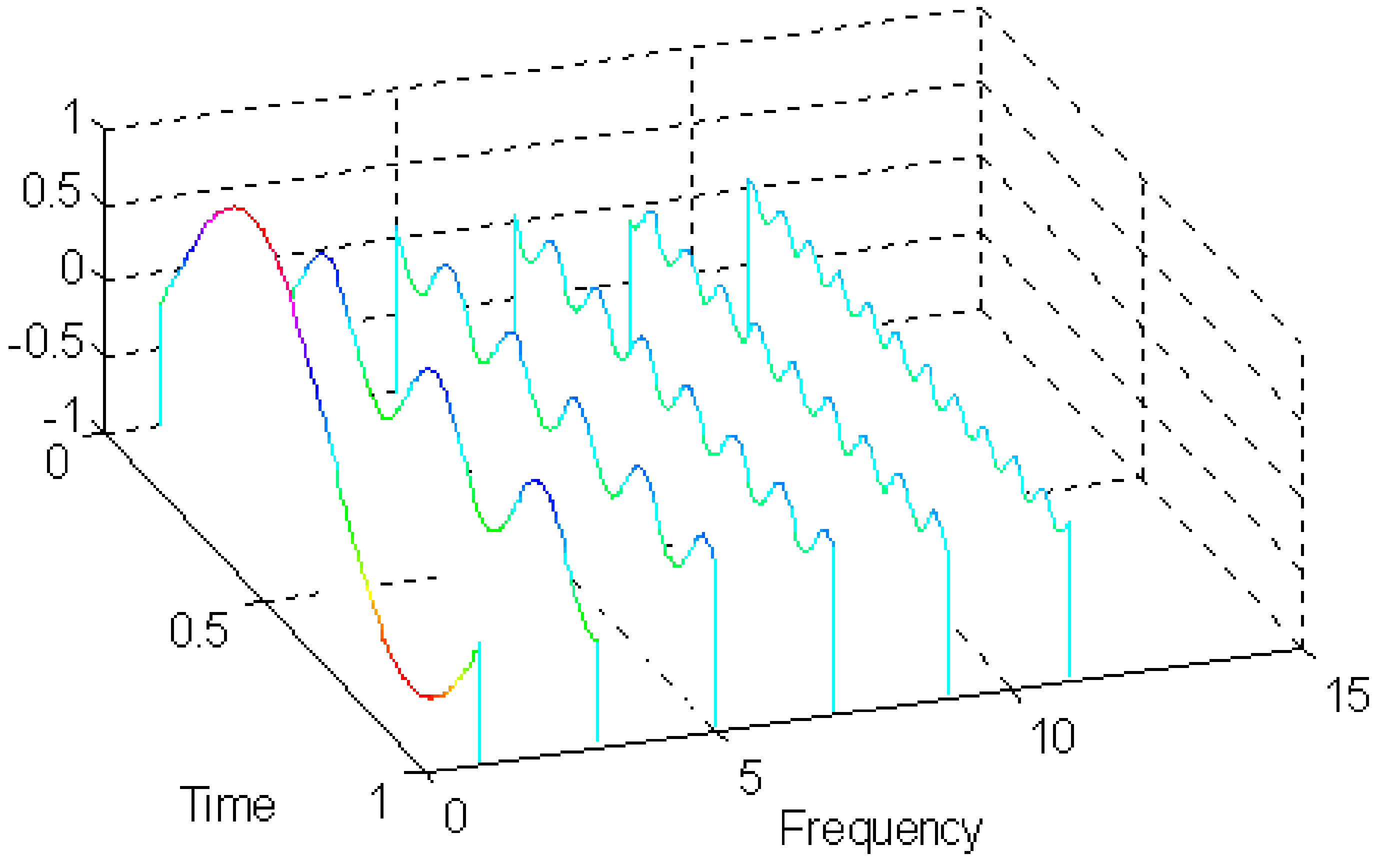
Multiple Waves



Multiple Waves

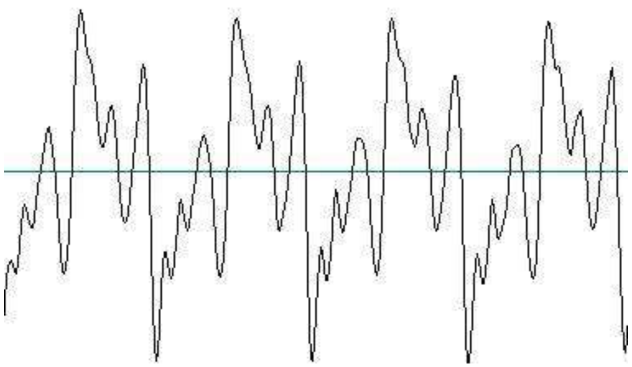


Both Domains

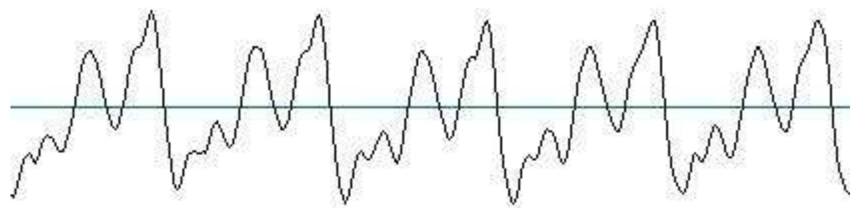


Harmonics

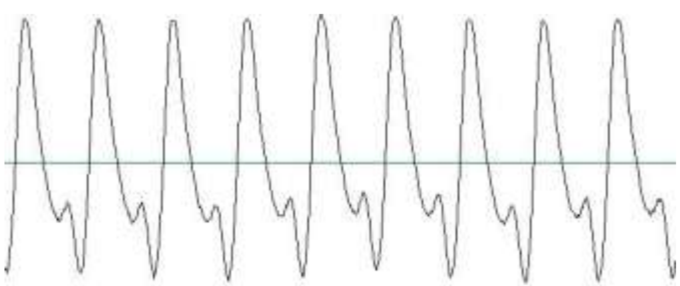
- See Spreadsheet



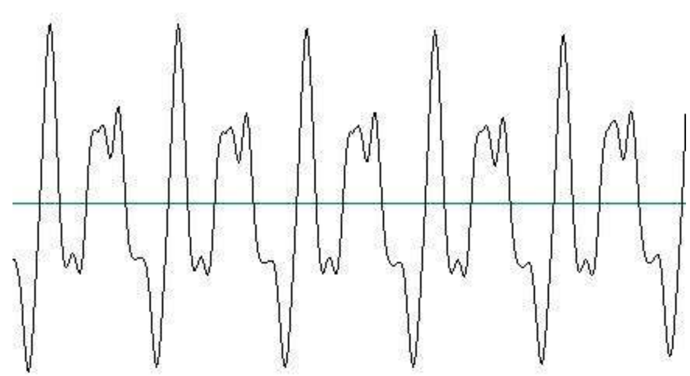
Clarinet



Flute



Horn



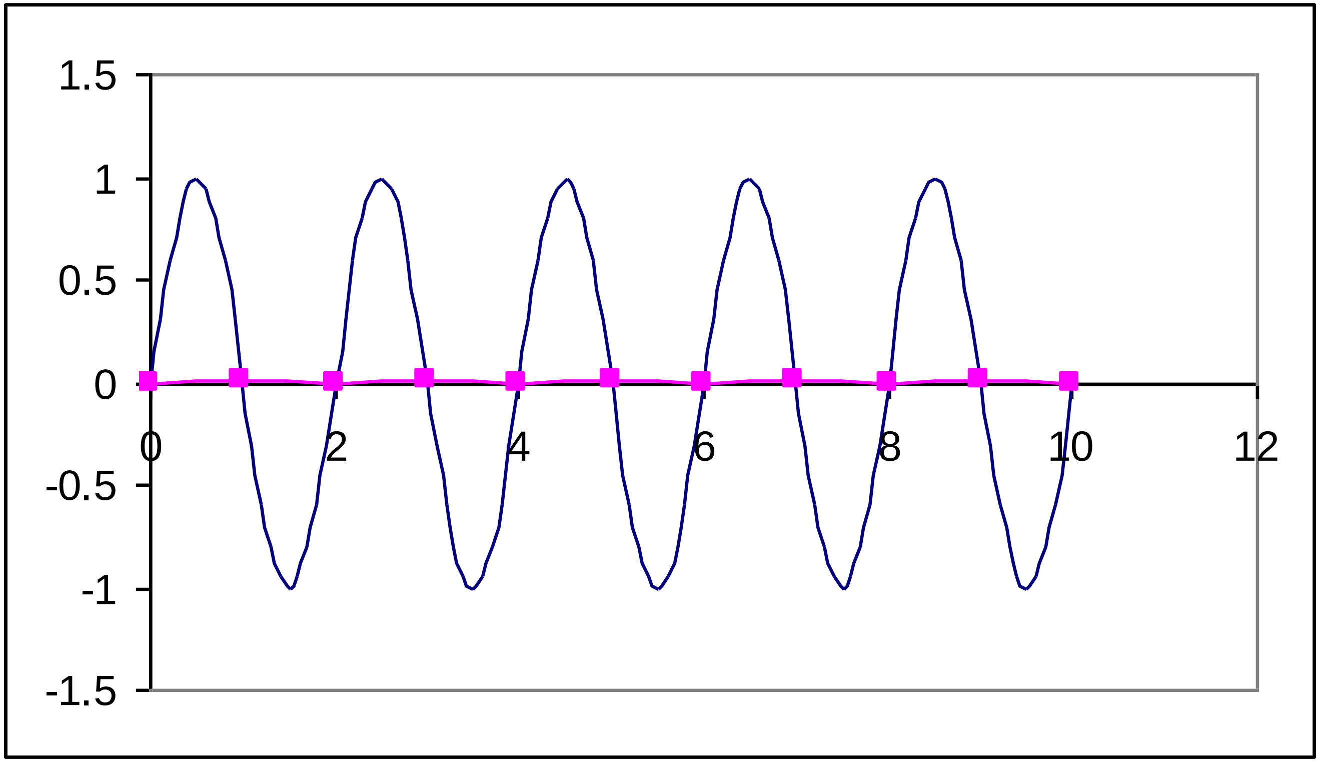
Guitar

Fourier Analysis

- The eardrum responds to a sum of all the waves arriving at a particular instant. Yet the individual sounds are "heard."

- Any waveform is composed of an infinite number of simple sine waves of various frequencies and amplitudes.

Poor Sampling



Sampling Frequency = 1/2 X Wave Frequency

Analog to Digital Conversion (A/D)

- In converting an analog signal to an equivalent sequence of “0’s” and “1’s”, we go through three processes:
  - Sampling:
    - converting continuous–time analog signals to discrete–time analog signals.
  - Quantization
    - converting discrete–time analog signals to discrete–time digital signals (finite set of amplitude levels).
  - Coding
    - Map each amplitude level to a binary sequence.

[1] Sampling: Mathematical Representation

- One sample of g(t) can be obtained from

$$g_s(t) = g(t)\cdot\delta(t - t_0) = g(t_0)\cdot\delta(t - t_0)$$

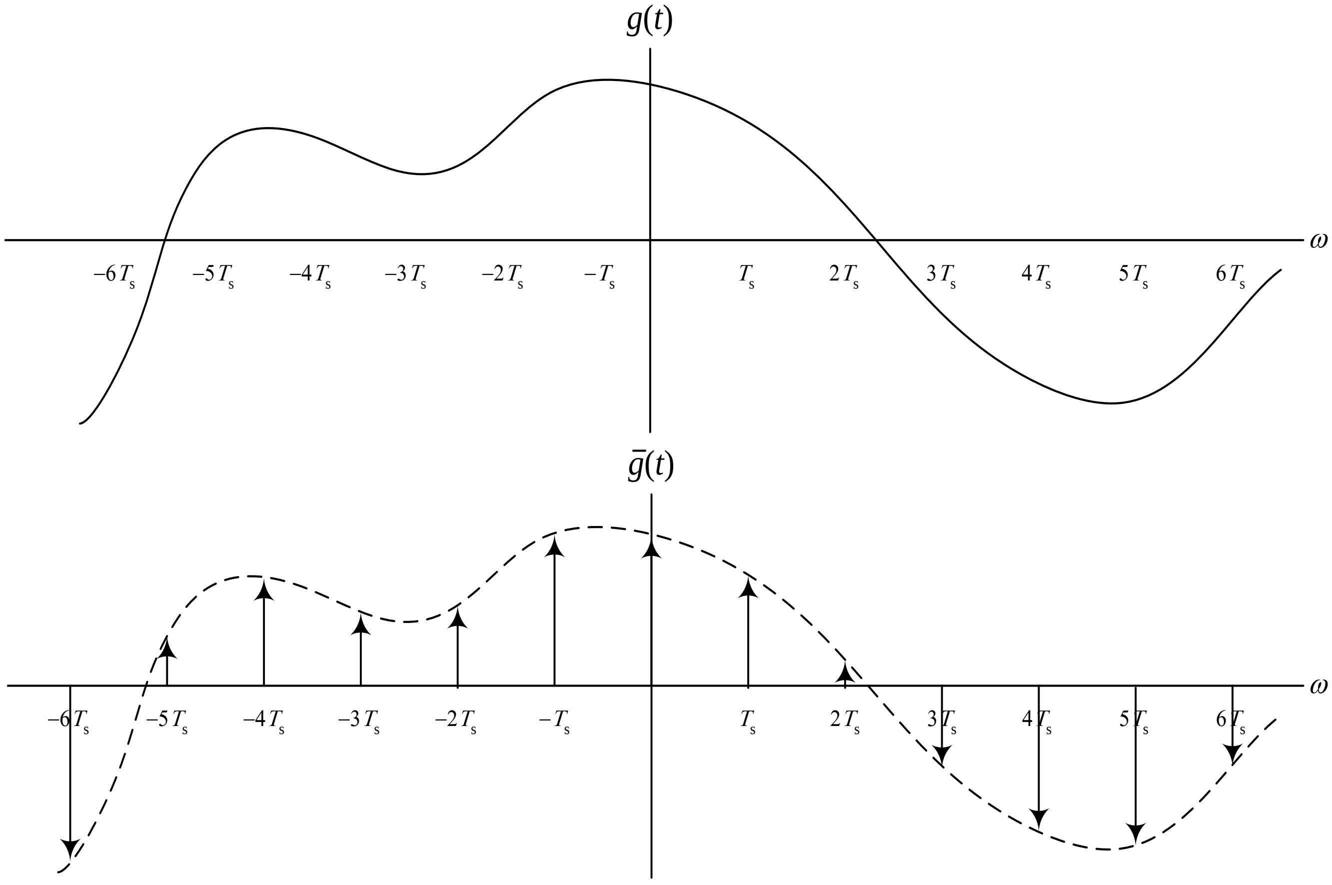
- If we want to sample $g(t)$ periodically every T_s sec then we can repeat this process periodically

$$\bar{g}(t) = g(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} g(t)\cdot\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s)\cdot\delta(t - nT_s).$$

Sampling: Time-Domain Plot



Sampling: Frequency-Domain Analysis (1/2)

$$\bar{g}(t) = g(t)\delta_{T_s}(t) \qquad \delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \qquad \omega_s = \frac{2\pi}{T_s}$$

$$= a_0 + a_1\cos(\omega_s t) + a_2\cos(2\omega_s t) + a_3\cos(3\omega_s t) + \ldots$$

$$+ b_1\sin(\omega_s t) + b_2\sin(2\omega_s t) + b_3\sin(3\omega_s t) + \ldots$$

$$a_0 = \frac{1}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta_{T_s}(t)dt = \frac{1}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)dt = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta_{T_s}(t)\cdot\cos(\omega_s t)dt = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)\cos(\omega_s t)dt = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)\cos(0)dt = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta_{T_s}(t)\cdot\sin(\omega_s t)dt = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)\sin(\omega_s t)dt = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)\sin(0)dt = 0$$

Sampling: Frequency-Domain Analysis (2/2)

$$\bar{g}(t) = g(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$

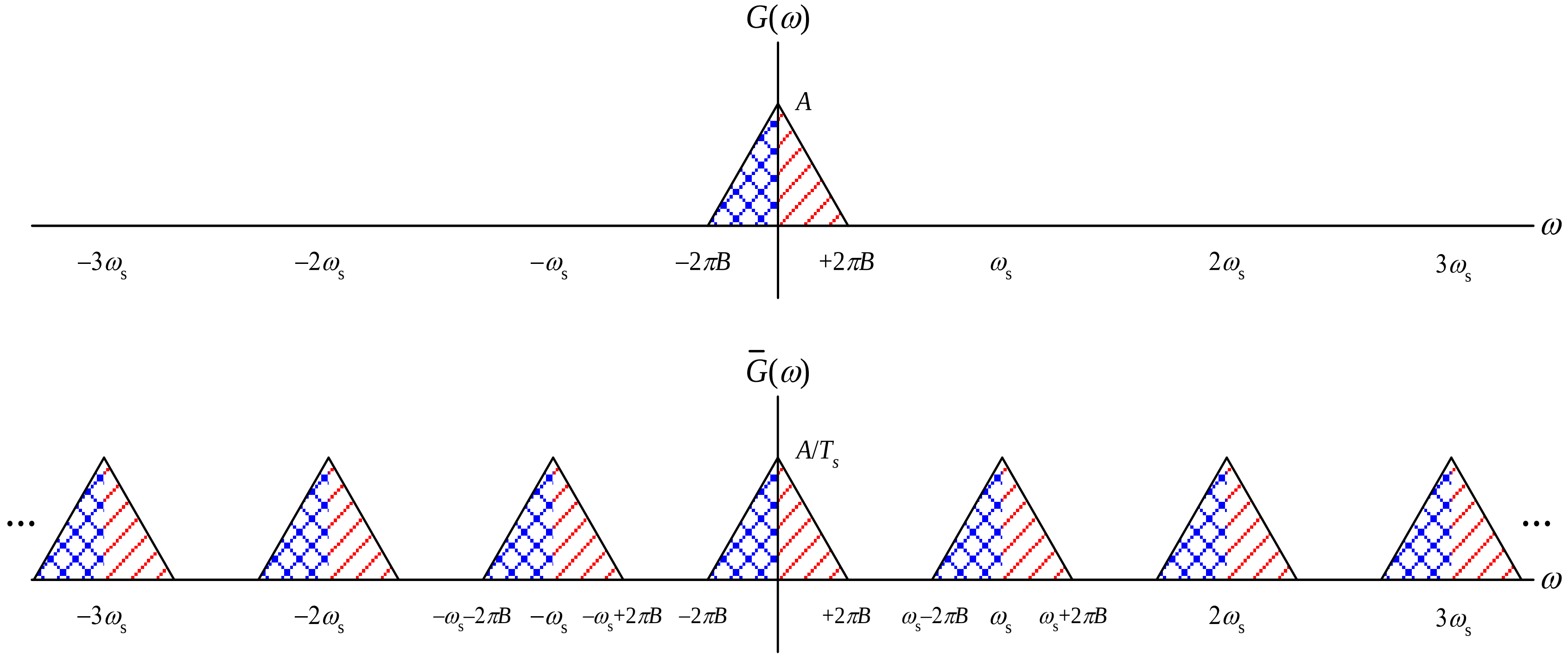
$$= \frac{1}{T_s}g(t) + \frac{2}{T_s}g(t)\cos(\omega_s t) + \frac{2}{T_s}g(t)\cos(2\omega_s t) + \frac{2}{T_s}g(t)\cos(3\omega_s t) + \ldots.$$

$$\bar{G}(\omega) = \frac{1}{T_s}G(\omega) + \frac{1}{T_s}\left[G(\omega-\omega_s) + G(\omega+\omega_s)\right] + \frac{1}{T_s}\left[G(\omega-2\omega_s) + G(\omega+2\omega_s)\right]$$

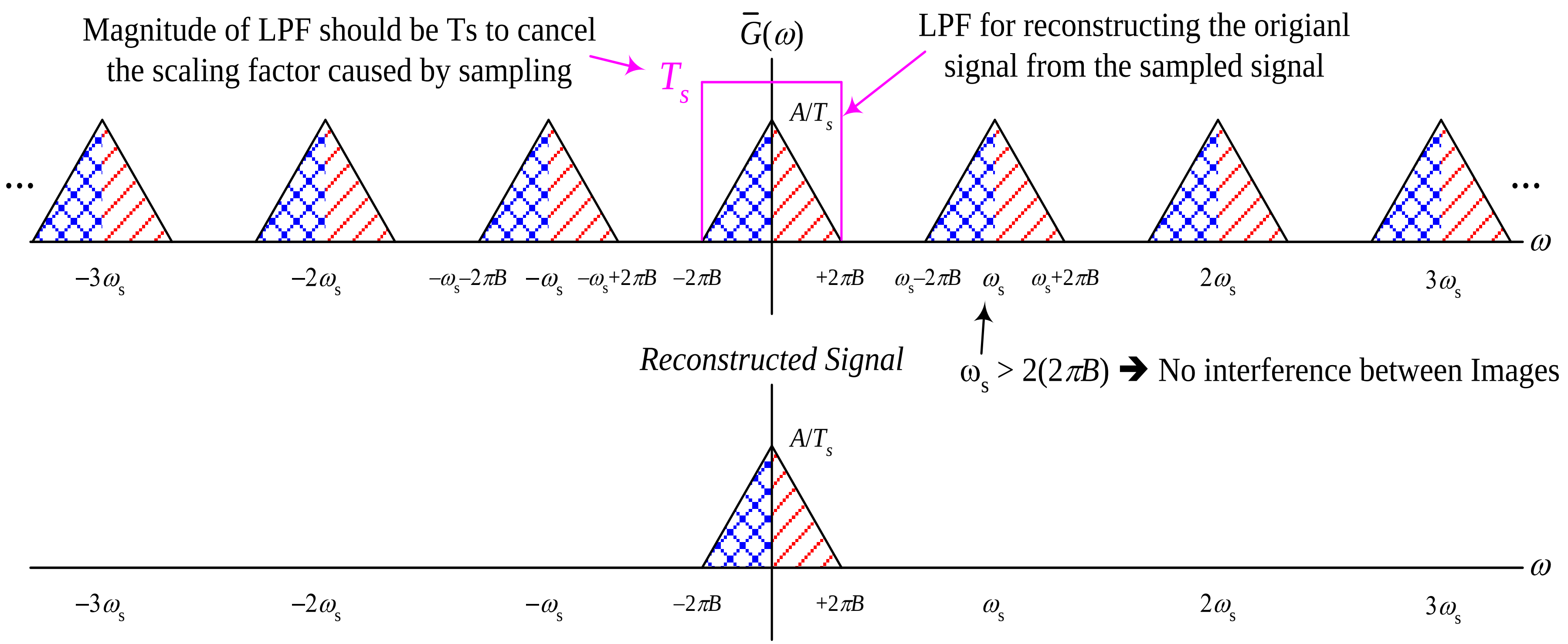
$$+\frac{1}{T_s}\left[G(\omega-3\omega_s) + G(\omega+3\omega_s)\right] + \ldots.$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty}G(\omega-n\omega_s)$$

Spectrum of Sampled Function



Recovering the Continuous-Time Signal



Sampling Theorem

- A baseband signal whose spectrum is band-limited to B Hz can be reconstructed exactly (without any error) from its samples taken uniformly at a rate $f_s \geq 2B$.
- $f_s \geq 2B$ is called Nyquist Criterion of sampling.
- $f_s = 2B$ is called the Nyquist rate of sampling.
- Does Sampling Theorem Make Sense?

