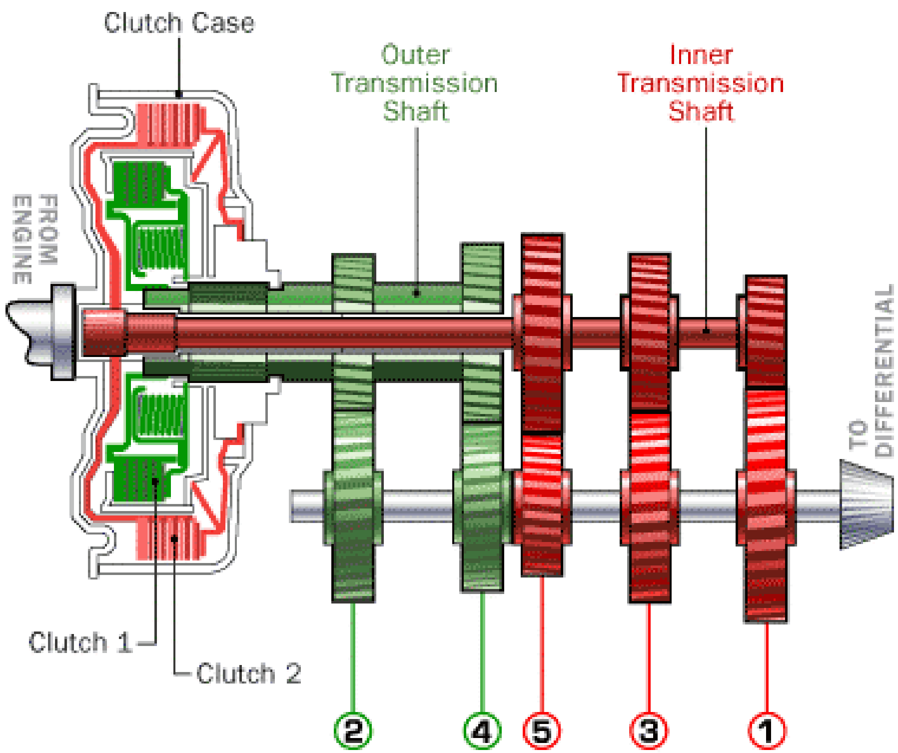
Transmission shaft

- A rotating machine element circular in cross section that supports transmission elements like gear, pulley and sprockets and transmit power.
- Shafts are made of medium carbon steel with a carbon content from 0.15 to 0.4 %.



Some specific shafts are:-

1. **Axle:** The term axle is used for a shaft that supports rotating elements like wheel, hoisting drum or rope sheave & that is fitted to the housing by means of bearings. e.g. rear wheel of railway wagon.
2. **Spindle:** A spindle is a short rotating shaft on which thread is twisted to attach other elements like bearing, gear etc.

3. **Counter shaft:** It is a secondary shaft which is driven by the main shaft and from which the power is supplied to the machine component.
4. **Line shaft:** A line shaft consists of a number of shafts which are connected in axial direction by means of couplings.

1. Shaft design on strength basis

- Transmission shaft are generally subjected to
  - Axial tensile force
  - Bending moment
  - Torsional moment
  - Or combination of these forces.

- Mostly shaft are subjected to bending and torsional moment.

Cont…

Case I. When the shaft is subjected to axial tensile force:

The tensile stress

$$\sigma_t = \frac{P}{\left(\pi/4\, d^2\right)} = \frac{4P}{\pi d^2}$$

Case II. When the shaft is subjected to pure bending moment:

The bending stress

$$\sigma_b = \frac{32M_b}{\pi d^3}$$

Case III. When the shaft is subjected to pure torsional moment:

Torsional shear stress

$$\tau = \frac{16M_t}{\pi d^3}$$

Shaft subjected to combination of loads

Case I. The shaft is subjected to combination of axial, bending and torsional moment-

$$\sigma_x = \sigma_t + \sigma_b$$

Case II. The shaft is subjected to combination of bending and torsional moment (without any axial force)-

$$\sigma_x = \sigma_b$$

Principal stress

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

Principal shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

Failure theories for designing shaft (applied to case II)

1. Maximum principal stress theory:

$$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \ldots\ldots\ldots\ldots(1)$$

