

Introduction to Thermal Radiation

Figures except for the McDonnell Douglas figures come from Incropera & DeWitt, Introduction to Heat and Mass Transfer or Cengel, Heat Transfer: A Practical Approach

Thermal Radiation

- Occurs in solids, liquids, and gases
- Occurs at the speed of light
- Has no attenuation in a vacuum
- Can occur between two bodies with a colder medium in between
- Applications: satellite temperature management, taking accurate temperature measurements, designing heaters for manufacturing, estimating heat gains through windows, infrared cameras, metal cooling during manufacturing, Greenhouse effect, lasers, insulating cryogenic tanks, thermos design, ice skating rink ceiling design

Background

- Electromagnetic radiation – energy emitted due to changes in electronic configurations of atoms or molecules
- where λ =wavelength (usually in μm), ν =frequency
- In a vacuum $c=c_0=2.998 \times 10^8$ m/s
- Other media: $c=c_0/n$ where n =index of refraction

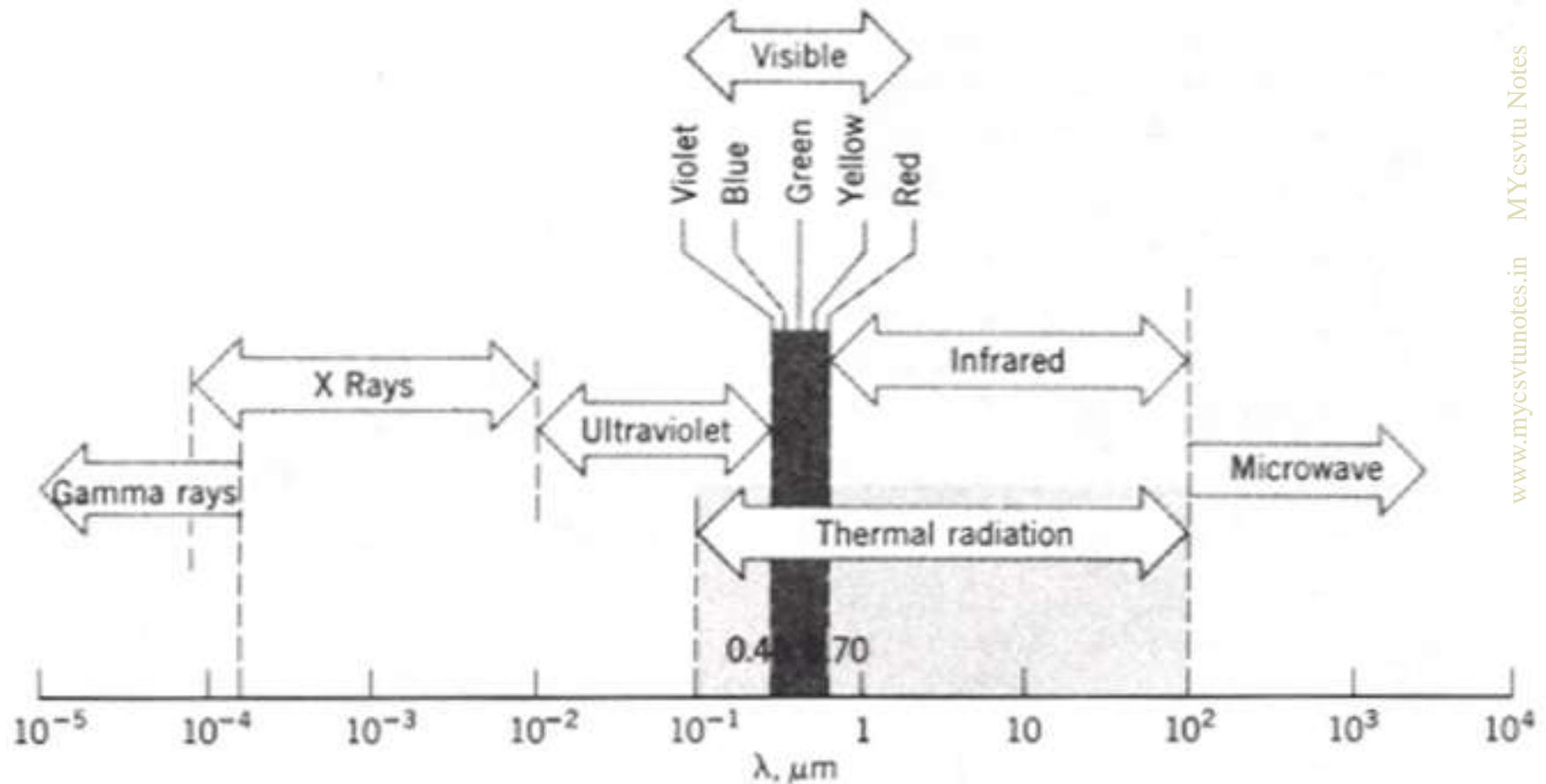
$$\lambda = c/\nu$$

Background, cont.

- Radiation – photons or waves?
- Max Planck (1900): each photon has an energy of
- h =Planck's constant= 6.625×10^{-34} Js
- Shorter wavelengths have higher energy

