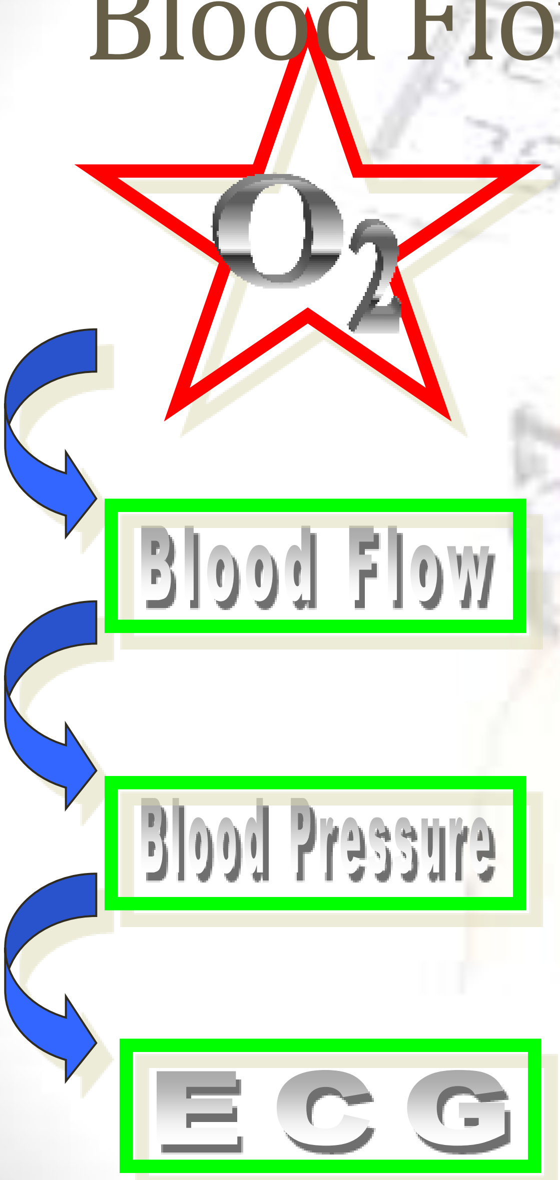
Indirect Methods
in
Blood Flow Measurements

Blood Flow



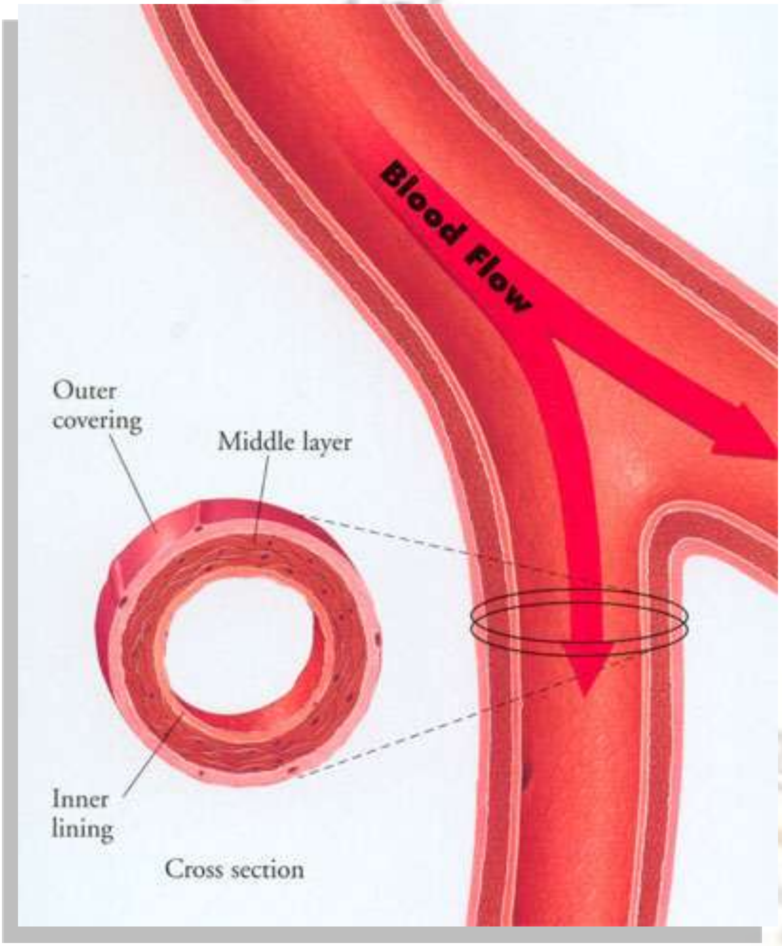
O_2 and other nutrition concentration in the cells are one of the primary measurements.

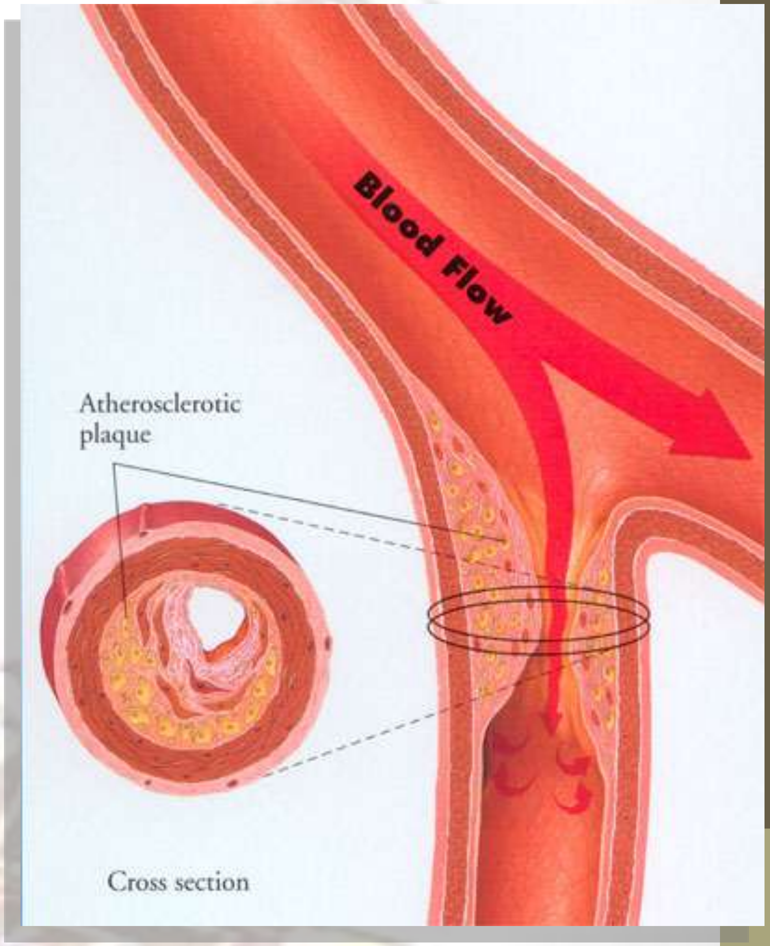
Blood flow helps to understand basic physiological processes and e.g. the dissolution of a medicine into the body.

It also helps to understand many pathological conditions, since many diseases alter the blood flow. Also the blood clots in the arterial system can be detected.

Usually the blood flow measurements are more invasive than blood pressure measurements / ECG

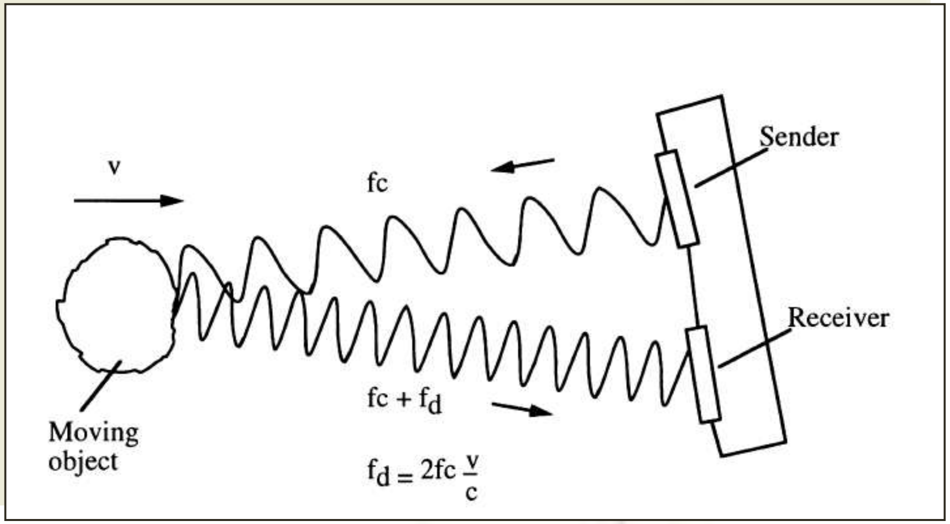
Blood Flow (2)





Normal blood flow velocity 0,5 m/s – 1 m/s (Systolic, large vessel)

Doppler Measurements (1) Ultrasound Doppler



The blood cells in the fluid scatter the Doppler signal diffusively.

In the recent years ultrasound contrast agents have been used in order to increase the echoes.

$$f_d = 2 f_c \frac{v}{c}$$

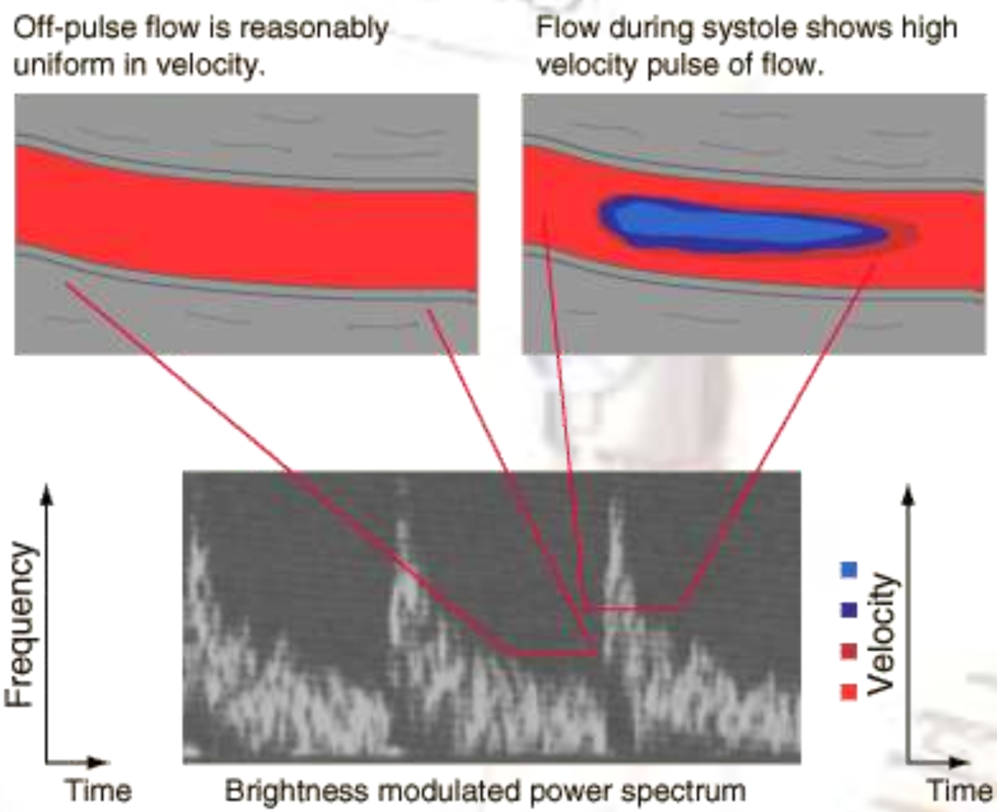
The ultrasound beam is focused by a suitable transducer geometry and a lens

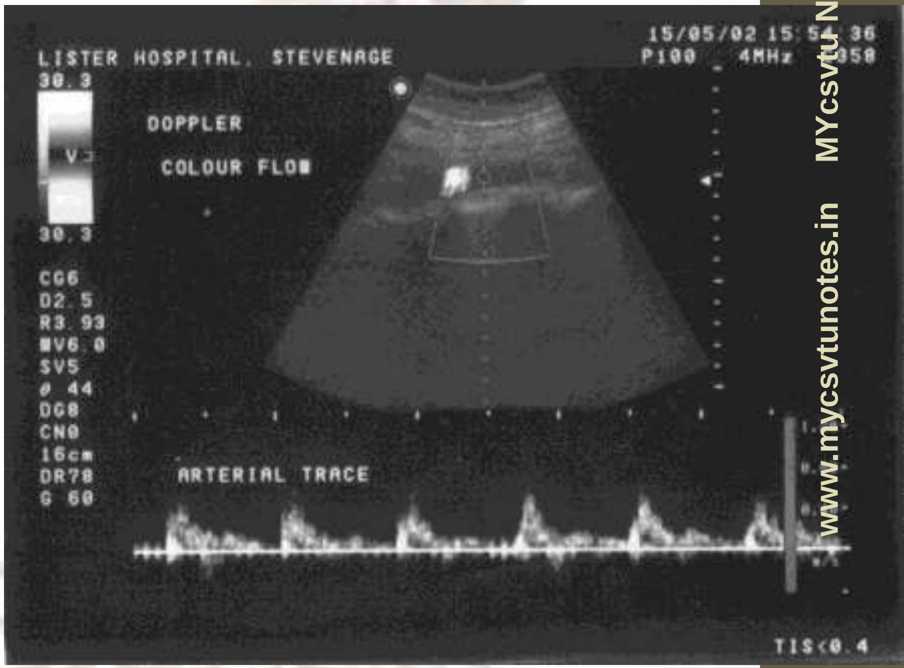
f_c = 2 – 10 MHz

c = 1500 - 1600 m/s (1540 m/s)

f_d = 1,3 – 13 kHz

Doppler Measurements (2)
Ultrasound Doppler





In order to know where along the beam the blood flow data is colledted, a pulsed Doppler must be used

The flow velocity is obtained from the spectral estimation of the received Doppler signal

Doppler Measurements (3) Ultrasound Doppler

The ultrasound Doppler device can be either ***a continuous wave*** or ***a pulsed Doppler***

CW DOPPLER	PULSED DOPPLER
+ No minimum range	+ Accuracy
+ Simpler hardware	+ No minimum flow
− Range ambiguity	− Minimum range
− Low flow cannot be detected	− (Maximum flow) x (range) = limited

Doppler Measurements (4) Ultrasound Doppler

GENERAL PARAMETERS

- the power decays exponentially because of the heating of the tissue. The absorption coefficient ~ proportional to frequency
- the far field operation should be avoided due to beam divergence.

$$d_{nf} = \frac{D^2}{4\lambda}$$

D = Transducer diameter (e.g. 1 – 5 mm)

- the backscattered power is proportional to f^4
- the resolution and SNR are related to the pulse duration. Improving either one of the parameters always affects inversely to the other

Doppler Measurements (5)
Laser Doppler Flowmetry

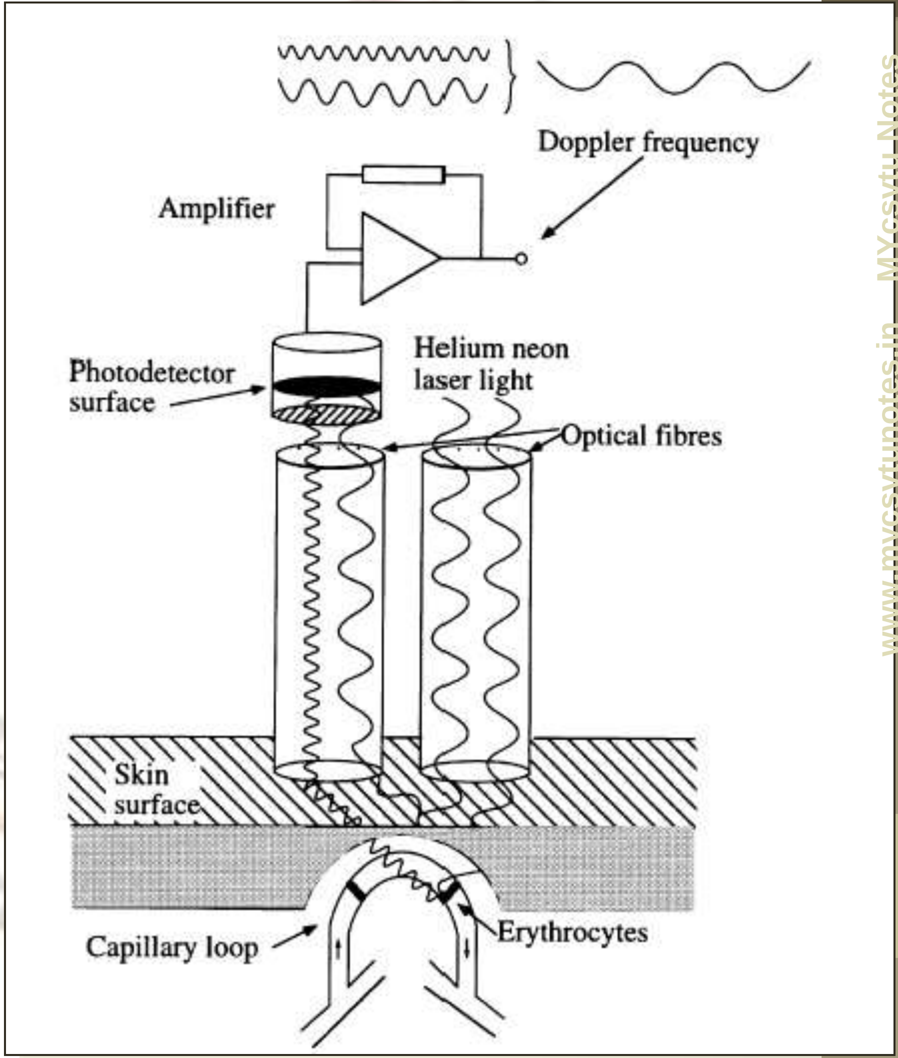
The principle of measurement is the same as with ultrasound Doppler

The laser parameter may have e.g. the following properties:

5 mW
He-Ne-laser
632,8 nm wavelength

The moving red blood cells cause Doppler frequency 30 – 12 000 Hz.