Reconstructing the Signal: Time-Domain Prespective

$$T_s \operatorname{rect}\left(\frac{\omega}{\omega_s}\right) \Leftrightarrow \operatorname{sinc}\left(\frac{\omega_s}{2} t\right)$$

$$\bar{g}(t) \leftrightarrow \overline{G}(\omega) \longrightarrow \boxed{\begin{array}{c}\text{LPF} \\ H(\omega) = T_s \operatorname{rect}(f/f_s)\end{array}} \longrightarrow g(t) \leftrightarrow G(\omega)$$

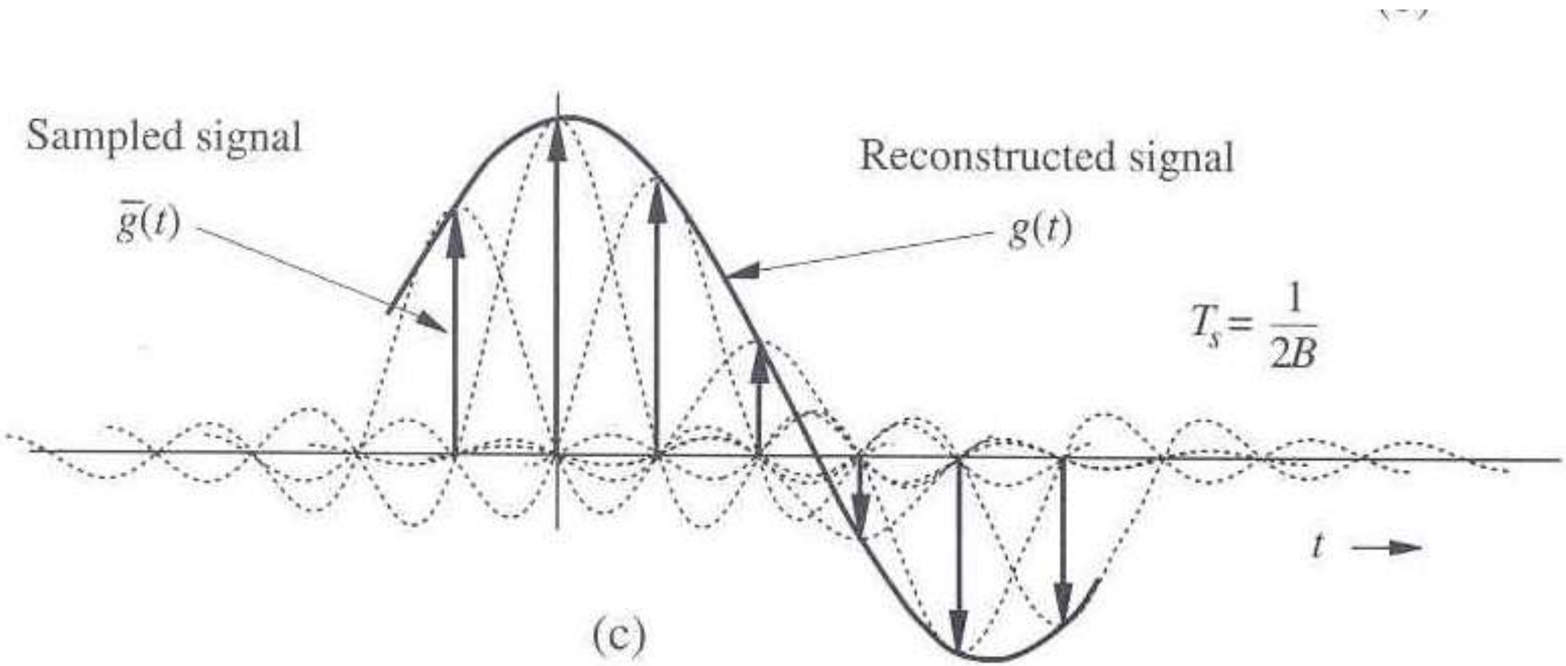
$$G(\omega) = \overline{G}(\omega) \cdot T_s \operatorname{rect}\left(\frac{\omega}{\omega_s}\right)$$

$$g(t) = \bar{g}(t) * \operatorname{sinc}\left(\frac{\omega_s}{2} t\right) = \left[g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right] * \operatorname{sinc}\left(\frac{\omega_s}{2} t\right)$$

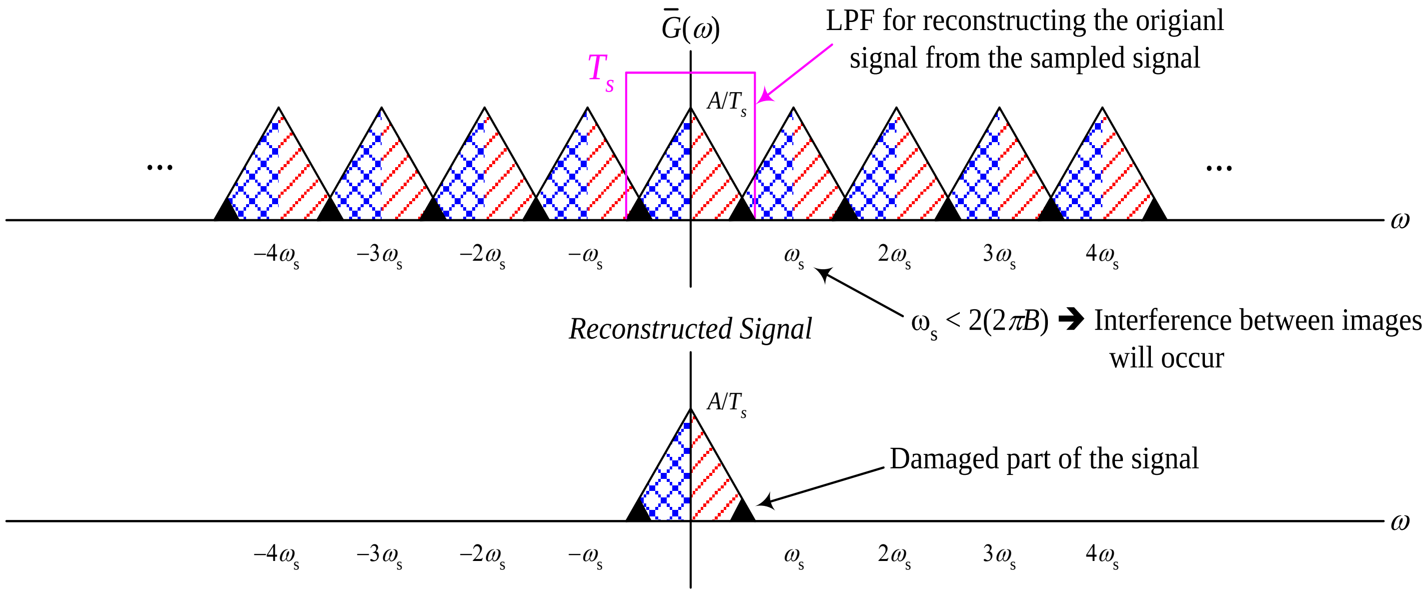
$$g(t) = \left[\sum_{n=-\infty}^{\infty} g(nT_s) \delta(t - nT_s) \right] * \operatorname{sinc}\left(\pi \frac{t}{T_s}\right)$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \operatorname{sinc}\left(\pi \frac{t - nT_s}{T_s}\right)$$

Graphical Illustration



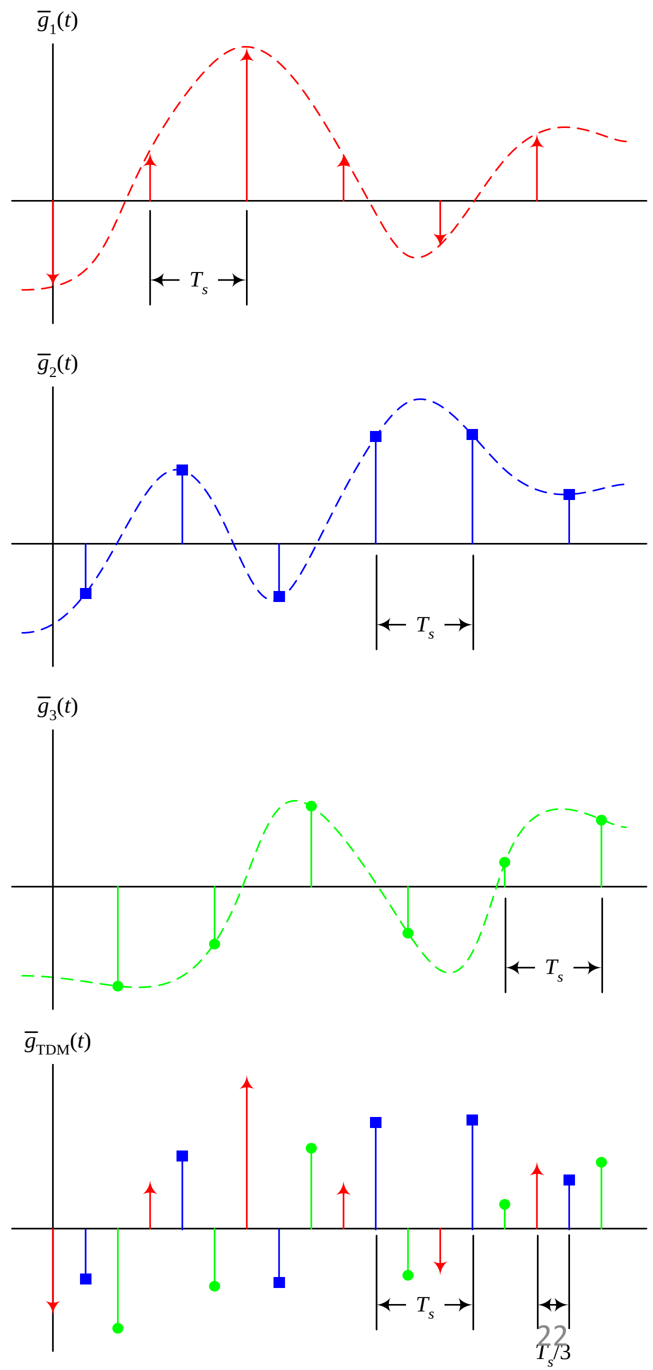
Aliasing



- Sampling a signal at a rate less that the Nyquist rate results in *Aliasing.*
- In aliasing, the higher frequency components take the identity of lower frequencies.
- Real life Example: Sampling a rotating wheel.

Time Division Multiplexing (TDM)

- Multiplexing: The process of sending two or more signals together
  - FDM: Sending them together at the same time over different bands using carrier modulation (AM & FM broadcasting)
  - TDM: Sending them together over the same band by sampling the signals and sending the samples at different time instants (interleaved).



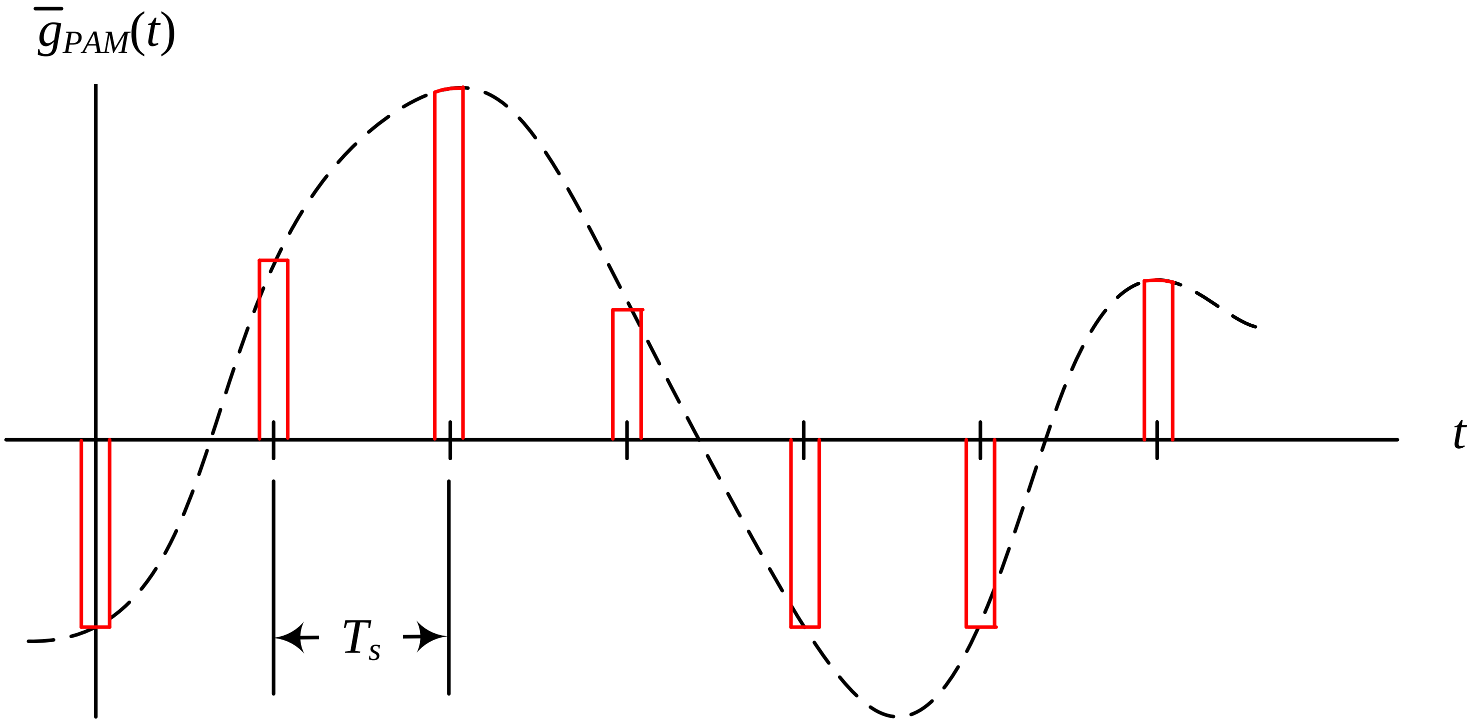
How to Transmit the Samples?

- Analog Pulse Modulation:

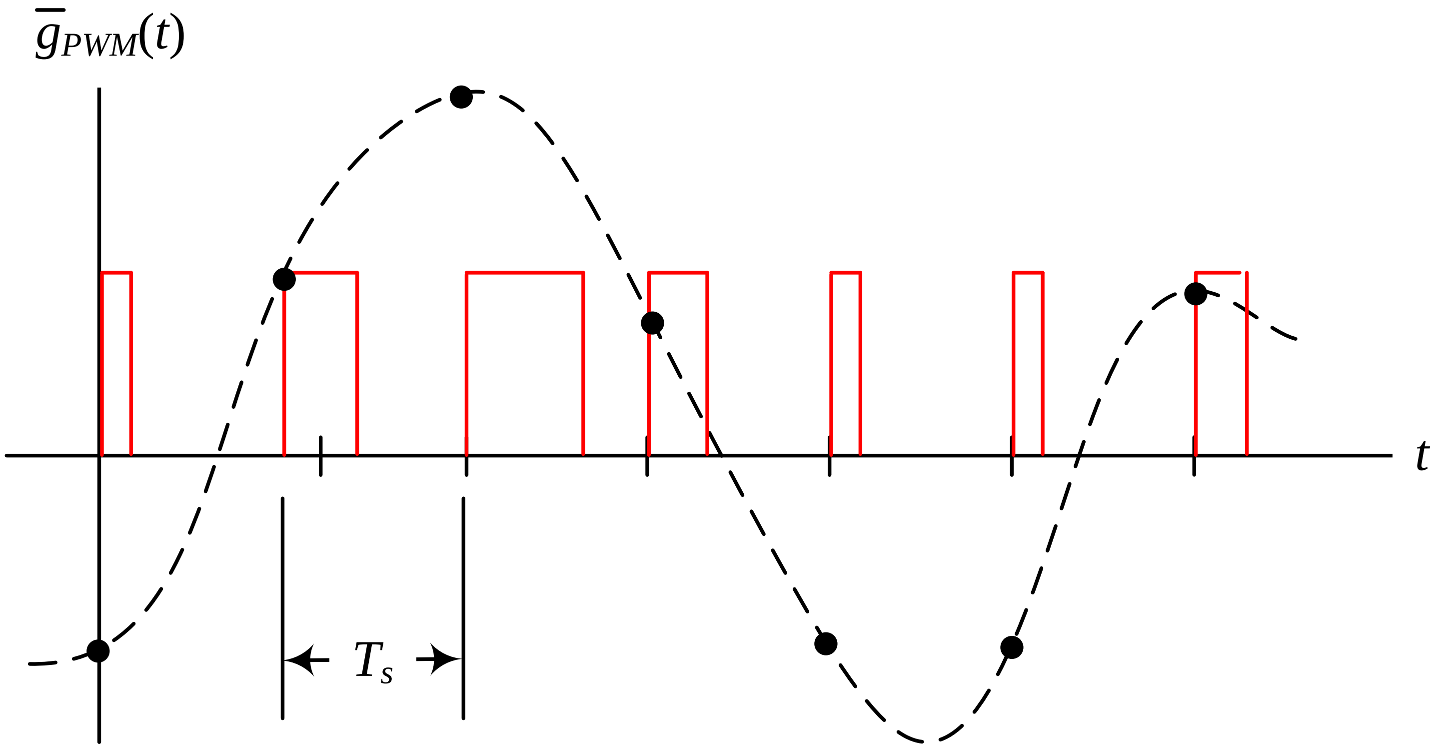
  Use the samples to modulate a carrier of pulses

  - Pulse Amplitude Modulation (PAM)
  - Pulse Width Modulation (PWM)
  - Pulse Position Modulation (PPM)
- Pulse Code Modulation (PCM)
  - Quantization of samples
  - Coding

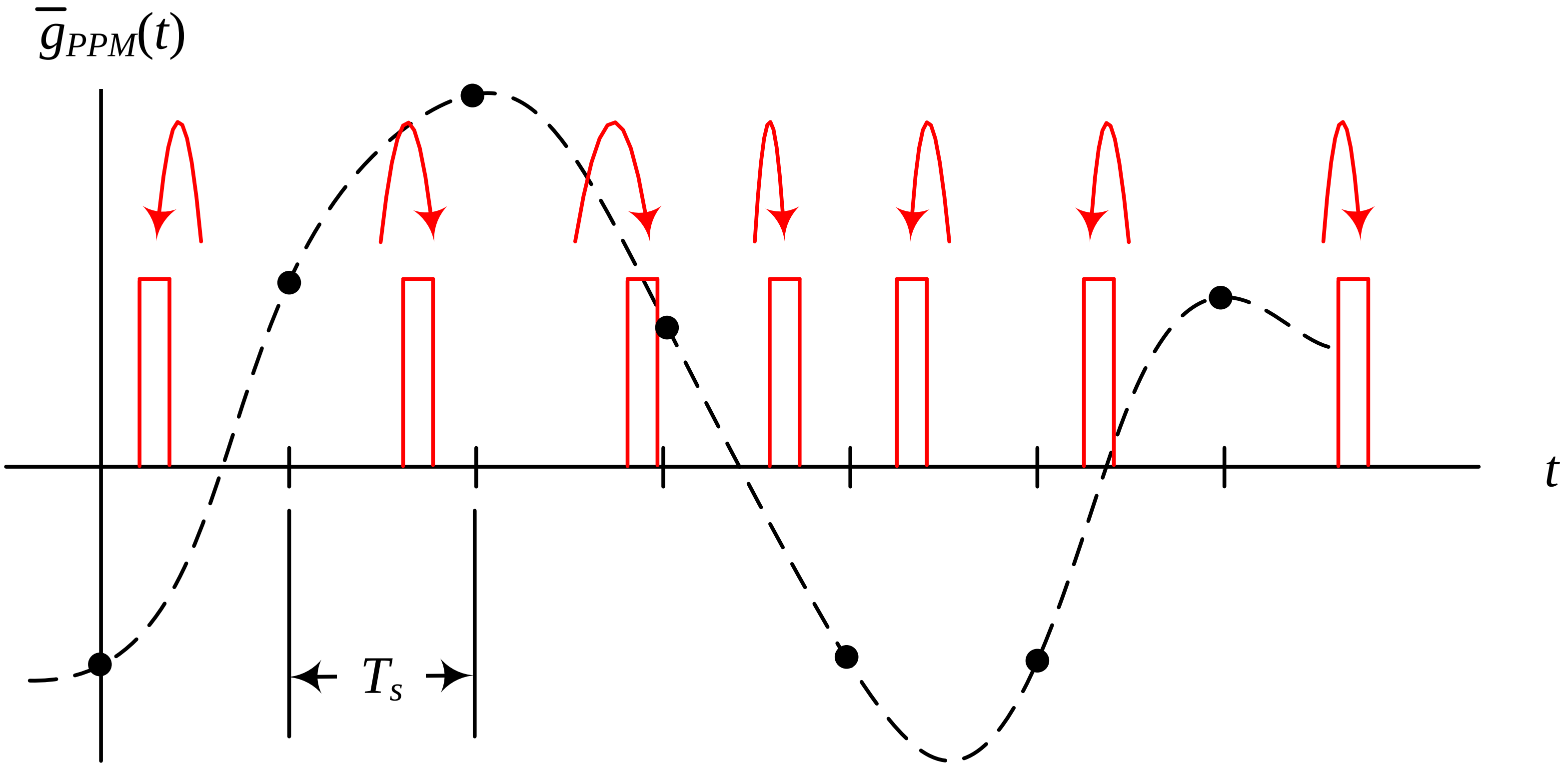
Pulse Amplitude Modulation (PAM)



Pulse Width Modulation (PWM)



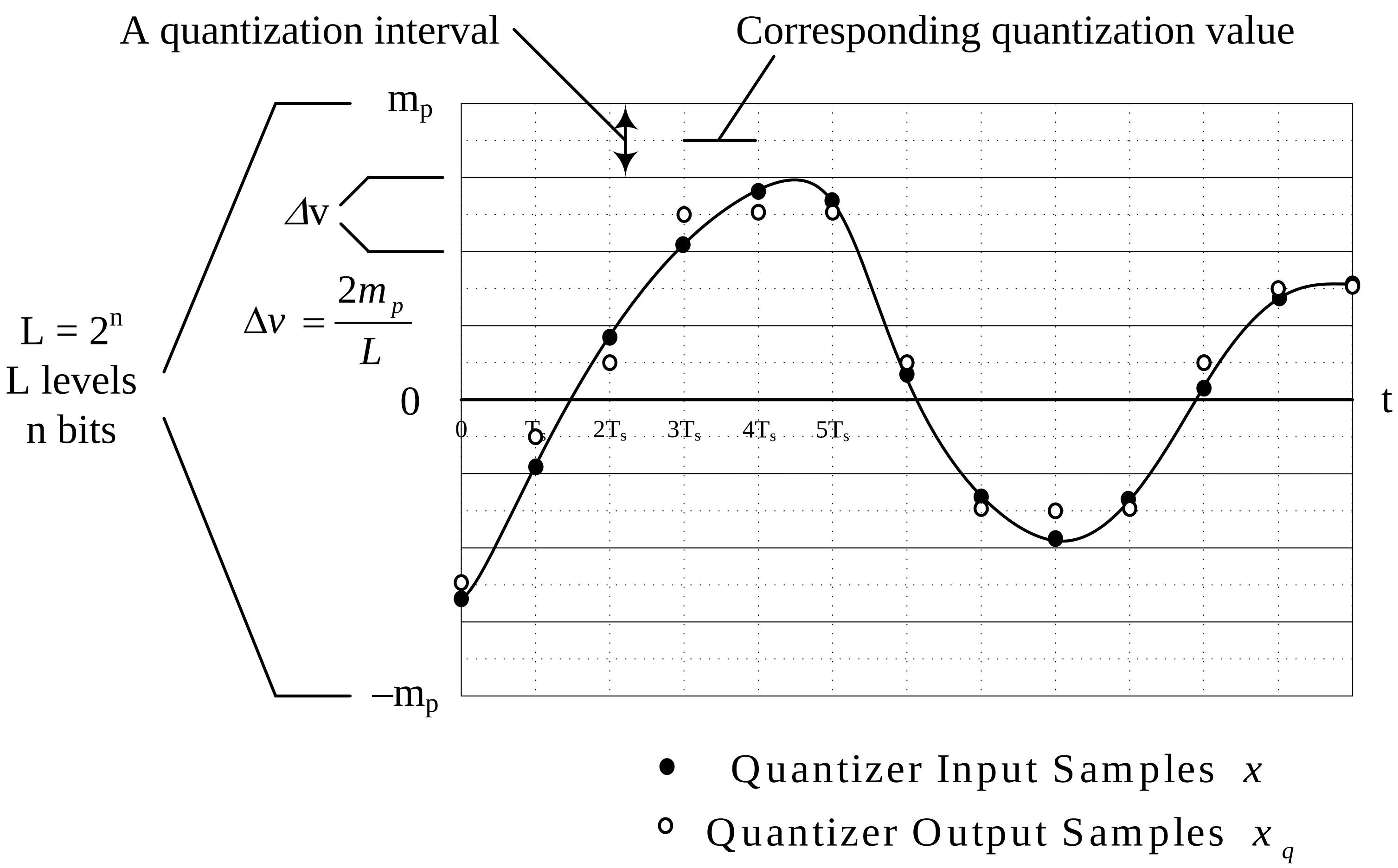
Pulse Position Modulation (PPM)



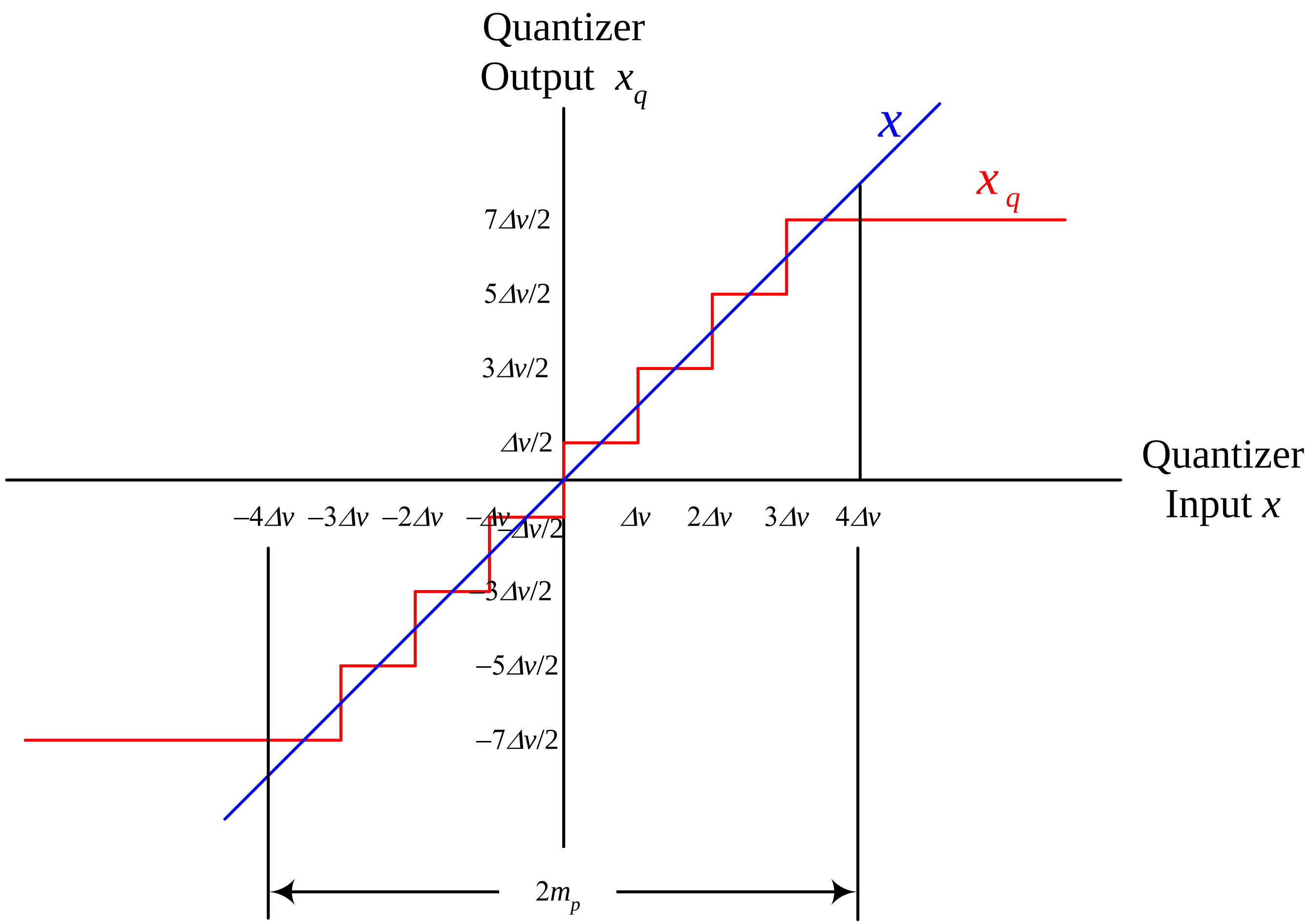
[2] Quantization

- *Analog* samples with an amplitude that may take value in a specific range are converted to a *digital* samples with an amplitude that takes one of a specific pre–defined set of values.
- The range of possible values of the analog samples is divide into *L* levels. L is usually taken to be a power of 2 ($L = 2^n$).
- The center value of each level is assigned to any sample that falls in that quantization interval.
- For almost all samples, the quantized samples will differ from the original samples by a small amount, called the *quantization error*.