$$\sigma_1 = \frac{S_{yt}}{fs} \ldots\ldots\ldots\ldots\ldots(2)$$

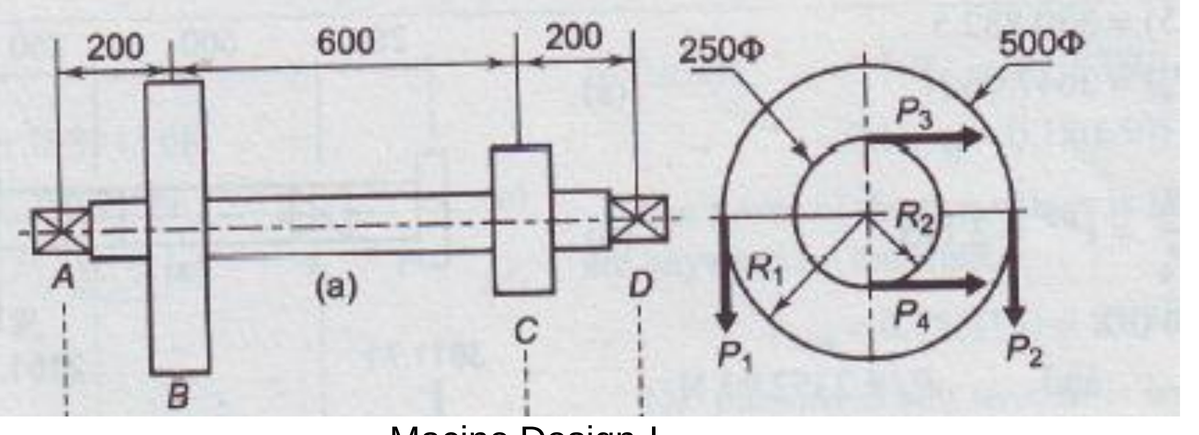
2. Maximum shear stress theory:

$$\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{(M_b)^2 + (M_t)^2}\right] \ldots\ldots\ldots\ldots(1)$$

$$\tau_{max} = \frac{S_{sy}}{fs} = \frac{0.5 S_{yt}}{fs} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

(most suitable for shaft design)

Problem 1. The layout of a transmission shaft carrying two pulleys B and C and supported on bearings A and D is shown in fig. Power is supplied to the shaft by means of a vertical belt on pulley B, that is then transmitted to pulley C carrying a horizontal belt. The maximum tension in belt on pulley B is 2.5 kN. The angle of wrap for both the pulley is 180^0 and the coefficient of friction 0.24. The shaft is made of plain carbon steel 30C8 (S_{yt}=400 N/mm^2) and the factor of safety is 3. Determine the shaft diameter on strength basis.



Problem 2. A shaft is supported by two bearings placed 1m apart. 600 mm pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directed below it with the help of belt having maximum tension 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt which is placed horizontally to the right. The angle of contact is 180^0 and $\mu = 0.24$. Find the diameter of shaft if

$\sigma_t = 60$ Mpa and $\zeta = 40$ Mpa. **(2009)**

Equivalent bending moment & equivalent torsional moment

$$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right] \text{.............(1)}$$

$$\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{(M_b)^2 + (M_t)^2}\right] \text{.............(2)}$$

- $\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$ is called equivalent bending moment, and is defined as the bending moment which when acting alone will produce the same bending stress in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t).

- $\left[\sqrt{(M_b)^2 + (M_t)^2}\right]$ is called equivalent torsional moment and it is defined as torsional moment which when acting alone will produce the same torsional shear stress in the shaft as under the combined action of bending moment (M_b) and torsional moment (M_t).

ASME Code for shaft designing

According to this:-

1. $\zeta_{max} = 0.3\ S_{yt}$ or $0.18\ S_{ut}$ (whichever is minimum)

   If key ways are present on the shaft

$$\zeta_{max} = 0.75\ (\zeta_{max})_{above}$$

2. The bending and torsional moments are to be designed by factors k_b and k_t respectively to account for shock and fatigue in operating condition.
3. ASME Code is based on maximum shear stress theory of failure-

$$\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{(k_b M_b)^2 + (k_t M_t)^2}\right]$$

where k_b= combined shock and fatigue factor applied to bending moment.