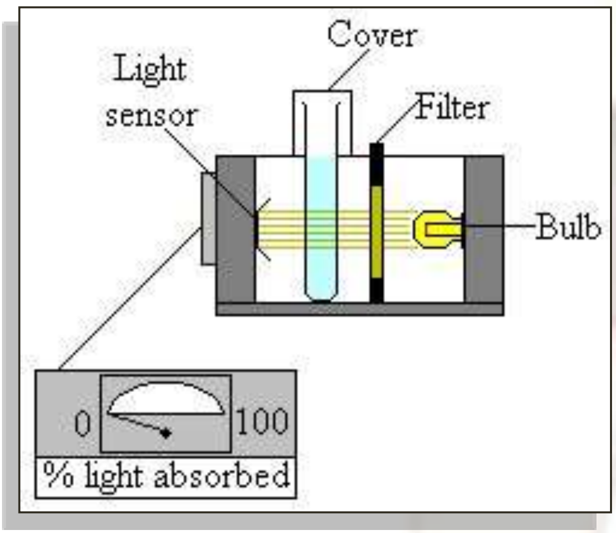
The method is used for capillary (microvascular) blood flow measurements



Direct Methods
in
Blood Flow Measurements

Indicator Dilution Methods (1)
Dye Dilution Method

A bolus of indicator, a colored dye *(indocyanine green),* is rapidly injected in to the vessel. The concentration is measured in the downstream



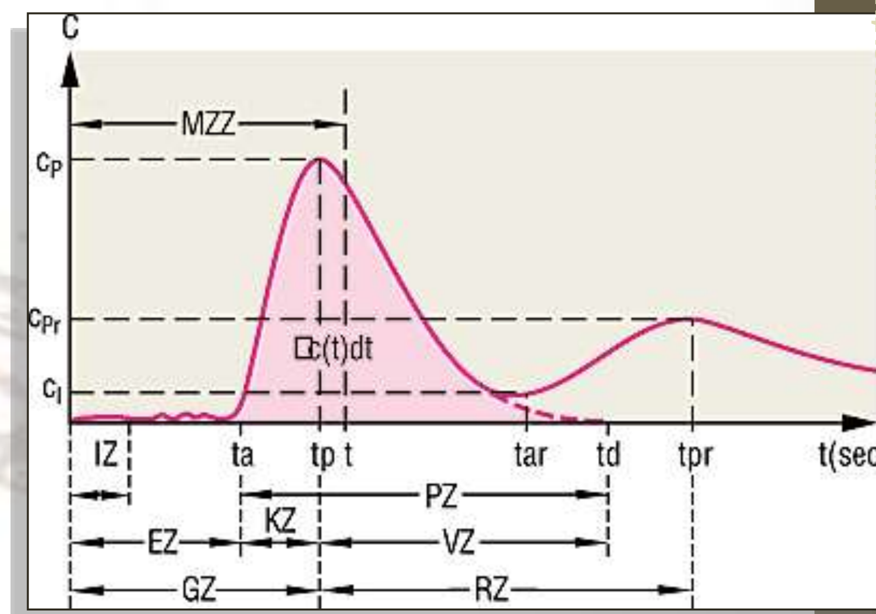
The blood is drawn through a colorimetric cuvette and the concentration is measured using the principle of absorption photometry

Avg. flow

amount of dye

1% peak C

$$F = \frac{m}{\int_0^{t_1} [\Delta C(t)]\,dt}$$



Indicator Dilution Methods (2)
Thermal Dilution Method

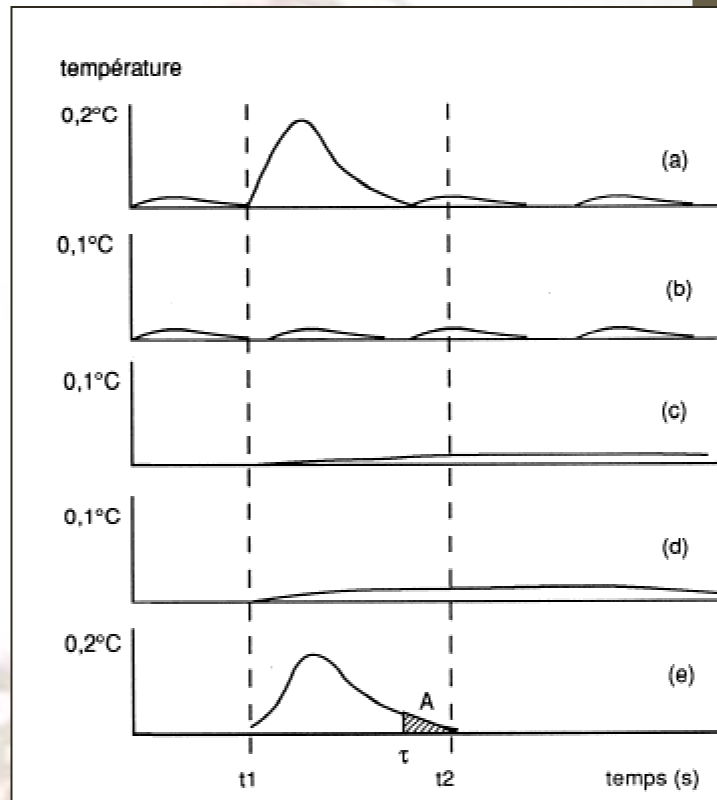
A bolus of chilled saline solution is injected into the blood circulation system (right atrium). This causes decrease in the pulmonary artery temperature.

$$F = \frac{Q}{\rho_b c_b \int_0^{t_1} \Delta T_b(t)\,dt}$$

heat content of injectate (Q)

density of blood (ρ_b) (e.g. 1060 kg/m3)

specific heat of blood (c_b) (e.g. 3640 J/(kg*K))



An artery puncture is not needed in this technique

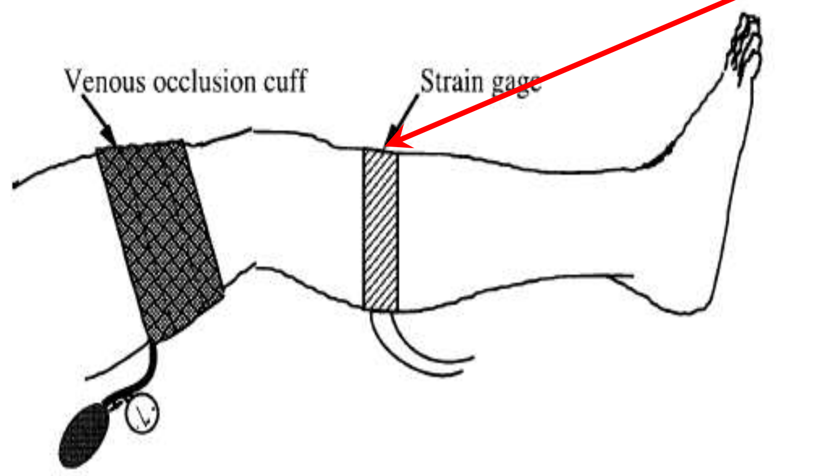
Several measurements can be done in relatively short time

A standard technique for measuring cardiac output in critically ill patients

Plethysmography
in
Blood Flow Measurements

Plethysmography (1)
Strain Gage Method

Plethysmography means the methods for recording volume changes of an organ or a body part (e.g. a leg)

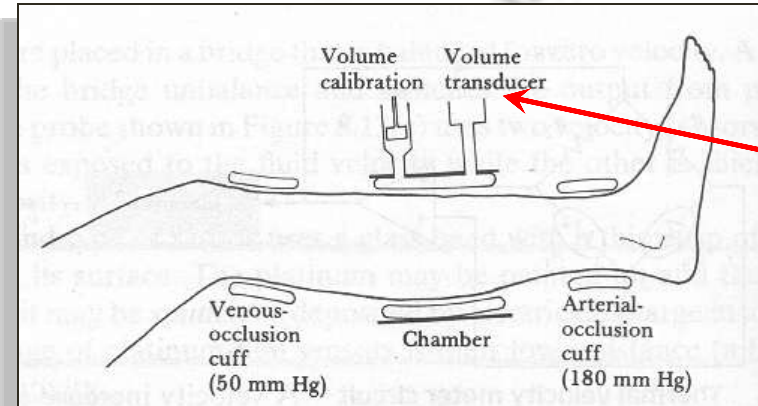


Strain gage is made of silicone rubber tubes, which are filled with conductive liquid (e.g. mercury) whose impedance changes with volume.

Venous occlusion cuff is inflated to 40 – 50 mmHg. In this way there will be the arterial inflow into the limb but no venous outflow.

If only a segment of limb is measured, there is a need for arterial occlusion cuff also.

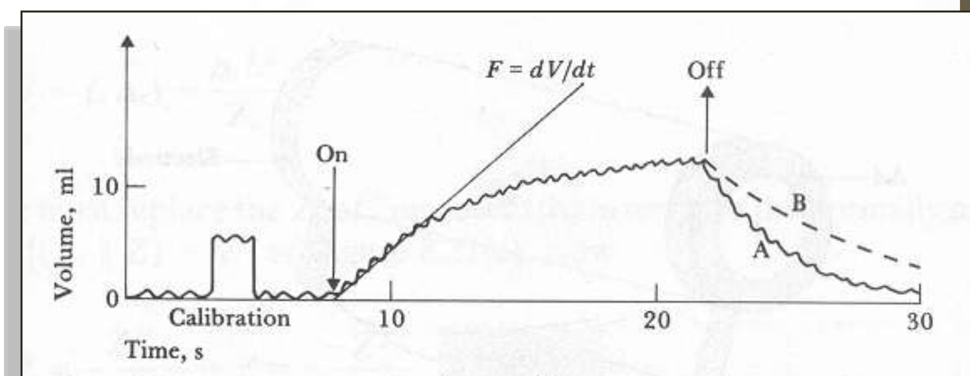
Plethysmography (2)
Chamber Method



As the volume of the leg increases, the leg squeezes some kind of bladder and decreases its volume

Volume transducer can be e.g. water filled tube (level) or air (pressure)

The speed of the return of the venous blood is measured



Chamber plethysmograph is the only accurate non-invasive way to measure changes in the blood volume

Plethysmography (3)
Electric-Impedance Method

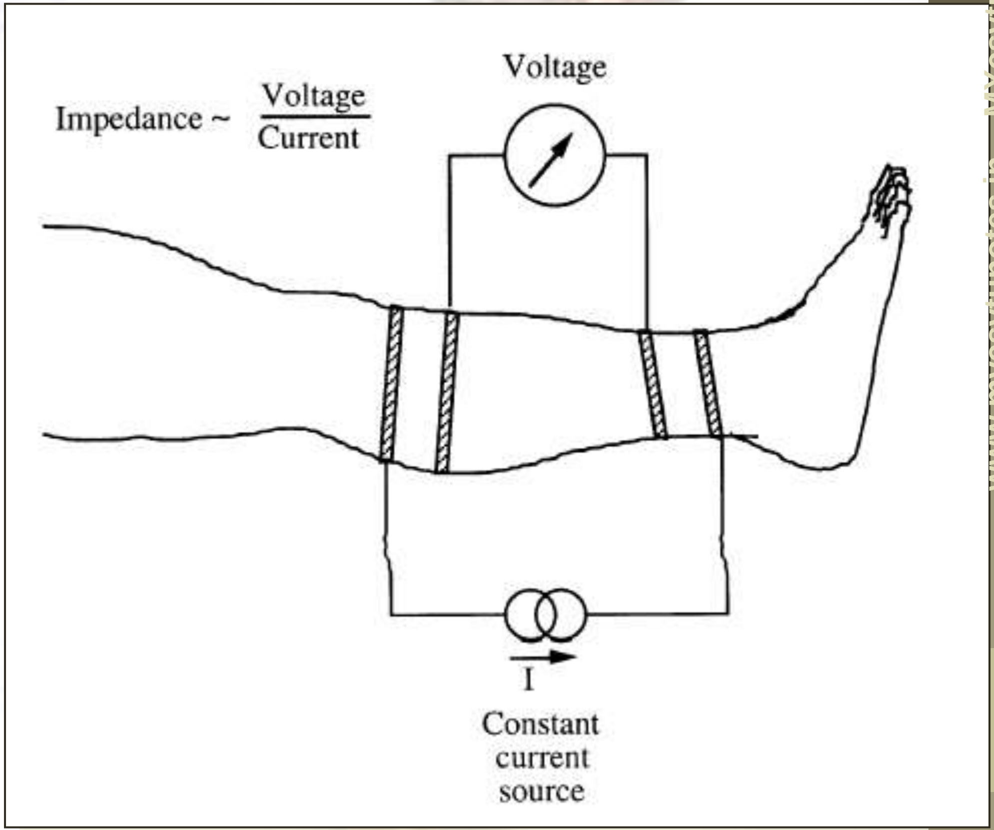
Different tissues in a body have a different resistivity. Blood is one of the best conductors in a body (ρ = 1,5 Ωm)

A constant current is applied via skin electrodes

- *I = 0,5 – 4 mA rms (SNR)*
- *f = 50 – 100 kHz*
- *($Z_{skin\text{-}electrode}$+shock)*

The change in the impedance is measured

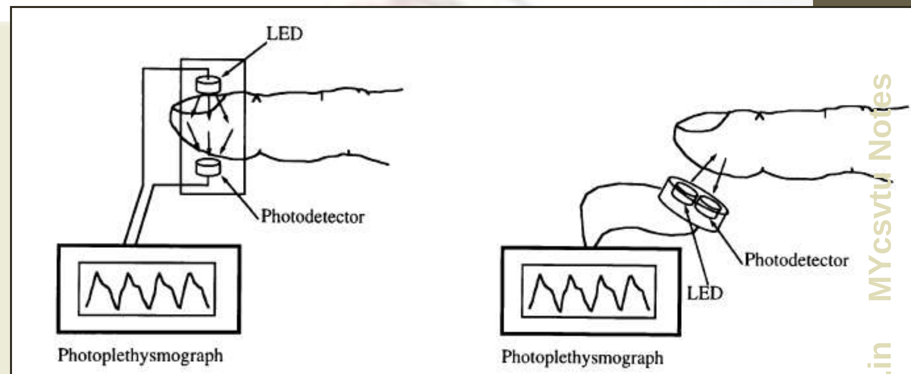
$$\Delta Vol = \rho \frac{L^2}{Z_0^2} \Delta Z$$

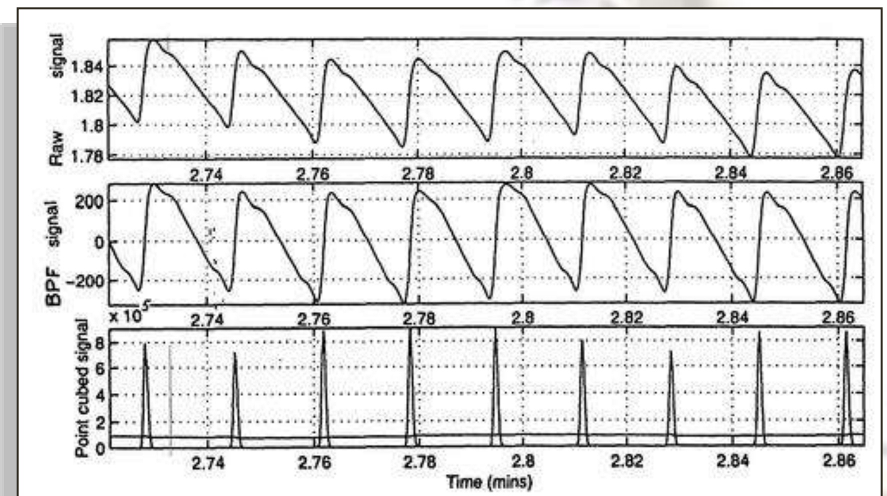


The accuracy is often poor or unknown

Plethysmography (4)
Photoelectric Method

A beam of IR-light is directed to the part of the tissue which is to be measured for blood flow (e.g. a finger or ear lobe)





The blood flow modulates the attenuated / reflected light which is recorded.

The light that is transmitted / reflected is collected with a photodetector

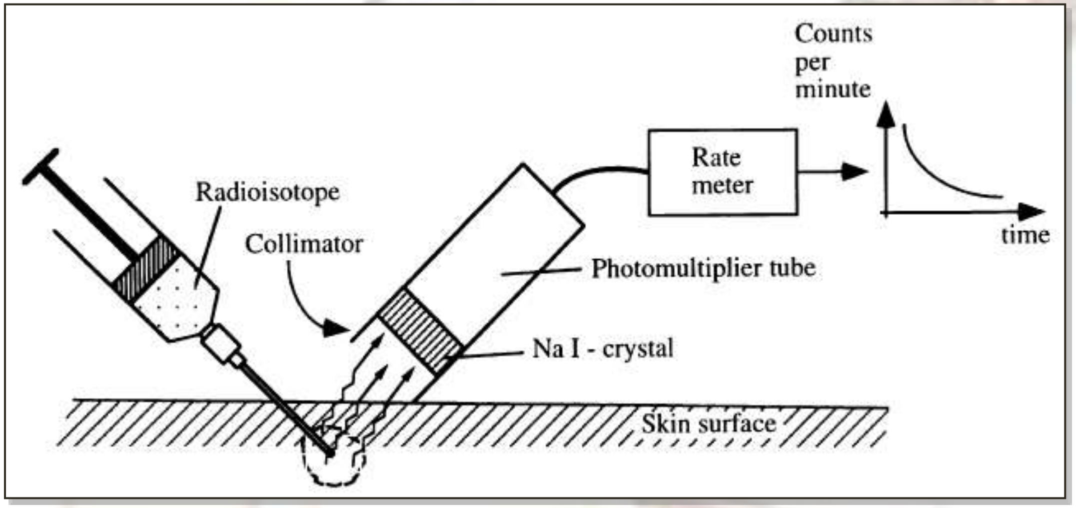
+ Method is simple
Heart rate is clearly seen

− Poor measure for changes in volume
Very sensitive to motion artefacts

Other Methods in Blood Flow Measurements

Radioisotopes

A rapidly diffusing, inert radioisotope of lipid-soluble gas (^{133}Xe or ^{85}Kr) is injected into the tissue or passively diffused



The elimination of the radioisotope from microcirculatory bed is related to the blood flow:

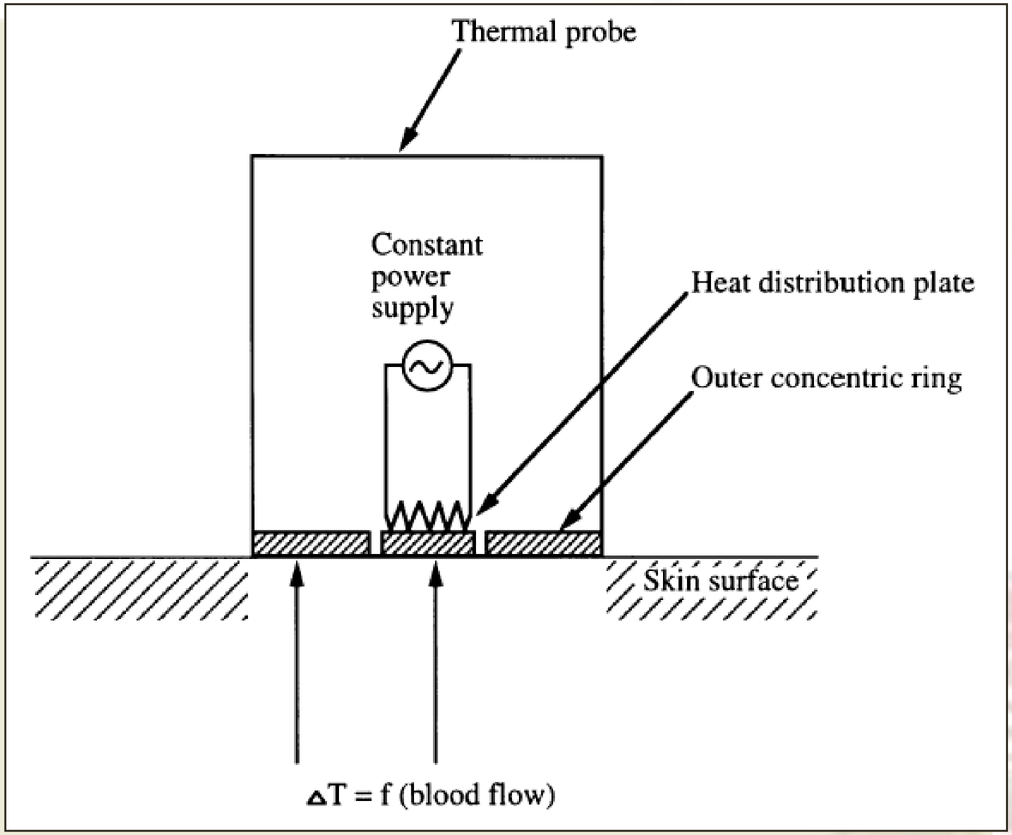
$$C(t) = C_0 \exp(-kt)$$

$$k = \ln 2 / t_{1/2}$$

Thermal Convection Probe

This is one of the earliest techniques for blood flow measurements

The rate of heat removal from the tissue under probe is measured



The concentric rings are isolated thermally & electrically from each other

The central disk is heated 1 – 2 ^{o}C over the temperature of tissue

A temperature difference of 2- 3 ^{o}C is established between the disks

The method is not very common due extreme nonlinear properties and difficulties in practical use (e.g. variable thermal characteristics of skin)

ary (1)

BLOOD PRESSURE

Describes the physiology and pathology of cardiocvascular system

”Normal” values are 120 / 80 mmHg
High values may lead to heart attack and strokes
Low values may lead to low oxygen perfusion

Almost all **indirect methods** rely on an occlusive cuff which is placed on the bracial artery. The actual measurement is done when the cuff is deflated

All **direct methods** require skin punctuation and a use of catheter. Methods are used only when continuous and accurate measurements are needed.