$$e = h\nu = hc/\lambda$$

Radiation Spectrum



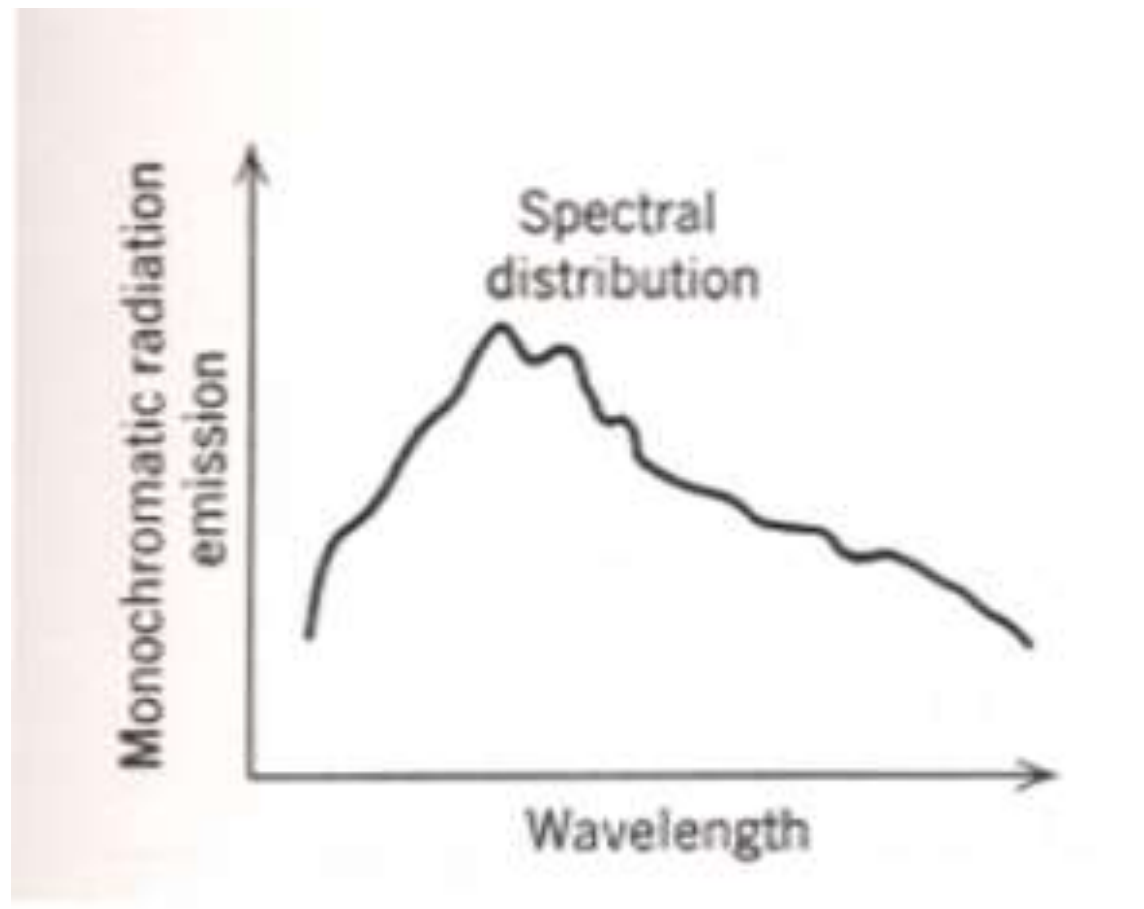
Types of Radiation

- Two categories
 - Volumetric phenomenon – radiation emitted or absorbed throughout gases, transparent solids, some fluids
 - Surface phenomenon – radiation to/from solid or liquid surface
- Thermal radiation – emitted by all substances above absolute zero
- Includes visible & infrared radiation & some UV radiation.

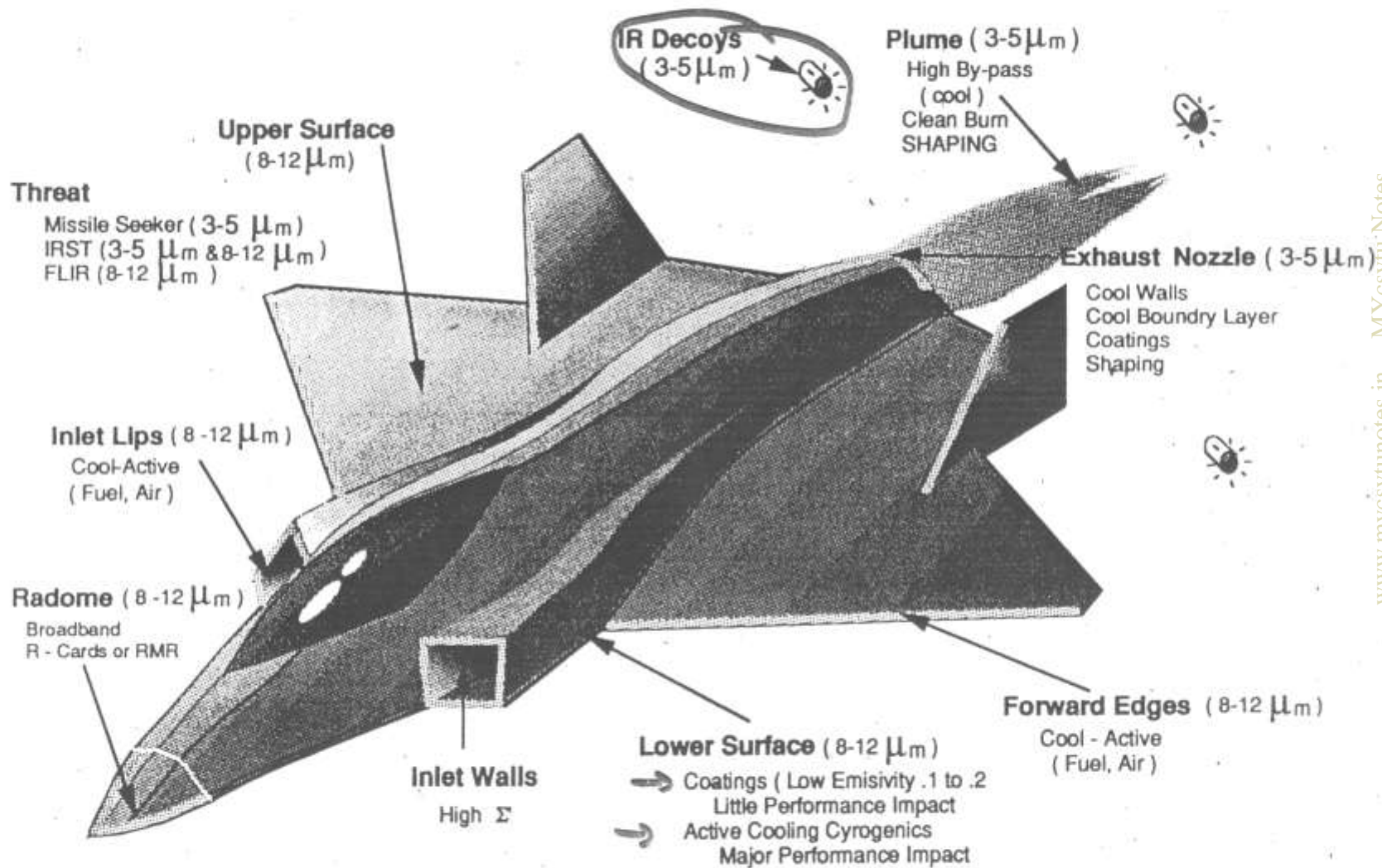
Radiation Properties

- Magnitude of radiation varies with wavelength – it's spectral.
 - The wavelength of the radiation is a major factor in what its effects will be.
 - Earth/sun example
- Radiation is made up of a continuous, nonuniform distribution of monochromatic (single-wavelength) components.
- Magnitude & spectral distribution (how the radiation varies with wavelength) vary with temp & type of emitting surface.