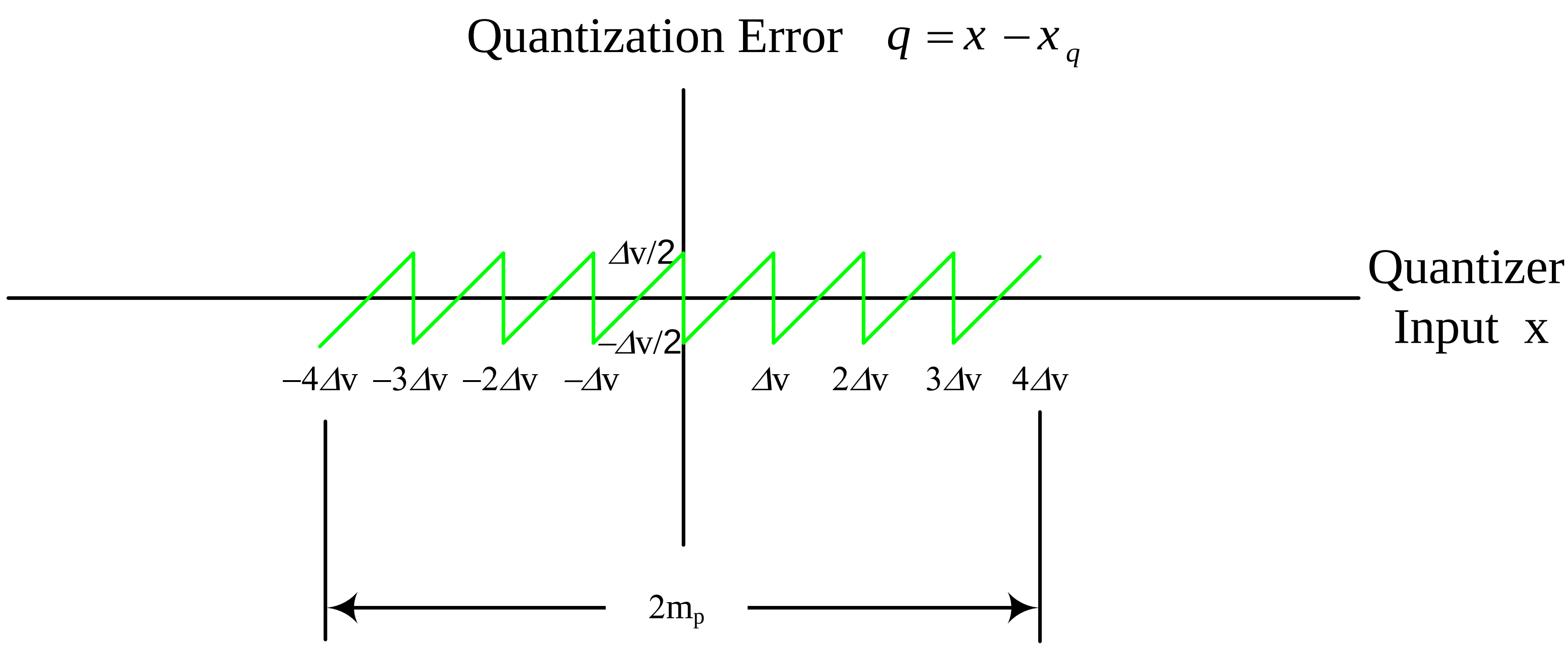
Quantization: Illustration



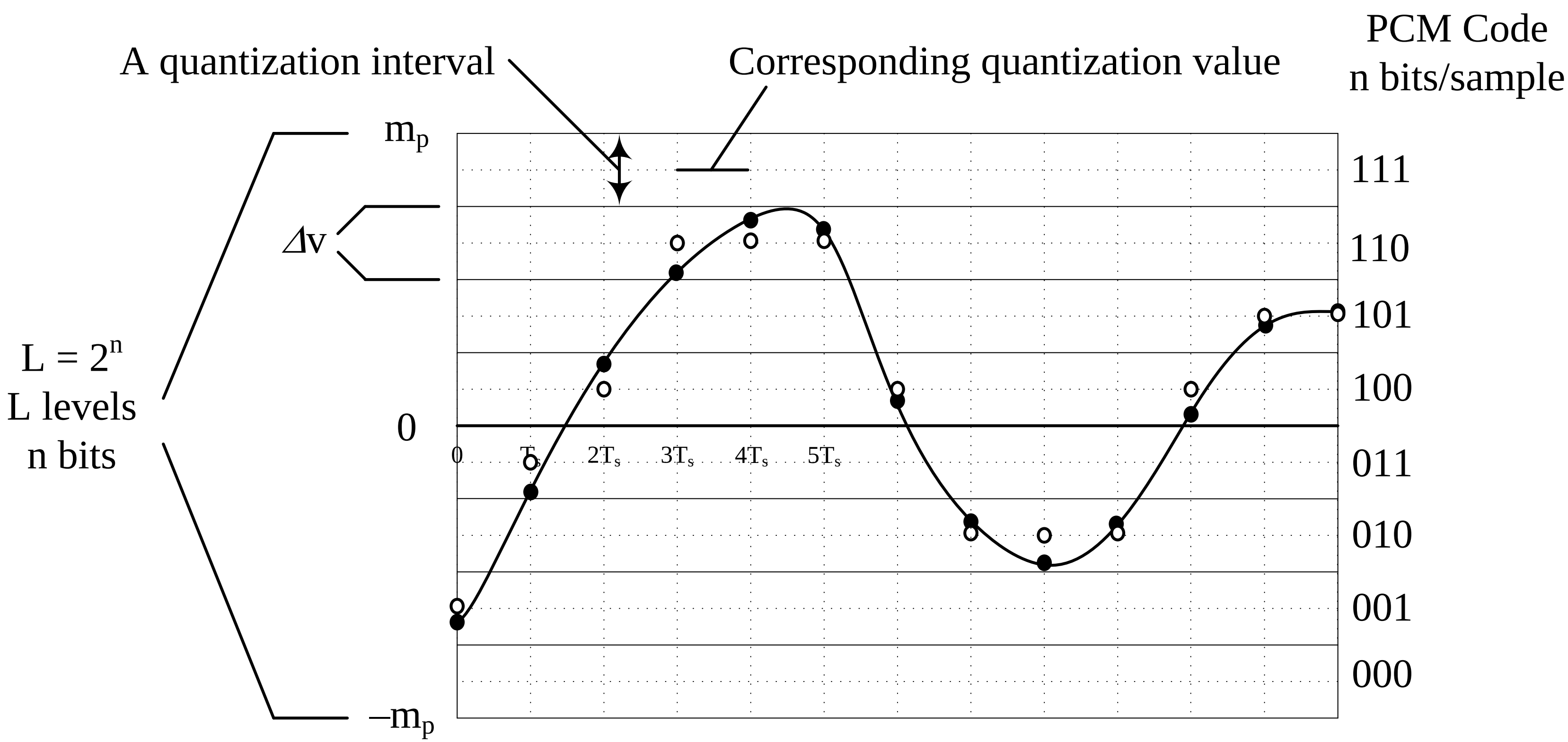
Input-Output Characteristics of Quantizer



Quantization Error

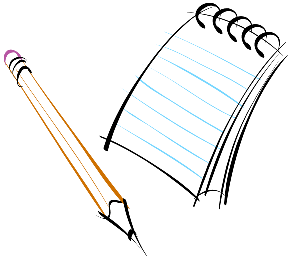


[3] Coding



- Quantizer Input Samples x
- Quantizer Output Samples x_q

001 011 100 110 110 110 100 010 010 010 100 101 101



- We want to scan and send a black-and-white image of height 11 inches and width 8.5 inches (Letter size paper). The resolution of the scanner is 600×600 dots per inch square. The picture will be quantized using 256 levels. Find the size of the scanned image and the time it takes to transmit the image using a modem of speed 56 kbps.
  - Size of image = 11(in)×8.5(in)×600×600(samples/in^2)×8bits/sample = 269280000 bits = 269 Mbits
  - Time to transmit = 269280000 / 56,000 = 4808 sec = 80 min

How would 0’s and 1’s be transmitted?

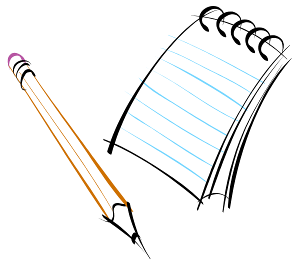
- The simplest form is to send a +ve pulse for a “1” and a –ve pulse for a “0”.
- Transmitting the message $g(t)$ would translate into sending a a long sequence of +ve and –ve pulses.

Advantages of Digital Communications

- Rugged: Can withstand channel noise and distortion much better.
- Use of repeaters (travels as far as needed).
- Use of TDM
- Can be encrypted (Security and Privacy)
- Can be encoded for error correction (reliability).
- Easy to process, store and search.

Nyquist Theorem for Transmission

- Note that the larger the transmission rate (pulses/sec) the narrower the pulse, the wider its spectrum, the higher the channel bandwidth required for transmission.
- **The minimum theoretical bandwidth required to transmit *R* pulses/sec is *R*/2 Hz.** (To be demonstrated later)



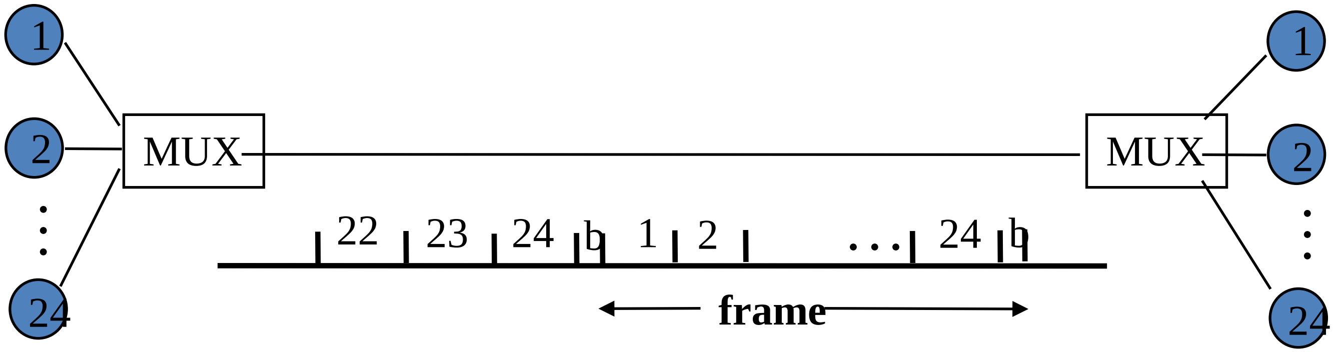
- A signal $m(t)$ band-limited to 3 kHz is sampled at a rate 33.33% higher than the Nyquist rate, quantized and coded. The maximum acceptable quantization error is 0.5% of m_p. Find the minimum bandwidth required for transmission? How is that compared to SSB?
- Ans: 32 kHz.

TDM Revisited

- Time axis is divided into frames. Frame rate is determined by sampling rate.
- Each frame is divided into slots.
- Each user is assigned a slot (periodically in each frame).
- A user uses the full bandwidth during his slot.
- The transmission rate of the multiplexed channel is the sum of the rates of individual channels plus the control bits.
- Can be used with digital signals only.

TDM in Telephony (T1 & E1 Systems)

- T1:
  - Introduced in 1960s
  - North America and Japan



- E1 system (Europe): 30 voice channels + 2 syn channels

T1 System

- Multiplexes 24 voice channels
- Voice bandwidth is approximately 3.4 kHz
- Nyquist rate of sampling = 6800 samples/sec
- Actual sampling rate = 8000 samples/sec
- 8 bits/sample (256 levels)
- Frame duration = 1/8000 = 125 μsec
- Number of bits/frame = $24 \times 8+1=193$
- Bit duration = 0.647 μsec
- Transmission rate:
  $(24 \times 8+1)$ bits/frame $\times$ 8000 frames/sec = 1.544 Mbps

Quantization Noise

- The quantization error is assumed to be uniformly distributed over the range ($-\Delta v/2, \Delta v/2$).

$$P_q = \int_{-\Delta v/2}^{\Delta v/2} q^2 \frac{1}{\Delta v} dq = \frac{1}{\Delta v}\left[\frac{q^3}{3}\right]_{q=-\Delta v/2}^{\Delta v/2}$$

$$= \frac{1}{\Delta v}\left[\frac{(\Delta v/2)^3}{3} - \frac{(-\Delta v/2)^3}{3}\right] = \frac{1}{\Delta v}\left[\frac{(\Delta v)^3}{24} + \frac{(\Delta v)^3}{24}\right]$$

$$= \frac{(\Delta v)^2}{12} \qquad = \frac{(2m_p/L)^2}{12} = \frac{m_p^2}{3L^2}$$

Signal-to-Quantization-Noise Ratio

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_s}{P_q}$$

$$= \frac{3L^2}{m_p^2} P_s .$$

$$SNR_{dB} = 10 \cdot \log_{10}\left(\frac{3L^2}{m_p^2} P_s \right) = 10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s \right) + 10 \cdot \log_{10}\left(2^{2n} \right)$$

$$= \underbrace{10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s \right)}_{\alpha} + \underbrace{20n \cdot \log_{10}(2)}_{6n}$$

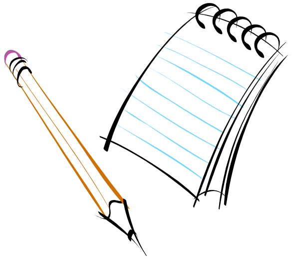
$$= \alpha + 6n \qquad \text{dB.}$$

SNR-Bandwidth Exchange

- More bits/sample for the same message results in more quantization levels, less quantization step, less quantization noise, higher SNR.
- On the other hand, more bits/sample results in bandwidth expansion

$$SNR = 3\frac{P_s}{m_p}(2)^{2n}; (SNR)_{dB} = \alpha + 6n$$

- One added bit results in multiplying SNR by a factor of 4 (6 dB), but multiplying the transmission bandwidth by a factor of (*n*+1)/*n*

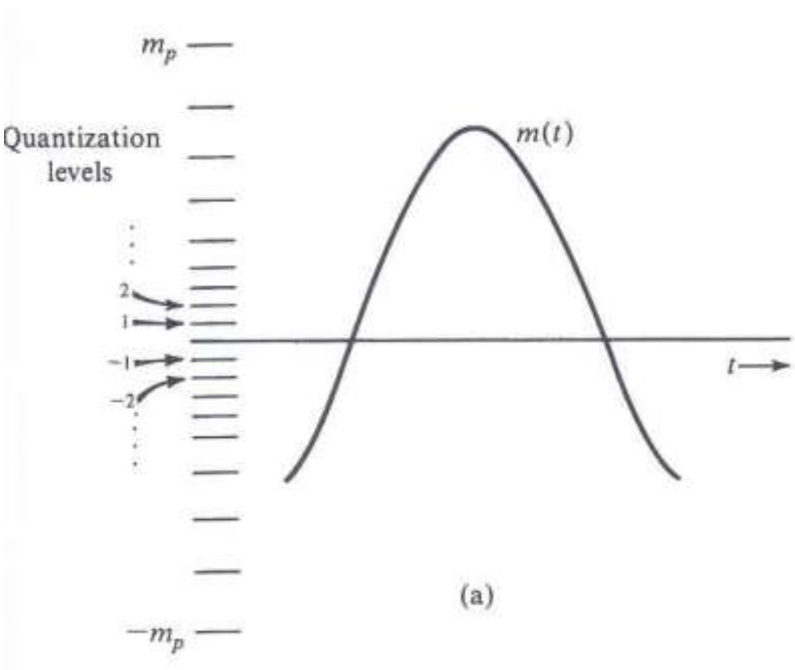


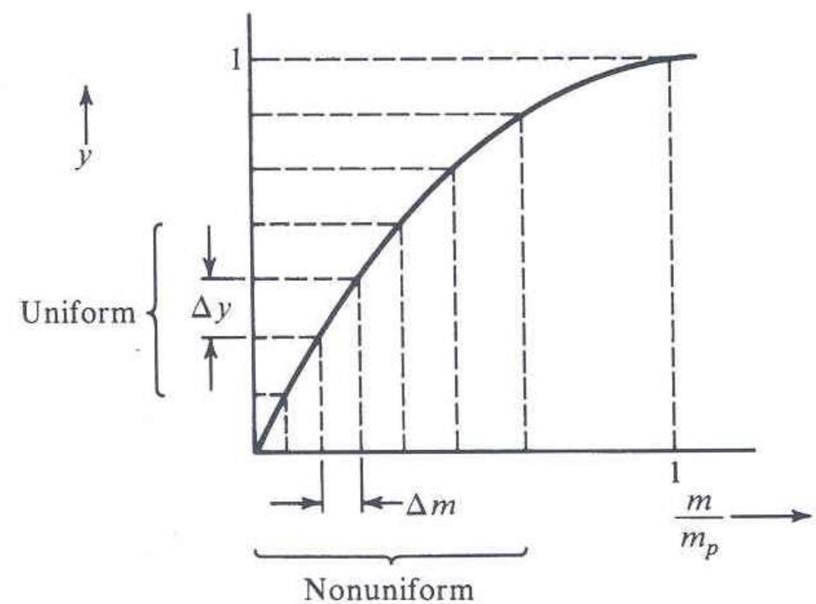
- A signal of bandwidth 4 kHz is samples at Nyquist rate and transmitted using PCM with uniform quantization. If the number of quantization levels L is increased from 64 to 256, find the change in SNR and transmission bandwidth.
  - Number of bits/sample has been increased from 6 to 8.
  - SNR improved by 12 dB (16 times)
  - B_T expanded by a factor of 1.33 (33% increase).
    From 24 kHz to 32 kHz.

Non-Uniform Quantization

- There is a huge variation in voice signal level from user to user, and for the same use from call to call as well as within the call (sometimes of the order of 1000:1)
- Uniform quantization provides same degree of resolution for low and high values.
- Designing the step size for the low values results in too many levels, and designing them for the high values destroys the low values.

Non-Uniform Quantizers





Compressors and Expanders

- It is practically more feasible to compress the signal logarithmically then apply it to a uniform quantizer.
- A reciprocal process takes place at the receiver by an expander.
- The compressor/expander system is called *compander*.
- There are two standard laws for companders, the ***μ*-law** (North America and Japan) and the ***A*-law** (Europe and rest of the world).

μ-Law and A-Law Characteristics

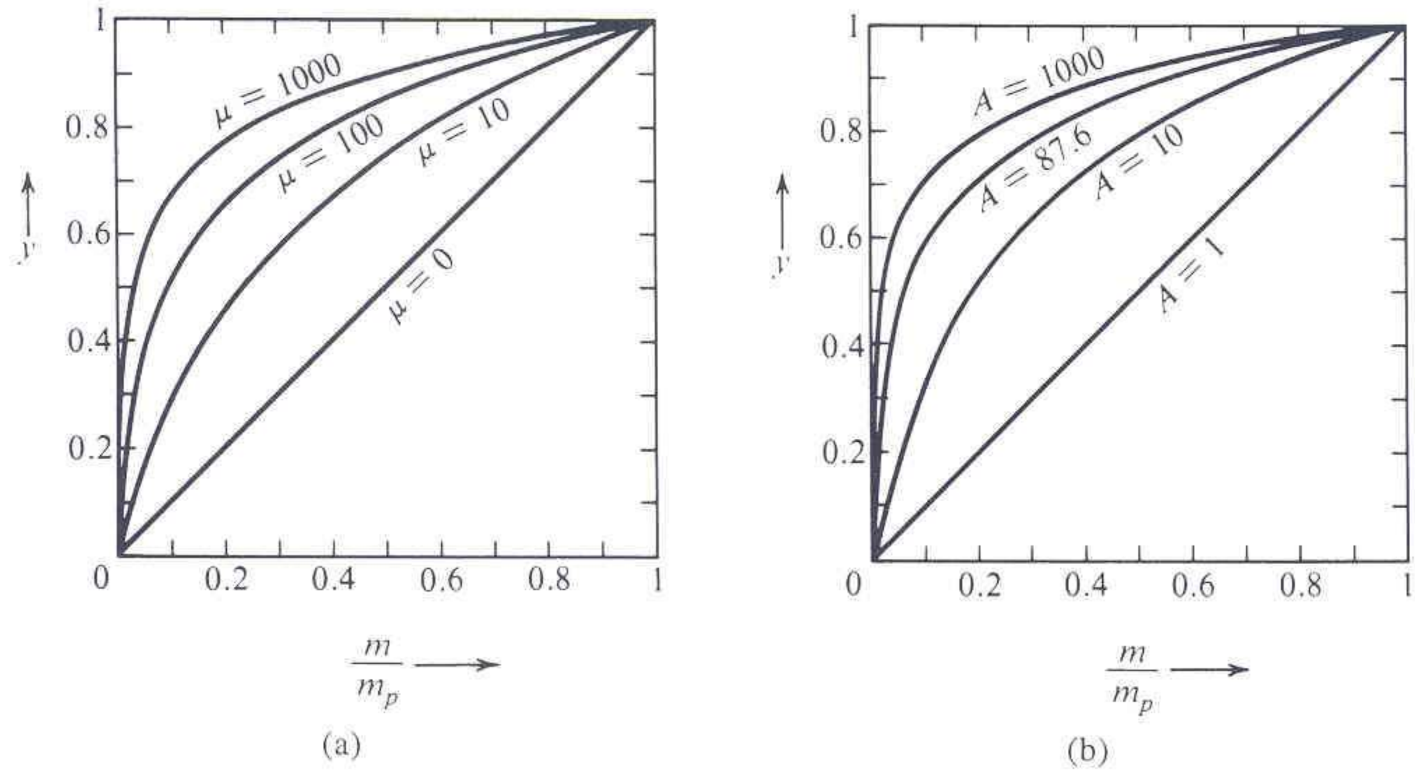


Figure 6.12 (a) μ-law characteristic. (b) A-law characteristic.

Introduction

- *Pulse modulation* consists essentially *of* sampling analog information signals and then converting those samples into discrete pulses and transporting the pulses from a source to a destination over a physical transmission medium.

- The four predominant methods *of* pulse modulation:
  - *pulse width modulation* (PWM)
  - *pulse position modulation* (PPM)
  - *pulse amplitude modulation* (PAM)
  - *pulse code modulation* (PCM).

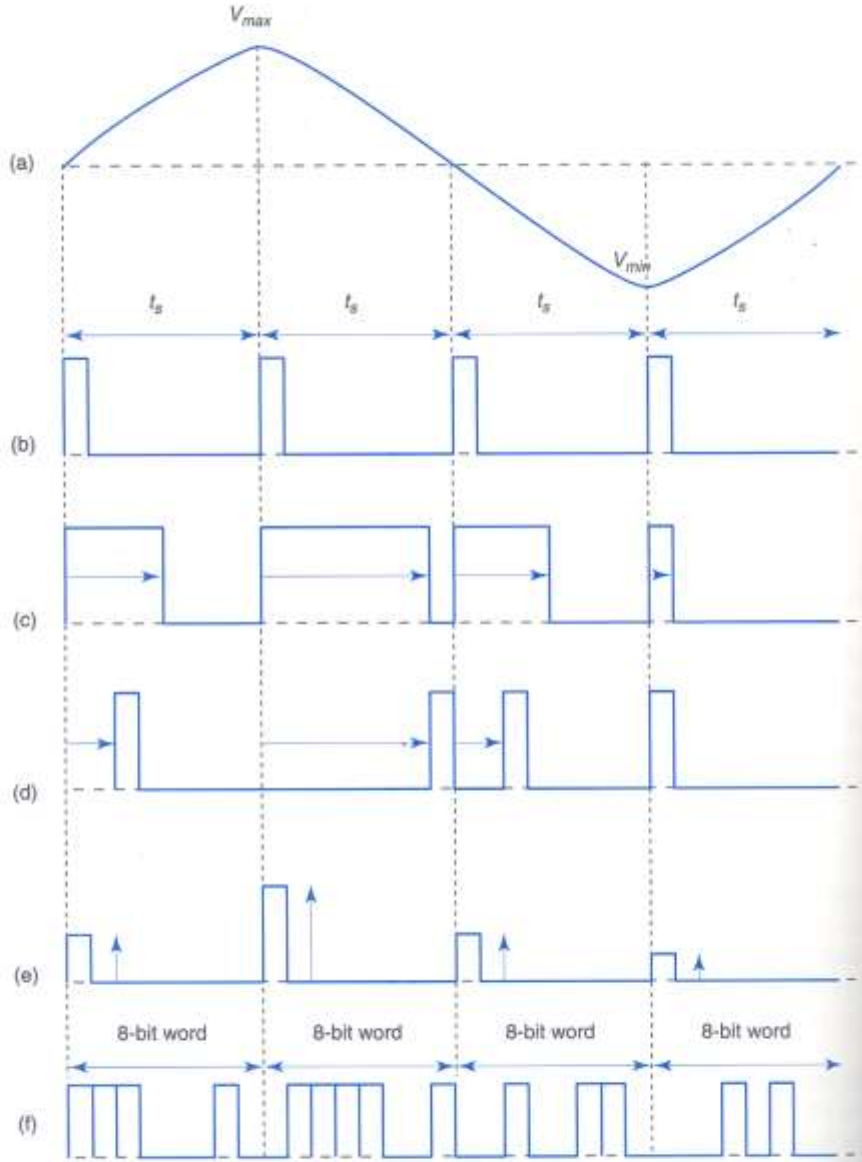


FIGURE 10-1 Pulse modulation: (a) analog signal; (b) sample pulse; (c) PWM; (d) PPM; (e) PAM; (f) PCM

Pulse Width Modulation

- PWM is sometimes called *pulse duration modulation* (PDM) or *pulse length modulation* (PLM), as the width (active portion *of* the duty cycle) *of* a constant amplitude pulse is varied proportional to the amplitude *of* the analog signal at the time the signal is sampled.

- The maximum analog signal amplitude produces the widest pulse, and the minimum analog signal amplitude produces the narrowest pulse. Note, however, that all pulses have the same amplitude.

Pulse Position Modulation

- With PPM, the position *of* a constant-width pulse within a prescribed time slot is varied according to the amplitude *of* the sample *of* the analog signal.

- The higher the amplitude *of* the sample, the farther to the right the pulse is positioned within the prescribed time slot. The highest amplitude sample produces a pulse to the far right, and the lowest amplitude sample produces a pulse to the far left.

Pulse Amplitude Modulation

- With PAM, the amplitude of a constant width, constant-position pulse is varied according to the amplitude *of* the sample *of* the analog signal.

- The amplitude *of* a pulse coincides with the amplitude *of* the analog signal.

- PAM waveforms resemble the original analog signal more than the waveforms for PWM or PPM.

Pulse Code Modulation

- With PCM, the analog signal is sampled and then converted to a serial n-bit binary code for transmission.

- Each code has the same number *of* bits and requires the same length *of* time for transmission

Pulse Modulation

- PAM is used as an intermediate form *of* modulation with PSK, QAM, and PCM, although it is seldom used by itself.
- PWM and PPM are used in special-purpose communications systems mainly for the military but are seldom used for commercial digital transmission systems.
- PCM is by far the most prevalent form *of* pulse modulation and will be discussed in more detail.

Pulse Code Modulation

- PCM is the preferred method *of* communications within the public switched telephone network because with PCM it is easy to combine digitized voice and digital data into a single, high-speed digital signal and propagate it over either metallic or optical fiber cables.

- The term *pulse code modulation* is somewhat of a misnomer, as it is not really a type of modulation but rather a form of digitally coding analog signals.