Summary (2)

BLOOD FLOW

Usually more invasive methods are used than with blood pressure measurements

Used for understanding physiological processes (e.g. medicine dissolution). Also used for locating clots in arteries

Normal velocity is 0,5 - 1 m/s

Indirect measurements are done by using ultrasound or plethysmographic method

Direct measurements are done by dilution methods (dye / thermal)

Temperature Measurements

- There are a few instruments that can be used to collect daily temperature measurements
- Recommended is a digital multi-day thermometer

  - logs six days of max/min temperatures and also measures soil temperature
- A U-Tube max/min thermometer can also be used

  - NOTE: contains mercury
- Automated Weather Stations can also be used

Why do we Measure Air Temperature?

- Provide a denser network of observations than is available using only official weather stations
- Provide finer resolution data crucial for investigating localized variations (e.g., urban heat islands, microclimates)
- Augment data needed for regional forecasts and climate records in areas of the world where there are few official weather stations

Digital Multi-day Max / Min Thermometer

- Has two sensors:

  - one sensor measures air temperature

  - the other measures soil temperature at 10 cm depth
- The thermometer stores six days worth of max/min temperatures
- The thermometer is must be reset to around the time of local solar noon <u>ONCE </u>when it is first setup
- The exact time that it is reset is known as the *Time of Reset*

Calibrating the Max/Min Thermometer

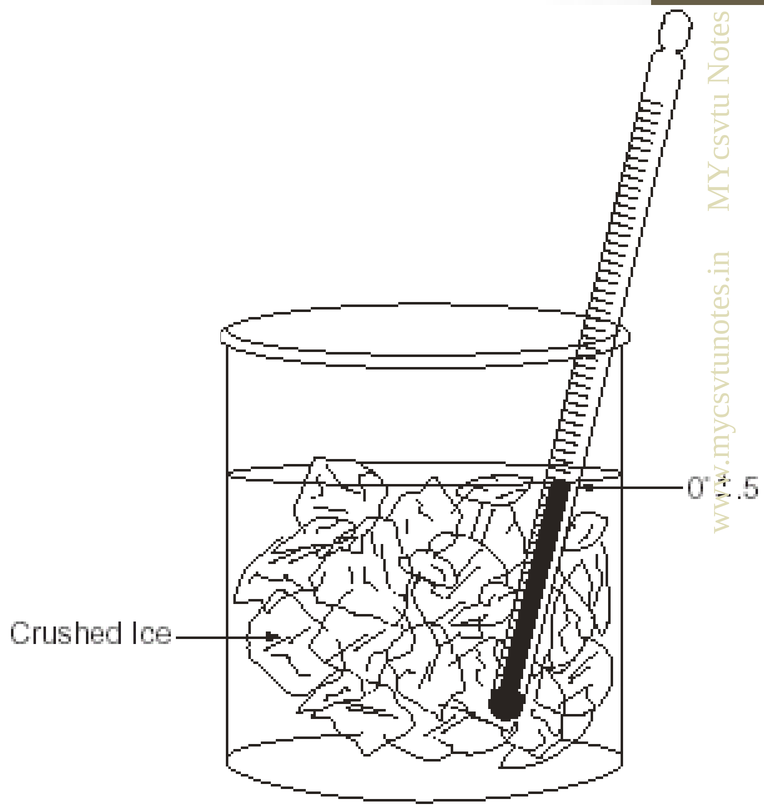
Basically, we compare the readings from the max/min thermometer to the readings from a calibration thermometer, and report the data to GLOBE.

First, we need to check the calibration thermometer!!

…by dipping it in an ice bath.

Checking Calibration Thermometer

- Submerge thermometer in ice-water bath
- Let sit for 10-15 minutes, stirring thermometer occasionally
- Read the thermometer. If it reads between -0.5° C and +0.5° C, the thermometer is fine.
- If the thermometer reads greater than +0.5° C, check to make sure that there is more ice than water in your ice-water bath.
- If the thermometer reads less than -0.5° C, check to make sure that there is no salt in your ice-water bath.



Digital Multi-Day Max/Min Thermometer Calibration

- Hang a calibration thermometer in the instrument shelter with the max/min thermometer
- Read five sets of readings from the two thermometers

Digital Multi-Day Max/Min Thermometer Calibration

- **Fill out the calibration table on the Digital Max/Min Thermometer Calibration and Reset Data Sheet**
- **Report data to GLOBE (the temperature offset will be calculated by GLOBE and automatically applied to data that you)**

Calibration

Thermometer Readings						
Reading Number	Date (Year/ Month/Day)	Local Time (Hour:Min)	UT Time (Hour:Min)	Calibration thermometer readings (°C)	Digital air sensor readings (°C)	Digital soil sensor readings (°C)
1						
2						
3						
4						
5						

Every six months…

- Recalibrate the air sensor of the max/min thermometer
- Check the soil sensor by comparing to a reading from a soil thermometer

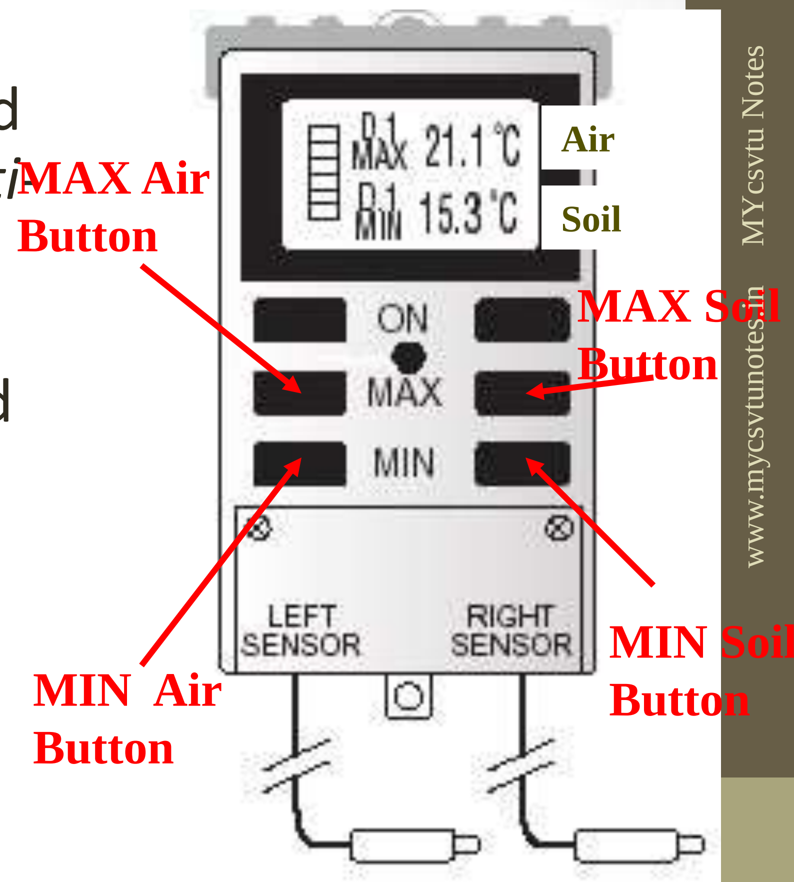
Reading Current Temperatures from the Multi-Day Max/Min Thermometer

- Press the ON button for air temperature (upper left) and read the current air temp (top of display)

- Press the ON button for soil temperature (upper right) and read the current soil temp (at bottom)

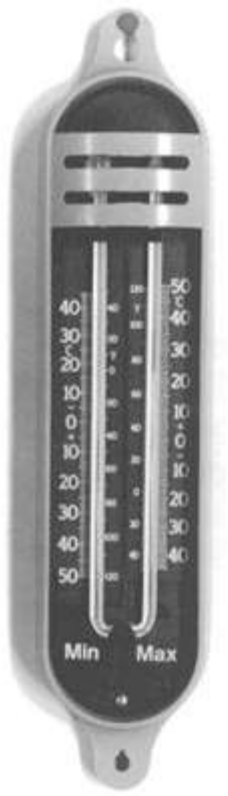
Reading Max/Min Temperature from the Multi-Day Max/Min Thermometer

- Press the MAX button for air temperature (middle left) **twice** and read the max air temperature labeled "D1" and record on your *Digital Multi-day Max/Min Thermometer Data Sheet*
- Press the MAX button again and read and record the max air temperature labeled "D2"
- Repeat until you read all the max air temps stored (up to "D6")
- Use the same procedure for reading min air temperatures and max/min soil temperatures

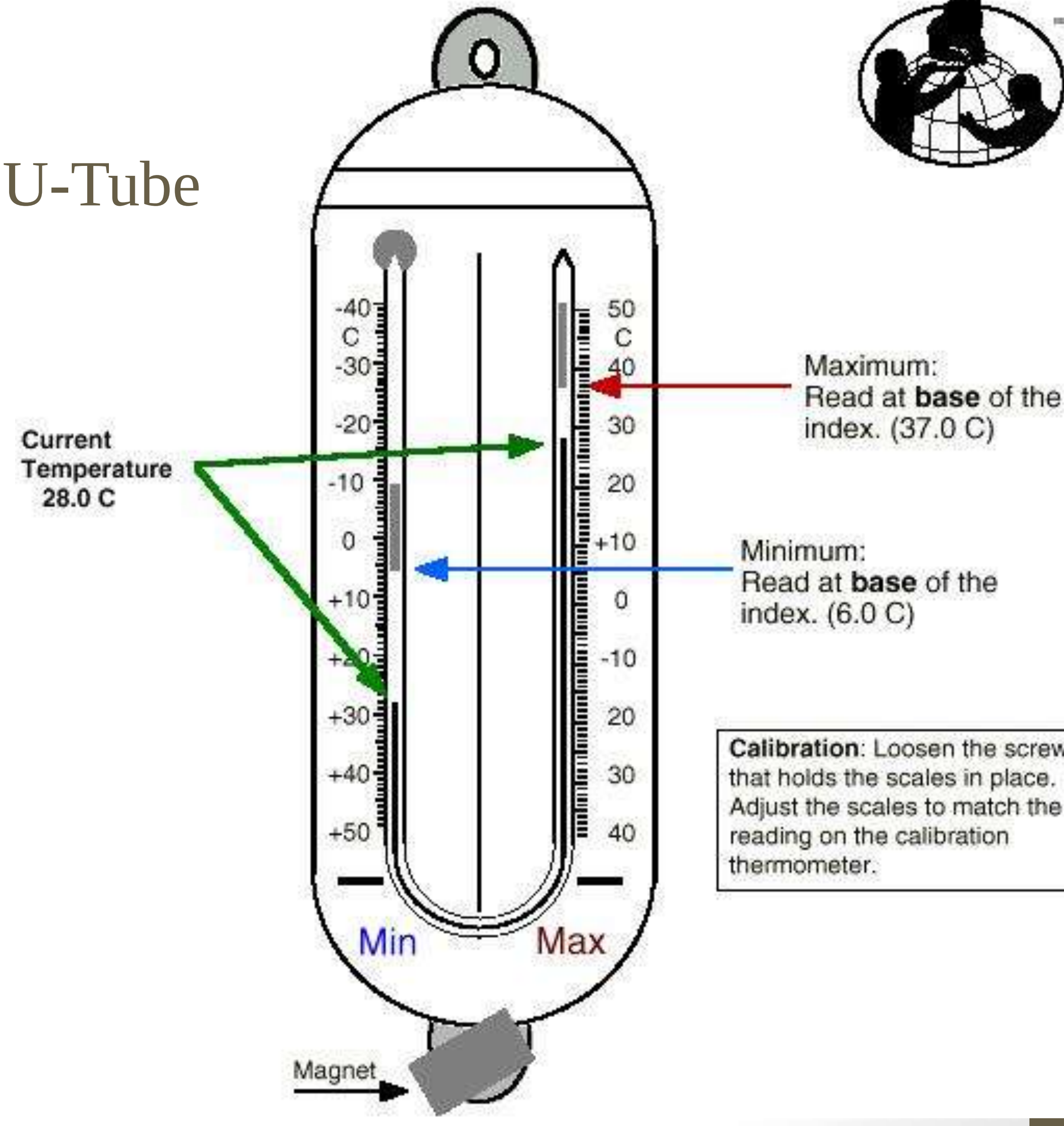


U-Tube Thermometer

- Max / Min Thermometer
  - Make sure mercury is continuous
  - Hold upright and shake
  - Tap bottom of the unit against the palm of your hand
  - Calibrate upon installation
  - Calibrate every 6 months



Reading the U-Tube

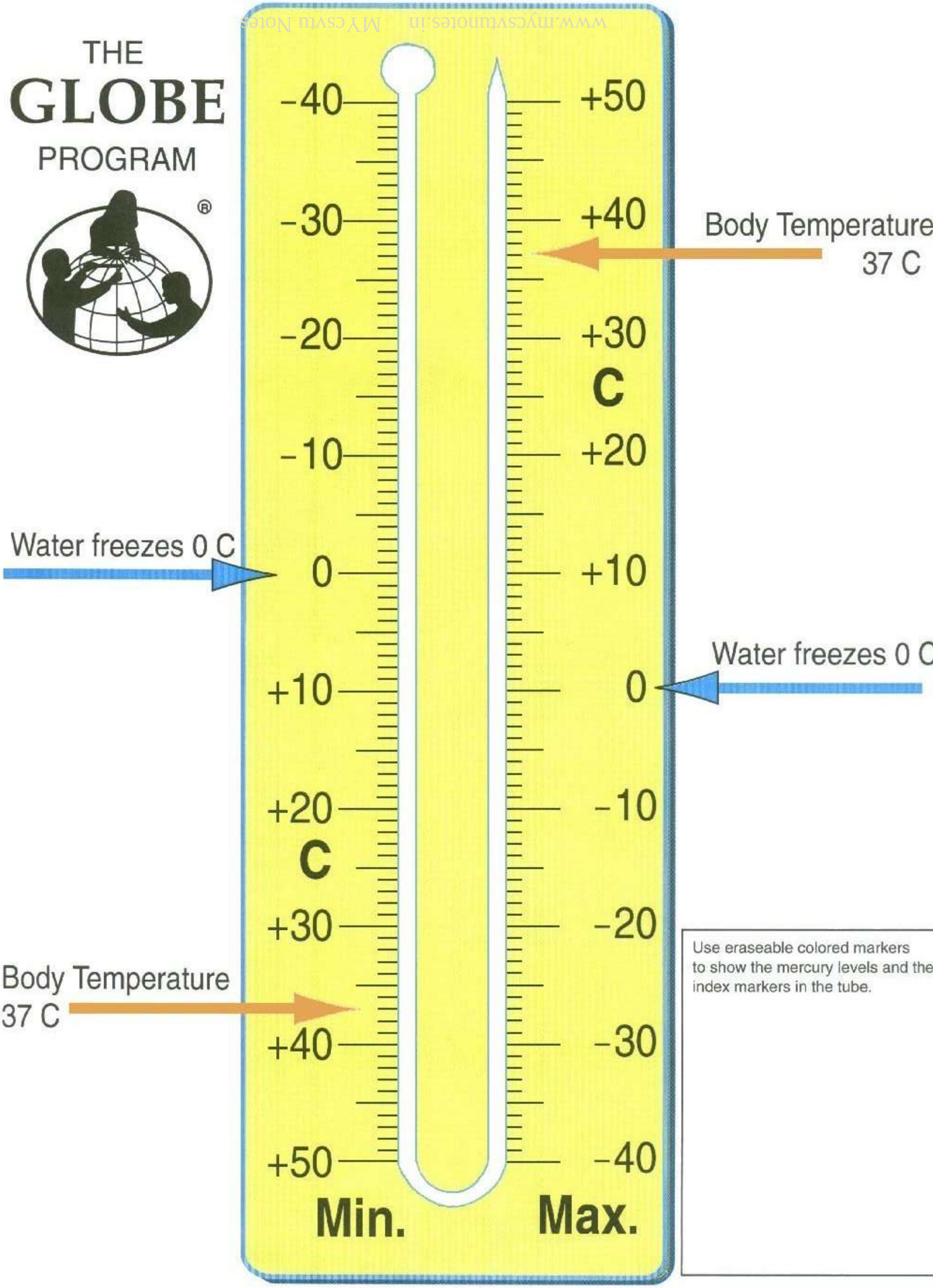


Calibration: Loosen the screw that holds the scales in place. Adjust the scales to match the reading on the calibration thermometer.

THE

GLOBE

PROGRAM ®



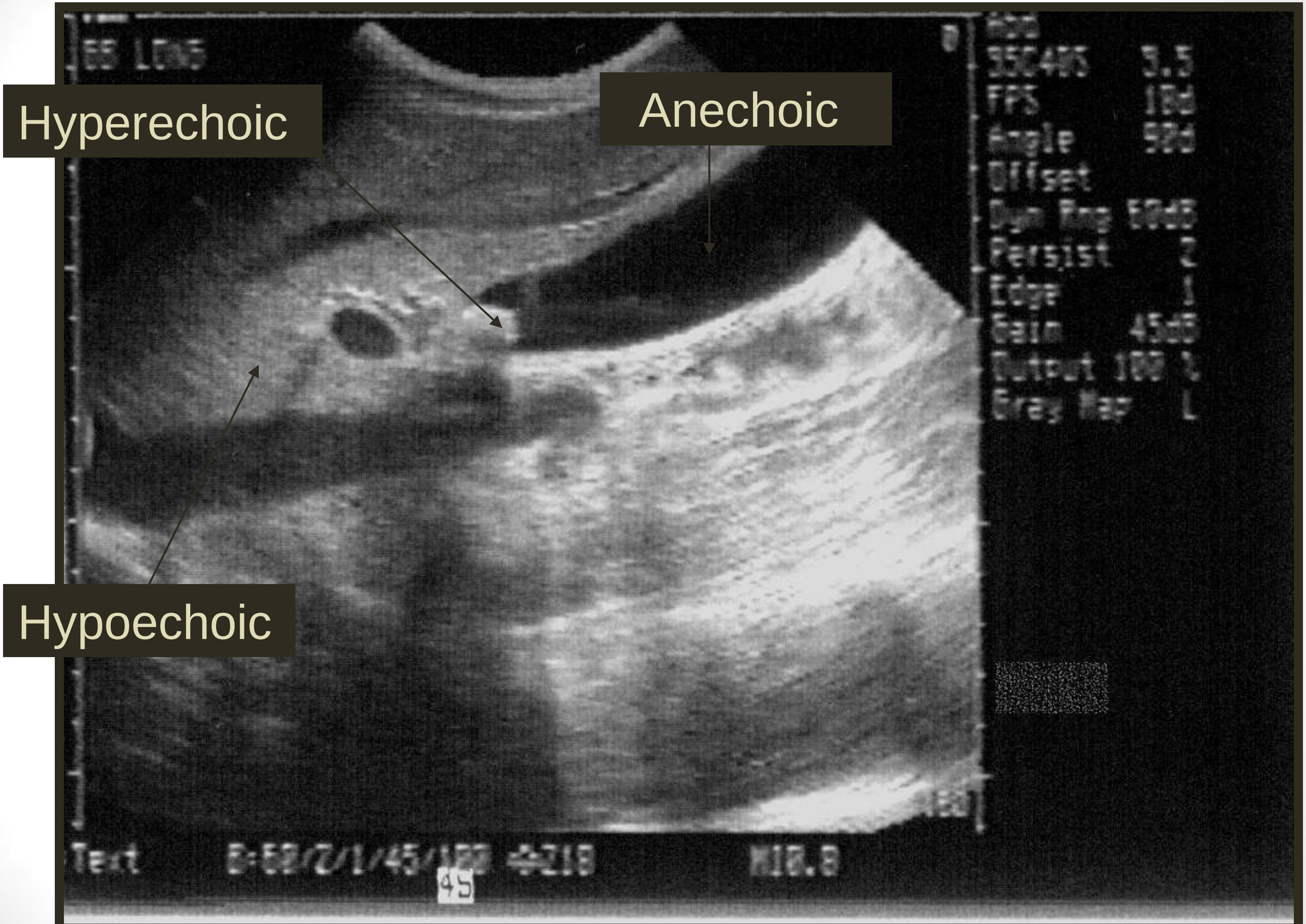
Placement of U-tube and Calibration Thermometers

- Place the U-tube and the calibration thermometers side-by side in the shelter
- Mount the U-tube on wooden blocks or soda-bottle tops to allow air flow behind