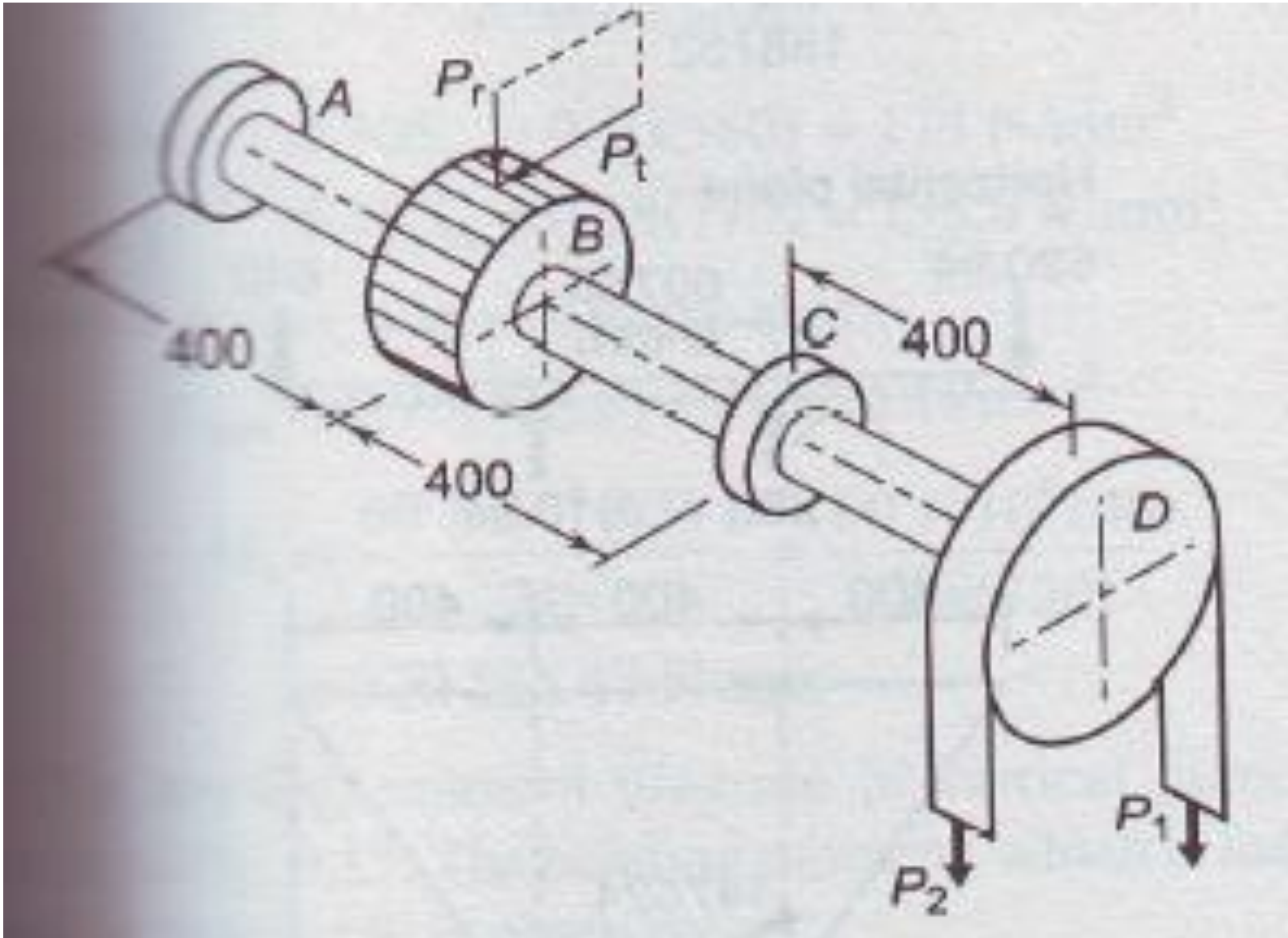
k_t= combined shock and fatigue factor applied to torsional moment.

Problem 3. A transmission shaft supporting a spur gear B and pulley D is shown in fig. The shaft is mounted two bearings A and C. The diameter of pulley and pitch circle diameter of gear are 450 and 300 mm respectively. The pulley transmits 20 kW power at 500 rpm to the gear. P_1 and P_2 are belt tensions in the tight and loose sides, while P_t and P_r are tangential and radial components of gear tooth force. Assume,

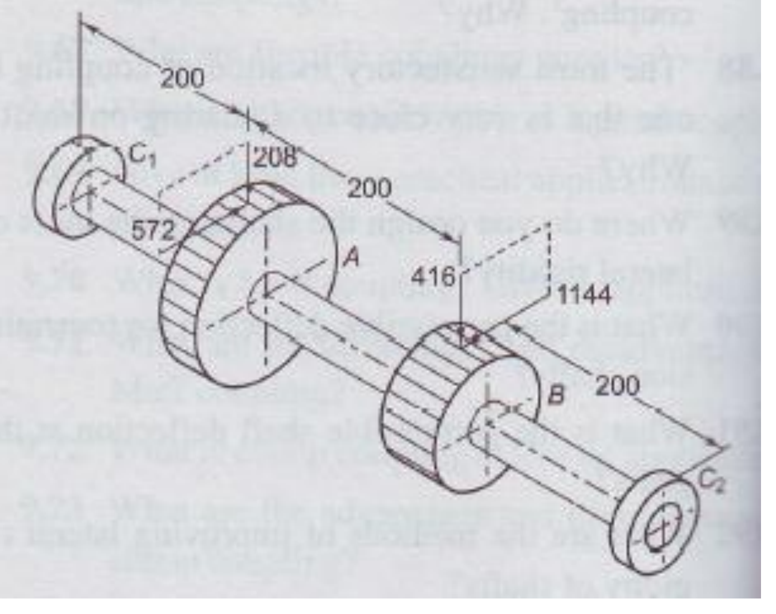
$$P_1 = 3P_2 \text{ and } P_r = P_t \tan(20^0)$$

The gear and pulley are keyed to the shaft. The material of the shaft is steel 50C4 (S_{ut}=700 and S_{yt}=460 N/mm^2). The factors k_b and k_t of ASME code are 1.5 each. Determine the shaft diameter using ASME code.

Cont…



Problem 4. An intermediate shaft of a gearbox, supporting two spur gears A and B and mounted between two bearings C_1 and C_2, is shown in fig. The pitch circle diameters of gears A and B are 500 and 250 mm respectively. The shaft is made of alloy steel 20MnCr5 (S_{ut}= 620 and S_{yt}= 480 N/mm^2). The factors k_b and k_t of the ASME code are 2 and 1.5 respectively the gears are keyed to the shaft. Determine the shaft diameter using the ASME code. (2008 supp.)