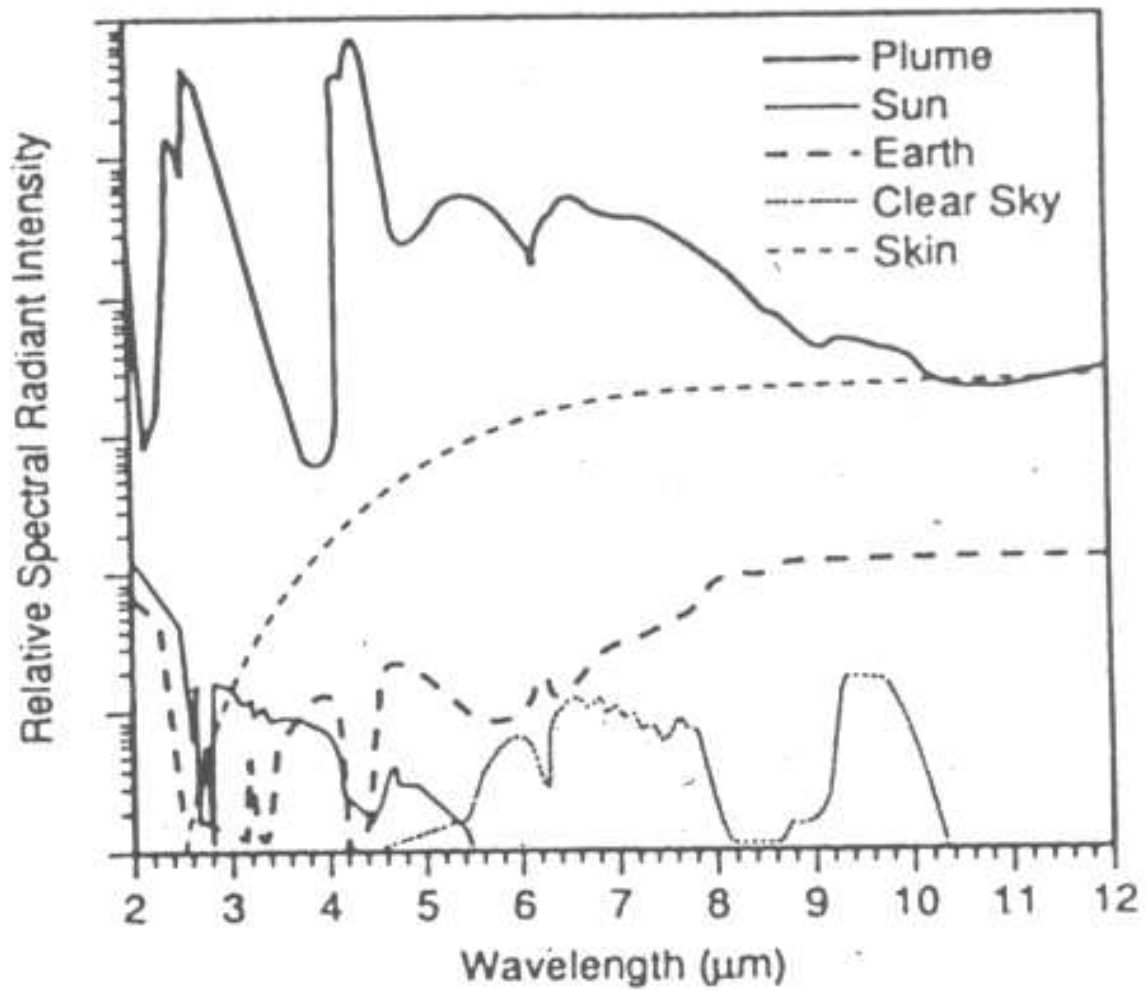
Emission Variation with Wavelength



IR REQUIREMENTS AND TREATMENTS



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The relative magnitude of typical target signature components versus wavelength for a jet aircraft at 90-deg aspect, M1.2 airspeed.

Blackbody Radiation

- Blackbody – a perfect emitter & absorber of radiation; it absorbs all incident radiation, and no surface can emit more for a given temperature and wavelength
- Emits radiation uniformly in all directions – no directional distribution – it's diffuse
- Example of a blackbody: large cavity with a small hole Joseph Stefan (1879)– total radiation emission per unit time & (area over all wavelengths and in all directions:)
$$E_b = \sigma T^4 \text{ (W/m}^2\text{)}$$

Planck's Distribution Law

- Sometimes we care about the radiation in a certain wavelength interval

- For a surface in a vacuum or gas

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad (\text{W/m}^2 \cdot \mu\text{m})$$

where

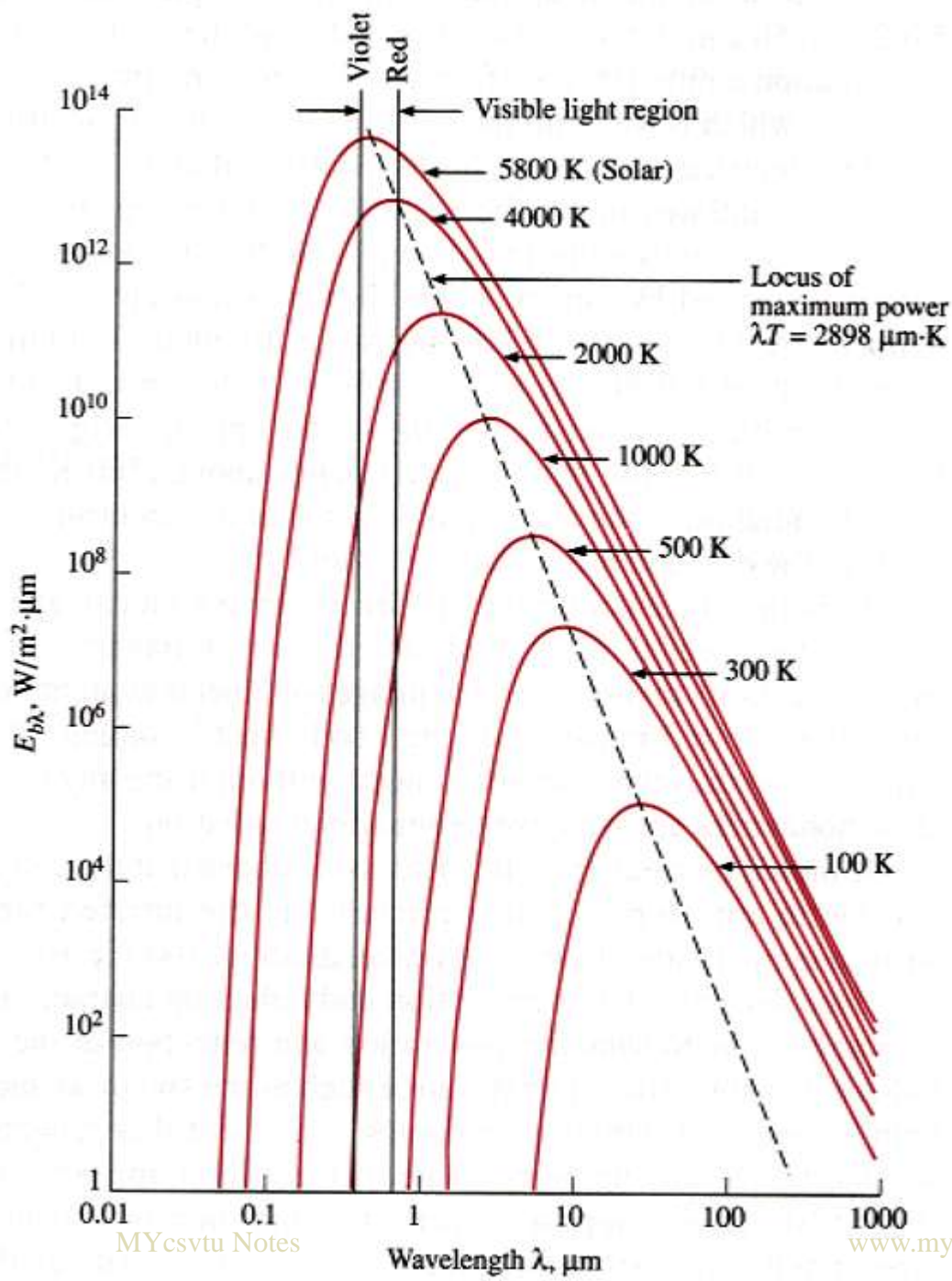
$$C_1 = 2\pi hc_o^2 = 3.742 \times 10^8 \text{ W} \cdot \mu\text{m}^4 / \text{m}^2$$