Ultrasonic Diagnosis

Image Resolution

- Image quality is dependent on
  - Axial Resolution
  - Lateral Resolution
  - Focal Zone
  - Probe Selection
  - Frequency Selection
  - Recognition of Artifacts


Hyperechoic
Anechoic
Hypoechoic

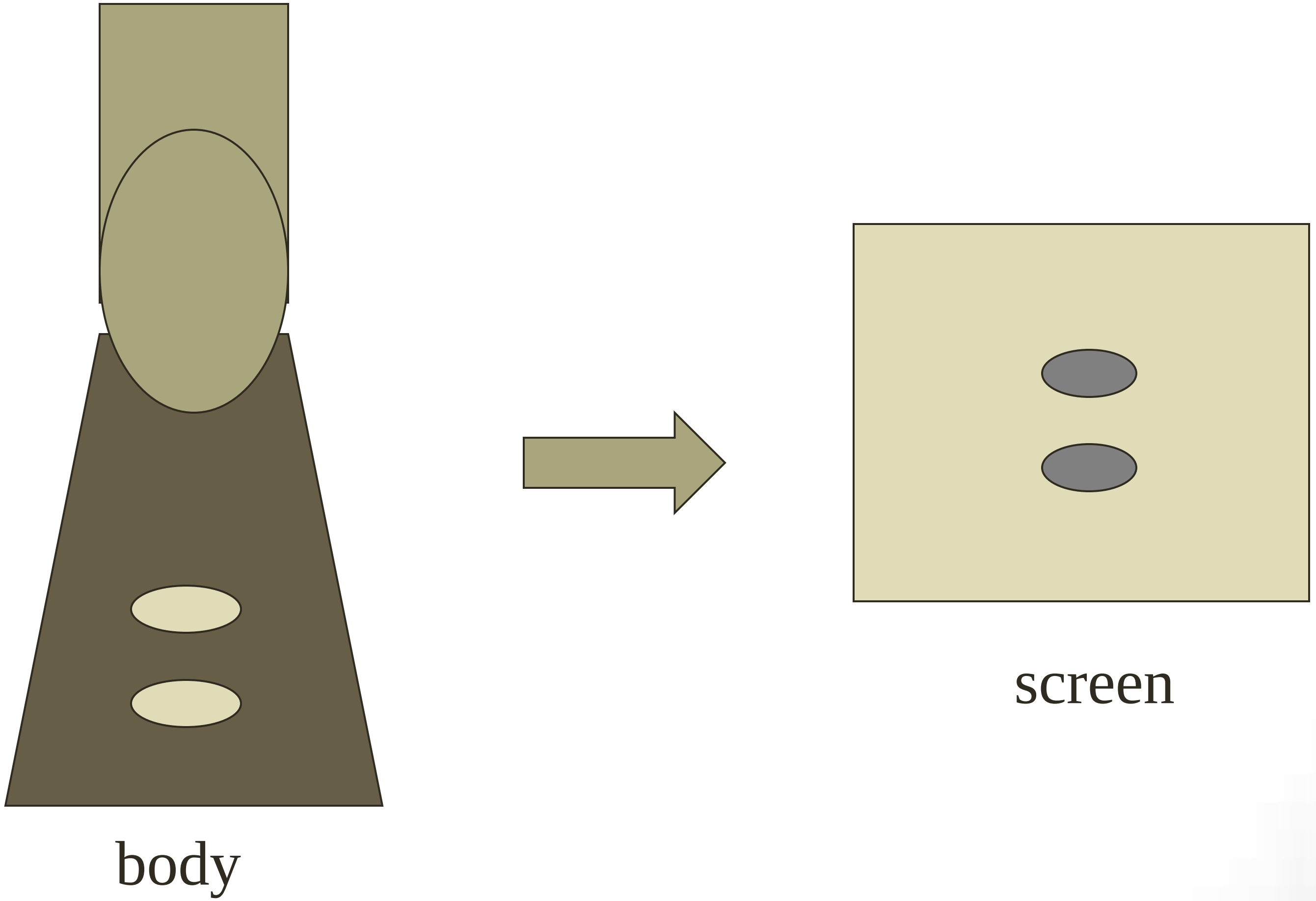
Axial Resolution

- Ability to differentiate two objects along the long axis of the ultrasound beam
- Determined by the pulse length
  - Product of wavelength λ and # of cycles in pulse
  - Decreases as frequency f increases
- Higher frequencies produce better resolution

Axial Resolution

- 5 MHz transducer
  - Wavelength 0.308mm
  - Pulse of 3 cycles
  - Pulse length approximately 1mm
  - Maximum resolution distance of two objects = 1 mm
- 10 MHz transducer
  - Wavelength 0.15mm
  - Pulse of 3 cycles
  - Pulse length approximately 0.5mm
  - Maximum resolution distance of two objects = 0.5mm

Axial Resolution



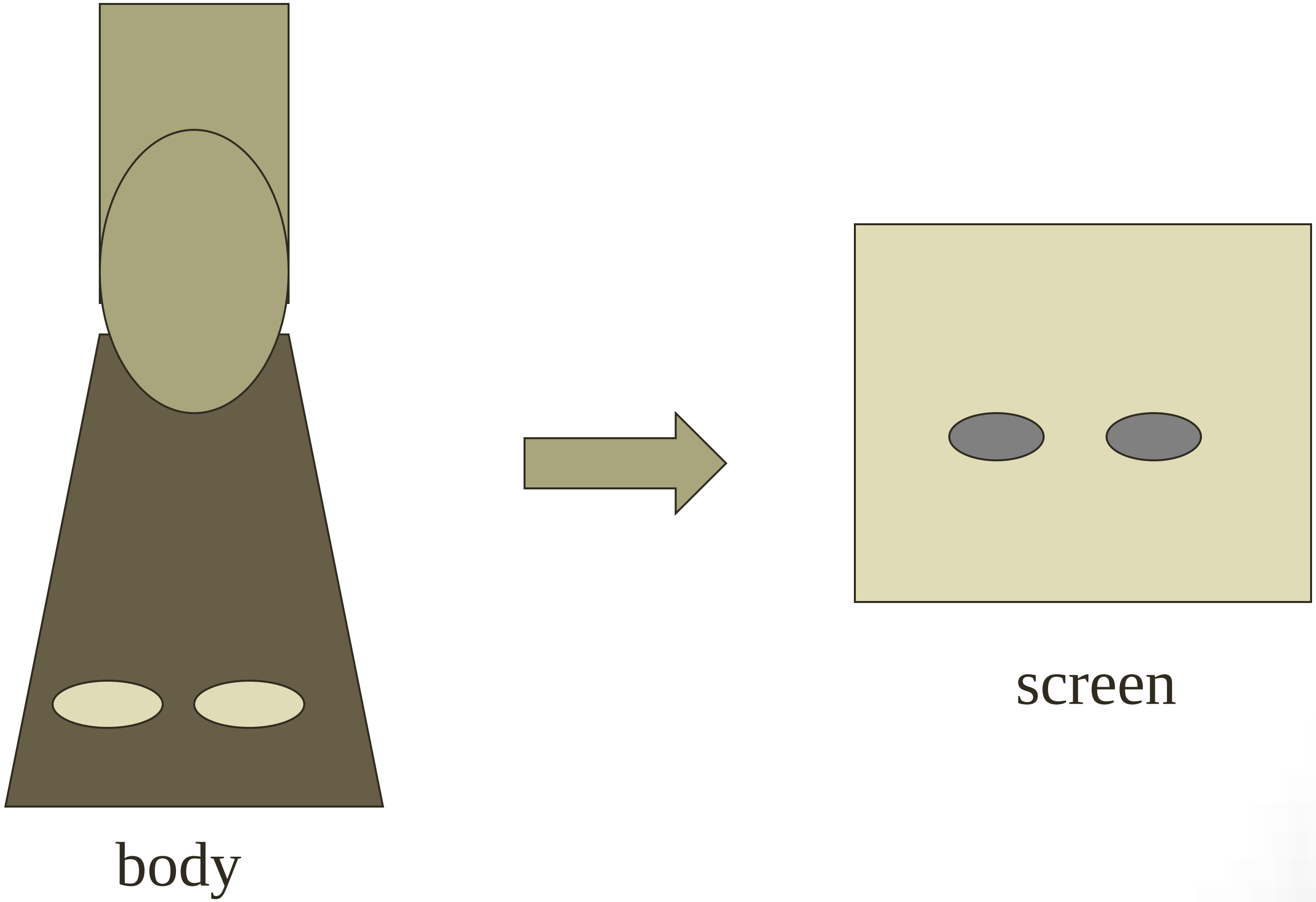
Lateral Resolution

- The ultrasound beam is made up of multiple individual beams
- The individual beams are fused to appear as one beam
- The distances between the single beams determines the lateral resolution

Lateral resolution

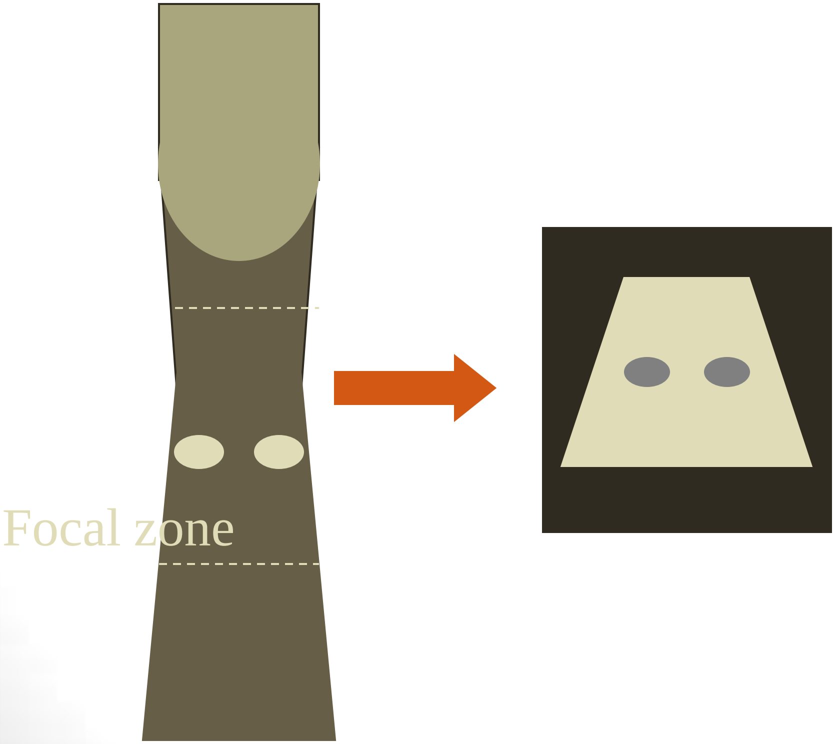
- Ability to differentiate objects along an axis perpendicular to the ultrasound beam
- Dependent on the width of the ultrasound beam, which can be controlled by focusing the beam
- Dependent on the distance between the objects

Lateral Resolution

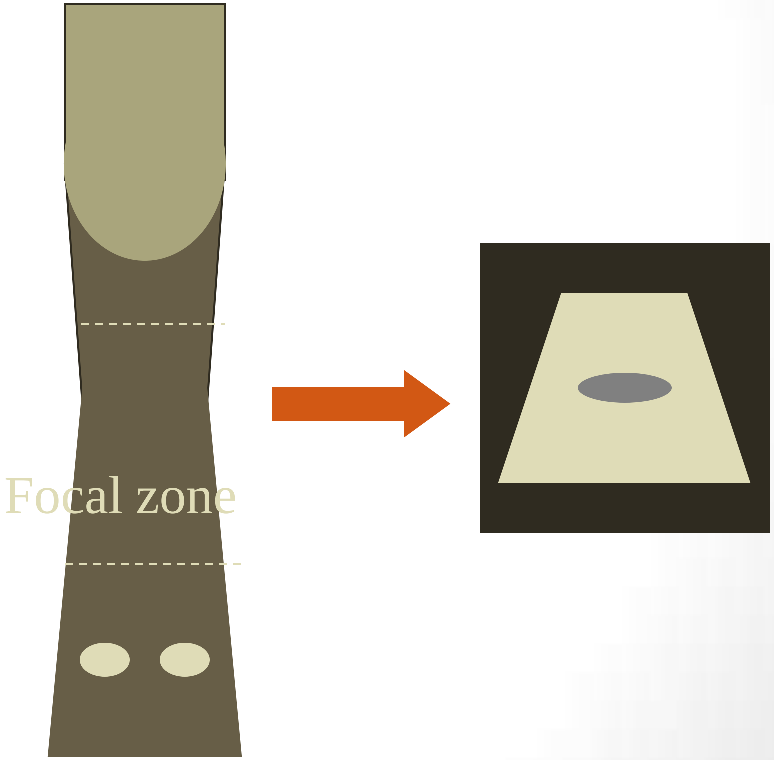


Focal Zone

- Objects within the focal zone

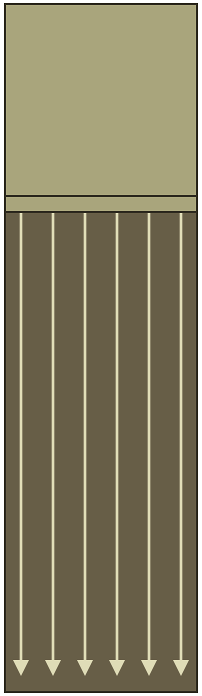


- Objects outside of focal zone

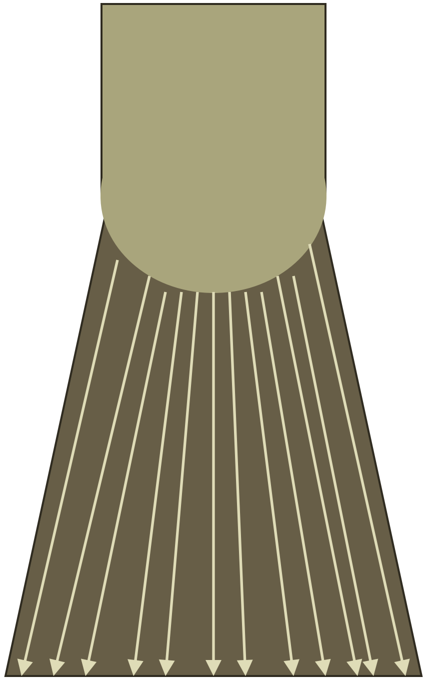


Probe options

- Linear Array



- Curved Array



Ultrasound Artifacts

- Can be falsely interpreted as real pathology
- May obscure pathology
- Important to understand and appreciate

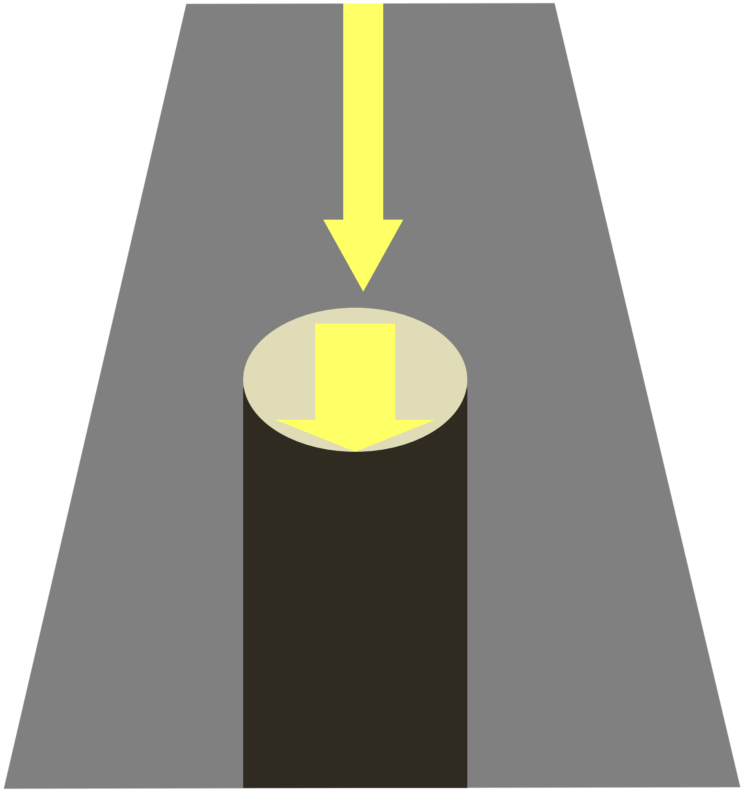
Ultrasound Artifacts

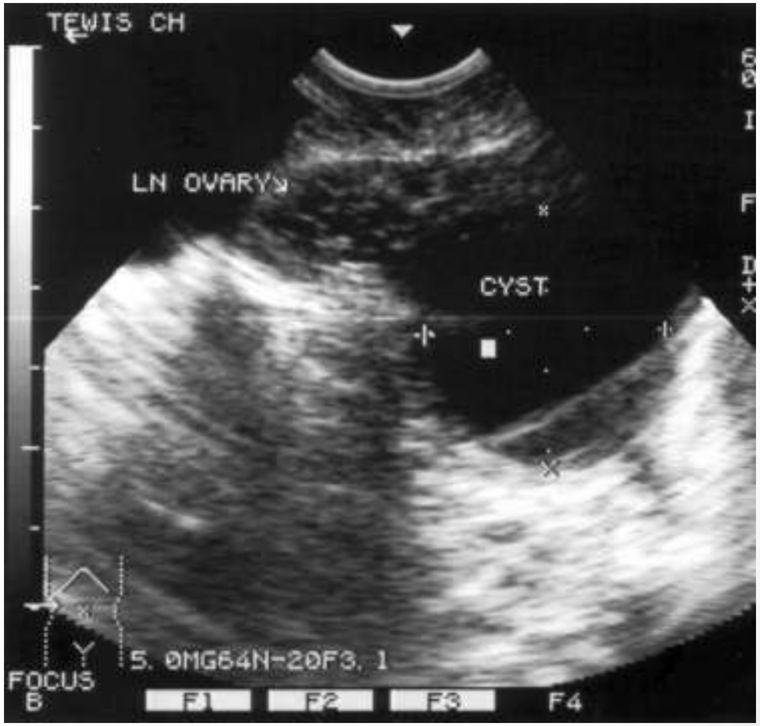
- Acoustic enhancement
- Acoustic shadowing
- Lateral cystic shadowing (edge artifact)
- Wide beam artifact
- Side lobe artifact
- Reverberation artifact
- Gain artifact
- Contact artifact

Acoustic Enhancement

- *<u>Opposite</u>* of acoustic shadowing
- Better ultrasound transmission allows enhancement of the ultrasound signal distal to that region

Acoustic Enhancement





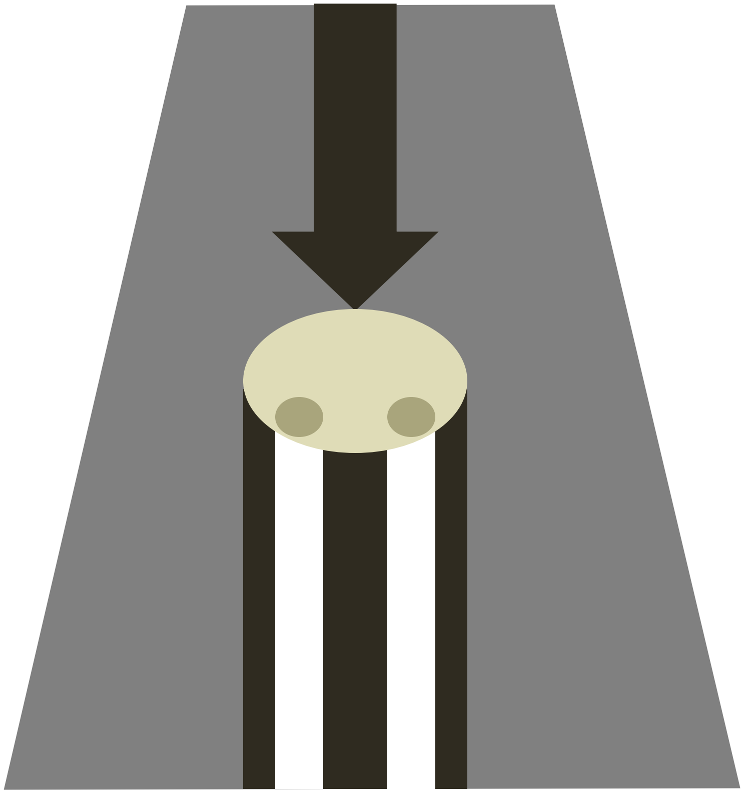
Acoustic Shadowing

- Occurs distal to any highly reflective or highly attenuating surface
- Important diagnostic clue seen in a large number of medical conditions
  - Biliary stones
  - Renal stones
  - Tissue calcifications