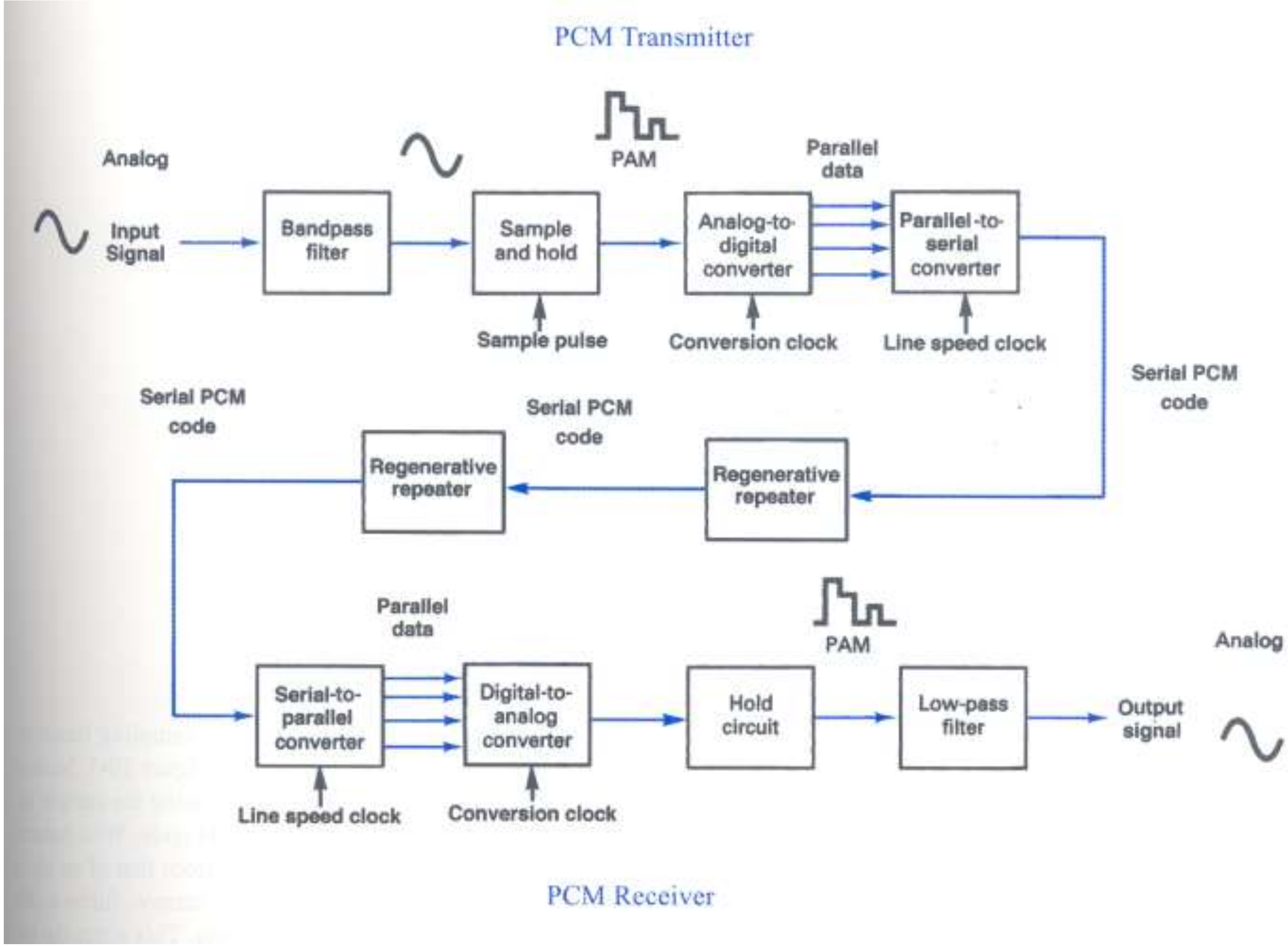
Pulse Code Modulation

- With PCM, the pulses are of fixed length and fixed amplitude.

- PCM is a binary system where a pulse or lack of a pulse within a prescribed time slot represents either a logic 1 or a logic 0 condition.

- PWM, PPM, and PAM are digital but seldom binary, as a pulse does not represent a single binary digit (bit).

PCM Transmitter / Receiver



PCM Sampling

- The function of a sampling circuit in a PCM transmitter is to periodically sample the continually changing analog input voltage and convert those samples to a series of constant-amplitude pulses that can more easily be converted to binary PCM code.

- For the ADC to accurately convert a voltage to a binary code, the voltage must be relatively constant so that the ADC can complete the conversion before the voltage level changes. If not, the ADC would be continually attempting to follow the changes and may never stabilize on any PCM code.

PCM Sampling

- Essentially, there are two basic techniques used to perform the sampling function
  - natural sampling
  - flat-top sampling

- Natural sampling is when tops of the sample pulses retain their natural shape during the sample interval, making it difficult for an ADC to convert the sample to a PCM code.

PCM Sampling

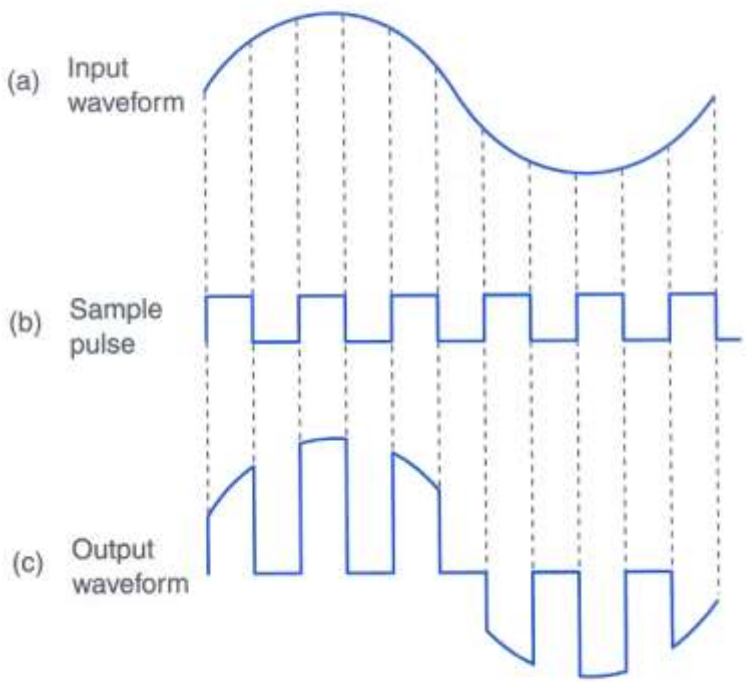


FIGURE 10-3 Natural sampling: (a) input analog signal; (b) sample pulse; (c) sampled output

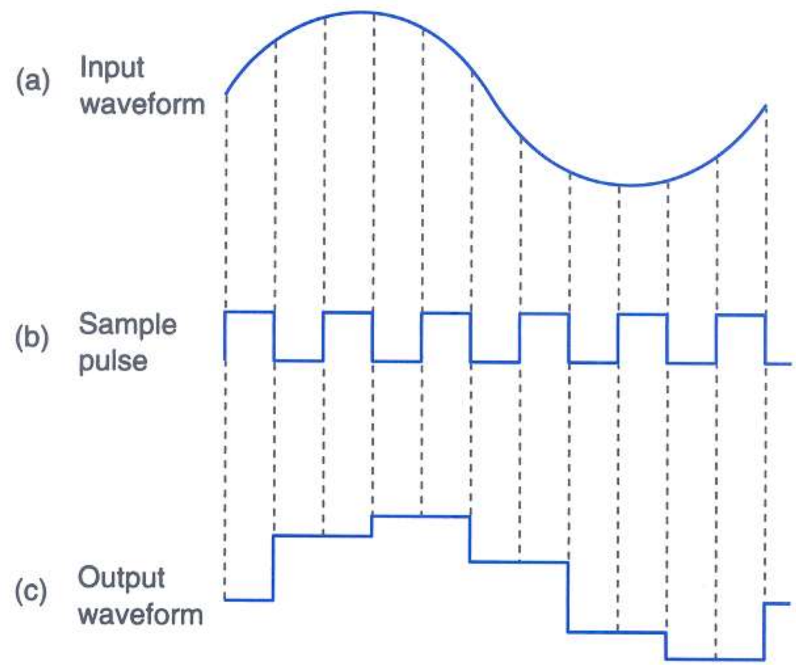
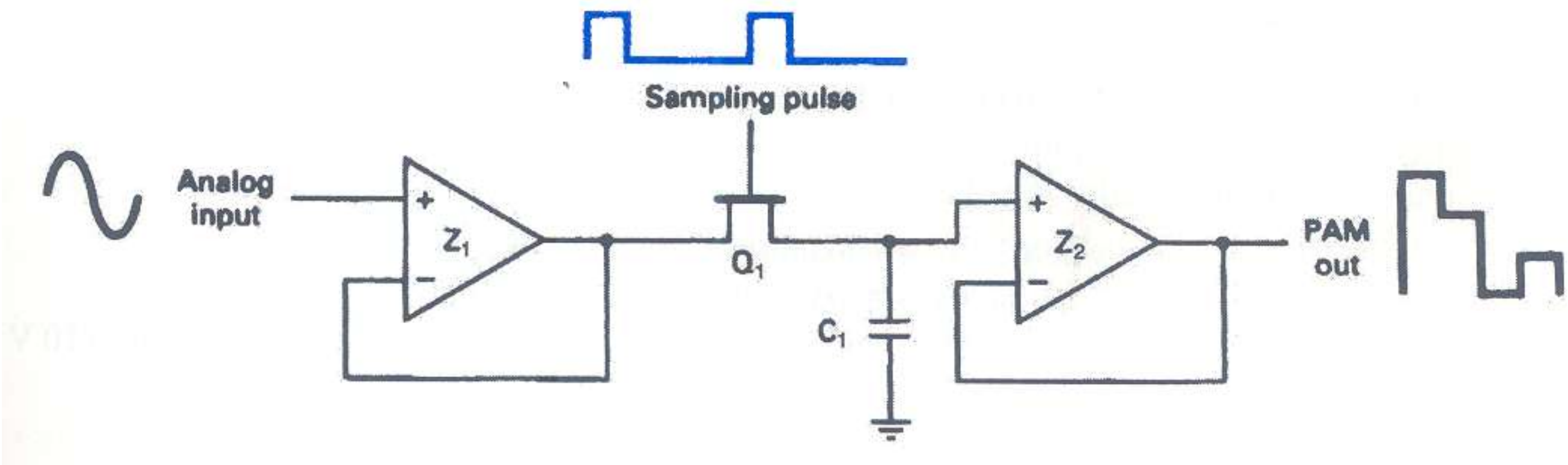


FIGURE 10-4 Flat-top sampling: (a) input analog signal; (b) sample pulse; (c) sampled output

Sample-and-Hold Circuit



PCM Sampling

- The most common method used for sampling voice signals in PCM systems is *flat- top sampling*, which is accomplished in a *sample-and-hold circuit.* The purpose of a sample-and-hold circuit is to periodically sample the continually changing analog input voltage and convert those samples to a series of constant-amplitude PAM voltage levels.

PCM Sampling Rate

- The *Nyquist sampling theorem* establishes the *minimum sampling rate (f_s)* that can be used for a given PCM system.

- For a sample to be reproduced accurately in a PCM receiver, each cycle of the analog input signal *(f_a)* must be sampled at least twice.

- Consequently, the minimum sampling rate is equal to twice the highest audio input frequency.

PCM Sampling Rate

- If f_s is less than two times f_a an impairment called *alias* *or* *foldover distortion* occurs.

- Mathematically, the min- imum Nyquist sampling rate is:

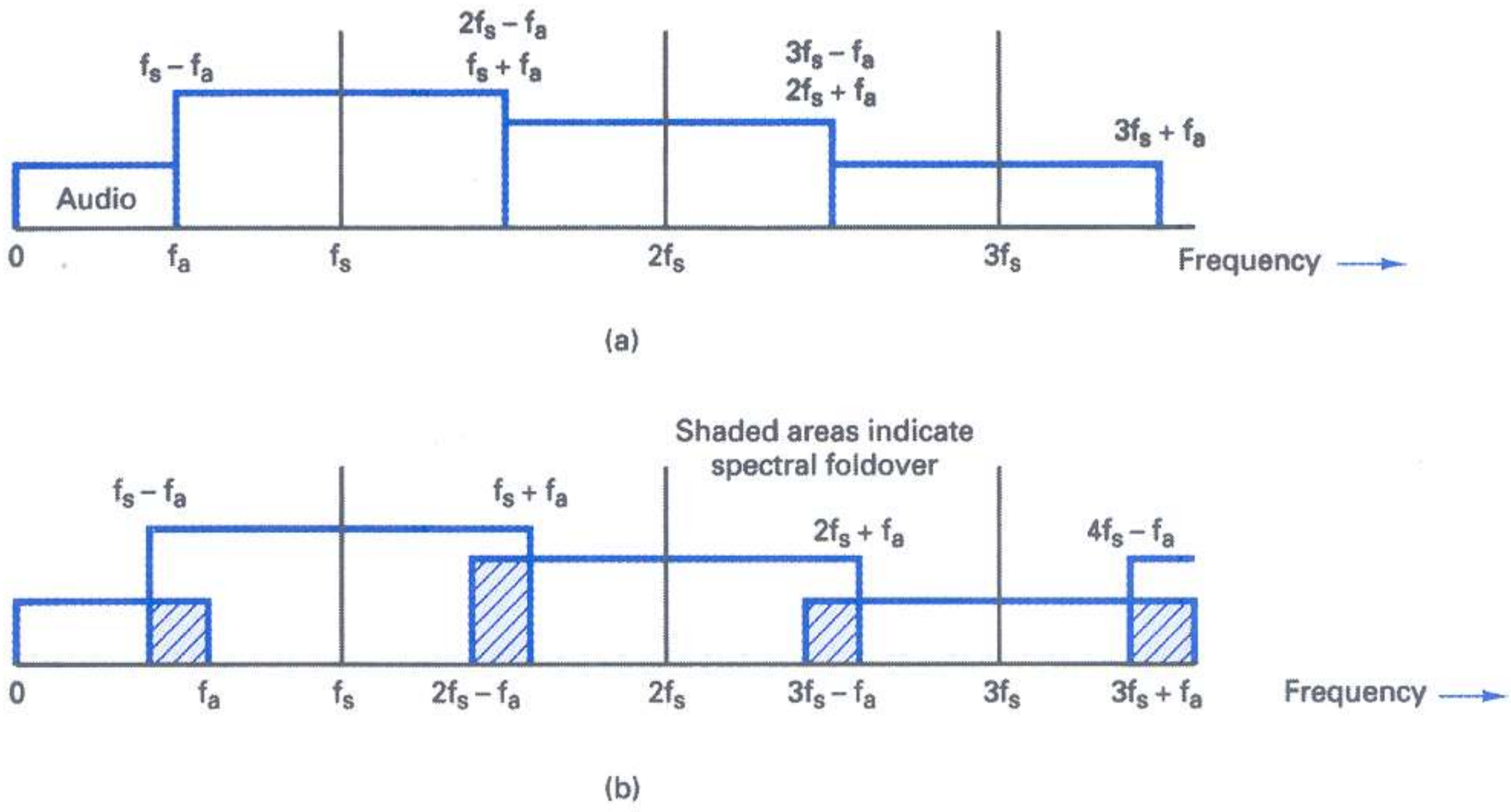
$$\boldsymbol{f_s \geq 2f_a}$$

PCM Sampling Rate

- A sample-and-hold circuit is a nonlinear device (mixer) with two inputs: the sampling pulse and the analog input signal. Consequently, nonlinear mixing occurs between these two signals.

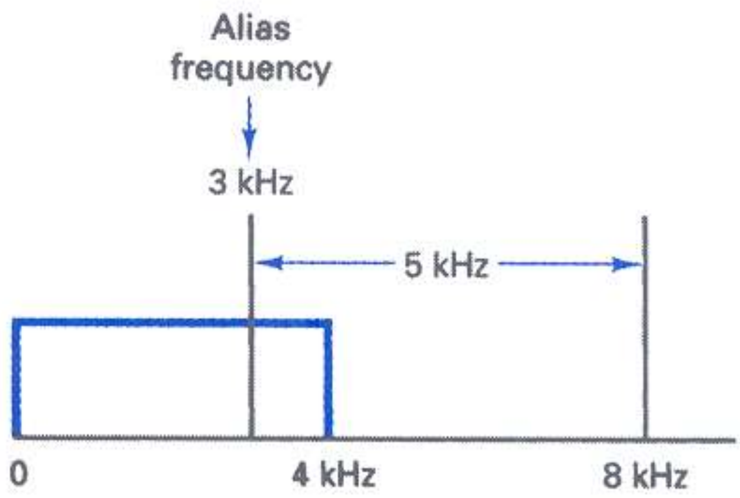
- The output includes the two original inputs (the audio and the fundamental frequency of the sampling pulse), their sum and difference frequencies $(f_s \pm f_a)$, all the harmonics of f_s and f_a $(2f_s, 2f_a, 3f_s, 3f_a,$ and so on), and their associated cross products $(2f_s \pm f_a, 3f_s \pm f_a,$ and so on).

PCM Sampling Rate



Example 1

- For a PCM system with a maximum audio input frequency of 4 kHz, determine the minimum sample rate and the alias frequency produced if a 5-kHz audio signal were allowed to enter the sample-and-hold circuit.

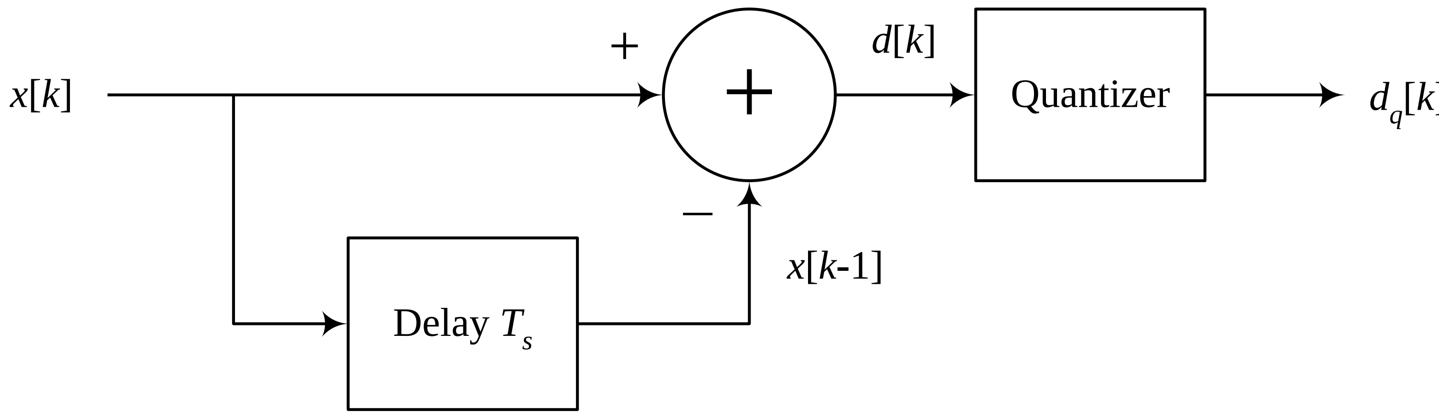


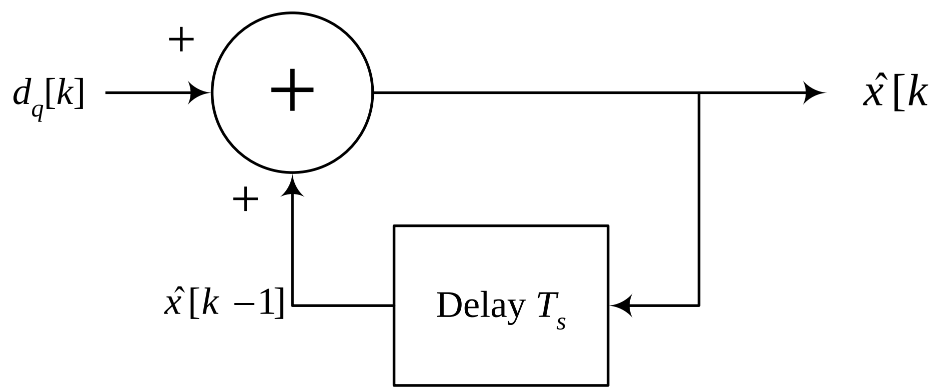
Differential Pulse Code Modulation (DPCM)

- In PCM we quantize the analog samples. Since the signal varies over a large range of amplitudes, we generally need a large number of levels (an hence bits).
- Note that neighboring samples are "*close*" to each other in values.
- If we instead quantize the difference between successive samples, we will be dealing with much smaller range of values.
- This will results in either:
  - Using less number of bits for the same SNR.
  - Obtaining smaller SNR for the same number of bits.
- Quantization noise will be reduced by a factor of $(m_p/m_d)^2$

Block Diagram of DPCM

$$d[k] = x[k] - x[k-1].$$





$$\hat{x}[k] = d_q[k] + \hat{x}[k-1].$$

Generalized DPCM

- We can get even a smaller range of values if we define the difference as:

$$d[k] = x[k] - \hat{x}[k]$$

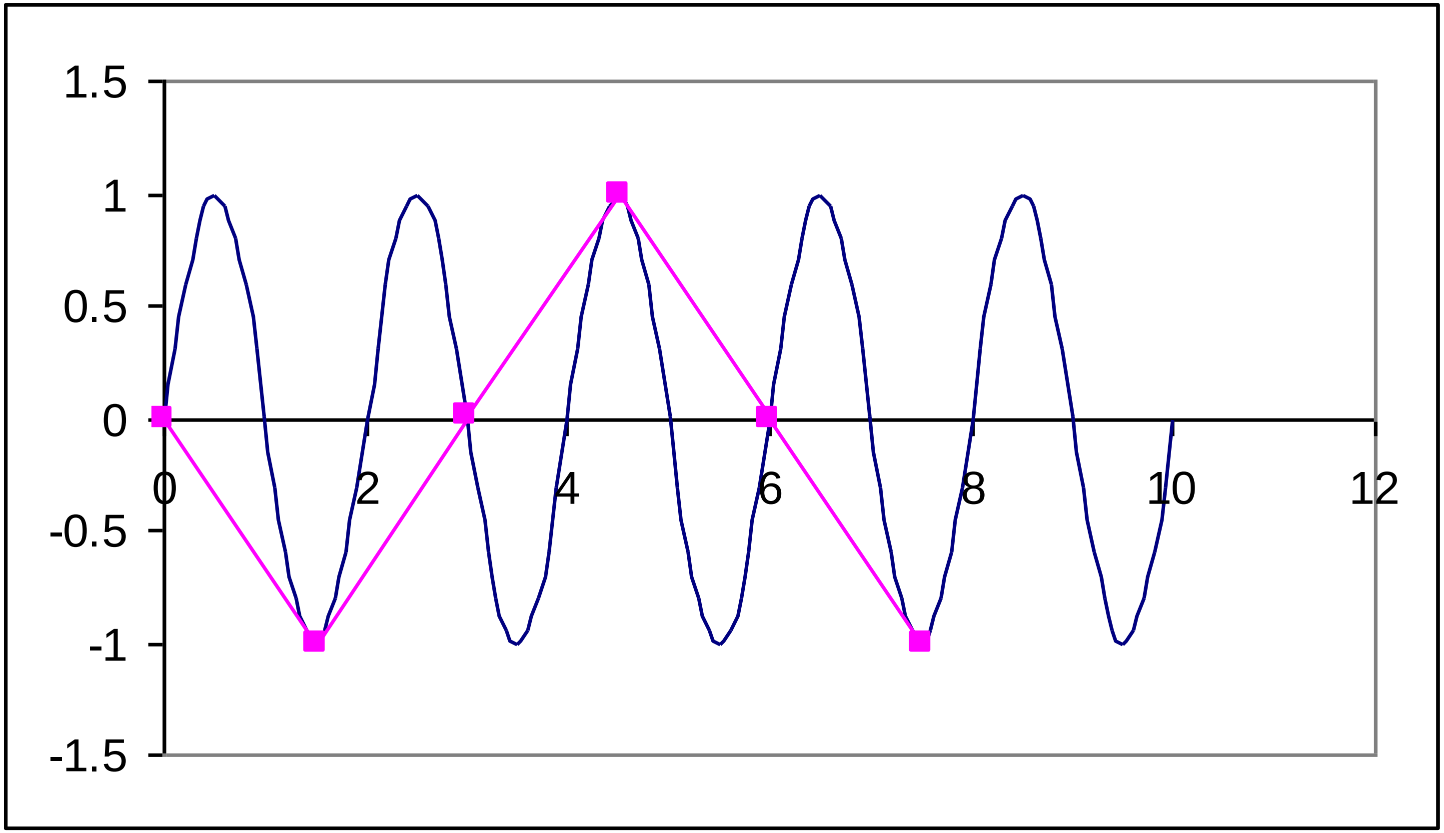
$\hat{x}(k)$ can be predicted from previous values of x,

$$\hat{x}(k) = a_1 x(k-1) + a_2 x(k-2) + a_3 x(k-3) + \cdots$$

- The more previous samples included, the better the approximation, the smaller the difference.
- The relation $d[k] = x[k] - x[k-1]$ is a special case where the previous sample is taken as a prediction of the current value.