Shaft design on torsional rigidity basis

- A transmission shaft is said to be rigid on the basis of torsional rigidity, if it does not twist too much under the action of external torque.
- Angle of twist in radian

$$\theta_r = \frac{M_t l}{JG}$$

- Converting from radian to degree

$$\theta = \frac{180}{\pi} \times \frac{M_t l}{JG}$$

- For solid circular shaft

$$J = \frac{\pi d^4}{32}$$

$$\boxed{\theta = \frac{584 M_t l}{G d^4}}$$

Problem 5. Assume the data of transmission shaft given in problem 2. For this shaft, the permissible angle of twist is 3^0 per meter length. The modulus of rigidity for the shaft material is 79300 N/mm^2. Calculate:

(i) The permissible angle of twist, and

(ii) The shaft diameter on the basis of torsional rigidity.

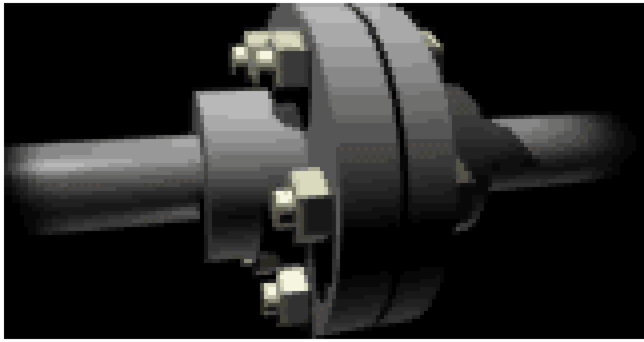
(Ans 2.4^0, 31.12mm)

Problem 6. Assume the data of the intermediate shaft illustrated in problem 3. The permissible angle of twist is 0.25^0 per meter length and the modulus of rigidity is 79300 N/mm^2. Determine the shaft diameter on the basis of torsional rigidity. (Ans= 45.3mm)

Coupling

- Shafts are usually available up to 7 m length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

- A Coupling can be defined as a mechanical device that permanently joins two rotating shafts to each other.



- **Uses:-**

➢ To Provide connections of shafts

➢ To provide mechanical flexibility for misalignment of shaft

➢ To reduce transmission of shock loads from one shaft to another shaft.

➢ To introduce protection against overloads.

Types of coupling

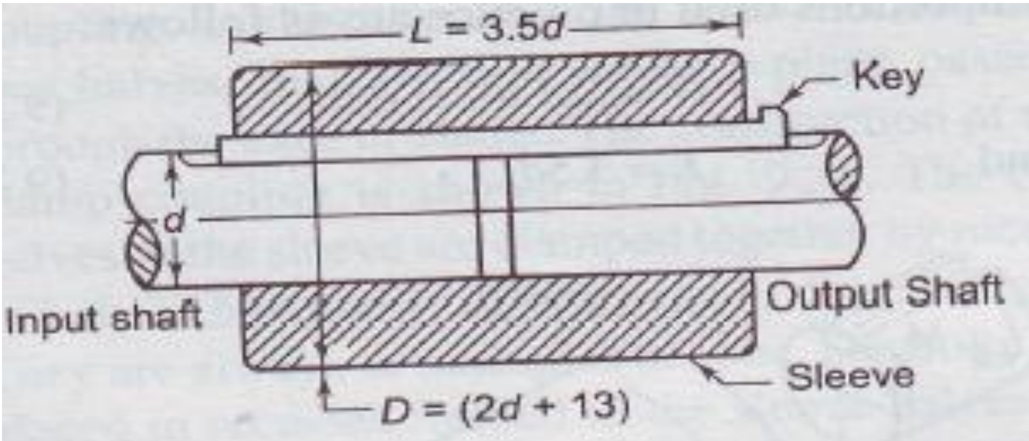
1. Rigid coupling- to connect two shafts which are perfectly aligned.

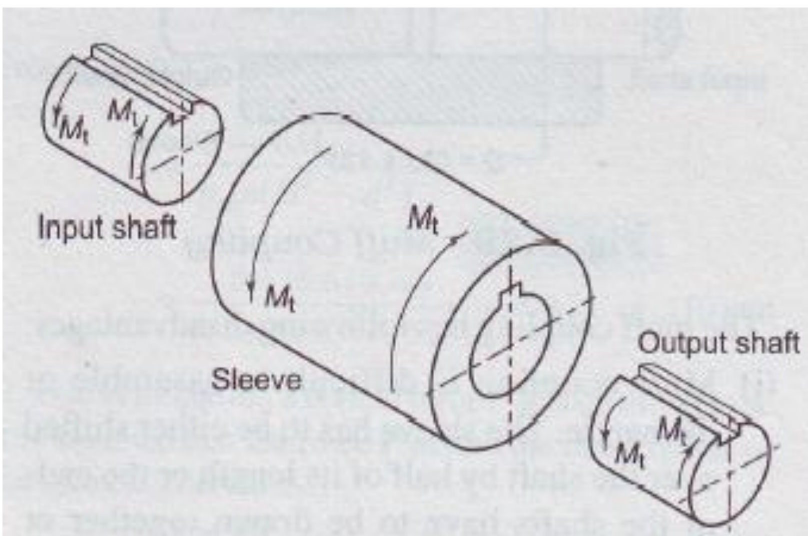
(a) Sleeve or muff coupling
(b) Clamp or compressive coupling
(c) Flange coupling