Acoustic Shadowing

- Shadow may be more prominent than the object causing it
- Failure to visualize the source of a shadow is usually caused by the object being outside the plane of the ultrasound beam

Acoustic Shadowing

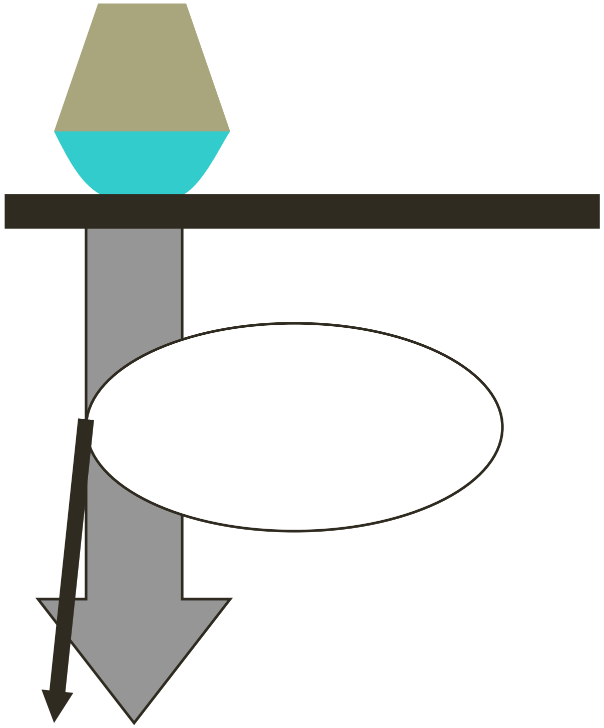


Acoustic Shadowing

Lateral Cystic Shadowing

- A type of refraction artifact
- Can be falsely interpreted as an acoustic shadow (similar to gallstone)

Lateral Cystic Shadowing



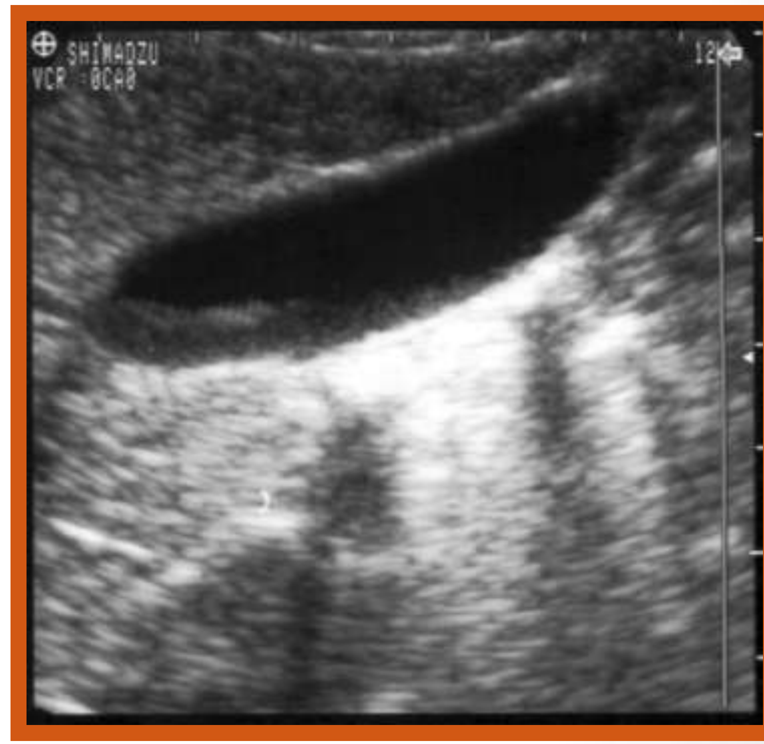
Beam-Width Artifact

- Gas bubbles in the duodenum can simulate a gall stone
- *Does not* assume a dependent posture
- *Do not* conform precisely to the walls of the gallbladder

Beam-Width Artifact

Beam-width artifact

Gas in the duodenum simulating stones



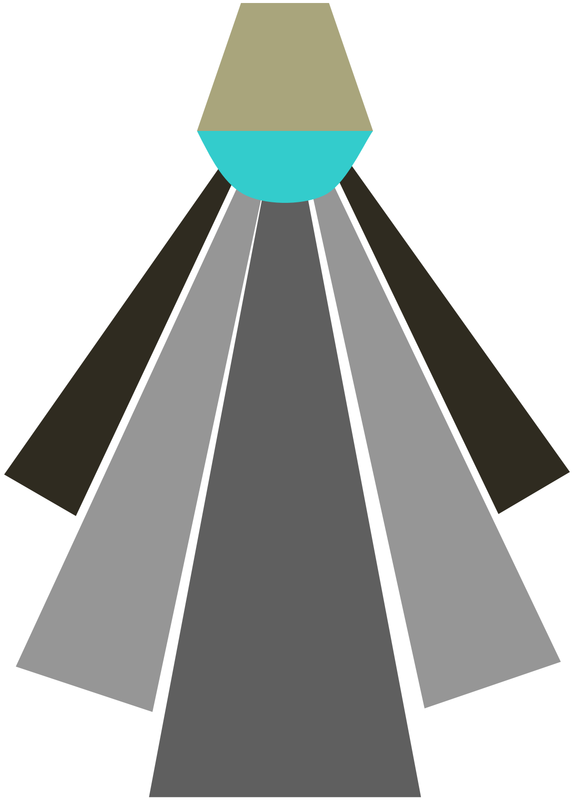
Side Lobe Artifact

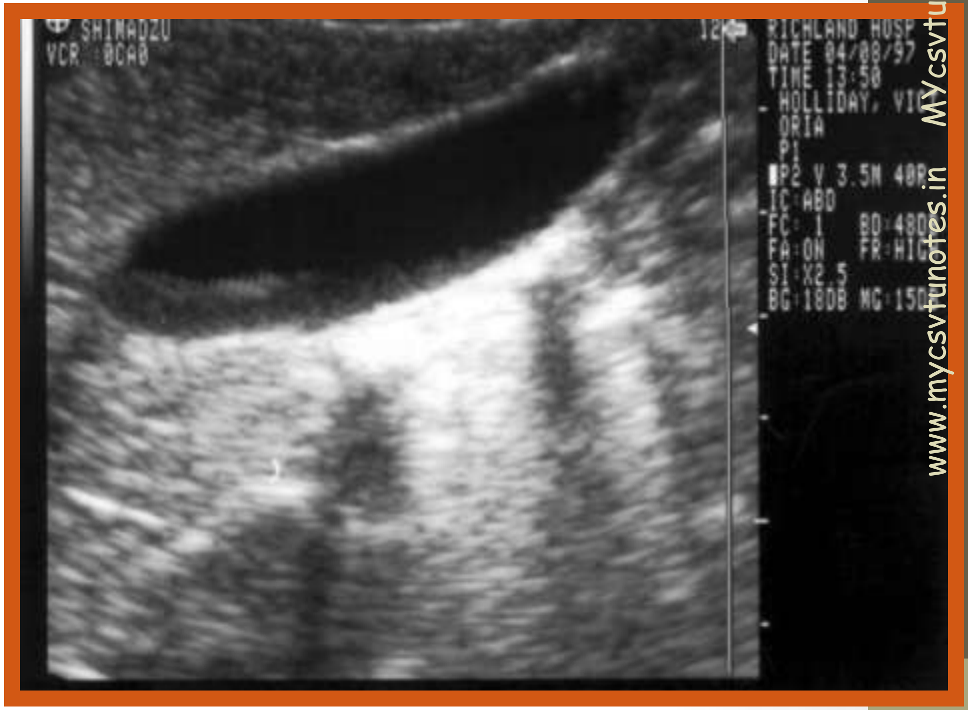
- More than one ultrasound beam is generated at the transducer head
- The beams other than the central axis beam are referred to as *side lobes*
- Side lobes are of low intensity

Side Lobe Artifact

- Occasionally cause artifacts
- The artifact by be obviated by alternating the angle of the transducer head

Side Lobe Artifact





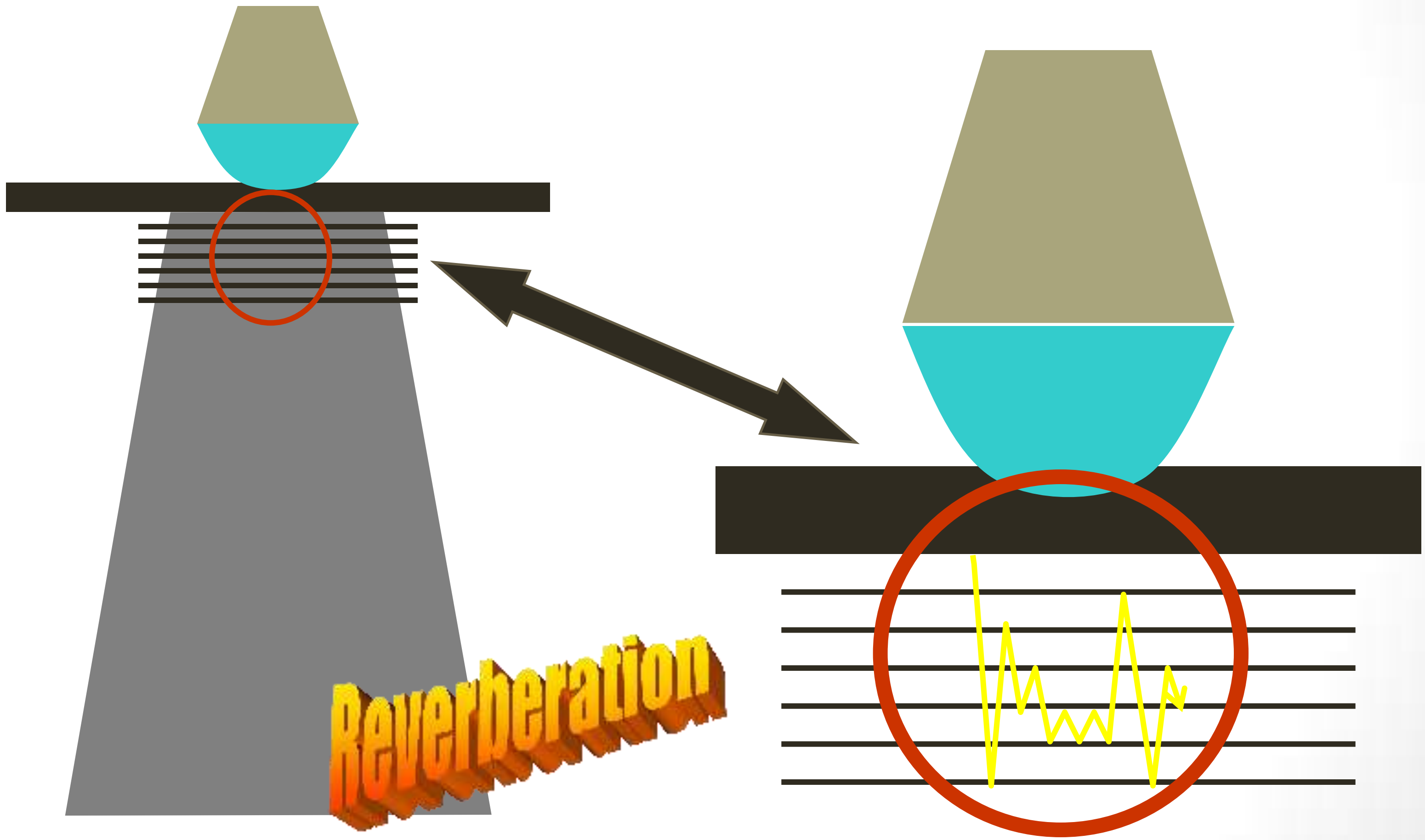
Reverberation Artifacts

- Several types
- Caused by the echo bouncing back and forth between two or more highly reflective surfaces

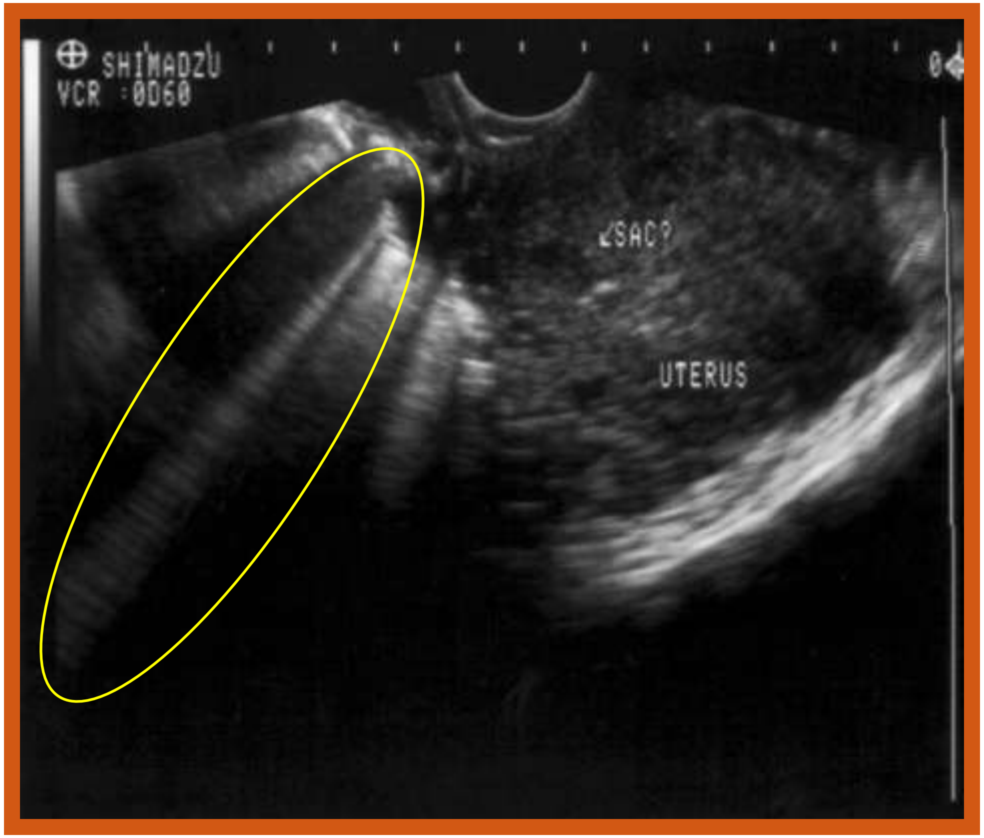
Reverberation Artifacts

- On the monitor parallel bands of *reverberation echoes* are seen
- This causes a *“comet-tail”* pattern
- Common reflective layers
  - Abdominal wall
  - Foreign bodies
  - Gas