$$C_2 = hc_o/k = 1.439 \times 10^4 \mu\text{m} \cdot \text{K}$$

$$k = 1.3805 \times 10^{-23} \text{ J/K} = \text{Boltzmann's constant}$$

- Integrating this function over all λ gives us $E_b = \sigma T^4$

radiation distribution



- Radiation is a continuous function of wavelength
- Magnitude increases with temp.
- At higher temps, more radiation is at shorter wavelengths.
- Solar radiation peak is in the visible range.

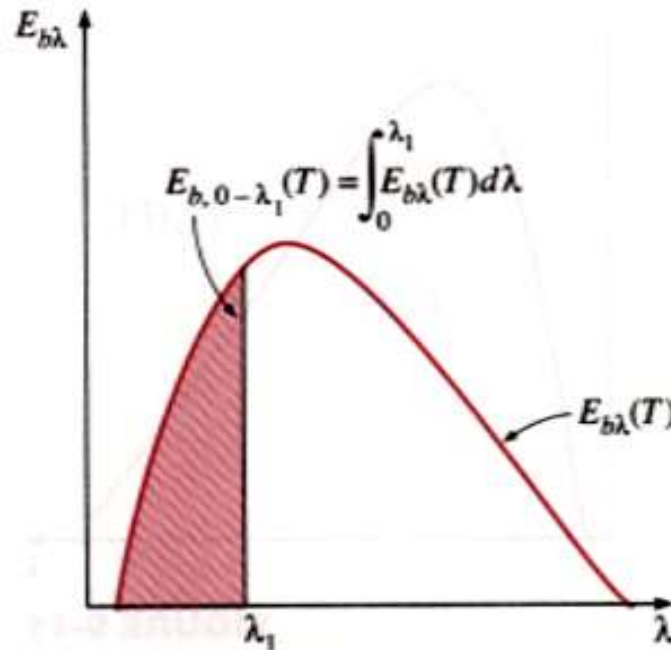
Wien's Displacement Law

- Wavelength of radiation with the largest magnitude can be found for different temps using Wien's Displacement Law:
- Note that color is a function of absorption & reflection, not emission

$$(\lambda T)_{\max \text{ power}} = 2897.8 \mu\text{m} \cdot \text{K}$$

Blackbody Radiation Function

- We often are interested in radiation energy emitted over a certain wavelength interval.



- This is a

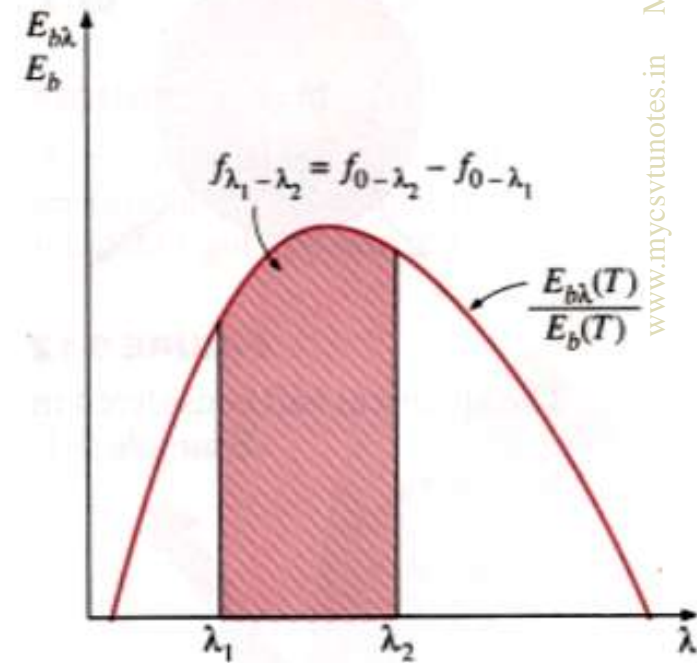
Blackbody Radiation Function

- Use blackbody radiation function, F_λ (often called the fractional function)

$$F_\lambda(T) = \frac{\int_0^\lambda E_{b\lambda}(T) d\lambda}{\sigma T^4}$$

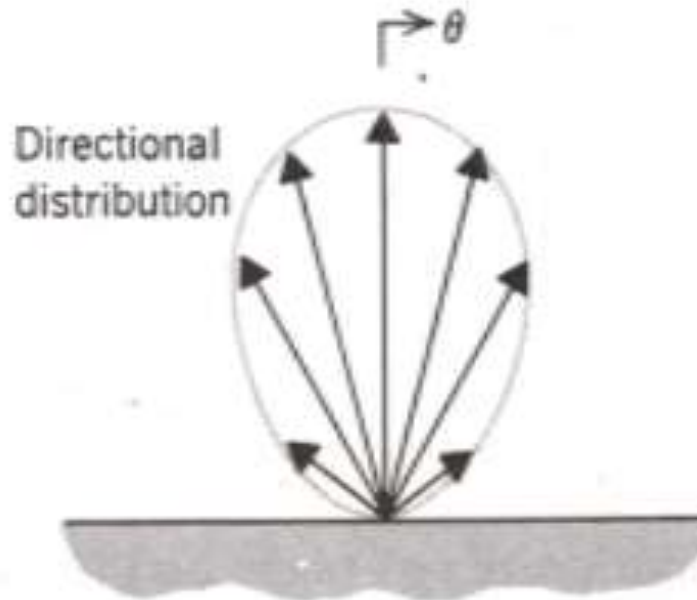
- If we want radiation between λ_1 & λ_2 ,