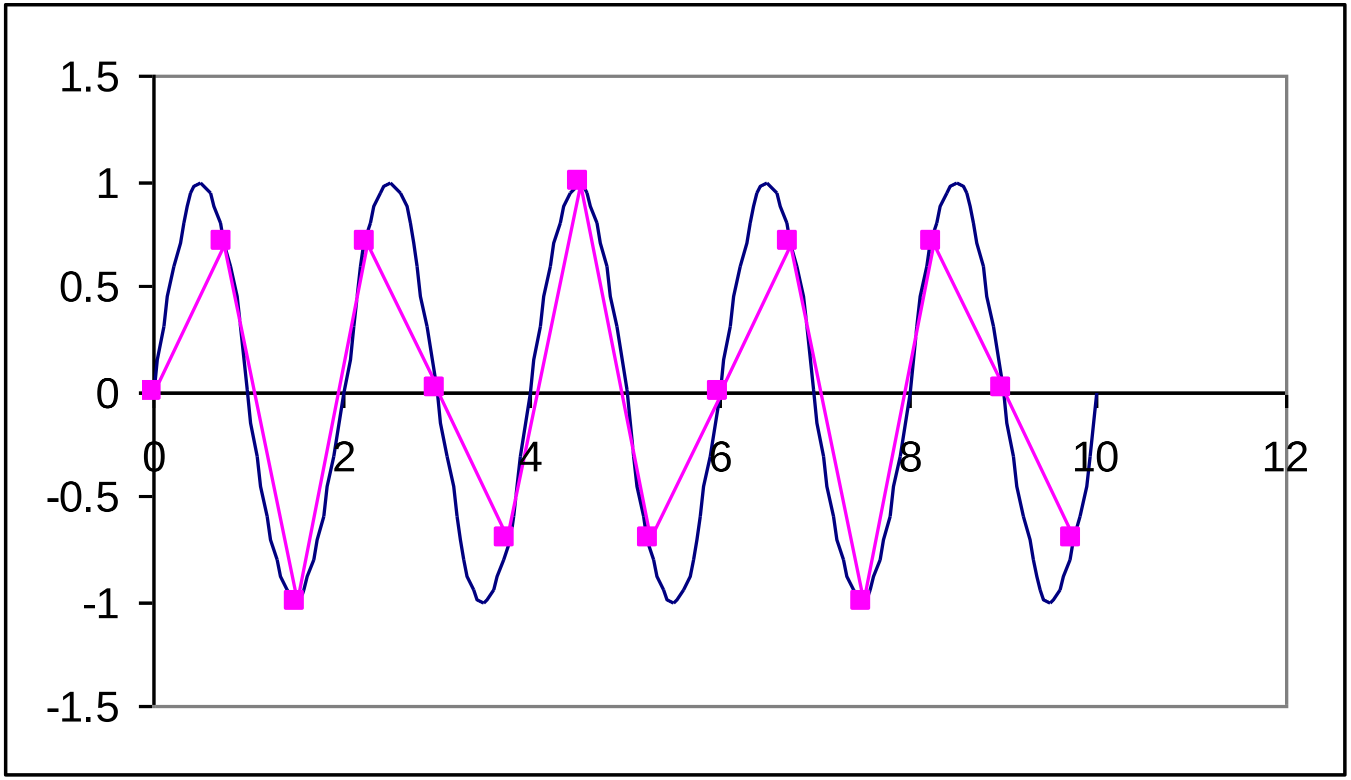
Sampling position

Even Worse



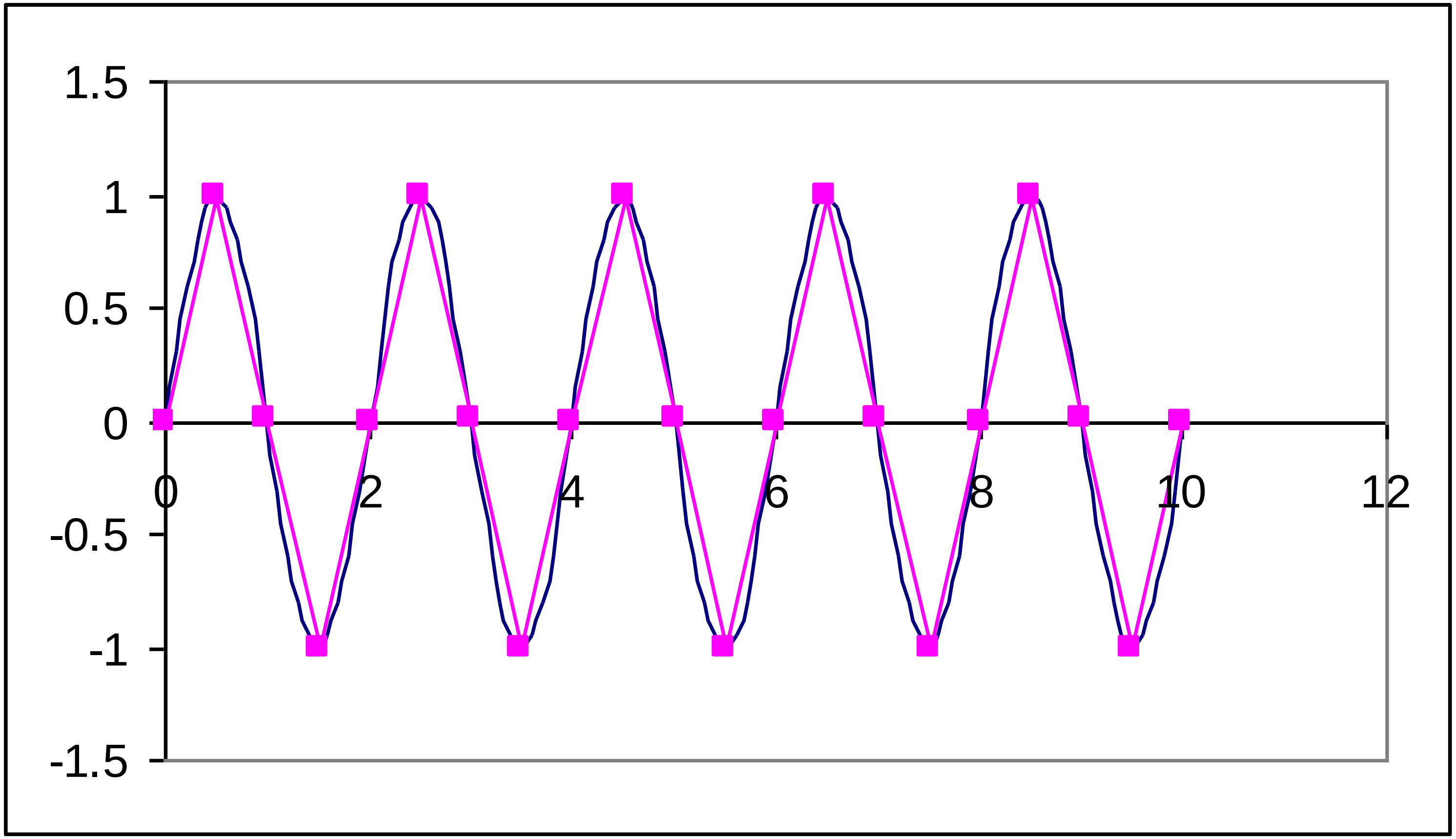
Sampling Frequency = 1/3 X Wave Frequency

Higher Sampling Frequency



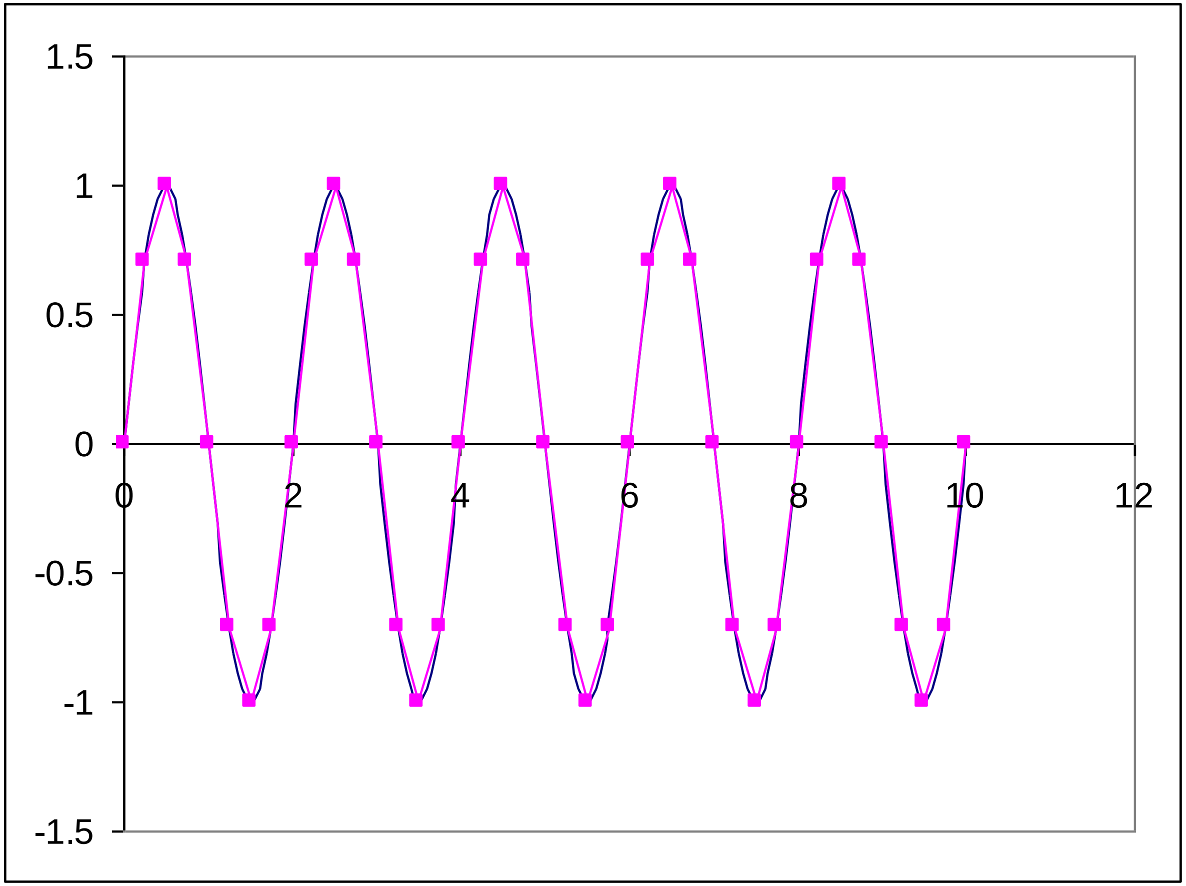
Sampling Frequency = 2/3 Wave Frequency

Getting Better



Sampling Frequency = Wave Frequency

Good Sampling



Sampling Frequency = 2 X Wave Frequency

Shannon-Nyquist's Sampling Theorem

- **A sampled time signal must not contain components at frequencies above half the sampling rate (The so-called Nyquist frequency)**

- **The highest frequency which can be accurately represented is one-half of the sampling rate**

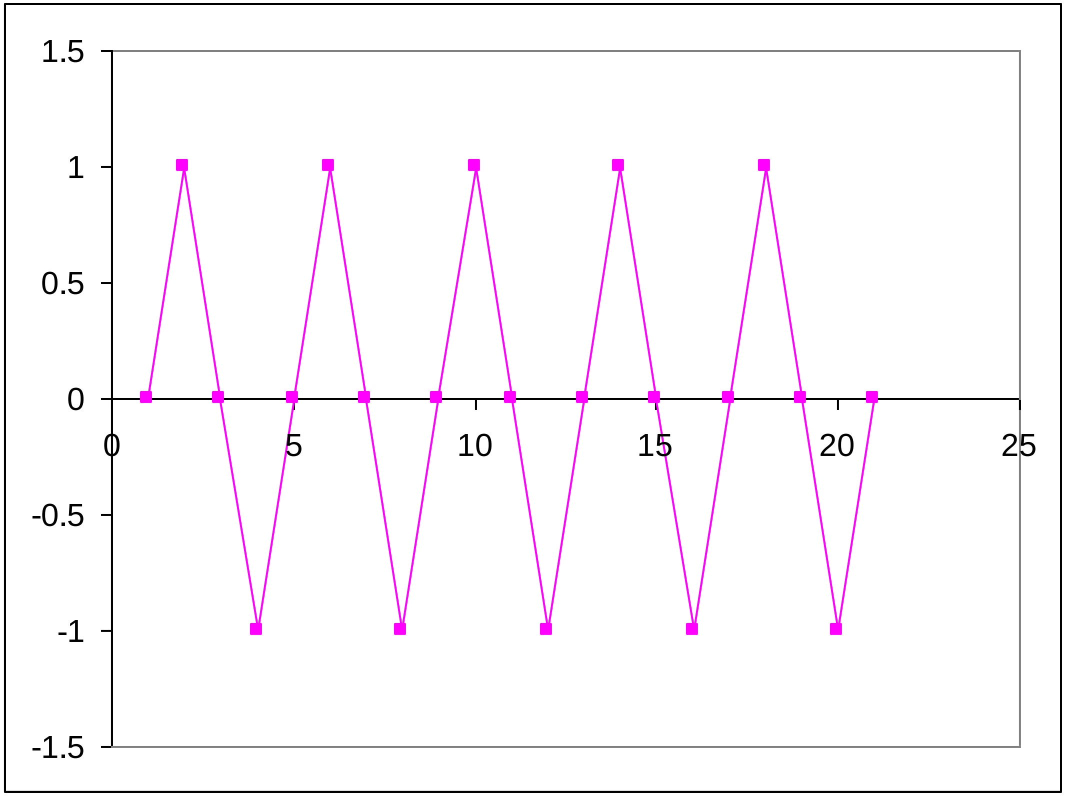
Range of Human Hearing

- 20 – 20,000 Hz
- We lose high frequency response with age
- Women generally have better response than men
- To reproduce 20 kHz requires a sampling rate of 40 kHz
  - Below the Nyquist frequency we introduce aliasing

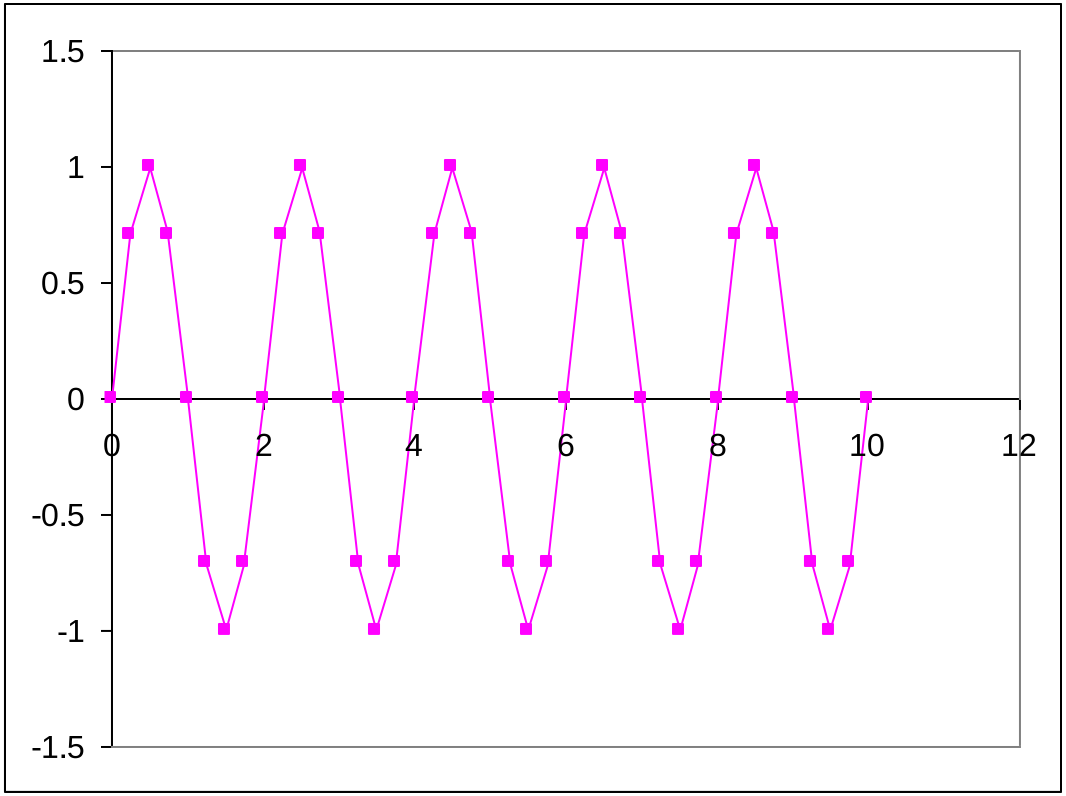
Effect of Aliasing

- Fourier Theorem states that any waveform can be reproduced by sine waves.
- Improperly sampled signals will have other sine wave components.

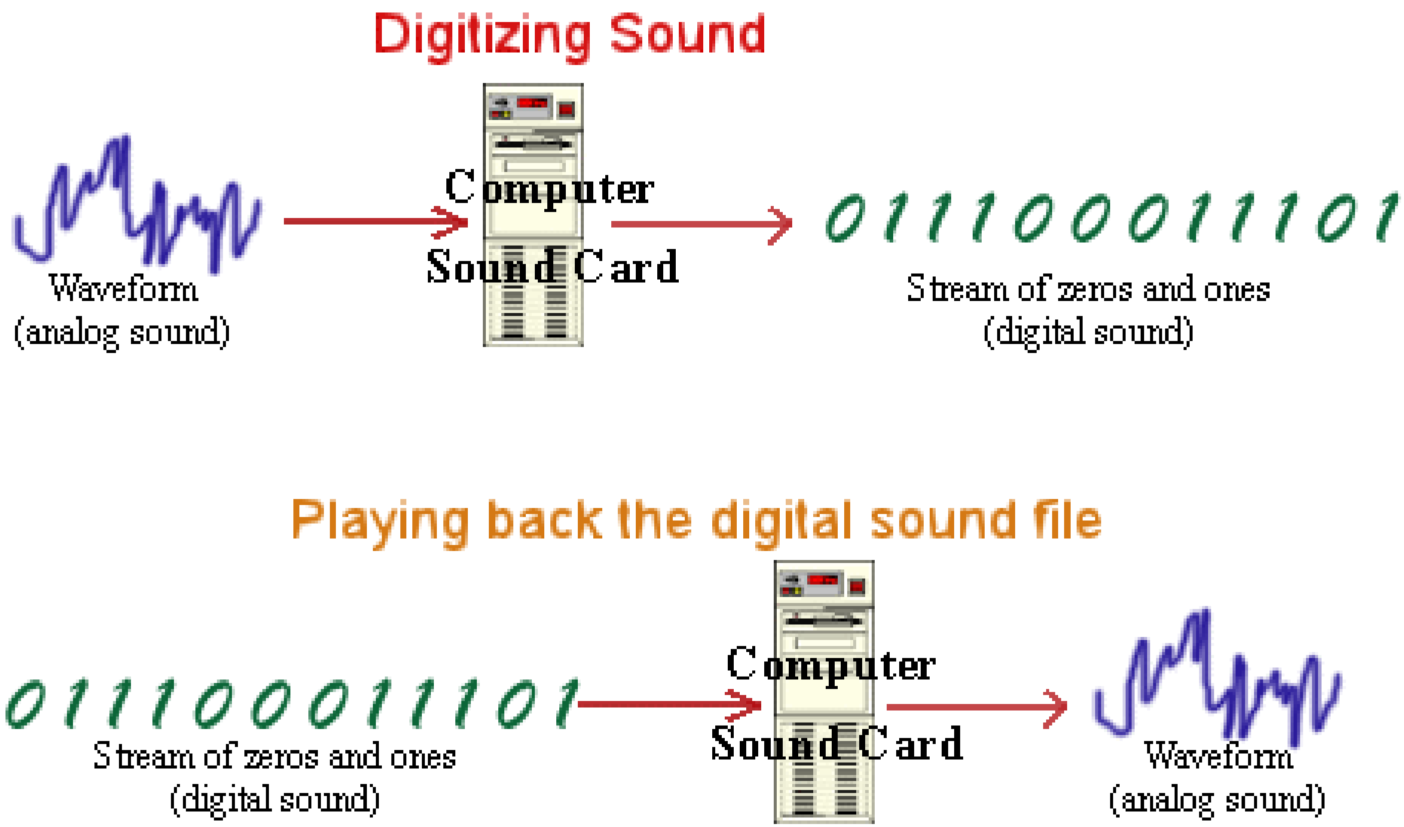
Half the Nyquist Frequency



Nyquist Frequency



Digitizing


Digitizing Sound
Computer
Sound Card
Waveform
(analog sound)
0 1 1 1 0 0 0 1 1 1 0 1
Stream of zeros and ones
(digital sound)
Playing back the digital sound file
0 1 1 1 0 0 0 1 1 1 0 1
Stream of zeros and ones
(digital sound)
Computer
Sound Card
Waveform
(analog sound)

Key Parameters

- Sampling frequency
  - 11.025kHZ or 22.05kHZ or 44.1kHZ
- Number of bits per sample
  - 8 bits (256 levels) or 16 bits (65,536 levels)

Digital Voice Telephone Transmission

- Voice data for telephony purposes is limited to frequencies less than 4,000 Hz.
- According to Nyquist, it would take 8,000 samples (2 times 4,000) to capture a 4,000 Hz signal perfectly.
- Generally, one byte is recorded per sample (256 levels). One byte is eight bits of binary data.
- (8 bits * 8,000 samples per second = 64K bps) over a circuit.

T-1 Transmisson

- T carrier circuits are designed around this requirement, since they are primarily designed to carry analog voice signals that have been digitalized.
- For example, look at the DS-1 signal which passes over a T-1 circuit. For DS-1 transmissions, each frame contains 8 bits per channel and there are 24 channels. Also, 1 "framing bit" is required for each of the 24 channel frames.

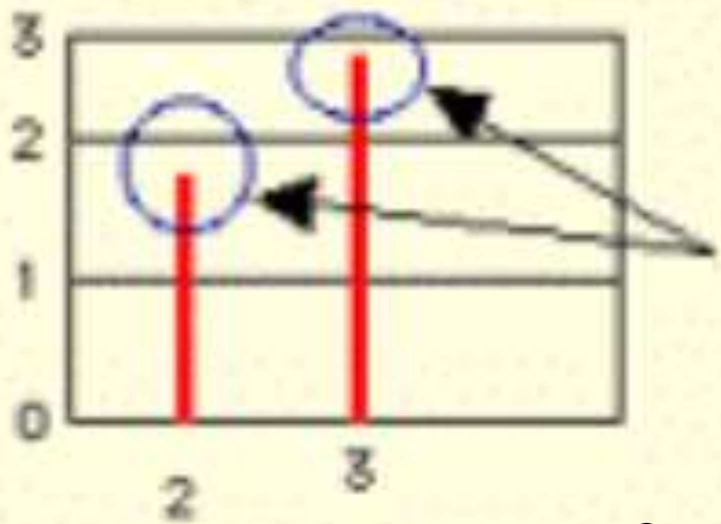
T-1 Transmissons

- (24 channels * 8 bits per channel) + 1 framing bit = 193 bits per frame.
  193 bits per frame * 8,000 "Nyquist" samples = 1,544,000 bits per second.
- And it just so happens that the T-1 circuit is 1.544 Mbps.--not a coincidence. Each of the 24 channels in a T-1 circuit carries 64Kbps.

Standards

- DS0 - 64 kilobits per second
- ISDN - Two DS0 lines plus signaling (16 kilobytes per second), or 128 kilobits per second
- T1 - 1.544 megabits per second (24 DS0 lines)
- T3 - 43.232 megabits per second (28 T1s)
- OC3 - 155 megabits per second (84 T1s)
- OC12 - 622 megabits per second (4 OC3s)
- OC48 - 2.5 gigabits per seconds (4 OC12s)
- OC192 - 9.6 gigabits per second (4 OC48s)

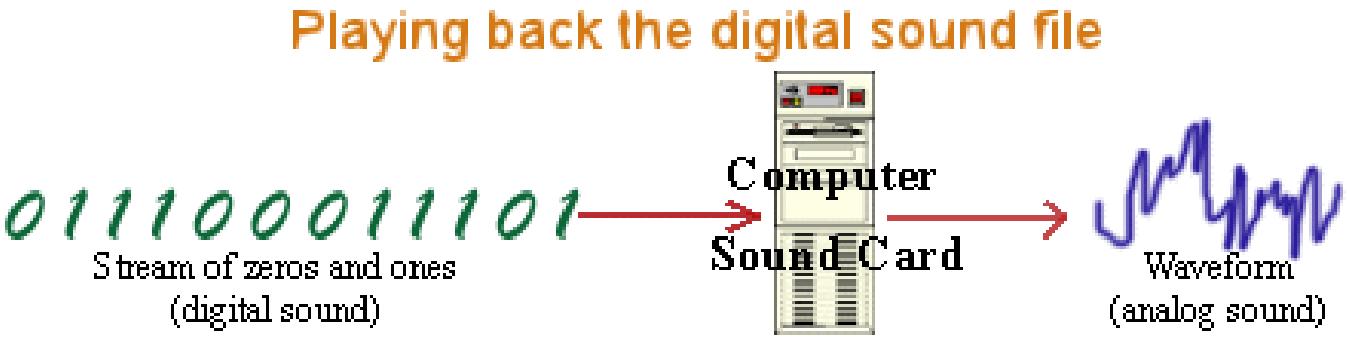
Quantization Error

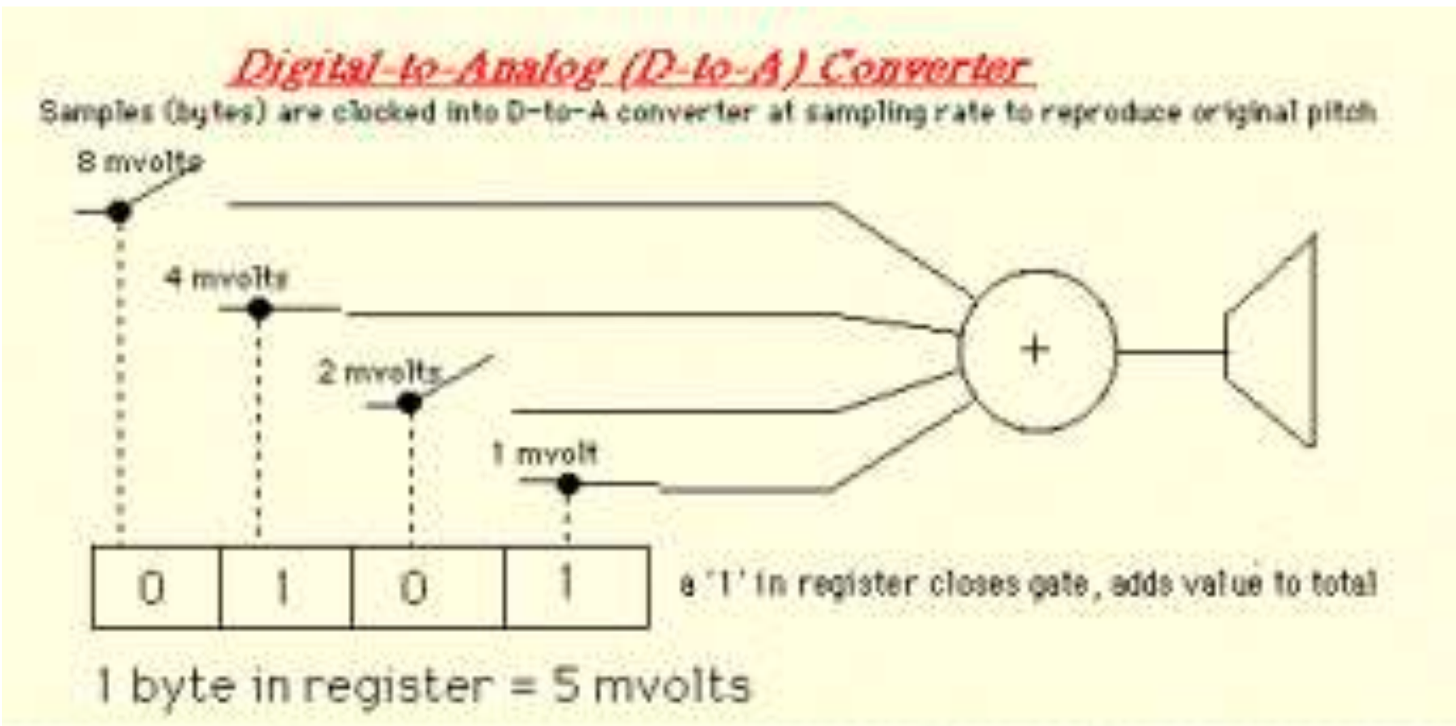


Approximation or quantizing error

Greater error = more noise

D/A Conversion





CD ROMS

- Sampling rate is 44.1 kHz
- Nyquist Theorem says that the highest reproduced frequency is 22.05 kHz.
  - Any frequency above 22.05 kHz will produce aliasing
- A low pass filter is used to block frequencies above 22.05 kHz.