2. Flexible coupling- to connect two shafts having both lateral and angular misalignment.

(a) Bushed pin coupling
(b) Universal coupling
(c) Odham coupling

1. Sleeve or Muff coupling





$$D = (2d + 13)\ \text{mm}$$

$$L = 3.5d$$

Design of sleeve or muff coupling

1. Design for shaft: find diameter of the shaft using equation

$$M_t = \frac{60 \times P}{2\pi n} \; and \;\; \tau = \frac{16 M_t}{\pi d^3}$$

2. Design for sleeve:

- Find outer dia. of sleeve using relation D = 2d +13 mm
- Check induced shear stress in the sleeve using equation

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

Where T= Torque to be transmitted by the coupling

τ_c = Permissible shear stress for the material of the sleeve which is cast iron

Cont…

3. Design for key:

- <u>For Parallel (rectangular and square sunk) keys</u>

- Width (b) and height (h) of the key:- Refer Data book page no. 5.16 and select width and height of the key for given shaft diameter (d).
- Length of the coupling key = 3.5 d

  length of the key in each shaft ℓ = L/2 = 3.5d/2

- After fixing dimensions induced shearing and crushing stress may be checked by following equations:-

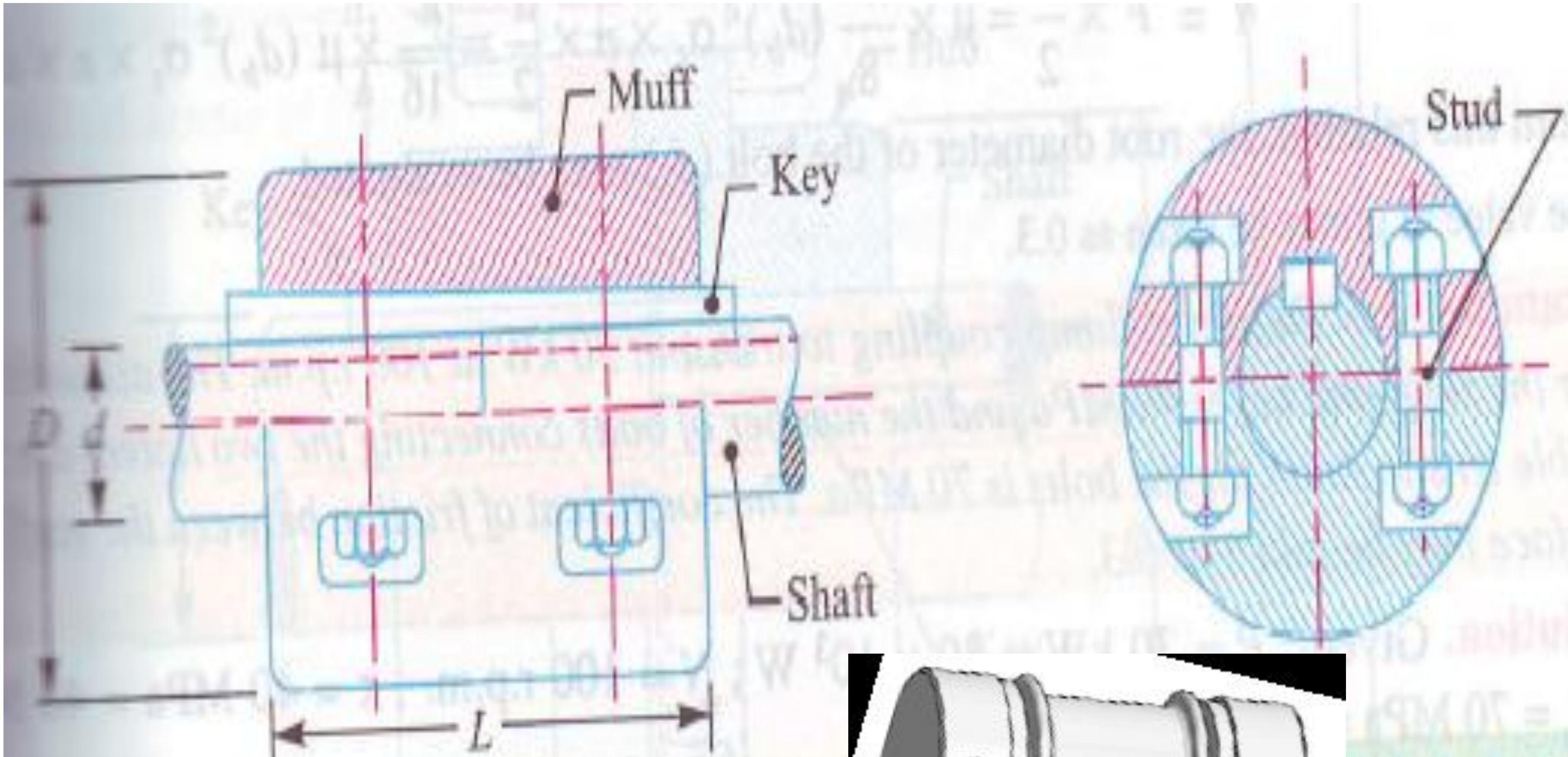
$$\tau = \frac{2M_t}{dbl}$$ (considering shear of key)