$$F_{\lambda_1-\lambda_2}(T) = F_{\lambda_2}(T) - F_{\lambda_1}(T)$$



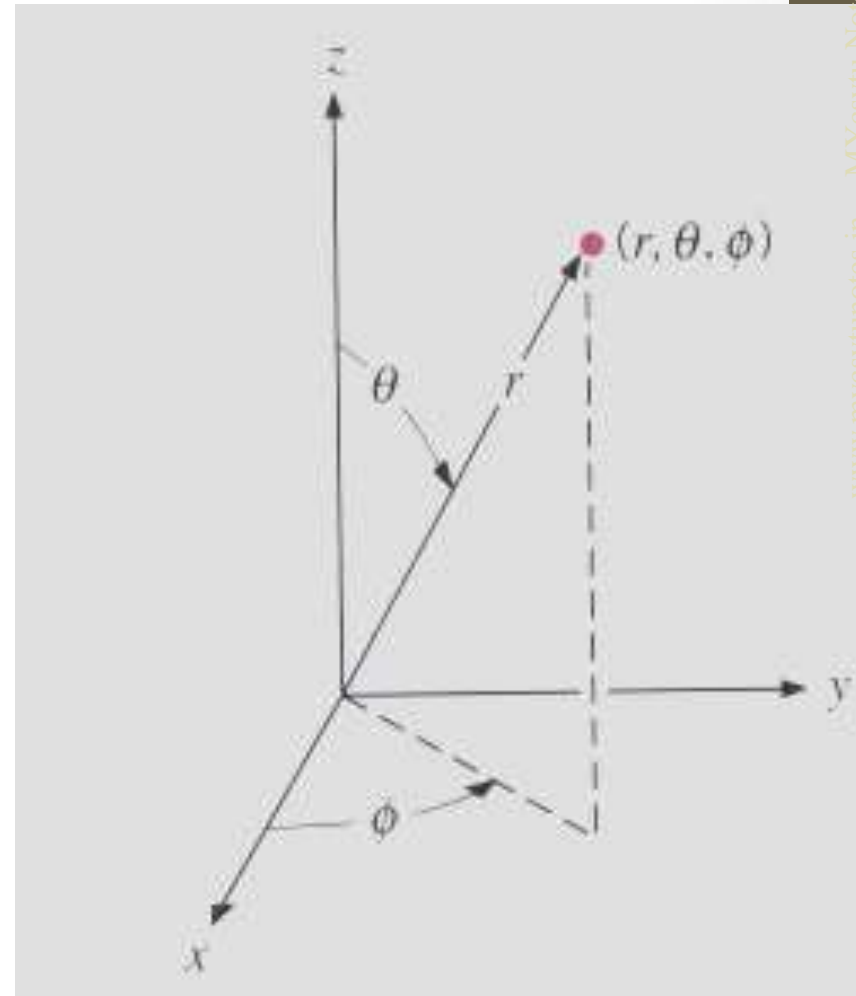
More Radiation Properties

- Directional distribution – a surface doesn't emit the same in all directions.
- Hemispherical – refers to all directions



Angle definitions

- zenith angle θ (up and down)
- azimuthal angle ϕ (side to side)



Solid Angle

- differential solid angle $d\omega = dA_n / r^2 = dA_1 \cos\theta / r^2 = \sin\theta d\theta d\phi$
- A solid angle is for a sphere what an angle is for a circle

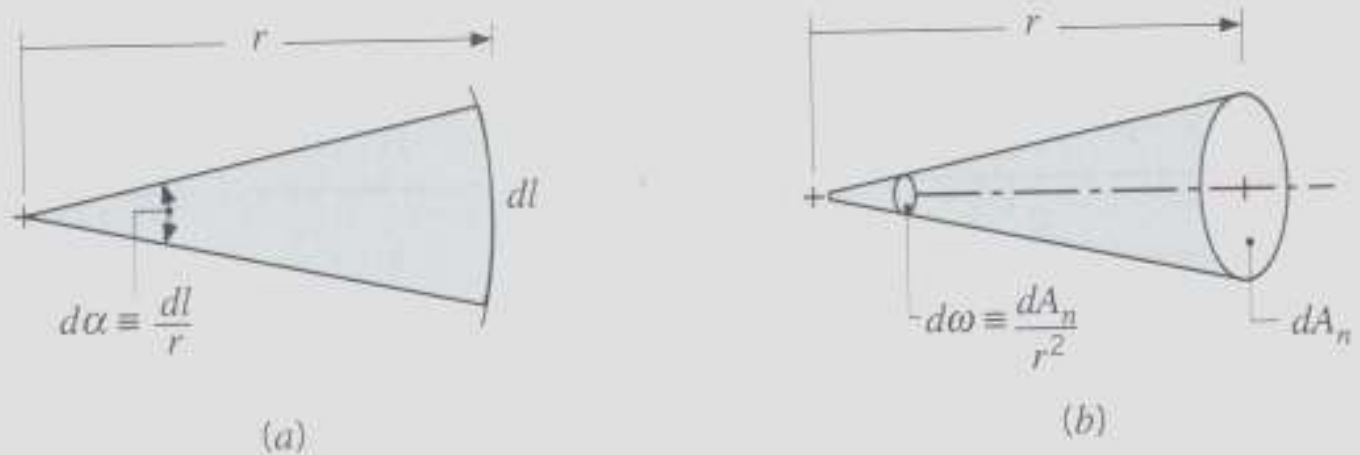
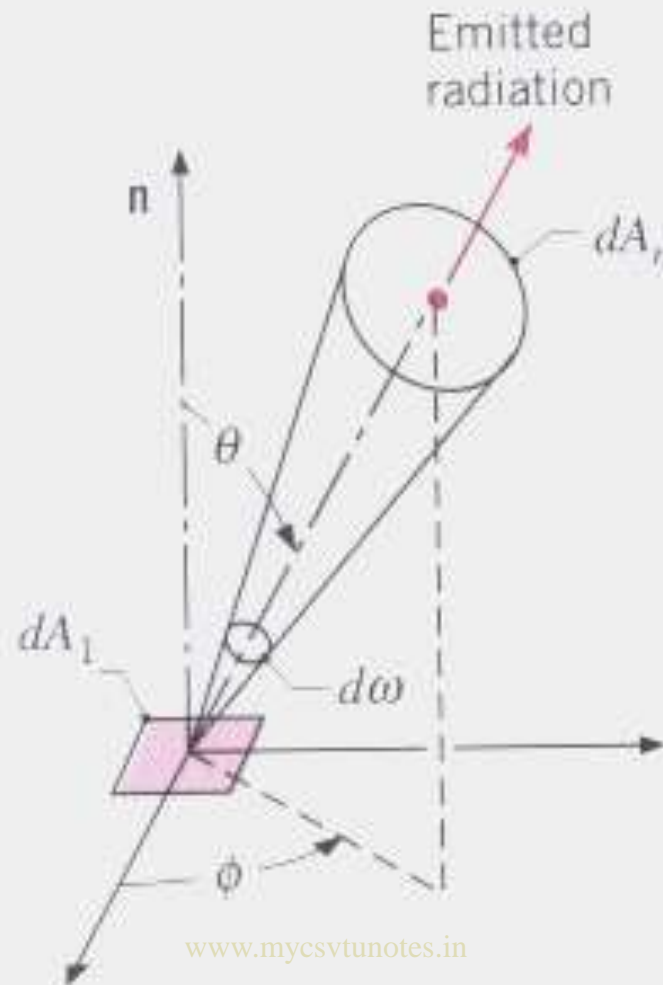


FIGURE 12.6 Definition of (a) plane and (b) solid angles.