Problems with D/A

- Imperfect low pass filters
- Ideally you want 0 dB attenuation at 20 kHz going up to 90 dB at 22 kHz
  - Very expensive
- Oversampling will help
  - Sample at 8 X 20 kHz = 160 kHz
    - Then the low pass filtering needs to be accomplished in 140 kHz not 2 kHz

Problems with D/A

- Finite word length
  - Most systems today do 16 bit digitizing
  - 65536 different levels
- The loudest sounds need room, so the normal sounds don’t use the entire range
  - Problems occur at the low levels where sounds are represented by only one or two bits. High distortions result.
- Dithering adds low level broadband noise

Problems with D/A

- Clock speed variation (Jitter)

DELTA MODULATION

- *Delta modulation* uses a single-bit PCM code to achieve digital transmission of analog signals.

- With conventional PCM, each code is a binary representation of both the sign and the magnitude of a particular sample. Therefore, multiple-bit codes are required to represent the many values that the sample can be.

- With delta modulation, rather than transmit a coded representation of the sample, only a single bit is transmitted, which simply indicates whether that sample is larger or smaller than the previous sample.

DELTA MODULATION

- The algorithm for a delta modulation system is quite simple.
- If the current sample is smaller than the previous sample, a logic 0 is transmitted.
- If the current sample is larger than the previous sample, a logic 1 is transmitted.

DELTA MODULATION
Transmitter

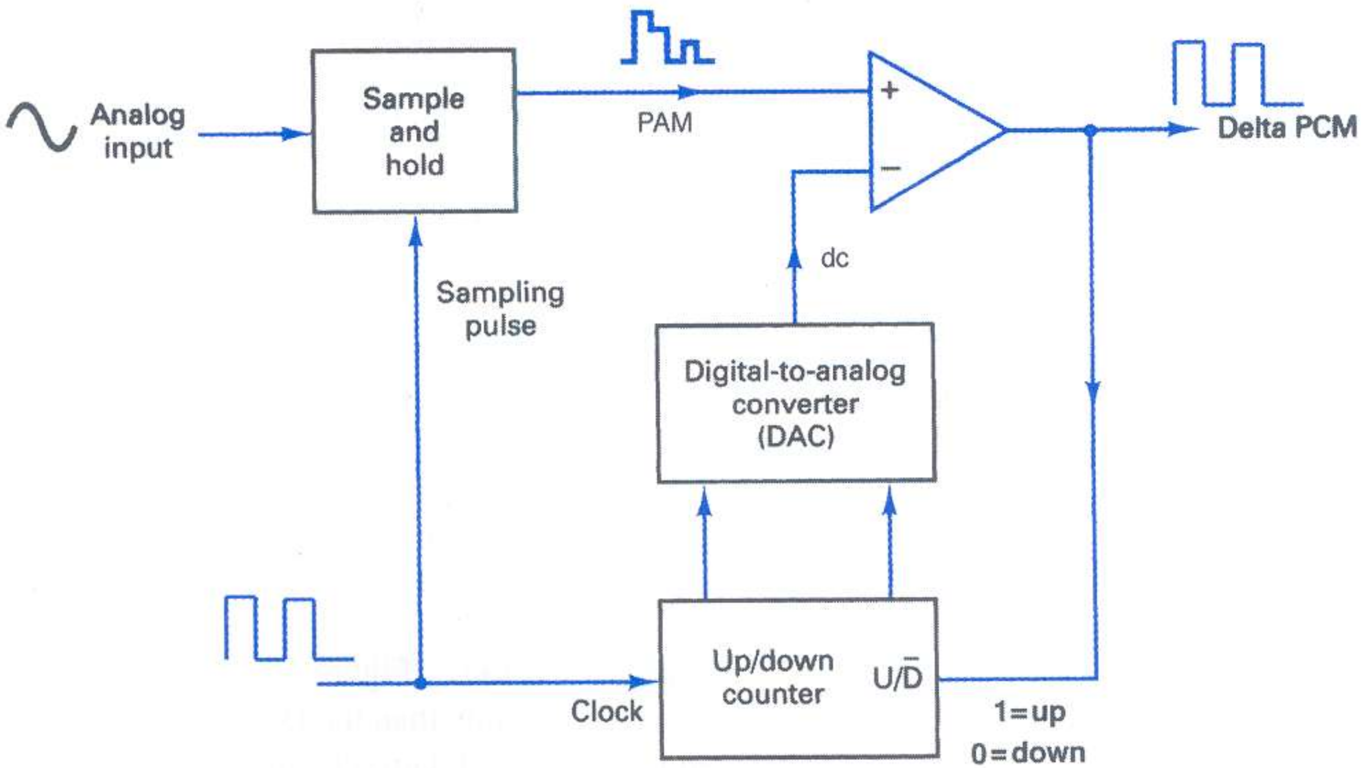


FIGURE 10-20 Delta modulation transmitter

DELTA MODULATION
Transmitter

- The input analog is sampled and converted to a PAM signal, which is compared with the output of the DAC.
- The output of the DAC is a voltage equal to the regenerated magnitude of the previous sample, which was stored in the up-down counter as a binary number.
- The up-down counter is incremented or decremented depending on whether the previous sample is larger or smaller than the current sample.

DELTA MODULATION
Transmitter

- The up-down counter is clocked at a rate equal to the sample rate. Therefore, the up-down counter is updated after each comparison.

- Initially, the up-down counter is zeroed, and the DAC is outputting 0 V. The first sample is taken, converted to a PAM signal, and compared with zero volts.

- The output of the comparator is a logic 1 condition (+ V), indicating that the current sample is larger in amplitude than the previous sample. On the next clock pulse, the up--down counter is incremented to a count of 1.

DELTA MODULATION

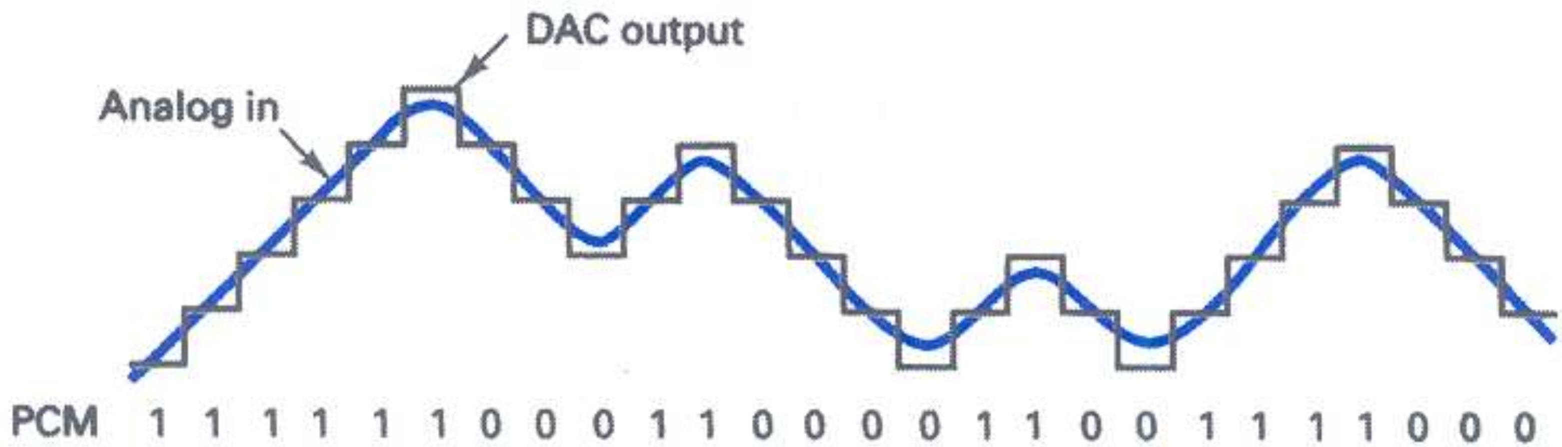


FIGURE 10-21 Ideal operation of a delta modulation encoder

DELTA MODULATION
Receiver

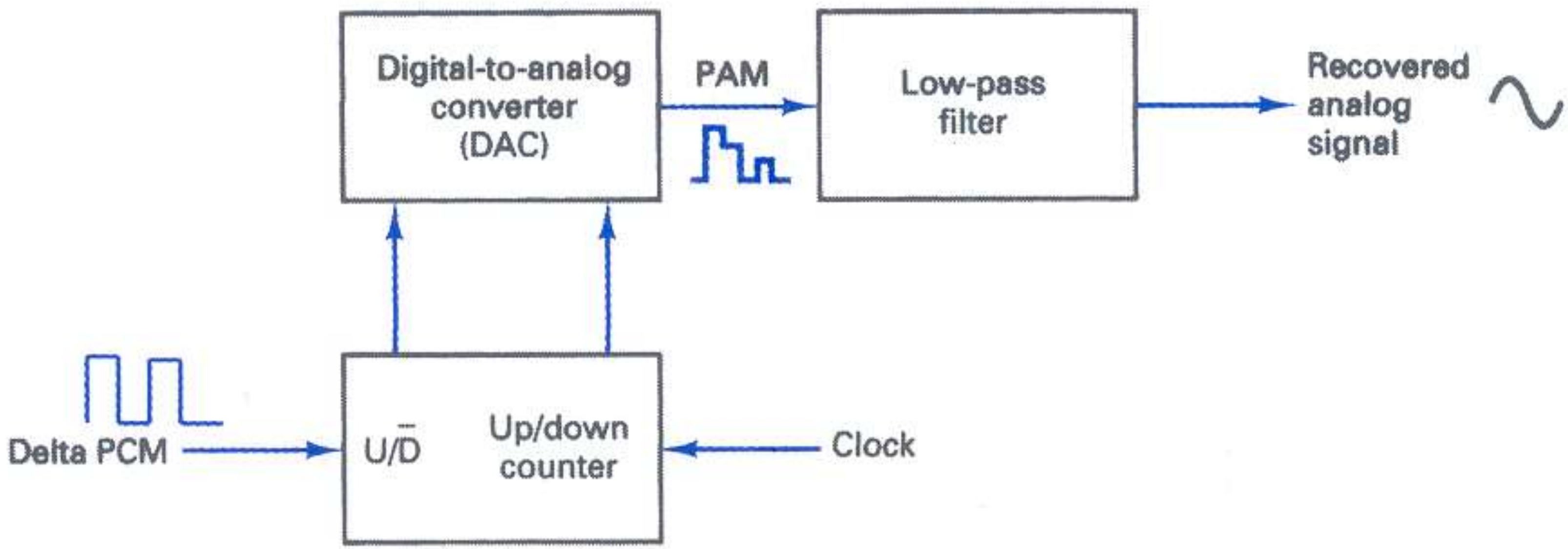


FIGURE 10-22 Delta modulation receiver

Problems with DM

- Slope overload - when the analog input signal changes at a faster rate than the DAC can maintain. The slope of the analog signal is greater than the delta modulator can maintain and is called *slope overload.*

- Increasing the clock frequency reduces the probability of slope overload occurring. Another way to prevent slope overload is to increase the magnitude of the minimum step size.

Problems with DM

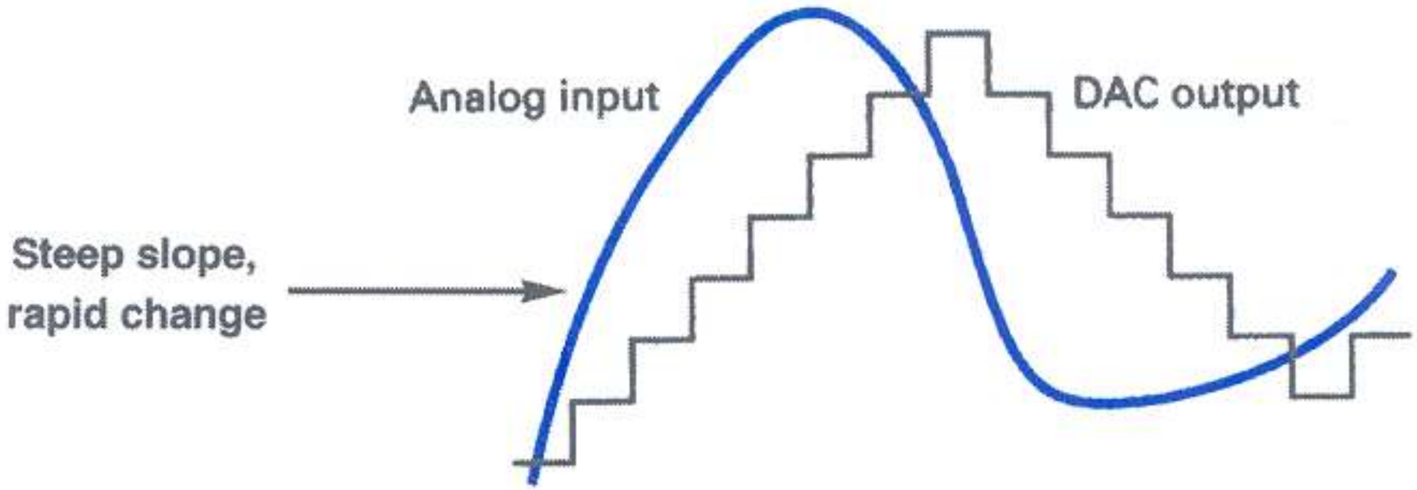


FIGURE 10-23 Slope overload distortion

Problems with DM

- Granular noise. Figure 10-24 contrasts the original and reconstructed signals associated with a delta modulation system. It can be seen that when the original analog input signal has a relatively constant amplitude, the reconstructed signal has variations that were not present in the original signal. This is called *granular noise.* Granular noise in delta modulation is analogous to quantization noise in conventional PCM.

Problems with DM

- Granular noise can be reduced by decreasing the step size. Therefore, to reduce the granular noise, a small resolution is needed, and to reduce the possibility of slope overload occurring, a large resolution is required. Obviously, a compromise is necessary.
-
- Granular noise is more prevalent in analog signals that have gradual slopes and whose amplitudes vary only a small amount. Slope overload is more prevalent in analog signals that have steep slopes or whose amplitudes vary rapidly.

Problems with DM

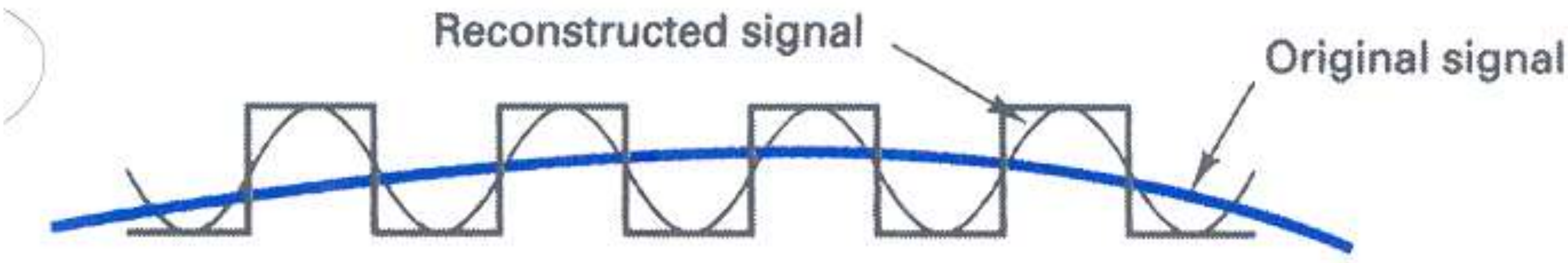


FIGURE 10-24 Granular noise

Adaptive DM

- *Adaptive delta modulation* is a delta modulation system where the step size of the DAC is automatically varied, depending on the amplitude characteristics of the analog input signal.

- Figure 10-25 shows how an adaptive delta modulator works.

Adaptive DM

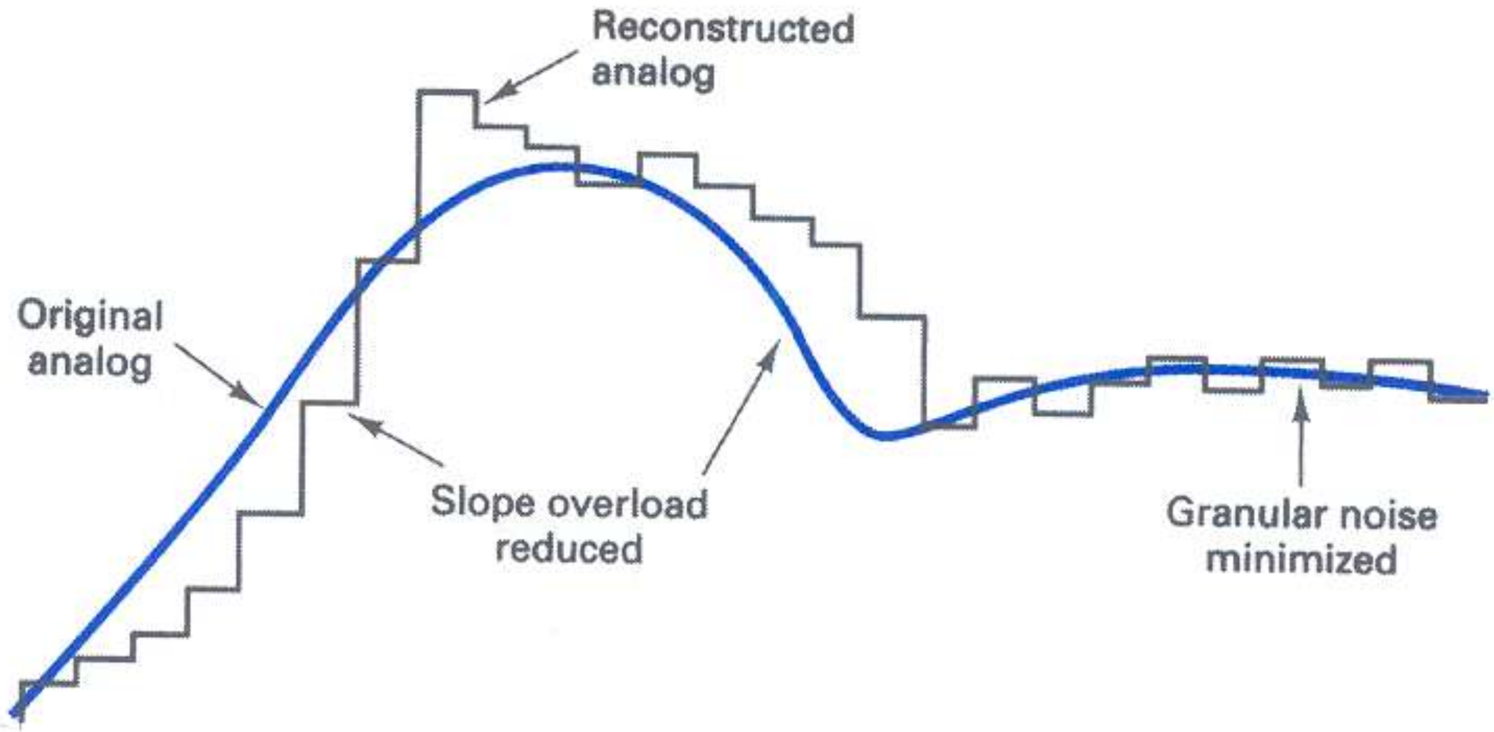


FIGURE 10-25 Adaptive delta modulation

Adaptive DM

- When the output of the transmitter is a string of consecutive Is or 0s, this indicates that the slope of the DAC output is less than the slope of the analog signal in either the positive or the negative direction. Essentially, the DAC has lost track of exactly where the analog samples are, and the possibility of slope overload occurring is high. With an adaptive delta modulator, after a predetermined number of consecutive 1s or 0s, the step size is automatically increased. After the next sample, if the DAC output amplitude is still below the sample amplitude, the next step is increased even further until eventually the DAC catches up with the analog signal.

Adaptive DM

- When an alternative sequence of 1s and 0s is occurring, this indicates that the possibility of granular noise occurring is high. Consequently, the DAC will automatically revert to its minimum step size and, thus, reduce the magnitude of the noise error.

- A common algorithm for an adaptive delta modulator is when three consecutive 1s or 0s occur, the step size of the DAC is increased or decreased by a factor of 1.5.

- Various other algorithms may be used for adaptive delta modulators, depending on particular system requirements

Differential DM

- In a typical PCM-encoded speech waveform, there are often successive samples taken in which there is little difference between the amplitudes of the two samples.

- This necessitates transmitting several identical PCM codes, which is redundant.

- Differential pulse code modulation (DPCM) is designed specifically to take advantage of the sample-to-sample redundancies in typical speech waveforms.

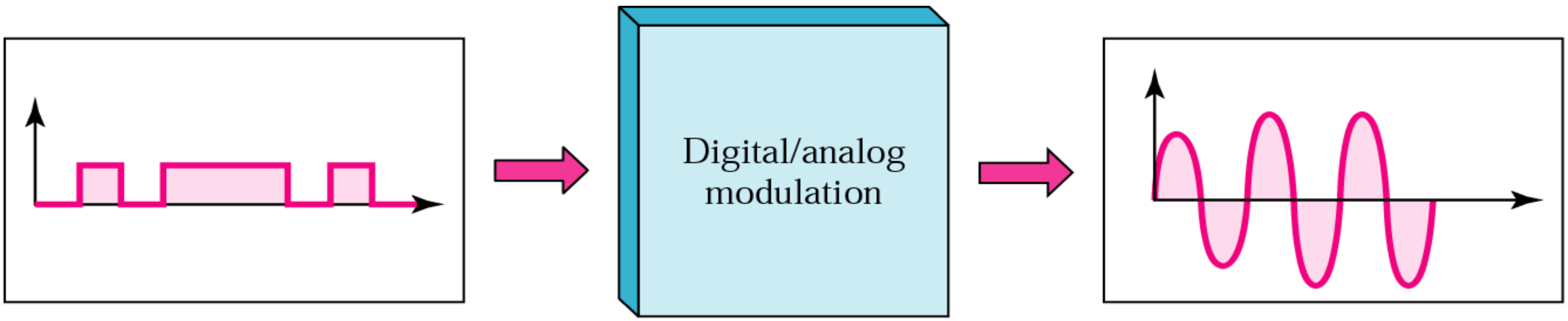
Differential DM

- With DPCM, the difference in the amplitude of two successive samples is transmitted rather than the actual sample. Because the range of sample differences is typically less than the range of individual samples, fewer bits are required for DPCM than conventional PCM.

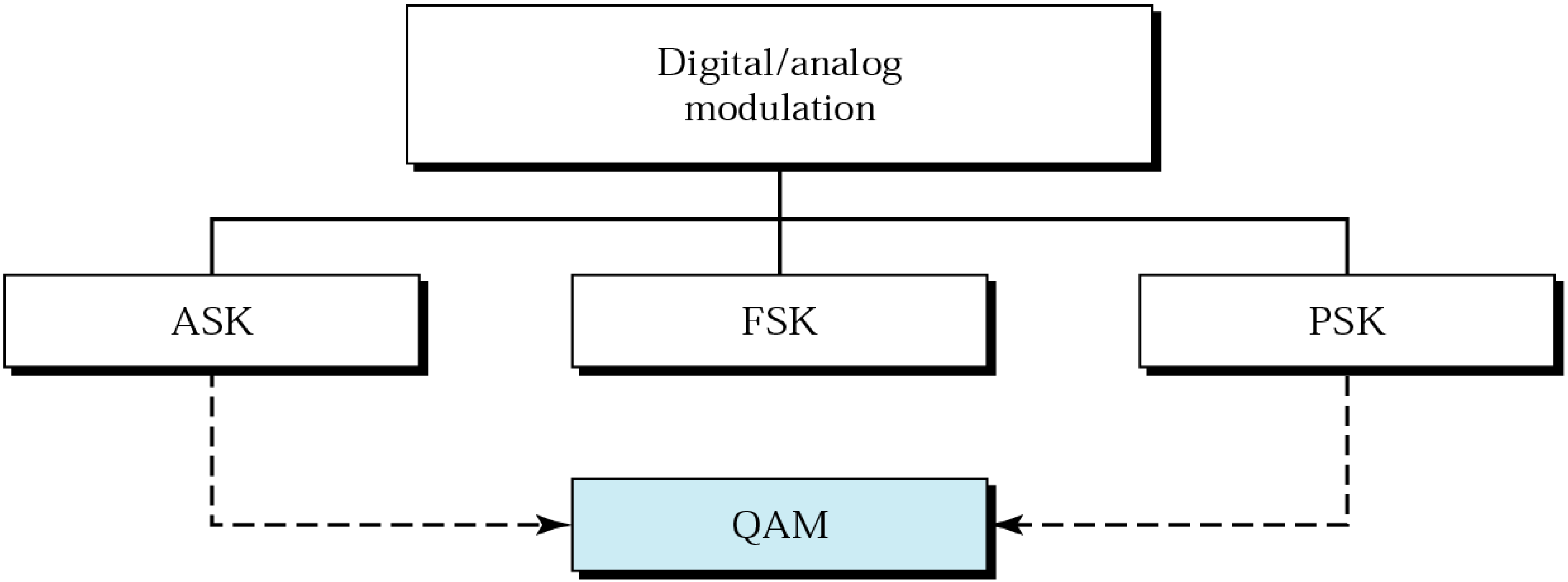
Modulation of Digital Data

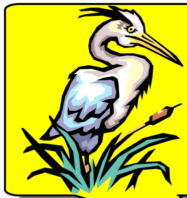
Digital-to-Analog Conversion
Amplitude Shift Keying (ASK)
Frequency Shift Keying (FSK)
Phase Shift Keying (PSK)
Quadrature Amplitude Modulation
Bit/Baud Comparison

Digital-to-analog modulation



Types of digital-to-analog modulation





Note:

Bit rate is the number of bits per second. Baud rate is the number of signal units per second. Baud rate is less than or equal to the bit rate.