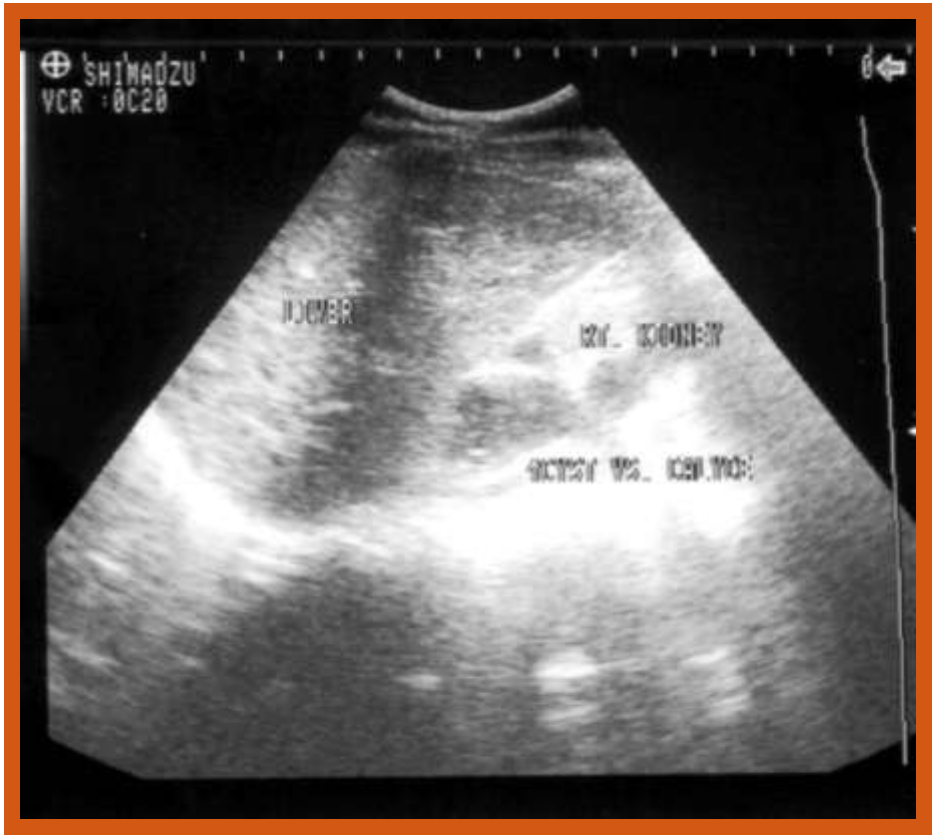
Reverberation Artifacts



Reverberation Artifacts



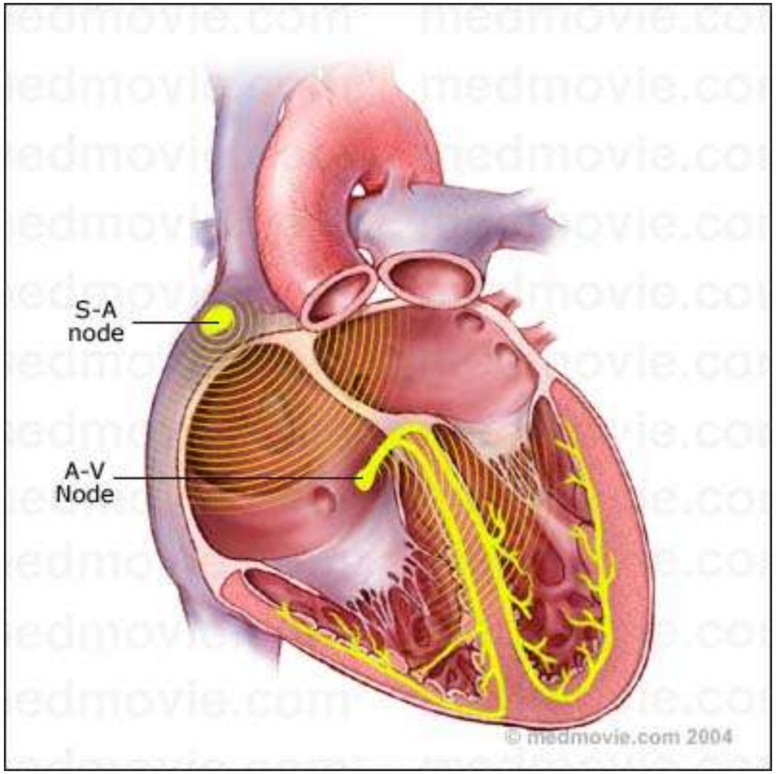
Gain Artifact

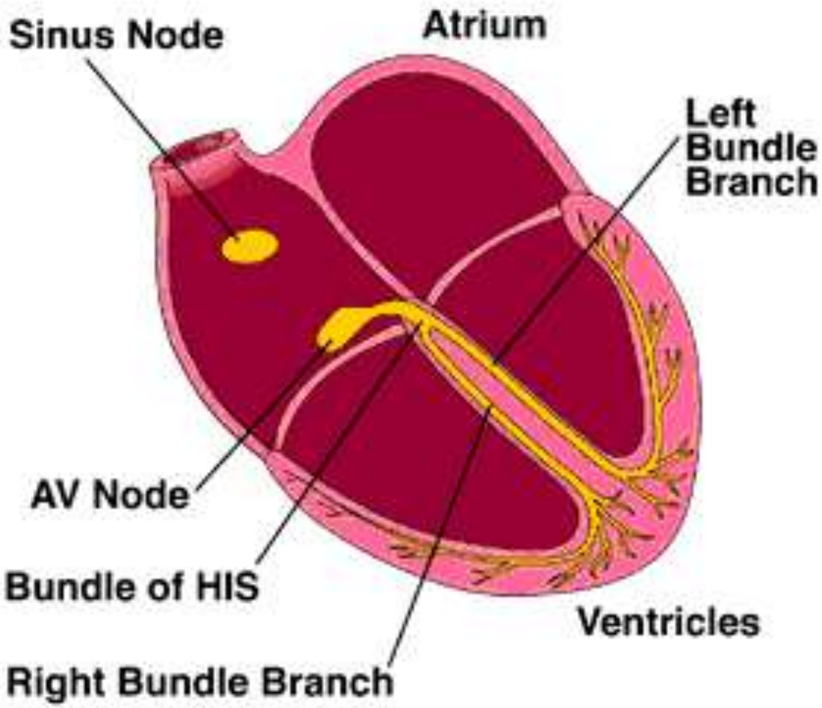


Contact artifact

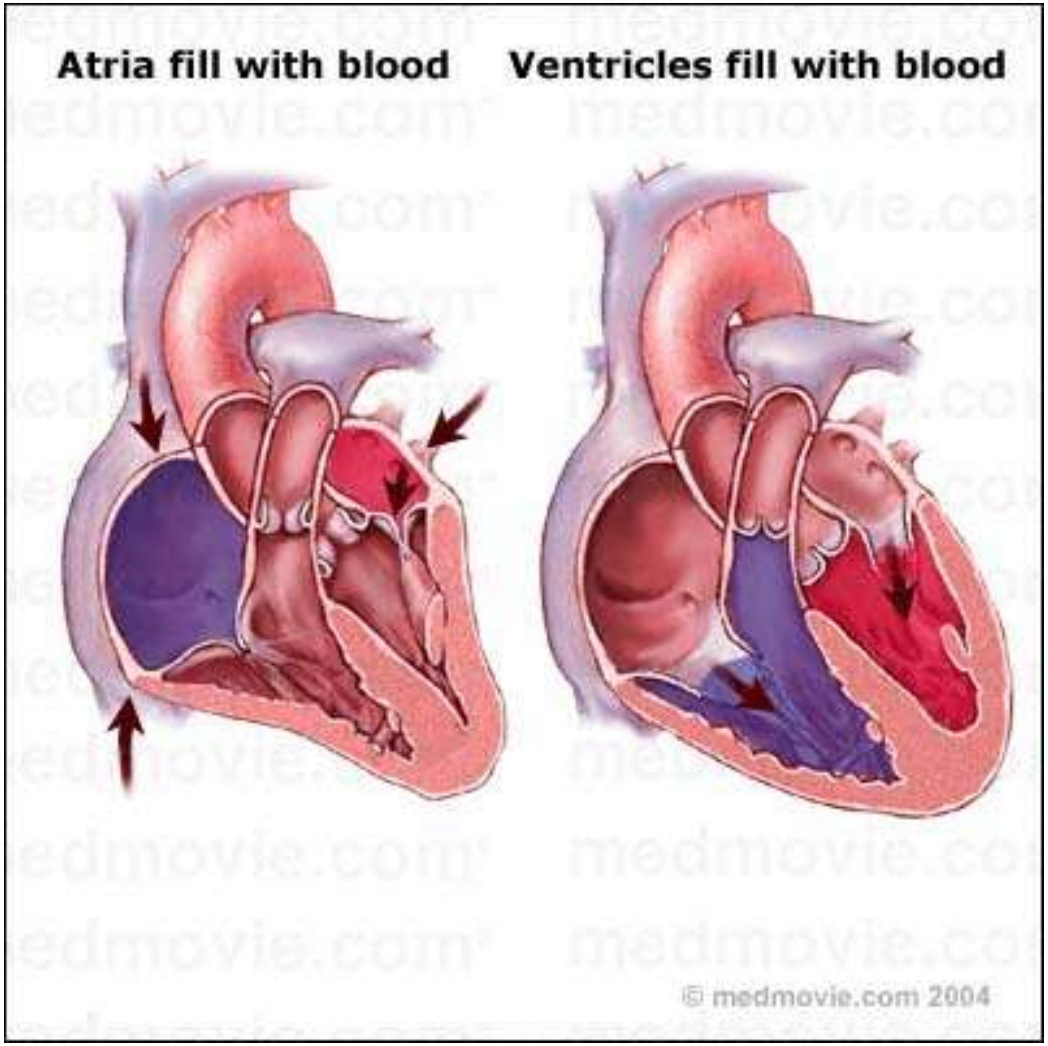
- Caused by poor probe-patient interface

The Electrical System





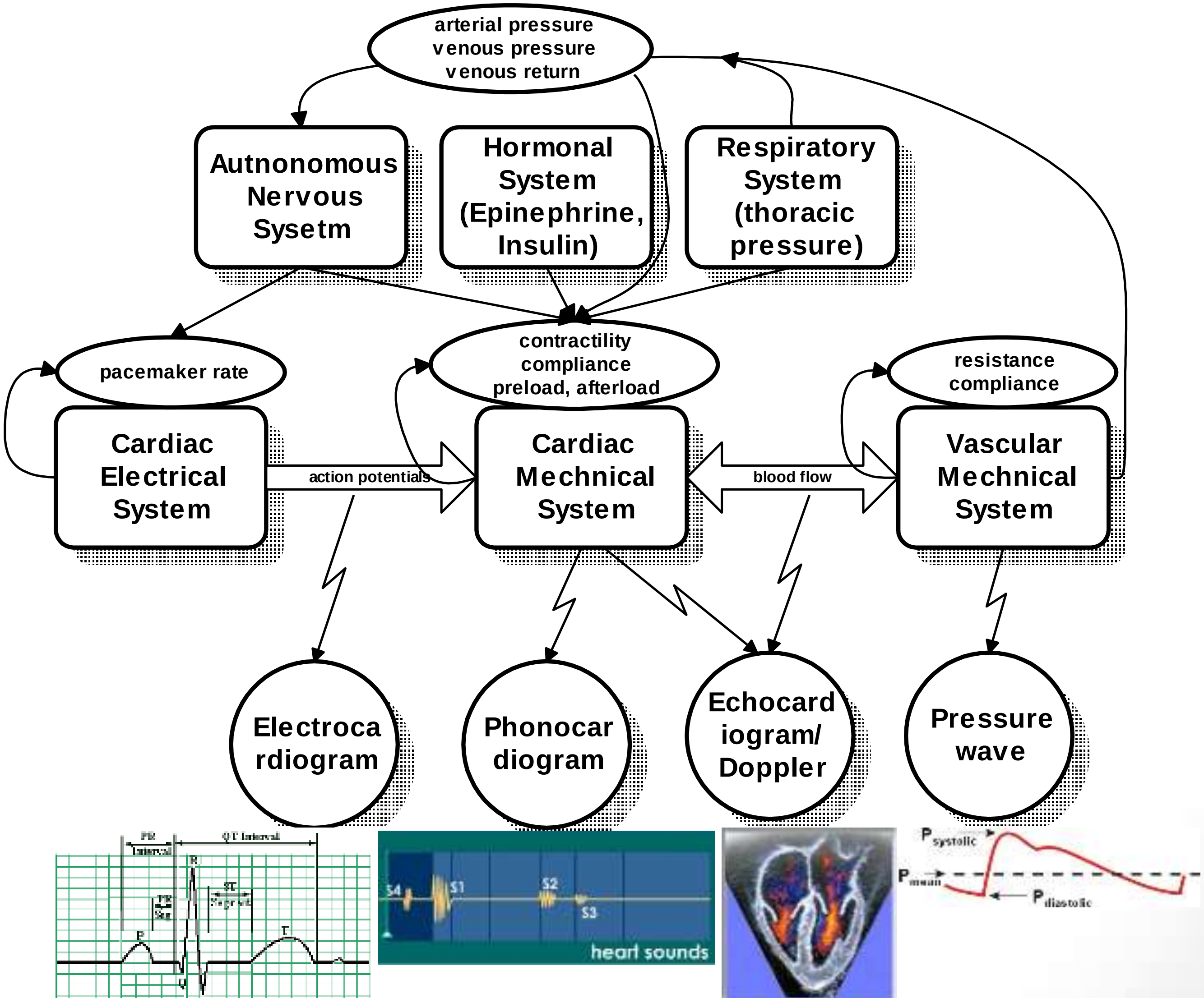
The Mechanical System



Modulating Systems

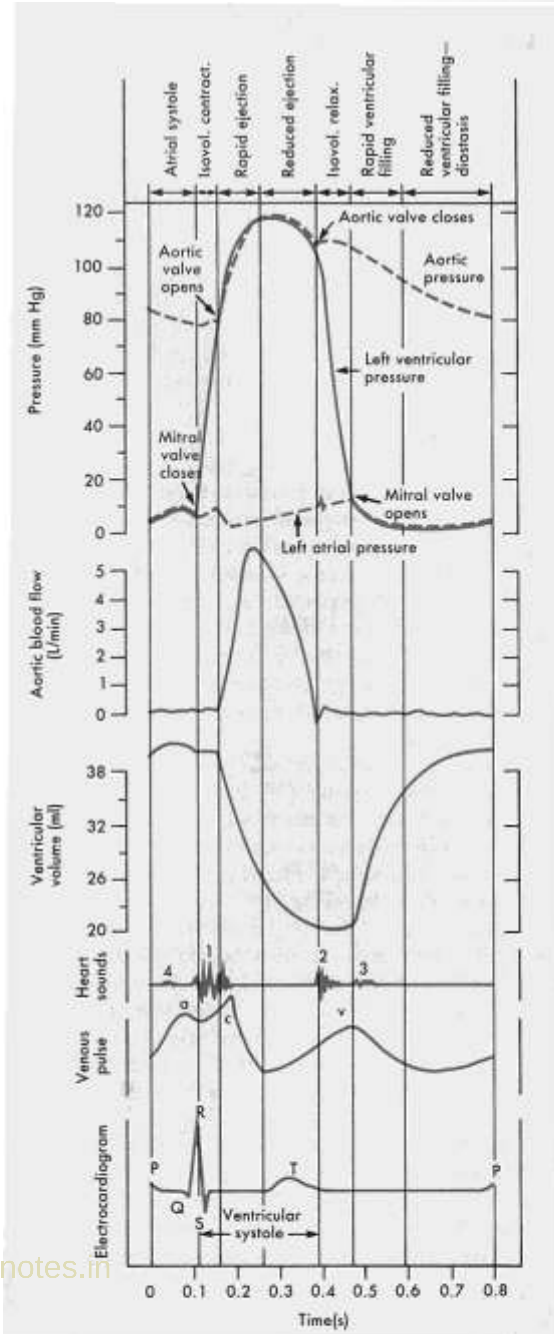
- The autonomous nervous system
- The hormonal system
- The respiratory system
- Mechanical factors
- Electrical factors

Multi-System Interactions



Multi-Signal Correlations

- Ventricular pressure
- Aortic pressure
- Atrial pressure
- Aortic blood flow
- Venous pulse
- Electrocardiogram
- Phonocardiogram



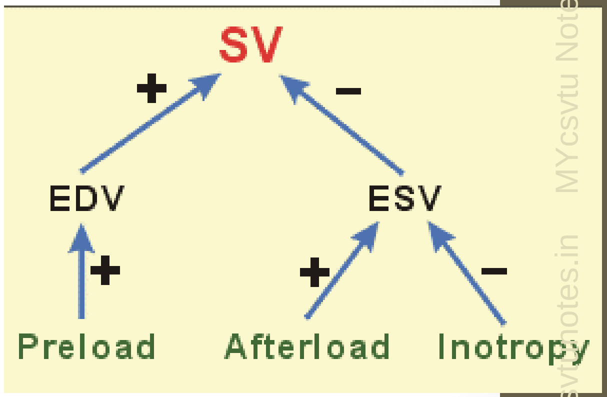
Berne R.M., Levy M.N.,
Cardiovascular Physiology, 6th edition

Heart Disease

- **Heart failure**
- Coronary artery disease
- Hypertension
- Cardiomyopathy
- Valve defects
- Arrhythmia

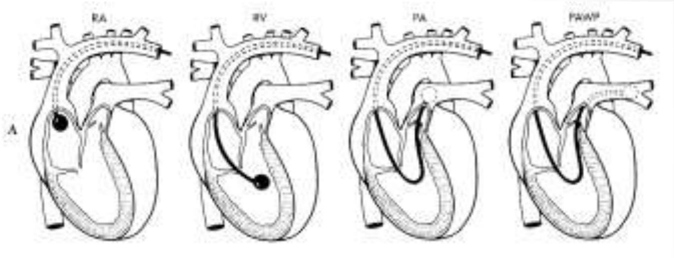
Cardiac Measurements

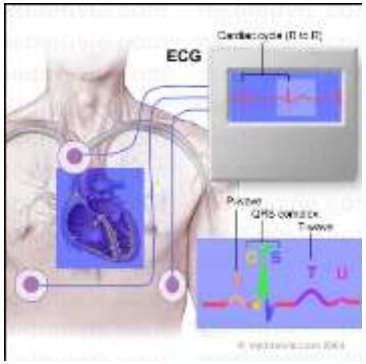
- Volumes:
  - Cardiac output CO=HR*SV
  - Stroke volume SV=LVEDV-LVESV
  - Ejection fraction EF=SV/LVEDV
  - Venous return
- Pressures:
  - Left ventricular end-diastolic pressure (preload)
  - Aortic pressure (afterload)
- Time intervals:
  - Pre-ejection period
  - Left ventricular ejection time

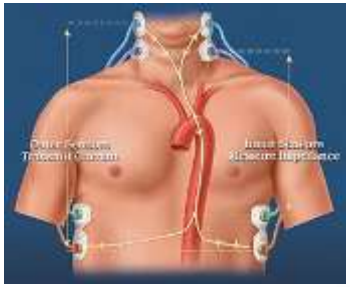


Cardiac Diagnosis

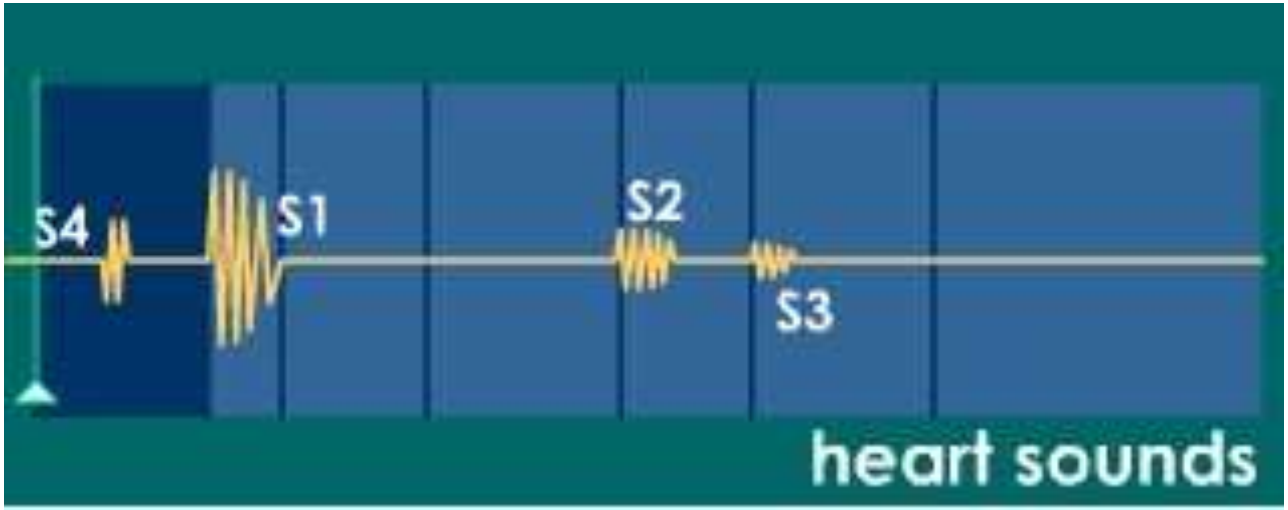
- Invasive
  - Right heart catheterization (Swan-Gan
  - Angiography
- Non-invasive
  - Electrocardiography
  - Echocardiography
  - Impedance cardiography
  - Auscultation & palpitation







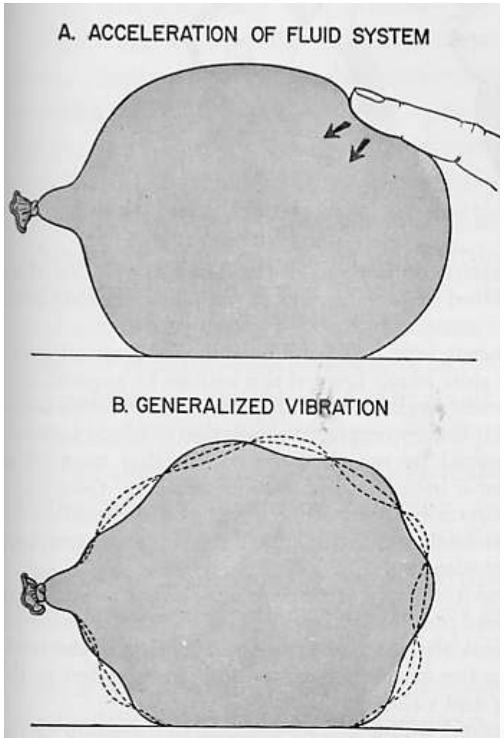
Heart Sounds



- **S1** – onset of the ventricular contraction
- **S2** – closure of the semilunar valves
- **S3** – ventricular gallop
- **S4** – atrial gallop
- **Other** – opening snap, ejection sound
- **Murmurs**

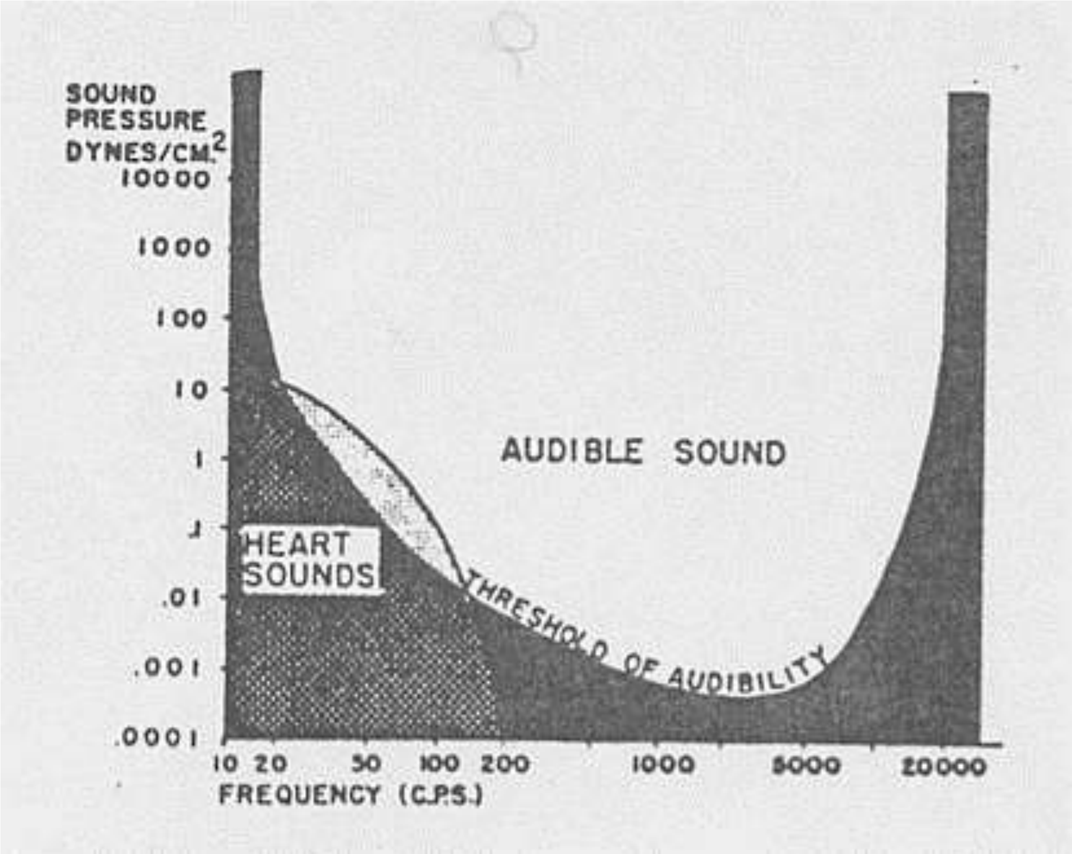
The Origin of Heart Sounds

- Valvular theory
  - Vibrations of the heart valves during their closure
- Cardiohemic theory
  - Vibrations of the entire cardiohemic system: heart cavities, valves, blood