Solid Angle, cont.

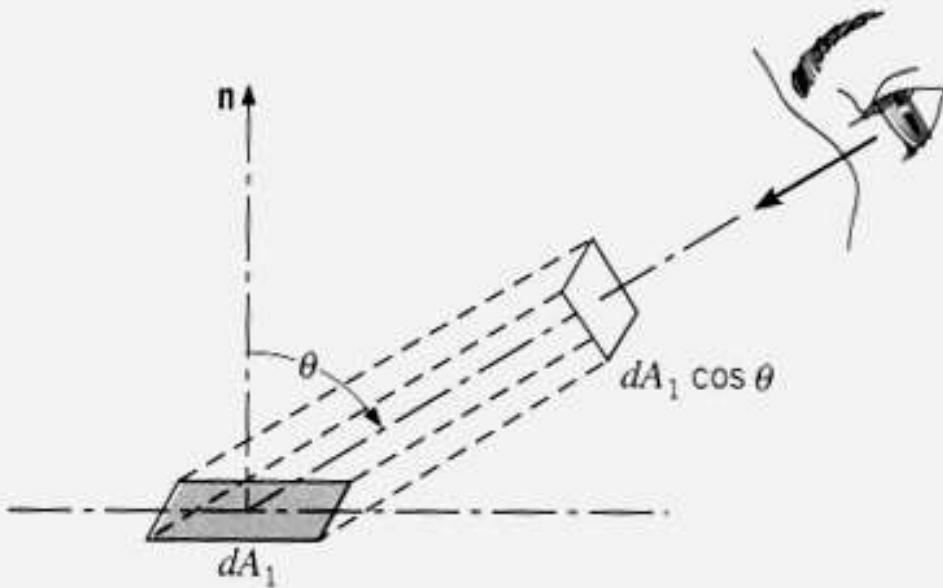


Spectral Intensity

- $I_{\lambda,e}$: rate at which radiant energy is emitted at the wavelength λ in the (θ, ϕ) direction, per unit area of the emitting surface normal to this direction, per unit solid angle about this direction, and per unit wavelength interval $d\lambda$ about λ
- Translation: rate of emission at a given wavelength, in a given direction, per unit area, solid angle, and wavelength interval
- Units: $W/(m^2 \text{ sr } \mu\text{m})$

Spectral Intensity

$$I_{\lambda,e}(\lambda, \theta, \phi) = \frac{dq}{dA_n \cdot d\omega \cdot d\lambda}$$



$$dA_n = dA_1 \cos \theta$$

FIGURE 12.8

The projection of dA_1 normal to the direction of radiation.

To solve for dq

$$d\dot{Q} = I_{\lambda,e}(\lambda, \theta, \phi) dA_1 \cos \theta \cdot d\varpi \cdot d\lambda$$

- Sometimes we know the intensity I_e rather than the spectral intensity (i.e., the rate of emission in a given direction, per unit area and solid angle). This is $I_{\lambda,e}$ integrated over all wavelengths. Then

$$d\dot{Q} = I_e(\theta, \phi) dA_1 \cos \theta \cdot d\varpi$$

Total heat flux

- To find the total heat flux, we must integrate over both angles and wavelength.

$$\dot{q}_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta d\theta d\phi \quad \text{spectral heat flux}$$

$$\dot{q} = \int_0^{\infty} \dot{q}_\lambda(\lambda) d\lambda \quad \text{total heat flux}$$

$$\text{OR} \quad \dot{q} = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta d\theta d\phi d\lambda \quad \text{total heat flux}$$

Emissive Power

- E: amount of radiation emitted per unit area
- Spectral hemispherical emissive power E_λ (often leave out the word “hemispherical”) $\text{W/m}^2\lambda$
 - Rate of emission per unit area of radiation of a given wavelength λ in all directions per unit wavelength interval
- Total (hemispherical) emissive power E (W/m^2)
 - Rate of emission per unit ~~area~~ ^{*of*} _{*emitted*} of radiation of all wavelengths and in all directions; this is

Diffuse emitters

- Diffuse emitter: intensity is the same in all directions; integration is easy!

$$E_{\lambda}(\lambda) = \pi I_{\lambda,e}(\lambda) \quad \text{and} \quad E = \pi I_e$$

$$\text{where} \quad I_e = \int_0^{\infty} I_{\lambda,e} d\lambda$$

Irradiation, G

- Irradiation: radiation incident on (hitting) a surface per unit area
- Use subscript i
- Same equations as E except replace E with G and $I_{\lambda,e}$ with $I_{\lambda,i}$ (intensity of incident radiation)

Radiosity, J

- Radiosity: all radiation leaving a surface per unit area, both emitted and reflected
- Use subscript e+r
- Same equations as E except replace E with J and $I_{\lambda,e}$ with $I_{\lambda,e+r}$ (intensity of emitted+reflected radiation)

Example 1

A small surface of area $A_1=3 \text{ cm}^2$ diffusely emits radiation with an emissive power of 7350 W/m^2 . Part of the radiation strikes another small surface of area $A_2=5 \text{ cm}^2$ oriented as shown on the board.

- a) Find the intensity of the emitted radiation.
- b) Find the solid angle subtended by A_2 when viewed from A_1 .
- c) Find the rate of radiation emitted by A_1 that strikes A_2 .

3 Introduction to Mass Transfer

Overview

- **Thermodynamics**
- **heat and mass transfer**
- **chemical reaction rate theory(chemical kinetics)**

Rudiments of Mass Transfer

- Open a bottle of perfume in the center of a room---mass transfer
- Molecular processes(e.g., collisions in an ideal gas)
- turbulent processes.

Mass Transfer Rate Laws

- Fick's law of Diffusion
- one dimension:

$$\dot{m}''_A = Y_A (\dot{m}''_A + \dot{m}''_B) - \rho D_{AB} \frac{dY_A}{dx}$$

Mass flow of species A per unit area

Mass flow of species A associated with bulk flow per unit area

Mass flow of species A associated with molecular diffusion per unit area

- The mass flux is defined as the mass flowrate of species A per unit area perpendicular to the flow:

- The units are kg/s-m^2
- D_{AB} is a property of the mixture and has units of m^2/s , the binary diffusivity.

$$\dot{m}_A = \dot{m}_A / A$$

- ***It means that species A is transported by two means: the first term on the right-hand-side representing the transported of A resulting from the bulk motion of the fluid, and the second term representing the diffusion of A superimposed on the bulk flow.***

- In the absence of diffusion, we obtain the obvious result that
- where \dot{m}'' is the mixture mass flux. The diffusion flux adds an additional component to the flux of species A

$$\dot{m}'' = Y_A (\dot{m}_A'' + \dot{m}_B'') = Y_A \dot{m}'' \equiv \text{Bulk flux of species A}$$

$$- \rho D_{AB} \frac{dY_A}{dx} \equiv \text{Diffusional flux of species A, } \dot{m}_{A,\text{diff}}''$$