Example 1

An analog signal carries 4 bits in each signal unit. If 1000 signal units are sent per second, find the baud rate and the bit rate

Solution

Baud rate = 1000 bauds per second (baud/s)
Bit rate = 1000 x 4 = 4000 bps

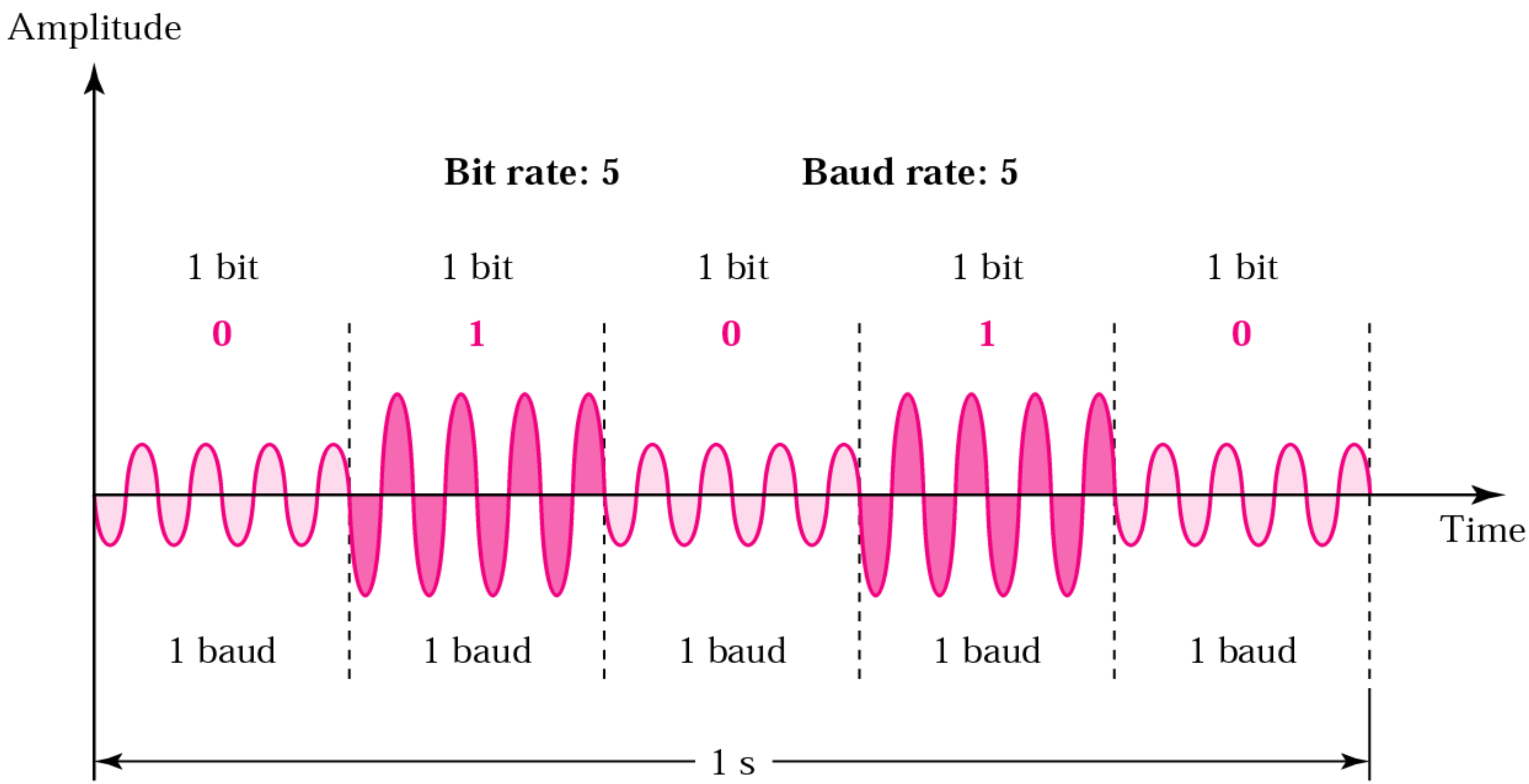
Example 2

The bit rate of a signal is 3000. If each signal unit carries 6 bits, what is the baud rate?

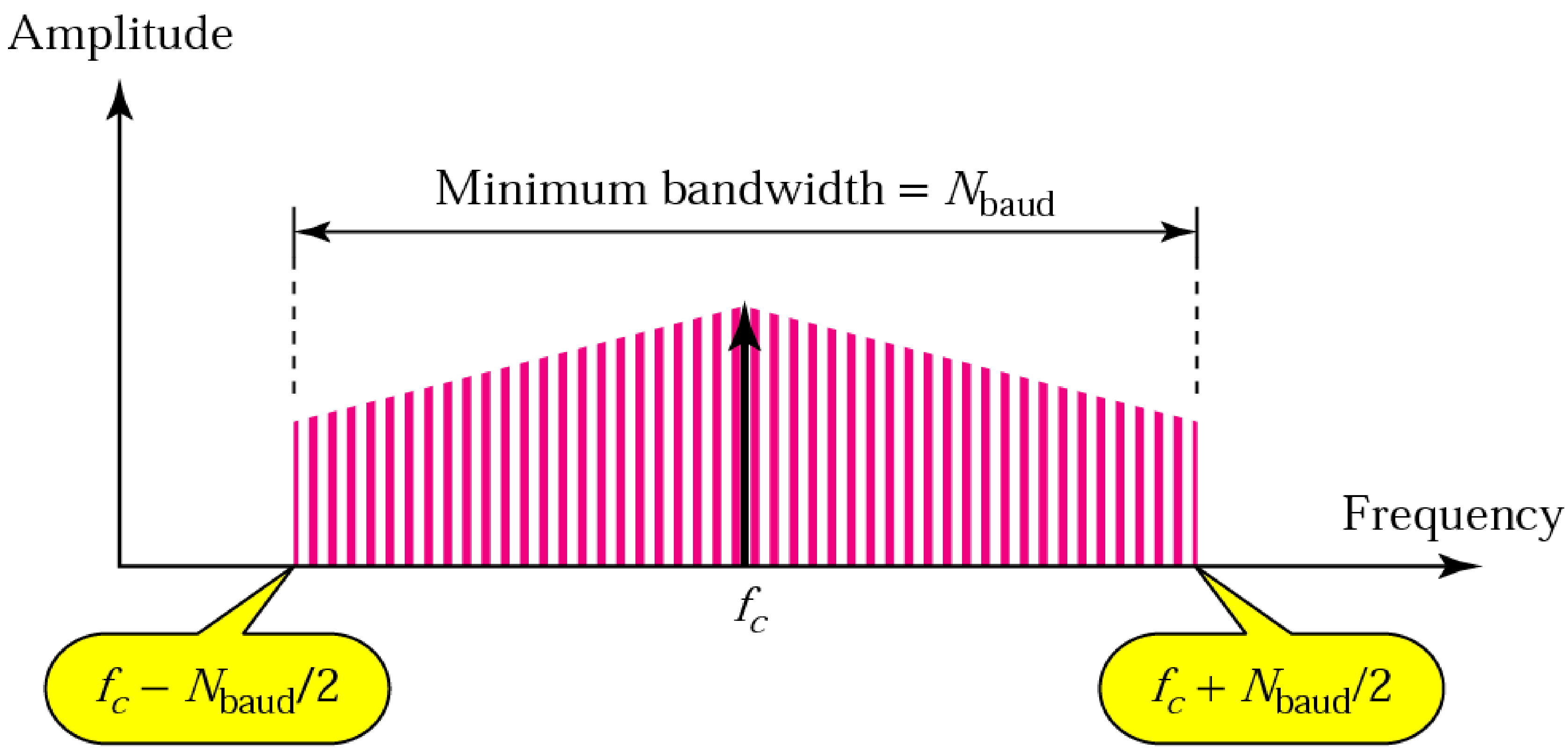
Solution

Baud rate = 3000 / 6 = 500 baud/s

ASK


Amplitude
Bit rate: 5
Baud rate: 5
1 bit
1 bit
1 bit
1 bit
1 bit
0
1
0
1
0
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 s

Relationship between baud rate and bandwidth in ASK



Example 3

Find the minimum bandwidth for an ASK signal transmitting at 2000 bps. The transmission mode is half-duplex.

Solution

In ASK the baud rate and bit rate are the same. The baud rate is therefore 2000. An ASK signal requires a minimum bandwidth equal to its baud rate. Therefore, the minimum bandwidth is 2000 Hz.

Example 4

Given a bandwidth of 5000 Hz for an ASK signal, what are the baud rate and bit rate?

Solution

In ASK the baud rate is the same as the bandwidth, which means the baud rate is 5000. But because the baud rate and the bit rate are also the same for ASK, the bit rate is 5000 bps.

Example 5

Given a bandwidth of 10,000 Hz (1000 to 11,000 Hz), draw the full-duplex ASK diagram of the system. Find the carriers and the bandwidths in each direction. Assume there is no gap between the bands in the two directions.

Solution

For full-duplex ASK, the bandwidth for each direction is

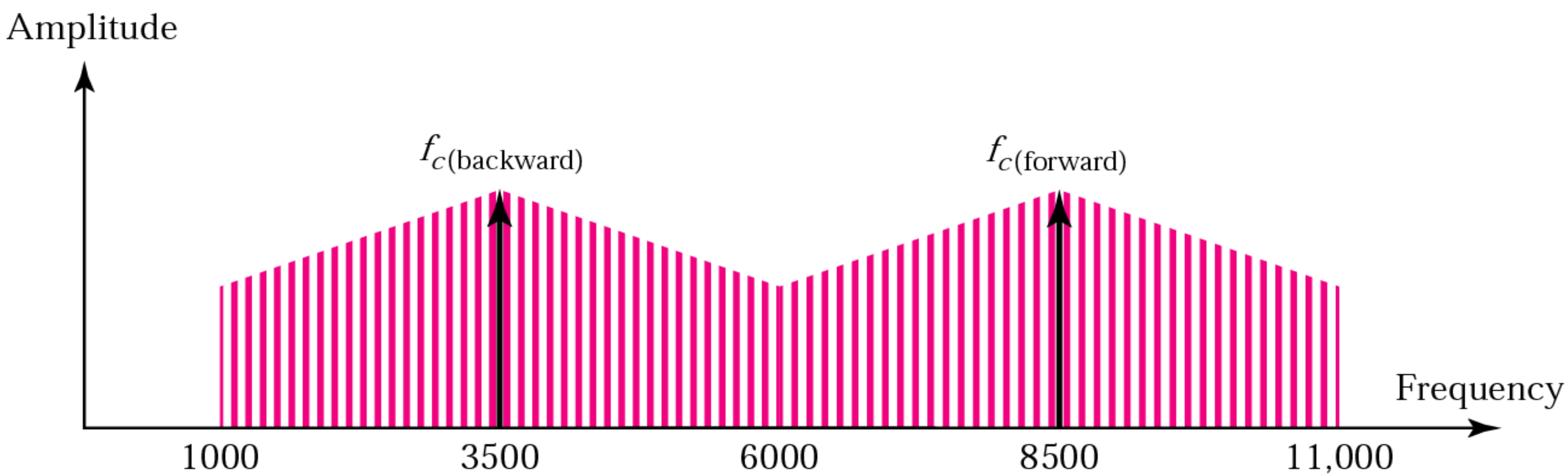
$$BW = 10000 / 2 = 5000 \text{ Hz}$$

The carrier frequencies can be chosen at the middle of each band (see Fig. 5.5).

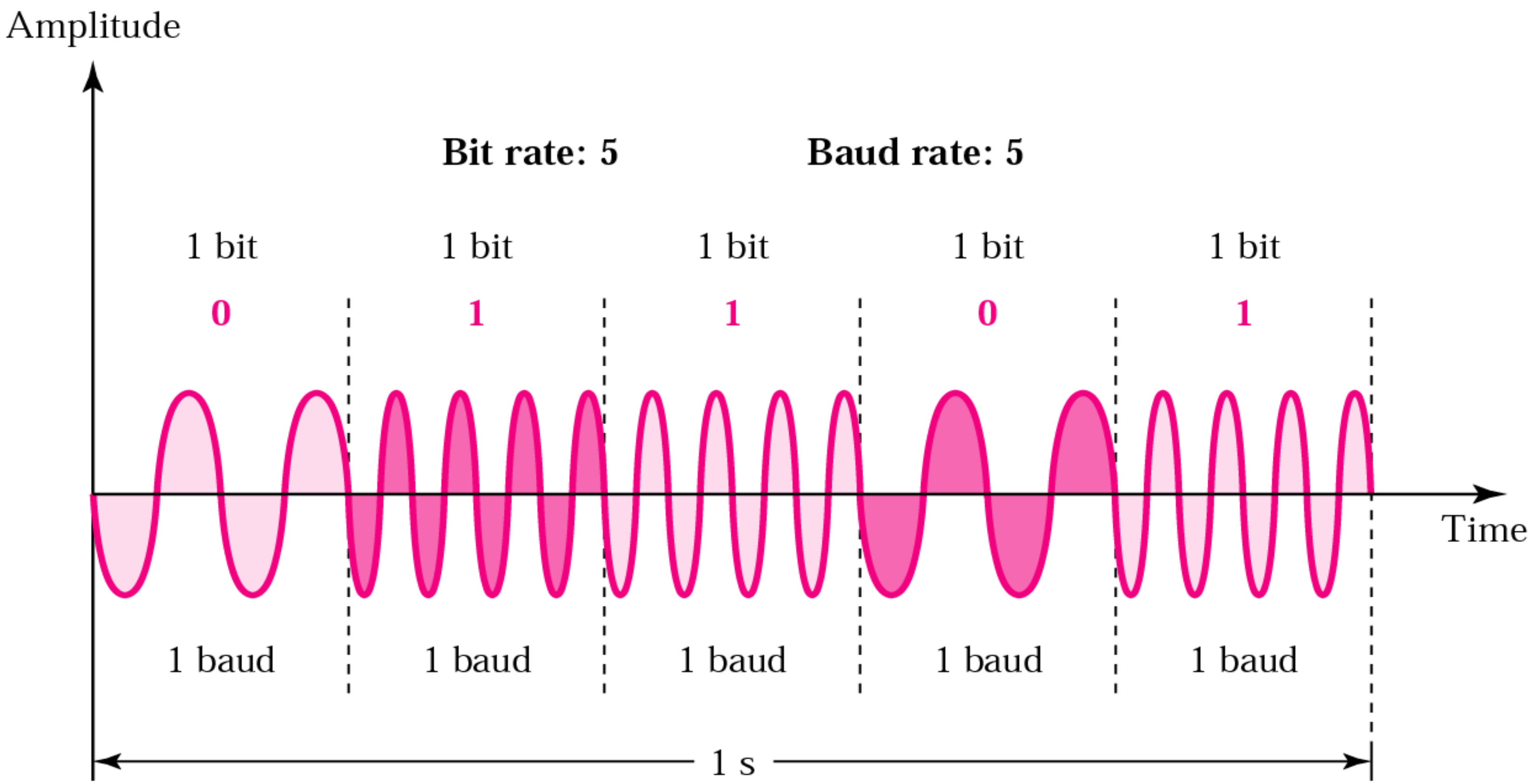
$$fc\ (\text{forward}) = 1000 + 5000/2 = 3500 \text{ Hz}$$

$$fc\ (\text{backward}) = 11000 - 5000/2 = 8500 \text{ Hz}$$

Solution to Example 5

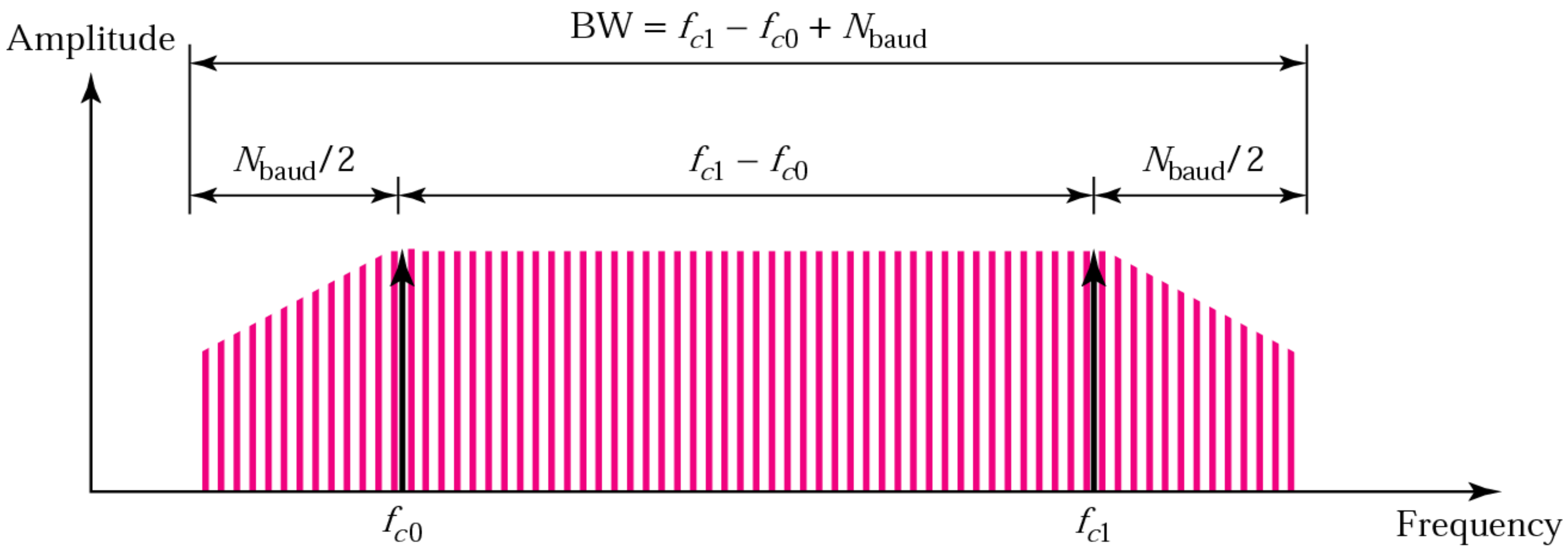


FSK



Amplitude
Bit rate: 5
Baud rate: 5
1 bit
1 bit
1 bit
1 bit
1 bit
0
1
1
0
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 s

Relationship between baud rate and bandwidth in FSK



Example 6

Find the minimum bandwidth for an FSK signal transmitting at 2000 bps. Transmission is in half-duplex mode, and the carriers are separated by 3000 Hz.

Solution

For FSK

$$BW = \text{baud rate} + \quad f_{c1} - f_{c0}$$

$$BW = \text{bit rate} + \quad fc1 - fc0 = 2000 + 3000 = 5000 \text{ Hz}$$

Example 7

Find the maximum bit rates for an FSK signal if the bandwidth of the medium is 12,000 Hz and the difference between the two carriers is 2000 Hz. Transmission is in full-duplex mode.

Solution

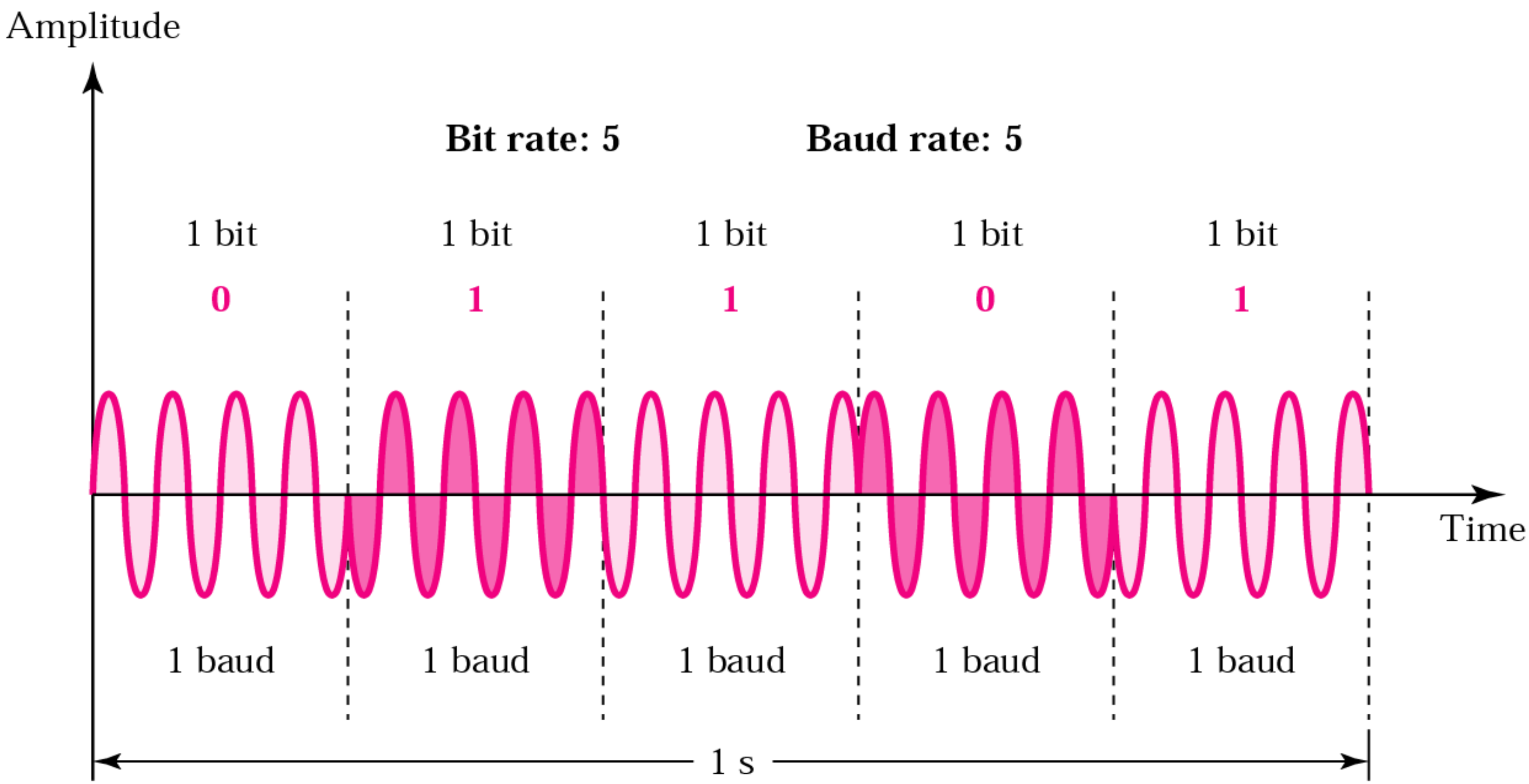
Because the transmission is full duplex, only 6000 Hz is allocated for each direction.

BW = baud rate + fc1 – fc0

Baud rate = BW – (fc1 – fc0) = 6000 – 2000 = 4000

But because the baud rate is the same as the bit rate, the bit rate is 4000 bps.

PSK

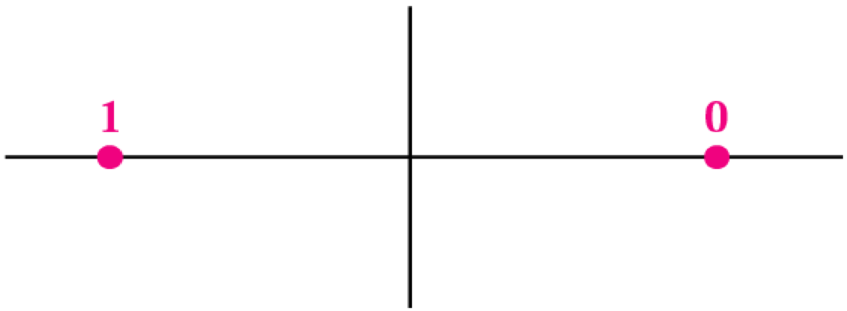


Amplitude
Bit rate: 5
Baud rate: 5
1 bit
1 bit
1 bit
1 bit
1 bit
0
1
1
0
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 s

PSK constellation

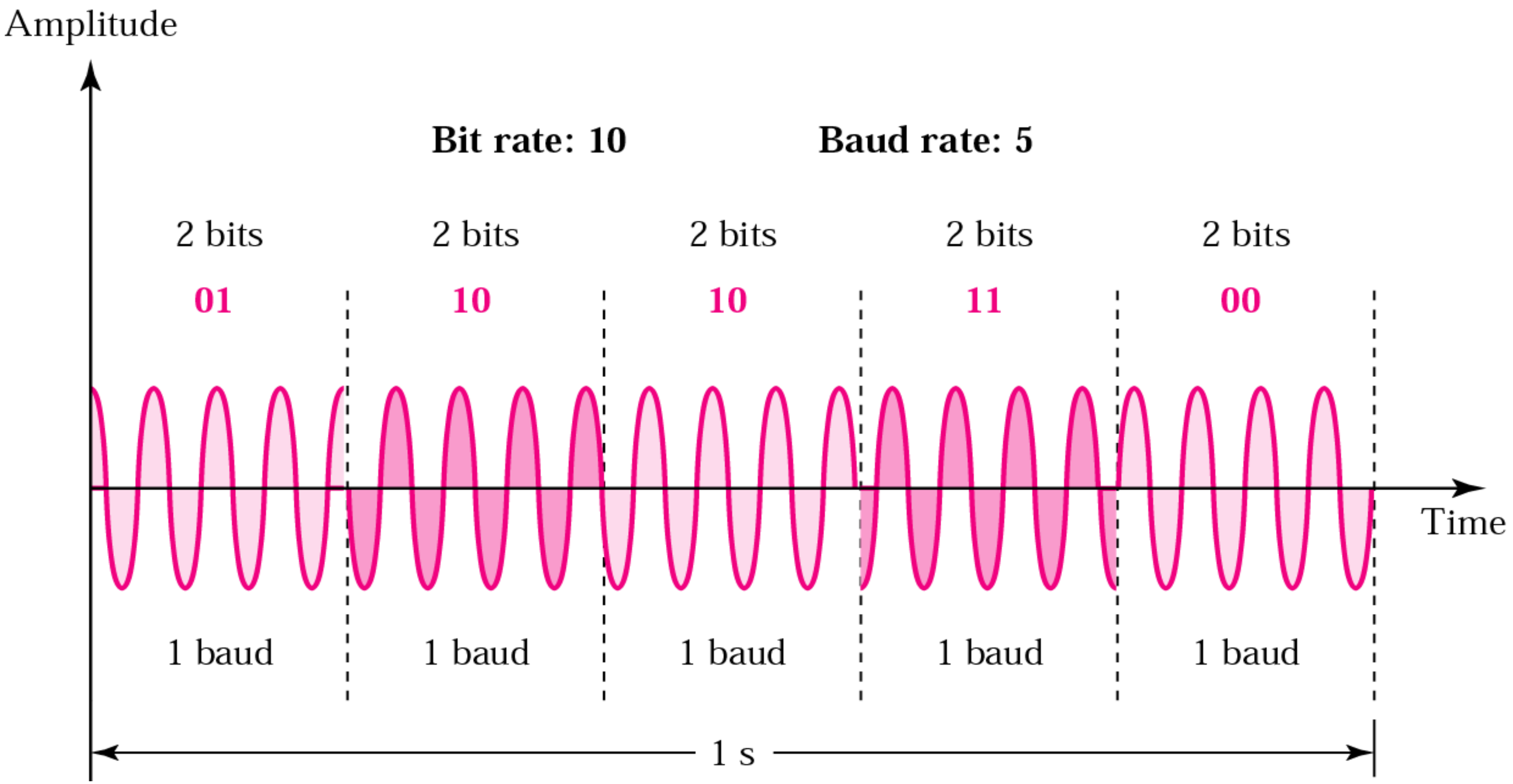
Bit	Phase
0	0
1	180

Bits



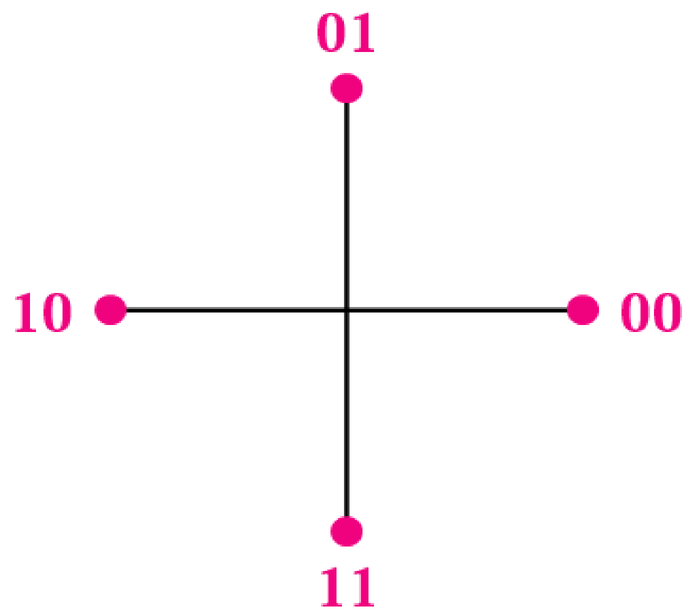
Constellation diagram

The 4-PSK method



Dibit	Phase
00	0
01	90
10	180
11	270

Dibit
(2 bits)