Rushmer, R.F., Cardiovascular Dynamics, 4yh ed. W.B. Saunders, Philadelphia, 1976

Audibility of Heart Sounds

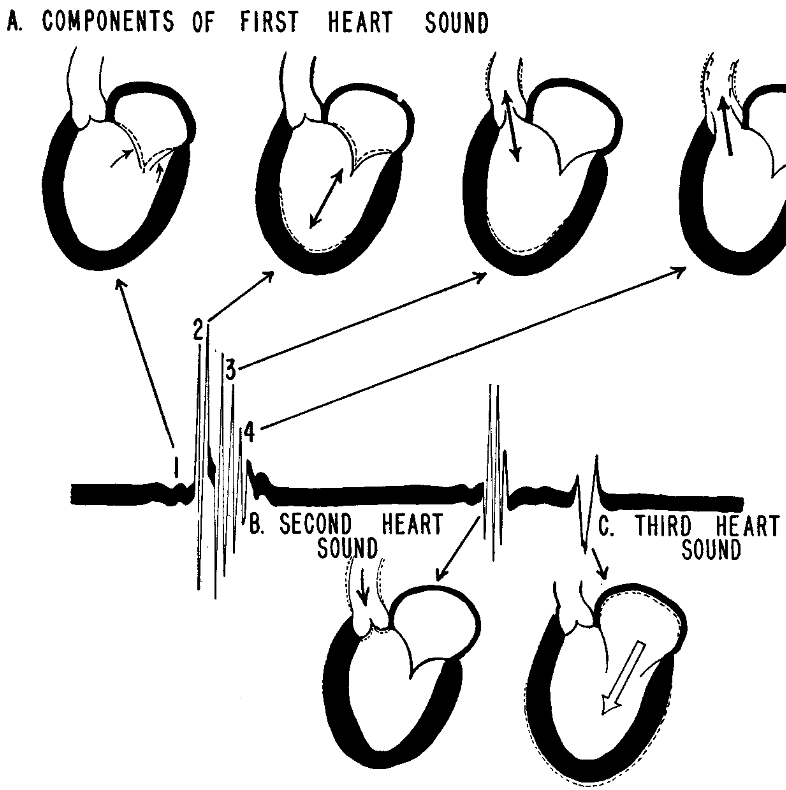


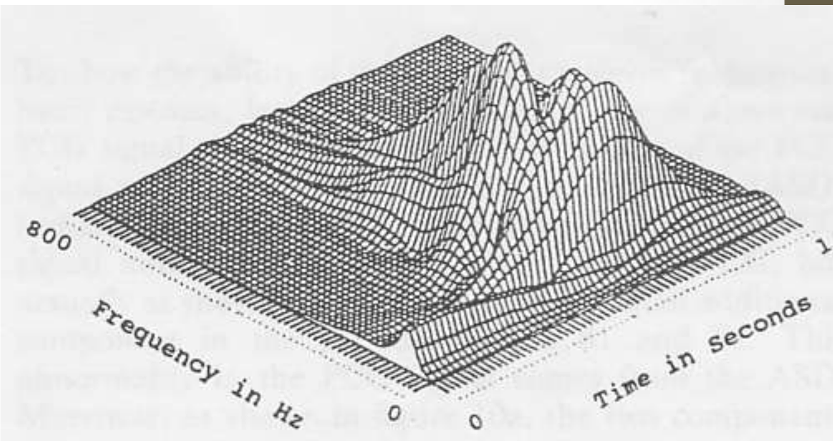
Rushmer, R.F., Cardiovascular Dynamics, 4yh ed. W.B. Saunders, Philadelphia, 1976

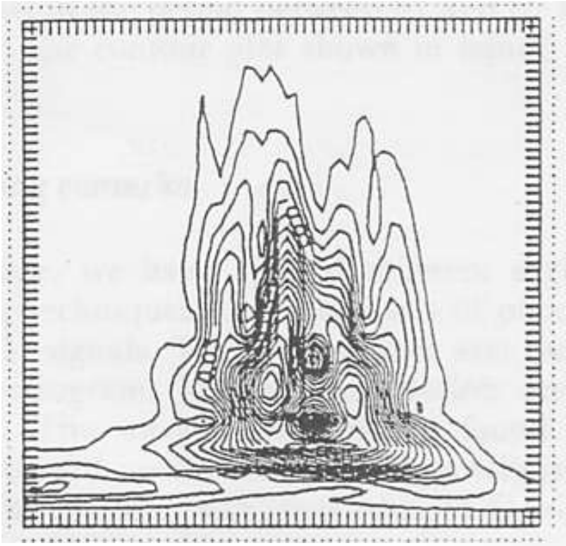
Heart Sounds as Digital Signals

- Low frequency
  - S1 has components in 10-140Hz bands
  - S2 has components in 10-400Hz bands
- Low intensity
- Transient
  - 50-100 ms
- Non-stationary
- Overlapping components
- Sensitive to the transducer's properties and location

Sub-Components of S1



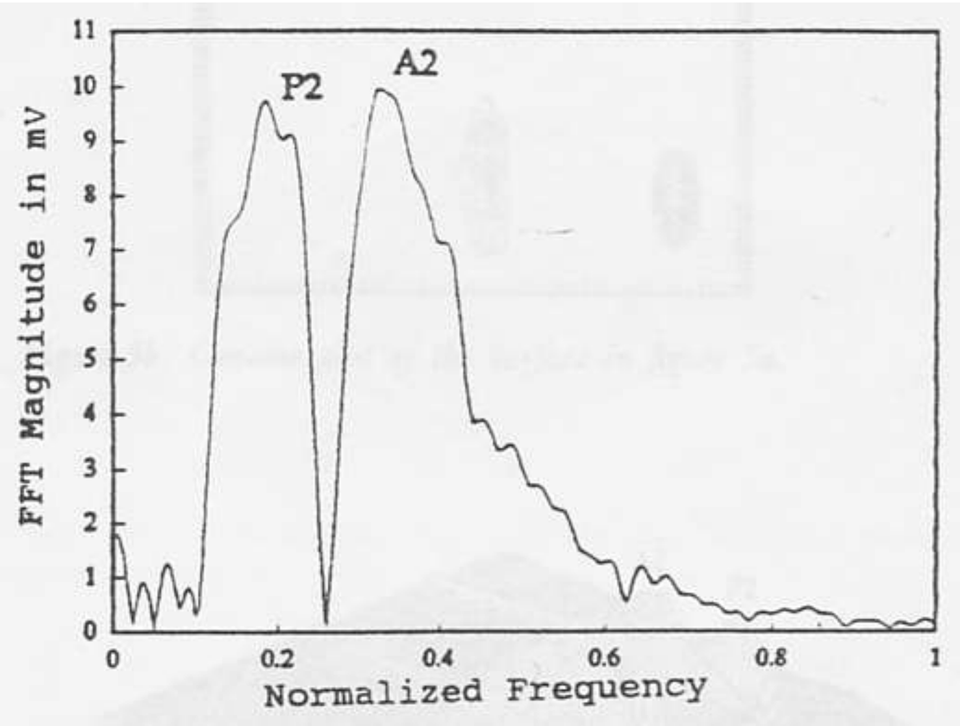


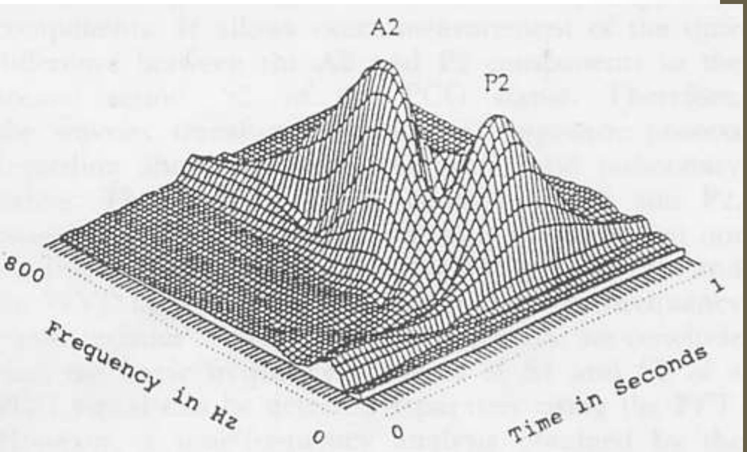


Rushmer, R.F., Cardiovascular Dynamics

Obaidat M.S., J. Med. Eng. Tech., 1993

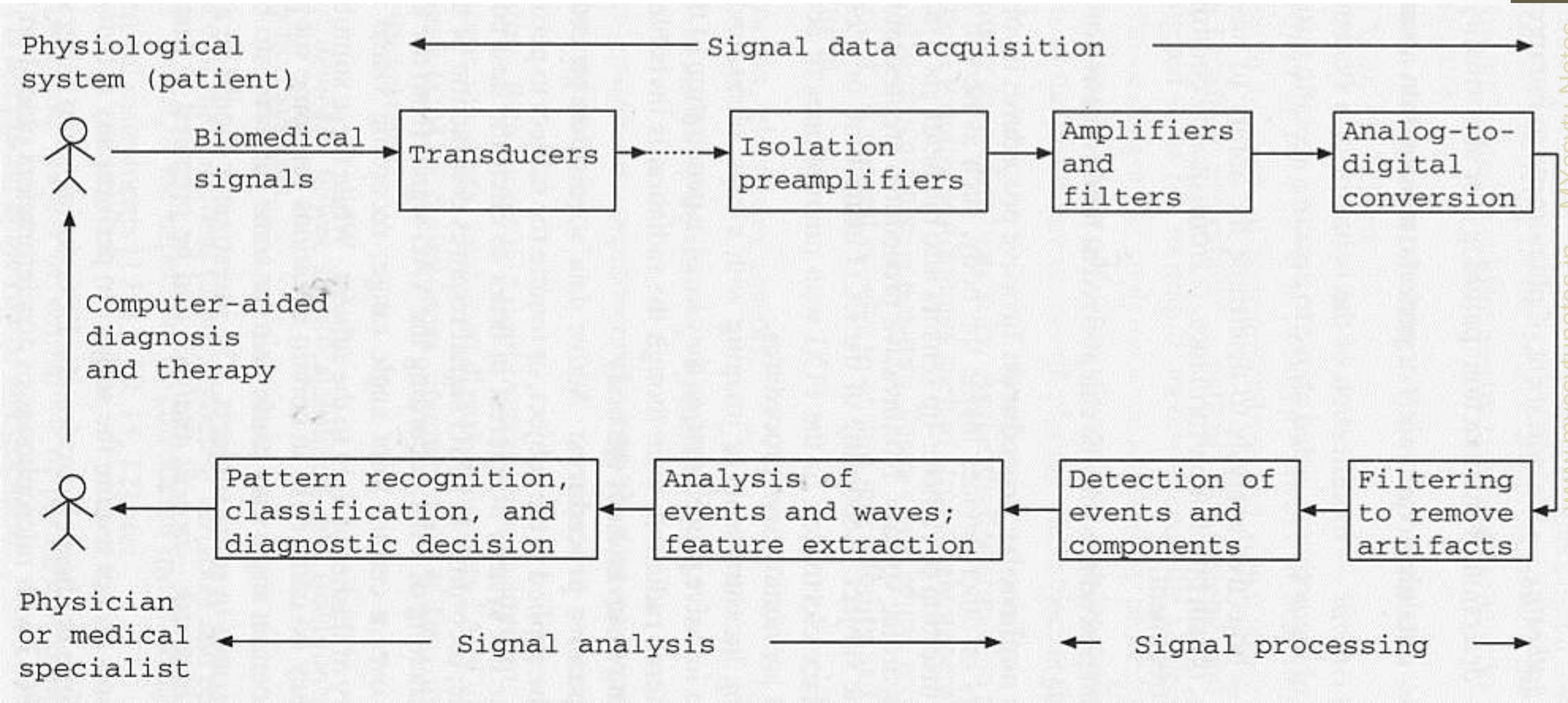
Sub-Components of S2





Obaidat M.S., J. Med. Eng. Tech., 1993

Heart Sound Analysis Techniques



R.M. Rangayyan, Biomedical Signal Analysis, 2002

Segmentation

- External references (ECG, CP)
- Timing relationship
- Spectral tracking
- Envelogram
- Matching pursuit
- Adaptive filtering

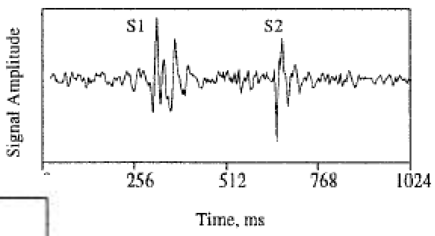
Decomposition (1)

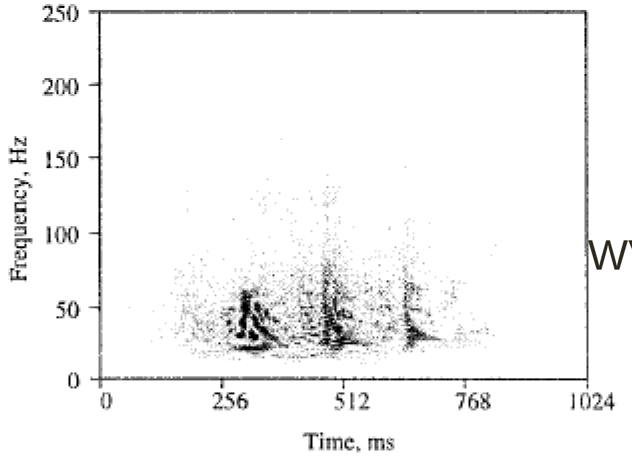
- Non-parametric time-frequency methods:
  - Linear
    - Short-Time Fourier Transform (STTF)
    - Continuous Wavelet Transform (CWT)
  - Quadratic TFR
    - Wigner-Ville Distribution (WVD)
    - Choi-Williams Distribution (CWD)

Decomposition (2)

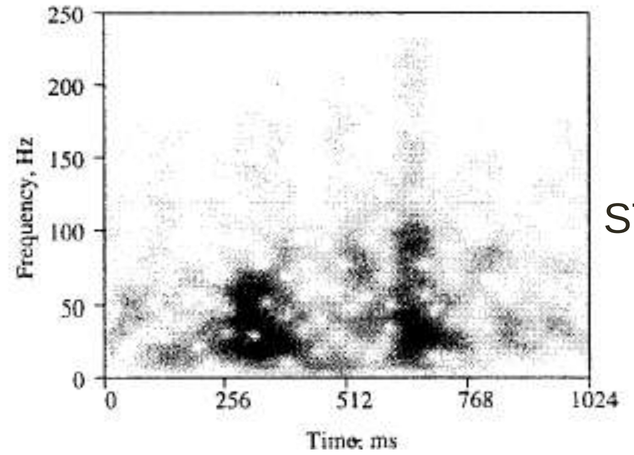
- Parametric time-frequency methods:
  - Autoregressive (AR)
  - Autoregressive Moving Average (ARMA)
  - Adaptive spectrum analysis

Decomposition - Example

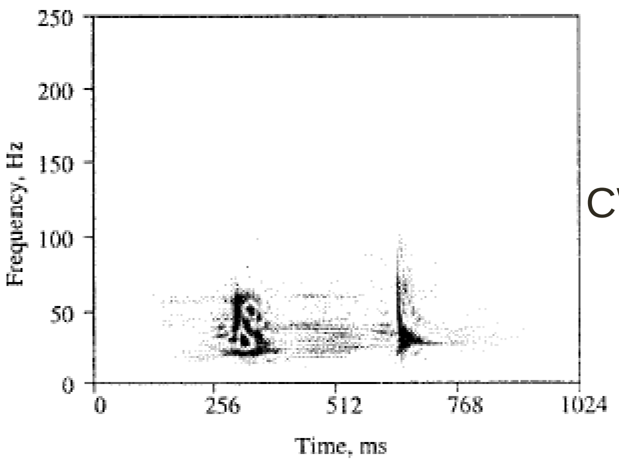




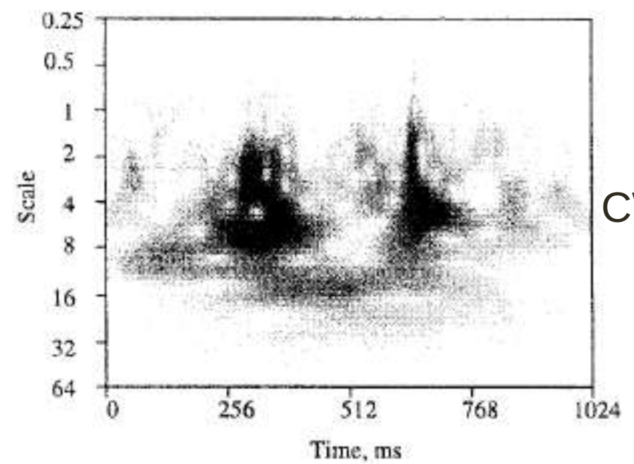
WVD



STFT



CWD



CWT

Bentley P.M. et al., IEEE Tran. BioMed. Eng., 1998

Feature extraction

- Morphological features
  - Dominant frequencies
  - Bandwidth of dominant frequencies (at -3dB)
  - Integrated mean area above -20dB
  - Intensity ration of S1/S2
  - Time between S1 and S2 dominant frequencies
- AR coefficients
- DWT-based features

Classification

- Methods:
  - Gaussian-Bayes
  - K-Nearest-Neighbor
  - Artificial Neural-Network
  - Hidden Markov Model
  - Rule-based
- Classes:
  - Normal/degenerated bioprosthetic valves
  - Innocent/pathological murmur
  - Normal/premature ventricular beat

Classification - Example

TABLE III
NUMBER OF PATTERNS PROVIDING ≥90% CORRECT CLASSIFICATIONS FOR NORMAL AND DEGENERATED MITRAL BIOPROSTHETIC VALVES

Type of Classifier	# of Features per Pattern	# of Patterns Providing ≥ 90% of Correct Classifications	Highest Performance	# of Patterns Providing the Highest Performance
Nearest Neighbor (Euclidean) (Distance)	2	0	85%	1
	3	1	90%	1
	4	2	90%	2
	5	2	92%	1
	6	2	92%	1
Nearest Neighbor (Normalized) (Euclidean) (Distance)	2	0	88%	1
	3	6	90%	6
	4	14	92%	3
	5	52	92%	12
	6	97	94%	1
Nearest Neighbor (Mahalanobis) (Distance)	2	1	90%	1
	3	6	94%	1
	4	25	94%	2
	5	31	94%	1
	6	48	94%	1
Bayes	2	1	90%	1
	3	0	88%	9
	4	3	90%	3
	5	7	90%	7
	6	500	98%	2

Durand L.G. et al., IEEE Tran. Biomed Eng., 1990

Heart Sound Analysis Applications

- Estimation of pulmonary arterial pressure
- Estimation of left ventricular pressure
- Measurement & monitoring of cardiac time intervals
- Synchronization of cardiac devices

Estimation of pulmonary artery pressure (Tranulis et al., 2002)