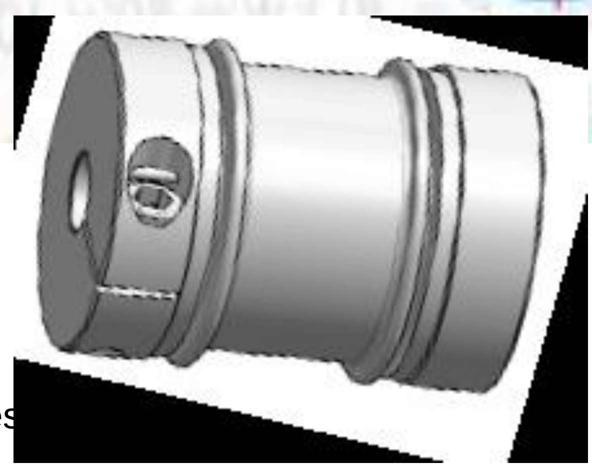
$$\sigma_c = \frac{4M_t}{dhl}$$ (considering crushing of key)

Problem 1. Design and make a neat dimensioned sketch of a muff coupling, which is used to connect two steel shafts transmitting 40 kW at 350 rpm. The material for the shaft and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed 15 MPa.

Problem 2. Design a muff coupling which is used to connect two steel shafts transmitting 25 kW power at 360 rpm. The shafts and key are made of plain carbon steel 30C8 (S_{yt}=S_{yc}= 400N/mm^2). The sleeve is made of grey cast iron FG 200 (S_{ut}= 200 N/mm^2). The factor of safety for the shafts and key is 4. For sleeve the factor of safety is 6 based on ultimate strength.

2. Clamp or compression coupling (split muff coupling)





Dimensions for clamp coupling

- For sleeve halves,

  Outer diameter of the sleeve D= 2.5 d,

  Length of the sleeve L= 3.5 d

- **<u>For clamping bolts,</u>**

<u>Diameter of the claming bolt</u> may be find out either

1. By using empirical equations:

   d_1= 0.2 d + 10 mm (when d< 55 mm)

And d_1= 0.15 d + 15 mm (when d> 55 mm)

or

2. By using following equations: (assuming that power is transmitted by friction)

$$P_1 = \frac{2M_t}{fdn} \quad and \quad P_1 = \frac{\pi}{4} d_1^{\,2} \sigma_t$$

Where P_1 = Tensile force on each bolt

d_1 = core diameter of clamping bolt

n = total no. of bolts & f= coefficient of friction

Design Procedure

Step 1. Calculate the diameter of each shaft.

Step 2. Calculate the main dimensions of the sleeve halves.

Step 3. Determine the standard cross-section of flat key from data book page no. 5.16.

Length of the key in each shaft is one half of the length of the sleeve.

with these dimensions of the key, check the shear stress and compressive stress by using,

$$\tau = \frac{2M_t}{dbl} \quad \text{and} \quad \sigma_c = \frac{4M_t}{dhl}$$

Step 4. calculate the diameter of clamping bolts by using

$$P_1 = \frac{2M_t}{fdn} \quad and \quad P_1 = \frac{\pi}{4} d_1^{\,2} \sigma_t$$

Problem 3. It is required to design a split muff coupling to transmit 50 kW power at 120 rpm. The shafts, key and coupling bolts are made of plain carbon steel 30C8 (S_{yt}= 400 N/mm^2). The yield strength in compression is 150% of tensile yield strength. The factor of safety for shafts key and bolts is 5. The number of clamping bolts is 8. The coefficient of friction between sleeve halves and the shaft is 0.3.

(i) Calculate the diameter of input and output shaft.

(ii) Specify length and outer diameter of sleeve halves.

(iii) Find out the diameter of clamping bolts assuming that the power is transmitted by friction.

(iv) Specify bolt diameter using standard empirical relations.

(v) Specify the size of key and check the dimensions for shear and compression criteria.

Problem 4. design a clamp coupling to transmit 30 kW at 100 rpm. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