- An analogy between the diffusion of mass and the diffusion of heat (conduction) can be drawn by comparing Fourier's law of conduction:

$$\dot{Q}_x'' = -k \frac{dT}{dx}$$

- The more general expression

$$\dot{m}_A'' = Y_A (\dot{m}_A'' + \dot{m}_B'') - \rho D_{AB} \nabla Y_A$$

- where the bold symbols represent vector quantities. In many instances, the molar form of the above equation is useful:

$$\dot{N}_A'' = x_A (\dot{N}_A'' + \dot{N}_B'') - \rho D_{AB} \nabla x_A$$

- Where N_A is the molar flux (kmol/s-m²) of species A, x_A is the mole fraction, and c is the mixture molar concentration (kmol_{mix}/m³)
- The meanings of bulk flow and diffusion flux become clearer if we express the total mass flux for a binary mixture as the sum of the mass flux of species A and the mass flux of species B:

$$\dot{m}'' = \dot{m}''_A + \dot{m}''_B$$

**Mixture
mass
flux**

**Species A
mass
flux**

**Species B
mass
flux**

- For one dimension:

- or
$$\dot{m}'' = Y_A \dot{m}'' - \rho D_{AB} \frac{dY_A}{dx} + Y_B \dot{m}'' - \rho D_{BA} \frac{dY_B}{dx}$$

$$\dot{m}'' = (Y_A + Y_B) \dot{m}'' - \rho D_{AB} \frac{dY_A}{dx} - \rho D_{BA} \frac{dY_B}{dx}$$

- For a binary mixture, $Y_A + Y_B = 1$, thus,

$$- \rho D_{AB} \frac{dY_A}{dx} - \rho D_{AB} \frac{dY_B}{dx} = 0$$

**Diffusional
flux of
species A**

**Diffusional
flux of
species B**

- **In general, overall mass conservation required that:**

$$\sum \dot{m}_{i,diff}'' = 0$$

- This is called ordinary diffusion.
- **Not binary mixture;**
- **thermal diffusion**
- **pressure diffusion.**

Molecular basis of Diffusion

- Kinetic theory of gases: Consider a stationary (no bulk flow) plane layer of a binary gas mixture consisting of rigid, nonattracting molecules in which the molecular mass of each species A and B is essentially equal. A concentration (mass-fraction) gradient exists in the gas layer in the x -direction and is sufficiently small that the mass-fraction distribution can be considered linear over a distance of a few molecular mean free paths, λ , as illustrated in Fig 3.1

- **Average molecular properties derived from kinetic theory:**

$$\bar{v} \equiv \frac{\text{mean speed of species A molecules}}{\text{species A molecules}} = \left(\frac{8k_B T}{\pi m_A}\right)^{1/2}$$

$$Z_A'' \equiv \frac{\text{Wall collision frequency of A molecules per unit area}}{\text{of A molecules per unit area}} = \frac{1}{4} \left(\frac{n_A}{V}\right) \bar{v}$$

$$\lambda \equiv \text{Mean free path} = \frac{1}{\sqrt{2}\pi \left(\frac{n_{tot}}{V}\right) \sigma^2}$$

$$a \equiv \frac{\text{Average perpendicular distance from plane of last collision to plane where next collision occurs}}{\text{last collision to plane where next collision occurs}} = \frac{2}{3} \lambda$$

- Where k_B is Boltzmann's constant;
- m_A the mass of a single A molecular,
- n_A/V is the number of A molecular per unit volume,
- n_{tot}/V is the total number of molecules per unit volume
- σ is the diameter of both A and B molecules.

- Assuming no bulk flow for simplicity, the net flux of A molecules at the x-plane is the difference between the flux of A molecules in the positive x-direction and the flux of A molecules in the negative x-direction:
- which, when expressed in terms of the collision frequency, becomes

$$\dot{m}''_A = \dot{m}''_{A,(+)x-dir} - \dot{m}''_{A,(-)x-dir}$$

$$\dot{m}_A'' = \left(Z_A'' \right)_{x-a} m_A - \left(Z_A'' \right)_{x+a} m_A$$

(Net mass flux of species A) = (Number of A molecules crossing plane at x originating from plane at x - a per unit time and area) (Mass of single molecule) - (Number of A molecules crossing plane at x originating from plane at x + a per unit time and area) (Mass of single molecule)

- We can use the definition of density
- ($\rho \equiv m_{\text{tot}}/V_{\text{tot}}$) to relate Z_A'' to the mass fraction of A molecules:

$$Z_A'' m_A = \frac{1}{4} \frac{n_A m_A}{m_{\text{tot}}} \rho \bar{v} = \frac{1}{4} Y_A \rho \bar{v}$$

- Substituting the above Equation into the early one, and treating the mixture density and mean molecular speeds as constants yields

$$\dot{m}''_A = \frac{1}{4} \rho \bar{v} (Y_{A,x-a} - Y_{A,x+a})$$

- With our assumption of a linear concentration distribution

- Solving the above equation for the concentration difference and substituting into equation 3.14, we obtain our final result:

$$\frac{dY_A}{dx} = \frac{Y_{A,x+a} - Y_{A,x-a}}{2a} = \frac{4\lambda}{3}$$

$$\dot{m}''_A = -\rho \frac{\bar{v} \lambda}{3} \frac{dY_A}{dx}$$

Comparing the above equation with the first equation, we define the binary diffusivity D_{AB} as

$$D_{AB} = \frac{\bar{v} \lambda}{3}$$

- Using the definitions of the mean molecular speed and mean free path, together with the ideal-gas equation of state $PV=nk_B T$, the temperature and pressure dependence of D_{AB} can easily be determined

- or

$$D_{AB} = \frac{2}{3} \left(\frac{k_B^3 T}{\pi^3 m_A} \right)^{1/2} \frac{T}{\sigma^2 P}$$

$$D_{AB} \propto T^{3/2} P^{-1}$$

- Thus, we see that the diffusivity depends strongly on temperature (to the 3/2 power) and inversely with pressure. The mass flux of species A, however, depends on the product ρD_{AB} , which has a square-root temperature dependence and is independent of pressure:

- In many simplified analyses of combustion processes, the weak temperature dependence is neglected and ρD is treated as a

constant

$$\rho D_{AB} \propto T^{1/2} P^0 = T^{1/2}$$

Comparison with Heat Conduction

- To see clearly the relationship between mass and heat transfer, we now apply kinetic theory to the transport of energy. We assume a homogeneous gas consisting of rigid nonattracting molecules in which a temperature gradient exists. Again, the gradient is sufficiently small that the temperature distribution is essentially linear over several mean free paths, as illustrated in Fig. 3.2.