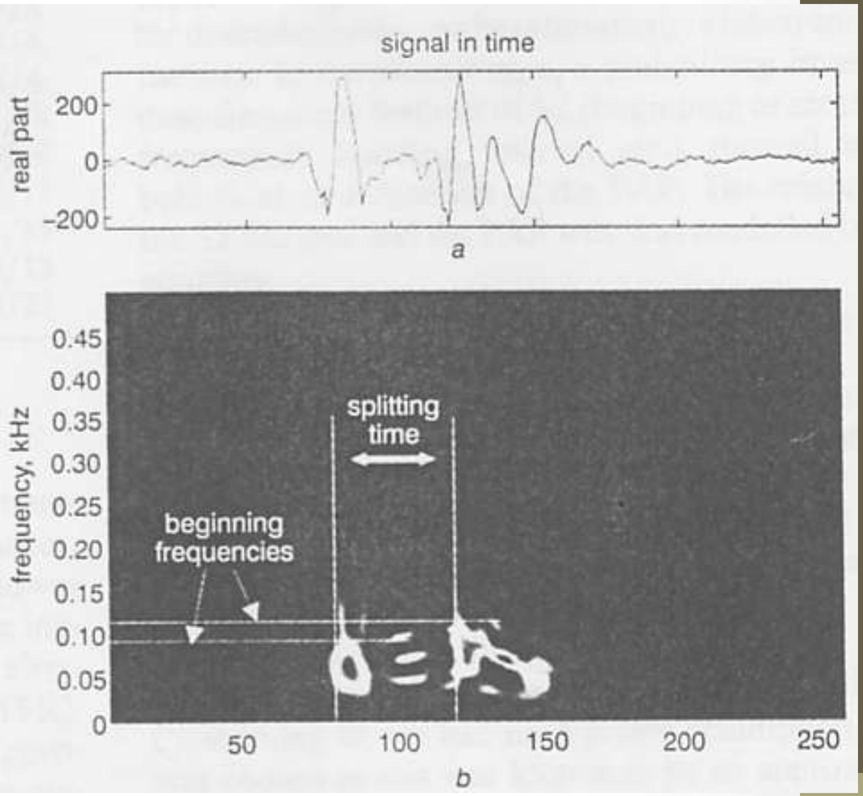
- Non-invasive method for PAP estimation and PHT diagnosis
- Feature-extraction using time-frequency representations of S2
- Learning and estimation using a neural network
- Comparison to invasive measurement and Doppler-echo estimation
- Animal model

Signal Processing

- **Filtering** the PCG signal:
  - 100Hz high-pass filter
  - 300Hz low-pass filter
- **Segmentation** of S2 by ECG reference
- **Decomposition** of S2 by TFR:
  - Smoothed Pseudo-Wigner-Ville distribution
  - Orthonormal wavelet transform

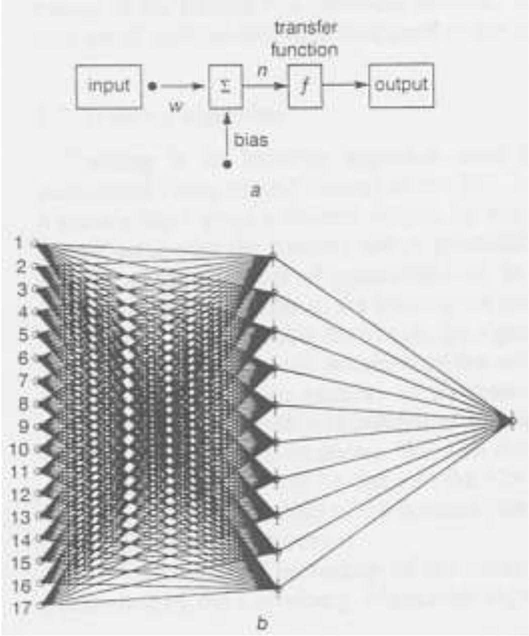
Feature Extraction

- SPWVD features:
  - Maximum instantaneous frequency of A2,P2
  - The splitting interval between A2 and P2
- OWT features (for each scale):
  - Maximum value
  - The position of the maximum value
  - The energy

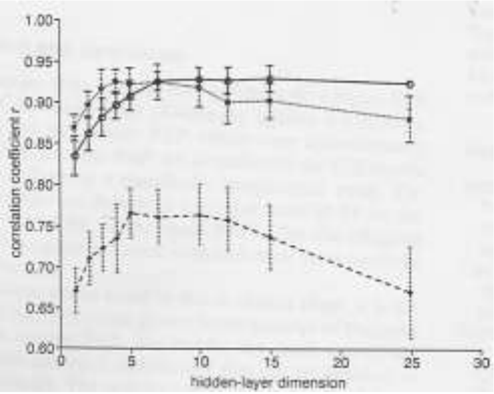


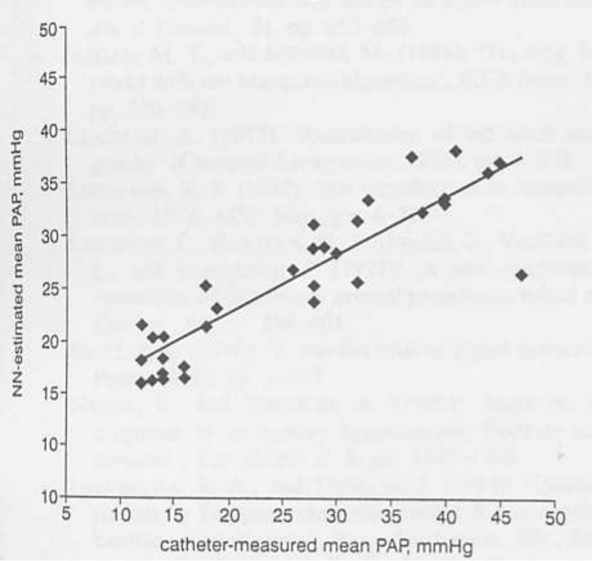
ANN Training and Testing

- A feed-forward, back-propagation ANN with one hidden layer
- The significance of the features and the size of the network were evaluated
- Training was conducted using 2/3 of the data using error-minimization procedure
- The NN estimations were averaged for series of beats and compared to the measured PAP



Results



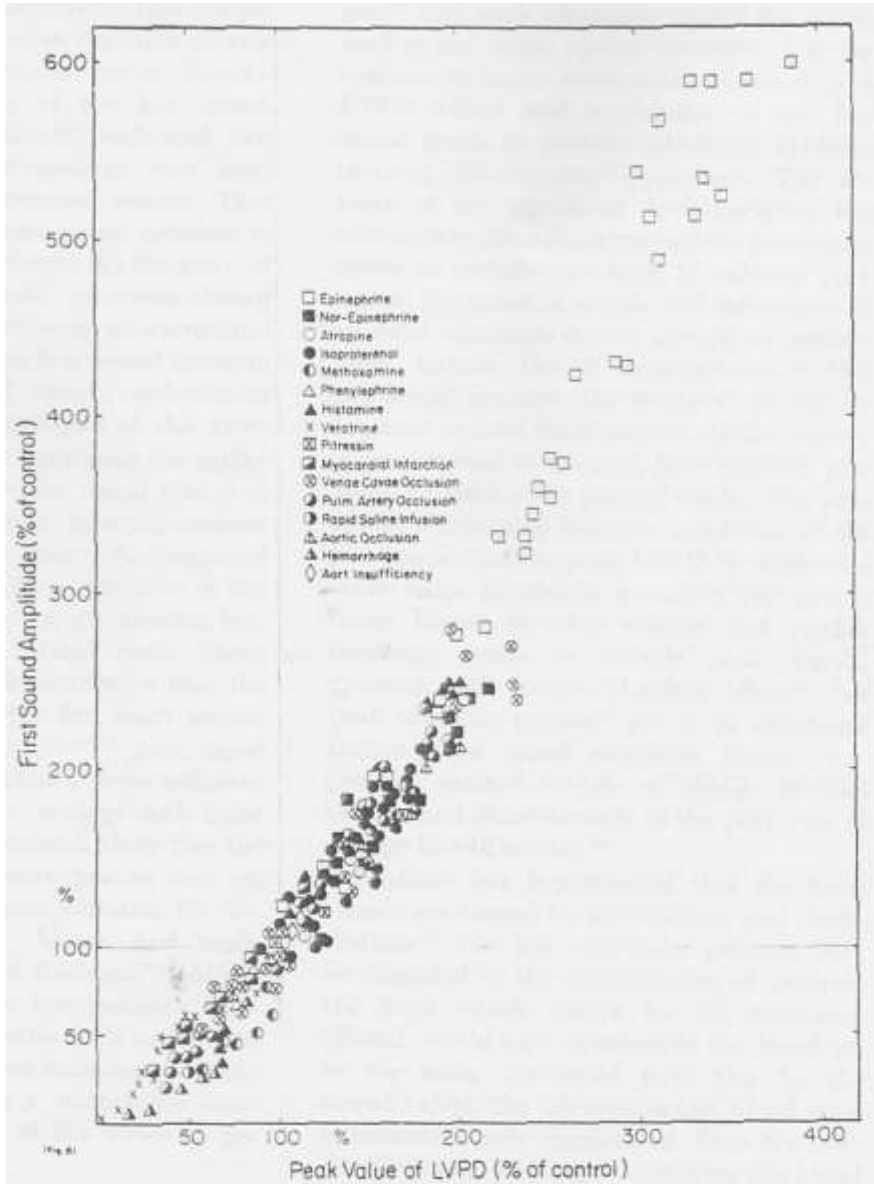


- A combination of TFR and OWT features gave the best results (r=0.89 SEE=6.0mmHg)
- The correct classification of PHT from the mean PAP estimate was 97% (sensitivity 100% ; specificity 93%)

Estimation of left ventricular pressure

- PCG and pressure tracing are different manifestations of cardiac energy
- The PCG is proportional to the acceleration of the outer heart wall => proportional to the changes of intra-ventricular pressure
- S3 is an indication of high filling pressure or/and stiffening of the ventricular wall

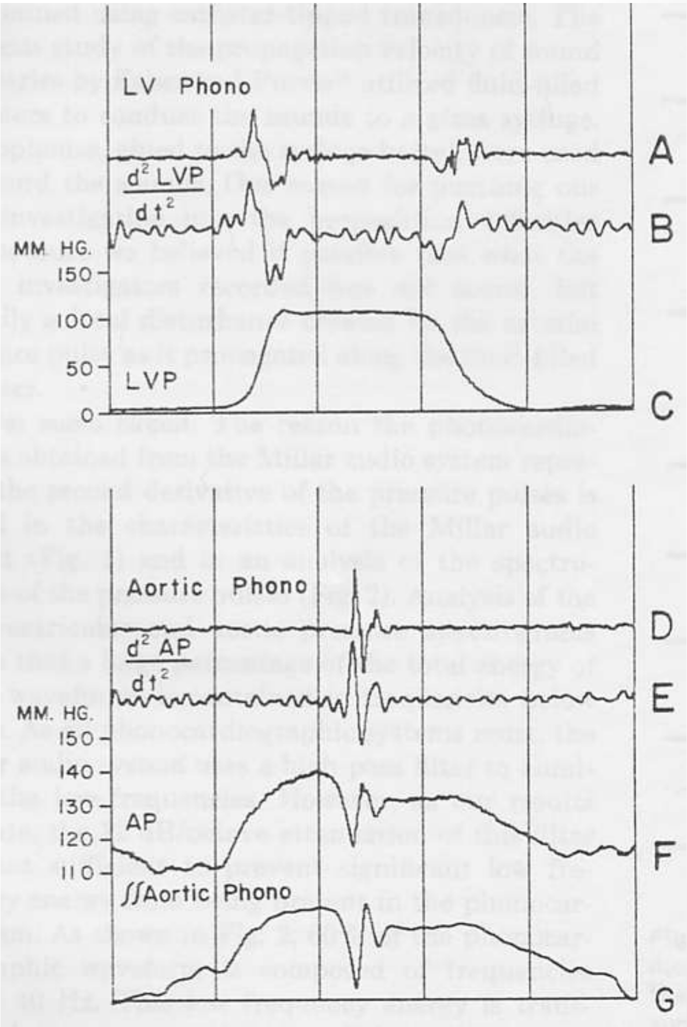
Amplitude of S1 and LV dP/dt



Sakamoto T. et al., Circ. Res., 1965

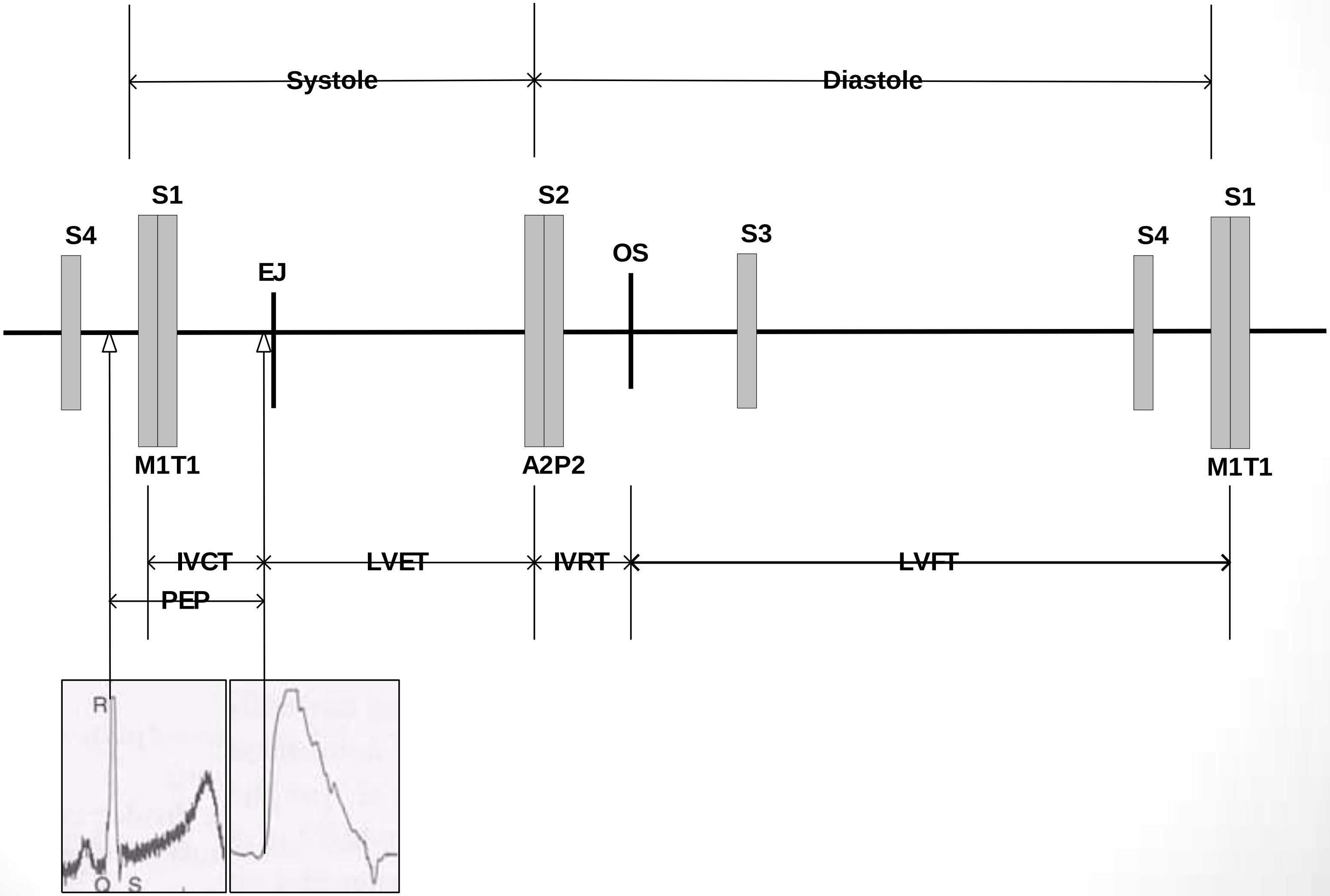
PCG as a Derivative of Pressure

- The transducer measures acceleration
- The acceleration is the second derivative of displacement/pressure
- Pressure can be estimated by integrating the PCG



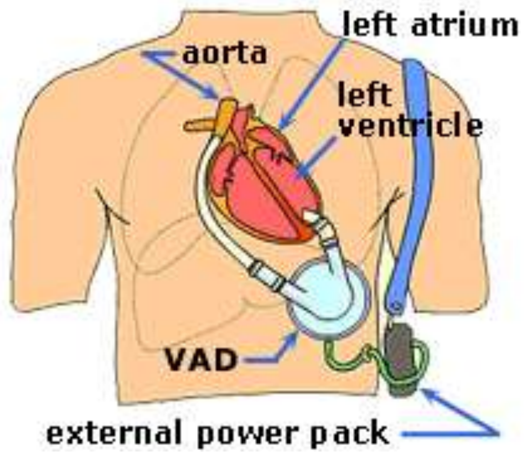
Heckman J.L., et al., Am. Heart J.,1982

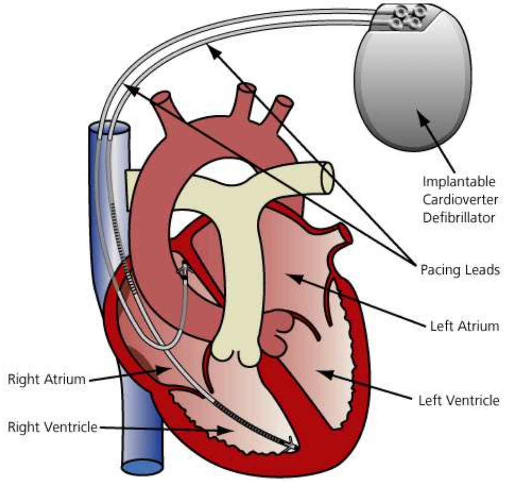
Measurement of cardiac time intervals



Synchronization of cardiac assist devices

- Left ventricular assist device (LVAD)
- Intra-aortic balloon pum
- Implantable Cardioverter Defibrillator





Summary

- Heart sounds/vibrations represent the mechanical activity of the cardiohemic system
- The heart sound signal can be digitally acquired and automatically analyzed
- Heart sound analysis can be applied to improve cardiac monitoring, diagnosis and therapeutic devices

Thank You !

Mathematical Appendix (1)

- STFT

$$S(t,\omega) = \int_{-\infty}^{\infty} s(\tau) w(\tau - t) e^{-i\omega t} d\tau$$

- CWT

$$S(t,a) = 1/\sqrt{a} \int s(t) g^*(\tau - t/a) d\tau$$

$$g(t) = e^{-t^2/2 + i\omega_0 t}$$

- WVD

$$S(t,\omega) = \int z(t + \tau/2) z^*(t - \tau/2) e^{-i\omega t}\, d\tau$$

$$z(t) = s(t) + iH(t)$$

- CWD

$$S(t,\omega) = \iint \frac{1}{\sqrt{\tau^2/\sigma}} s(u + \tau/2) s^*(u - \tau/2) e^{-[(u-t)^2/(4\tau^2/\sigma)] - i\omega t}\, du\, d\tau$$

Mathematical Appendix (2)

- AR $$y(n) = -\sum_{k=1}^{p} a_k y(n-k) + Gx(n)$$
- ARMA $$y(n) = -\sum_{k=1}^{p} a_k y(n-k) + G\sum_{l=0}^{q} b_l x(n-l)$$
- Adaptive spectrogram

$$AS(t, f) = 2\sqrt{\pi}\sum_{i} A_i^2 \sigma_i\, e^{[(t-t_i)^2/\sigma_i^2) + 2\pi\sigma_i^2 (f-f_i)^2]}$$

Mathematical Appendix (3)

- SPWVD

$$S(t,\omega) = \int_{-\infty}^{\infty} q(\tau)[\int_{-\infty}^{\infty} g(s-t)x(s+\tau/2)x(s-\tau/2)ds]e^{-i\omega\tau}d\tau$$

$$q(\tau) = h(\tau/2)h(-\tau/2)$$

- OWT $OWT(k,j) = 2^{-j/2}\int_{-\infty}^{\infty} x(s)\psi(2^{-j}s-k)ds$