3. Rigid flange coupling

(A) Unprotected type flange coupling

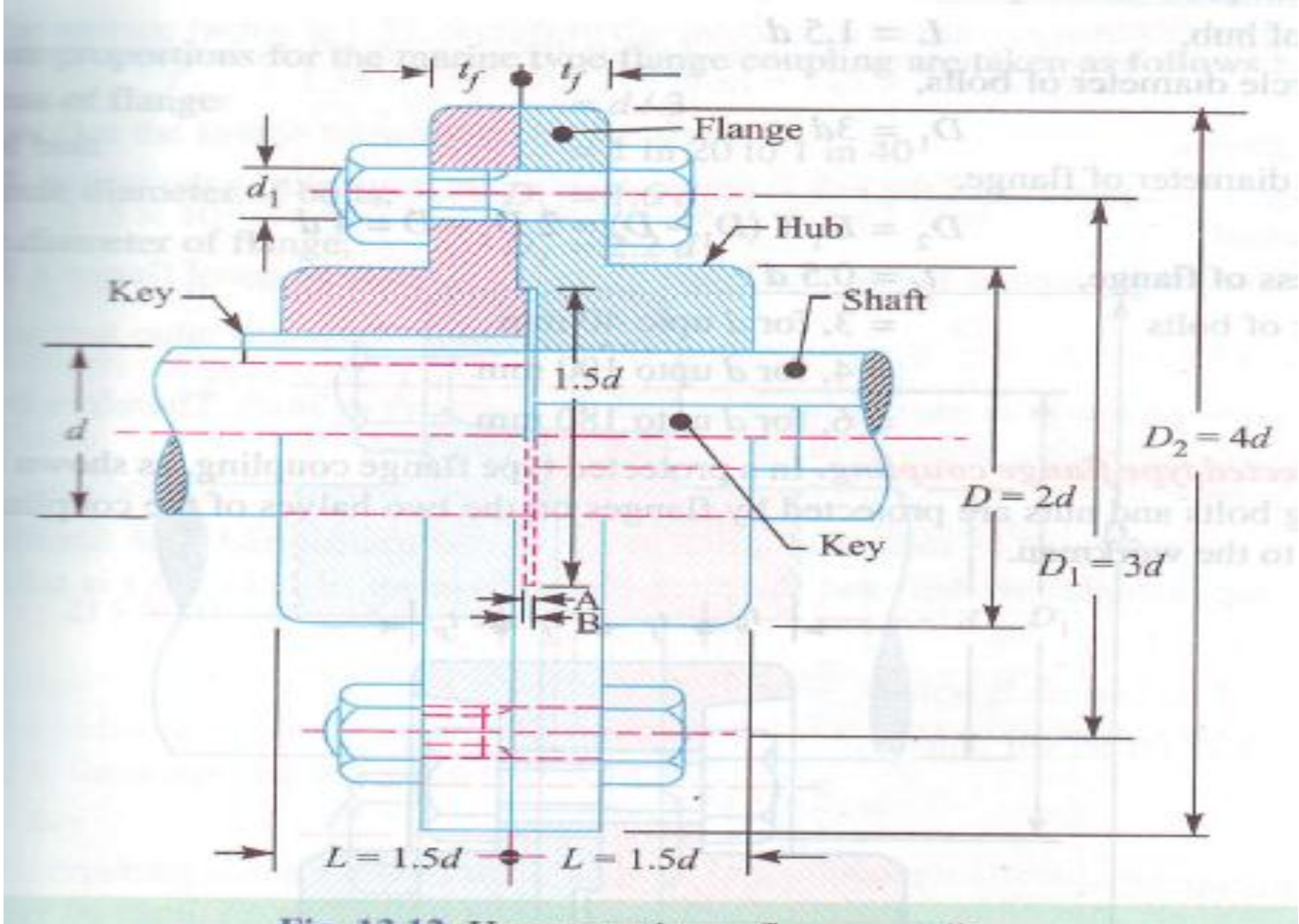


Fig. 13.12. Unprotected type flange coupling.

Dimensions for unprotected type flange coupling

- Outside diameter of hub D= 2d
- Length of hub L= 1.5 d
- Pitch circle diameter of bolts D_1= 3d
- Outside diameter of flange $D_2 = D_1 + (D_1 - D) = 2D_1 - D = 4d$
- Thickness of flange t_f= 0.5 d
- Number of bolts = 3, for d upto 40 mm

  = 4, for d upto 100 mm

  = 6, for d upto 180 mm

(A) Protected type flange coupling

- Thickness of the Circumferential flange $(t_p) = 0.25\ d$

- Other dimensions are the same as for unprotected type flange coupling.