- The mean molecular speed and mean free path have the same definitions as given in Eqns. 3.10a and 3.10c, respectively; however, the molecular collision frequency of interest is now based on the total number density of molecules, n_{tot}/V , i.e.,

$$Z'' \equiv \frac{\text{Average wall collision frequency per unit area}}{\text{frequency per unit area}} = \frac{1}{4} \left(\frac{n_{tot}}{V} \right) \bar{v}$$

- In our no-interaction-at-a-distance hard-sphere model of the gas, the only energy storage mode is molecular translational i.e., kinetic, energy. We write an energy balance at the x -direction is the difference between the kinetic energy flux associated with molecules traveling from $x-a$ to x and those traveling from $x+a$ to x

$$\dot{Q}_x'' = Z''(ke)_{x-a} - Z''(ke)_{x+a}$$

- Since the mean kinetic energy of a molecule is given by
- the heat flux can be related to the temperature as

$$ke = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$$

$$\dot{Q}'' = \frac{3}{2} k_B Z'' (T_{x-a} - T_{x+a})$$

- The temperature difference in Eqn 3.22 relates to the temperature gradient following the same form as Eqn. 3.15 i.e.,

- Substituting difference in Eqn. 3.22 employing the definition of Z'' and a , we obtain our final result for the heat flux:

$$\dot{Q}_x'' = -\frac{1}{2} k_B \left(\frac{n}{V}\right) \bar{v} \lambda \frac{dT}{dx}$$

- Comparing the above with Fourier's law of heat conduction (Eqn. 3.4), we can identify the thermal conductivity k as

- Expressed in terms of T and molecular mass and size, the thermal conductivity is $k = \frac{1}{2} k_B \left(\frac{n}{V} \right) \bar{v} \lambda$

$$k = \left(\frac{k_B^3}{\pi^3 m \sigma^4} \right)^{1/2} T^{1/2}$$

- The thermal conductivity is thus proportional to the square-root of temperature,

- as is the ρD_{AB} product. For real gases, the true temperature dependence is greater.

$$k \propto T^{1/2}$$

Species Conservation

- Consider the one-dimensional control volume of Fig. 3.3, a plane layer Δx thick.

- The net rate of increase in the mass of A within the control volume relates to the mass fluxes and reaction rate as follows:

$$\frac{dm_{A,cv}}{dt} = [\dot{m}''_A A]_x - [\dot{m}''_A A]_{x+\Delta x} + \dot{m}'''_A V$$

Rates of increase of mass of A within control volume

Mass flow of A into the control volume

Mass flow of A out of the control volume

Mass production rate of species A by chemical reaction

- \dot{m}_A is the mass production rate of species A per unit volume ($\text{kg}/\text{m}^3\text{-s}$). In Chapter 5, we specifically deal with how to determine

. Recognizing that the mass of A within the control volume is $m_{A,cv} = Y_A m_{cv} = Y_A \rho V_{cv}$ and that the volume $V_{cv} = A \Delta x$, Eqn. 3.28 can be written:

$$\dot{m}_A$$

$$A\Delta x \frac{\partial(\rho Y_A)}{\partial t} = A[Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}]_x$$

- Dividing through by $A\Delta x$ and taking the limit as $\Delta x \rightarrow 0$, Eqn. 3.29 becomes

$$-\frac{\partial}{\partial x} [Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}]_{x+\Delta x} + \dot{m}''_A A\Delta x$$

$$\frac{\partial(\rho Y_A)}{\partial t} = -\frac{\partial}{\partial x} [Y_A \dot{m}'' - \rho D_{AB} \frac{\partial Y_A}{\partial x}] + \dot{m}''_A$$

- Or, for the case of steady flow where

$$\partial(\rho Y_A) / \partial t = 0$$

- Equation 3.31 is the steady-flow, one-dimensional form of species conservation for a binary gas mixture, assuming species diffusion occurs only as a result of concentration gradients; i.e., only ordinary diffusion is considered. For the multidimensional case, Eqn. 3.31 can be generalized as

$$\dot{m}_A''' - \frac{d}{dx} \left[Y_A \dot{m}'' - \rho D_{AB} \frac{dY_A}{dx} \right] = 0$$

$$\dot{m}_A''' - \nabla \cdot \dot{m}_A''' =$$

**Net rate of
production of
species A by
chemical
reaction, per
unit volume**

**Net flow of
species A
out of
control
volume, per
unit volume**

Some application

- **The stefan Problem:**
- Consider liquid A, maintained at a fixed height in a glass cylinder as illustrated in Fig. 3.4.

- Mathematically, the overall conservation of mass for this system can be expressed as

- Since $\dot{m}''(x) = 0$, then $\dot{m}''(x) = \text{constant} = \dot{m}''_A + \dot{m}''_B$

$$\dot{m}''_B$$

$$\dot{m}''_A = \dot{m}''(x) = \text{constant}$$

- Equation 3.1 now becomes:

- Rearranging and separating variables, we obtain

$$\dot{m}_A'' = Y_A \dot{m}_A'' - \rho D_{AB} \frac{dY_A}{dx}$$

$$-\frac{\dot{m}_A''}{\rho D_{AB}} dx = \frac{dY_A}{1 - Y_A}$$

- Assuming the product ρD_{AB} to be constant, Eqn. 3.36 can be integrated to yield

- where C is the constant of integration. With the boundary condition:
$$\frac{\dot{m}''_A}{\rho D_{AB}} x = -\ln[1 - Y_A] + C$$

$$Y_A(x = 0) = Y_{A,i}$$

- We eliminate C and obtain the following mass-fraction distribution after removing the logarithm by exponentiation:

$$Y_A(x) = 1 - (1 - Y_{A,i}) \exp\left[\frac{\dot{m}''_A x}{\rho D_{AB}}\right]$$

- The mass flux of A, \dot{m}''_A , can be found by letting $Y_A(x=L)=Y_{A,\infty}$ in Eqn. 3.39. Thus,

$$\dot{m}''_A$$

$$\dot{m}''_A = \frac{\rho D_{AB}}{L} \ln \left[\frac{1 - Y_{A,\infty}}{1 - Y_{A,i}} \right]$$

- From the above equation, we see that the mass flux is directly proportional to the product of the density and the mass diffusivity and inversely proportional to the length, L . Larger diffusivities thus produce larger mass fluxes.
- To see the effects of the concentrations at the interface and at the top of the varying $Y_{A,i}$, the interface mass fraction, from zero to unity.

- Physically, this could correspond to an experiment in which dry nitrogen is blown across the tube outlet and the interface mass fraction is controlled by the partial pressure of the liquid, which in turn, is varied by changing the temperature. Table 3.1 shows that at small values of $Y_{A,i}$ the dimensionless mass flux is essentially proportional to $Y_{A,i}$. For $Y_{A,i}$ greater than about 0.5, the mass flux increases very rapidly.

Table 3.1 Effect of interface mass fraction on mass flux

| $Y_{A,i}$ | $\dot{m}_A'' / (\rho D_{AB} / L)$ |
|-----------|-----------------------------------|
| 0 | 0 |
| 0.05 | 0.0513 |
| 0.10 | 0.1054 |
| 0.20 | 0.2231 |
| 0.50 | 0.6931 |
| 0.90 | 2.303 |
| 0.99 | 6.908 |

Liquid-Vapor Interface Boundary Conditions