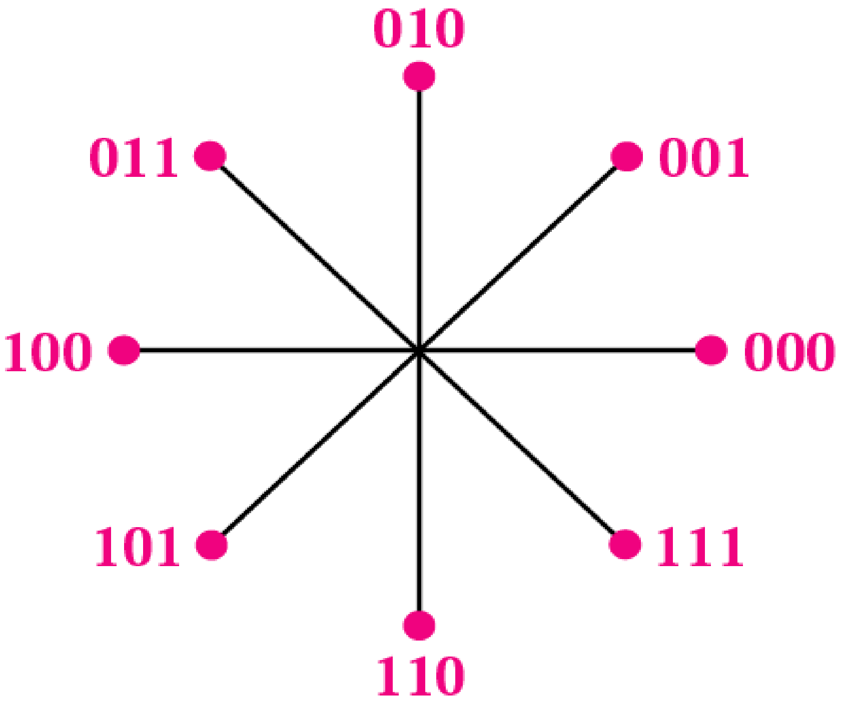


Constellation diagram

The 8-PSK characteristics

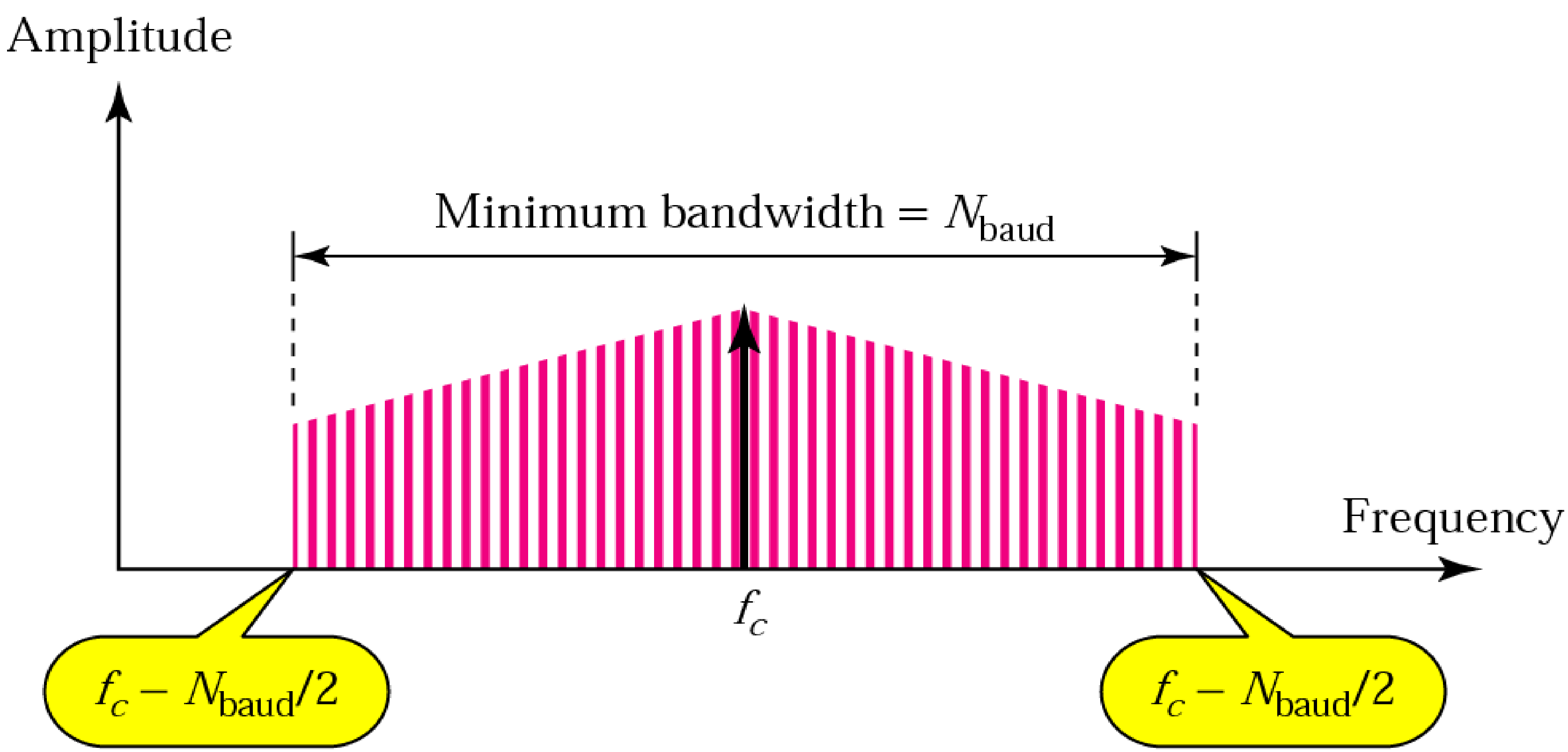
Tribit	Phase
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315

Tribits
(3 bits)



Constellation diagram

Relationship between baud rate and bandwidth in PSK



Example 8

Find the bandwidth for a 4-PSK signal transmitting at 2000 bps. Transmission is in half-duplex mode.

Solution

For PSK the baud rate is the same as the bandwidth, which means the baud rate is 5000. But in 8-PSK the bit rate is 3 times the baud rate, so the bit rate is 15,000 bps.

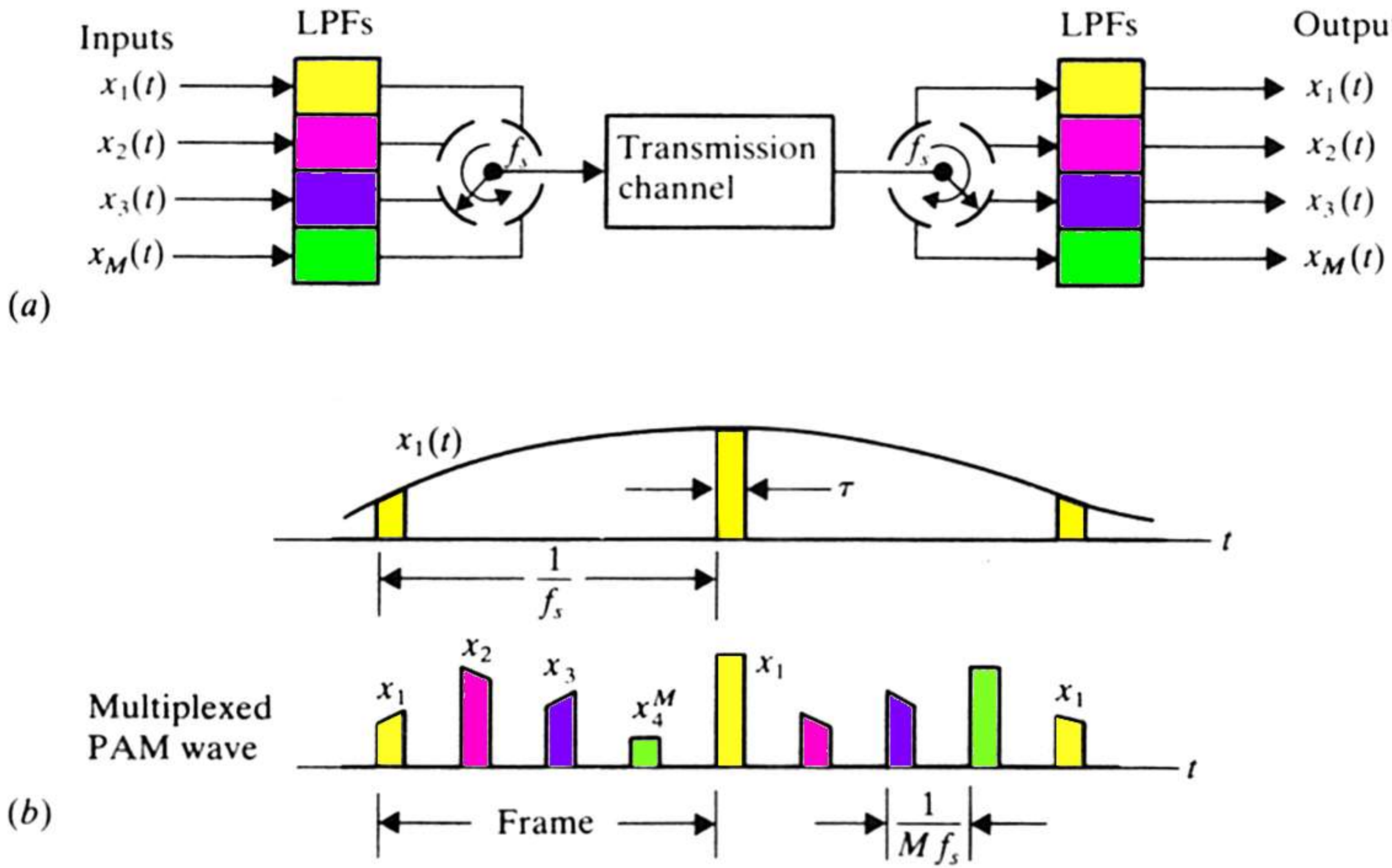
Example 9

Given a bandwidth of 5000 Hz for an 8-PSK signal, what are the baud rate and bit rate?

Solution

For PSK the baud rate is the same as the bandwidth, which means the baud rate is 5000. But in 8-PSK the bit rate is 3 times the baud rate, so the bit rate is 15,000 bps.

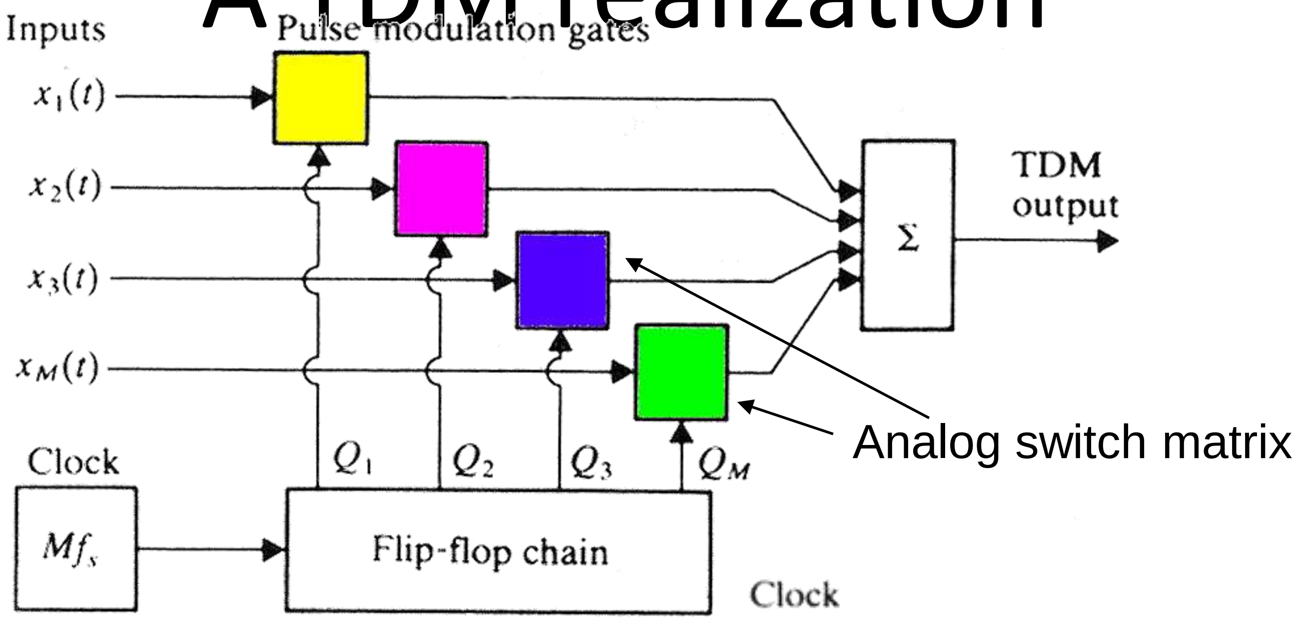
Time-division multiplexing (TDM) can be used to combine PAM or PCM signals



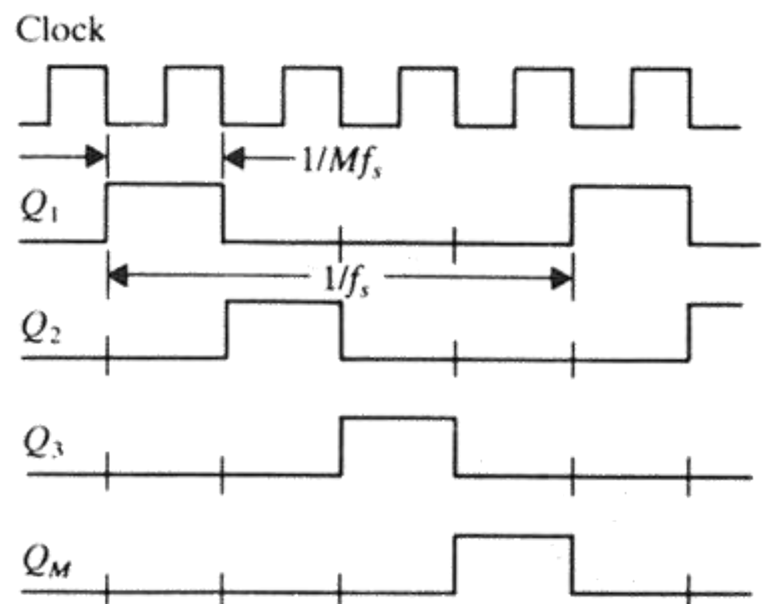
$r \geq Mf_s = M\,2W$ = total number of pulses / second

(With raised cos-pulses TDM baseband bandwidth at least $B_T \geq r/2 = MW$)

A TDM realization

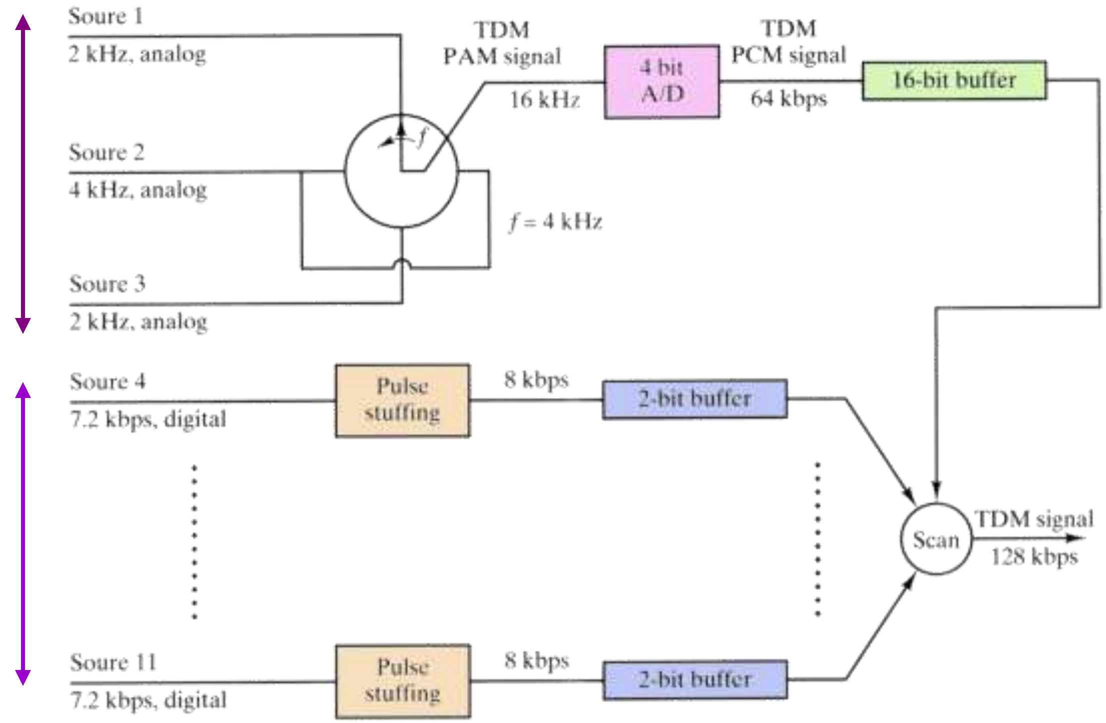


- TDM systems are critical in timing
- Timing can be arranged by
  - bit-stuffing
  - marker pulses
  - pilot tones
  - utilizing statistical properties of TDM signals



3 Analog sources

8 Digital sources

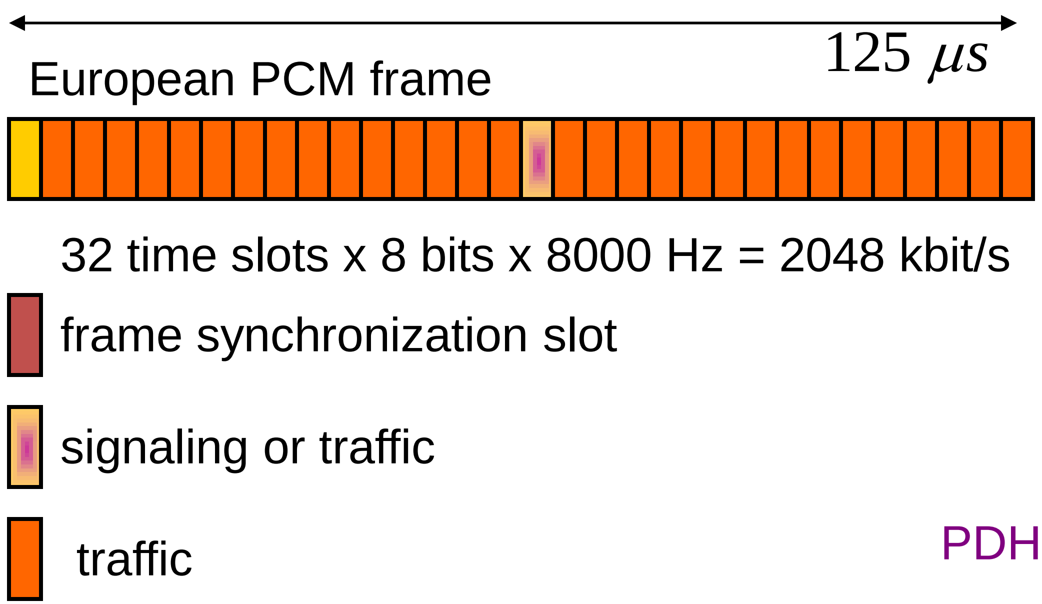


- *Analog sources* must be sampled at least at the rates: 1,3: 4kHz and 2: 8 kHz
- Hence PAM samples generated at the rate of 2x4+8=16 kHz
- After 4 bit-digitization data stream has the rate of 4x16=64 kbit/s
- For the digital sources pulse stuffing (clock rate shift/jitter compensation) increases the source rates to 8 kbit/s yielding output rate of 8x8 = 64 kbit/s
- Buffers enable constant rate for the second rotating switch

**W Stallings: Data and Computer Communications*

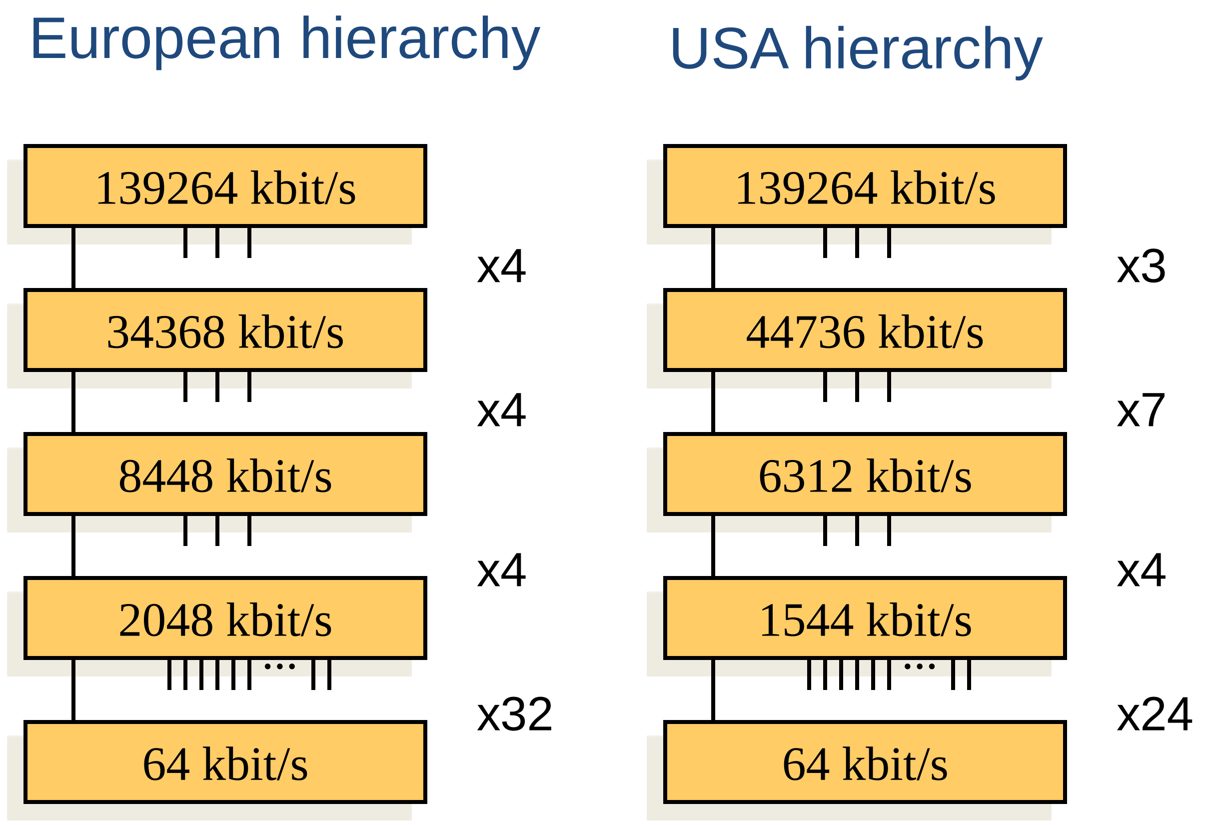
PCM systems and digital time division multiplexing (TDM)

- In digital multiplexing several messages are transmitted via same physical channel. For multiplexing 64 kbit/s channels in digital exchanges following three methods are popular:
  - PDH (plesio-synchronous digital hierarchy) (the dominant method today) (‘50-’60, G.702)
  - SONET (synchronous optical network) (‘85)
  - SDH (synchronous digital hierarchy) (CCITT ‘88)



PDH E-1 frame

PCM hierarchy in PDH



If one wishes to disassemble a tributary from the main flow the main flow must be demultiplexed step by step to the desired main flow level in PDH.

T1 and E1 Digital transmission systems

- In PSTN two PCM systems dominate:
  - T1, developed by Bell Laboratories, used in USA
  - E1, developed by CEPT* used in most of other countries
- In both data streams divided in frames of 8000 frames/sec
- In T1 (E1)
  - 24 (32) times slots and a framing (F) bit serves 24 channels
  - F bit (frame sync) repeats every 12 frames
  - Frame length: 1+ 8x24=193 bits
  - Rate 193x8000 bits/second=1544 kb/s
  - In E1 TS 0 holds a synchronization pattern and TS 16 holds signaling information
  - An E1 frame has 32x8=256 bits and its rate is 8000x256=2048 kb/s

PCM-method summarized

- Analog speech signal is applied into a LP-filter restricting its bandwidth into 3.4 kHz
- Sampling circuit forms a PAM pulse train having rate of 8 kHz
- Samples are quantized into 256 levels that requires a 8 bit-word for each sample (2^8=256).
- Thus a telephone signal requires 8x8 kHz = 64 kHz bandwidth
- The samples are line coded by using the HDB-3 scheme to alleviate synchronization problems at the receiver
- Usually one transmits several channels simultaneously following SDH hierarchy (as 30 pcs)
- Transmission link can be an optical fiber, radio link or an electrical cable
- At the receiver the PAM signal is first reconstructed where after it is lowpass filtered to yield the original-kind, analog signal