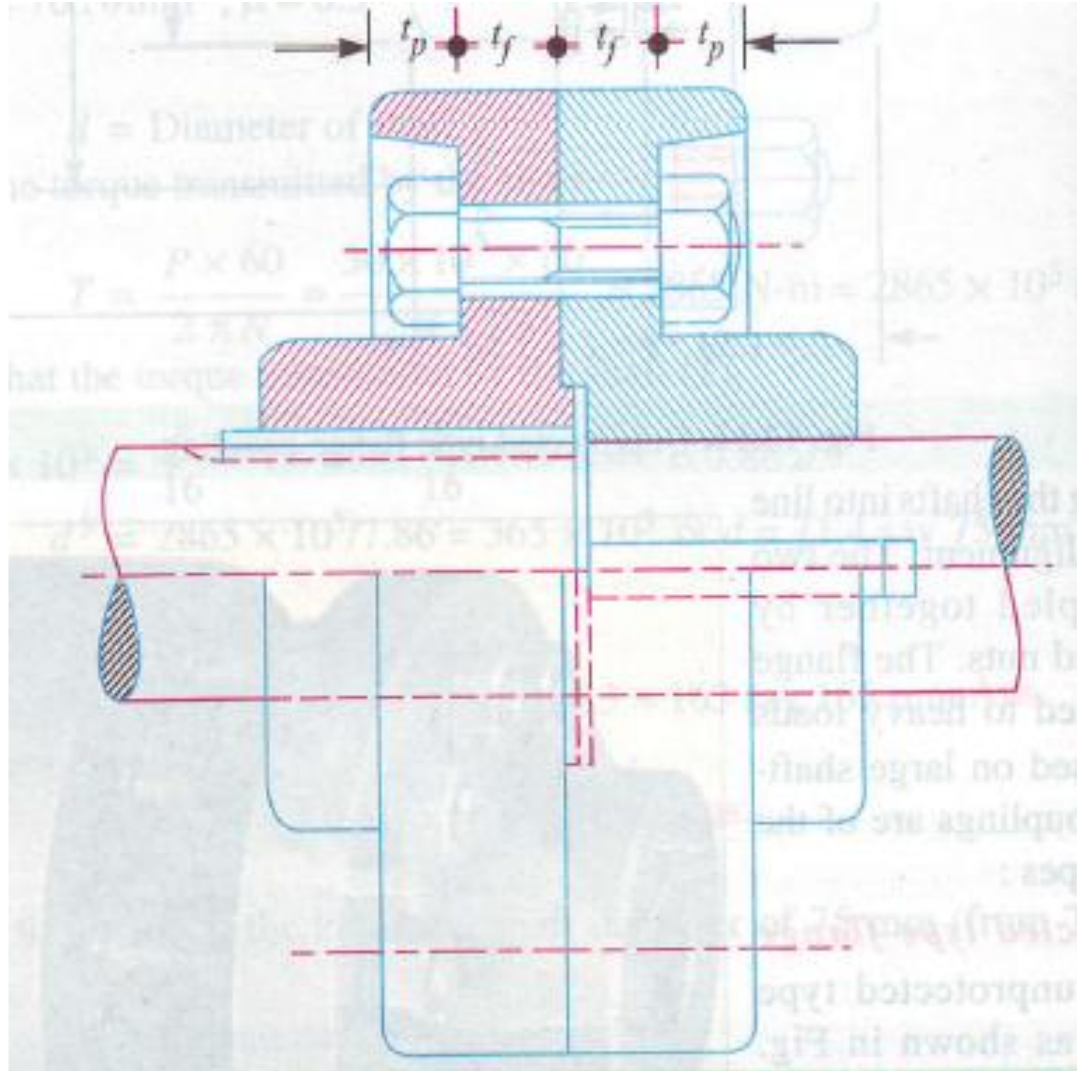


Fig. 13.13. Protective type flange coupling.

Design of flange coupling

- Let $\tau_s, \tau_b \; and \; \tau_k$ = allowable shear stress for shaft, bolt and key material respectively.
- τ_c= allowable shear stress for the flange material i.e cast iron.
- σ_{cb} and σ_{ck} = allowable crushing stress for bolt and key material respectively.

1. **Design for hub:**

D= 2d, L=1.5d

$$M_t = \frac{\pi}{16} \times \tau_c \times \left(\frac{D^4 - d^4}{D} \right)$$

From this equation induced shear stress in the hub may be checked.

2. Design for key:

Same procedure as in muff and clamp coupling.

Length of the key = length of hub = 1.5d

3. Design for flange:

Thickness of flange t_f= 0.5 d

$$M_t = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The induced shearing stress in the flange may be checked.

4. Design for bolts:

Pitch circle diameter of bolts D_1= 3d

$$M_t = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2}$$ ....(d_1 may be obtained)

$$M_t = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$$ ......(induced crushing stress in bolt may be checked.)

Problem 5. Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 rpm and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa. **(2008 Supp.)**

Problem 6. Design a cast iron protective type flange coupling to transmit 15 kW at 900 rpm from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used:

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw also a neat sketch of the coupling. **(2009,2011)**

Problem 7. A rigid flange coupling is used to transmit 15 kW power at 720 rpm between two steel shaft. The shaft key and bolt are made of plain carbon steel 80C8 (S_{yt}= 400 N/mm^2), FOS= 3. The yield strength in compression may be taken as 150 % of tensile yield strength. The flanges are made of grey cast iron of grade FG200 (S_{ut}= 200 N/mm^2), FOS= 6. The keys have square cross section. Design a coupling and specify the dimensions of its part.

Problem 8. Design a C.I. flange coupling to transmit 150 H.P. at 250 rpm. The following permissible stresses may be used:

Permissible shear stress for shaft, bolt and key material = 50 N/mm^2

Permissible crushing stress for bolt and key material = 150 N/mm^2

Permissible shear stress for Cast iron = 8 N/mm^2

(2010)

Problem 9. A rigid flange coupling is required to transmit 50 kW at 300 rpm. There are six bolts. The outer diameter is 150 mm. The coefficient of friction between the flanges is 0.15. The shaft and bolt are made of plain carbon steel 45C8 (S_{yt}= 380 N/mm^2), fos= 3. Determine the diameter of the bolts. Assume that the bolts are set in large clearance holes. **(2008)**

4. Bushed-pin flexible coupling

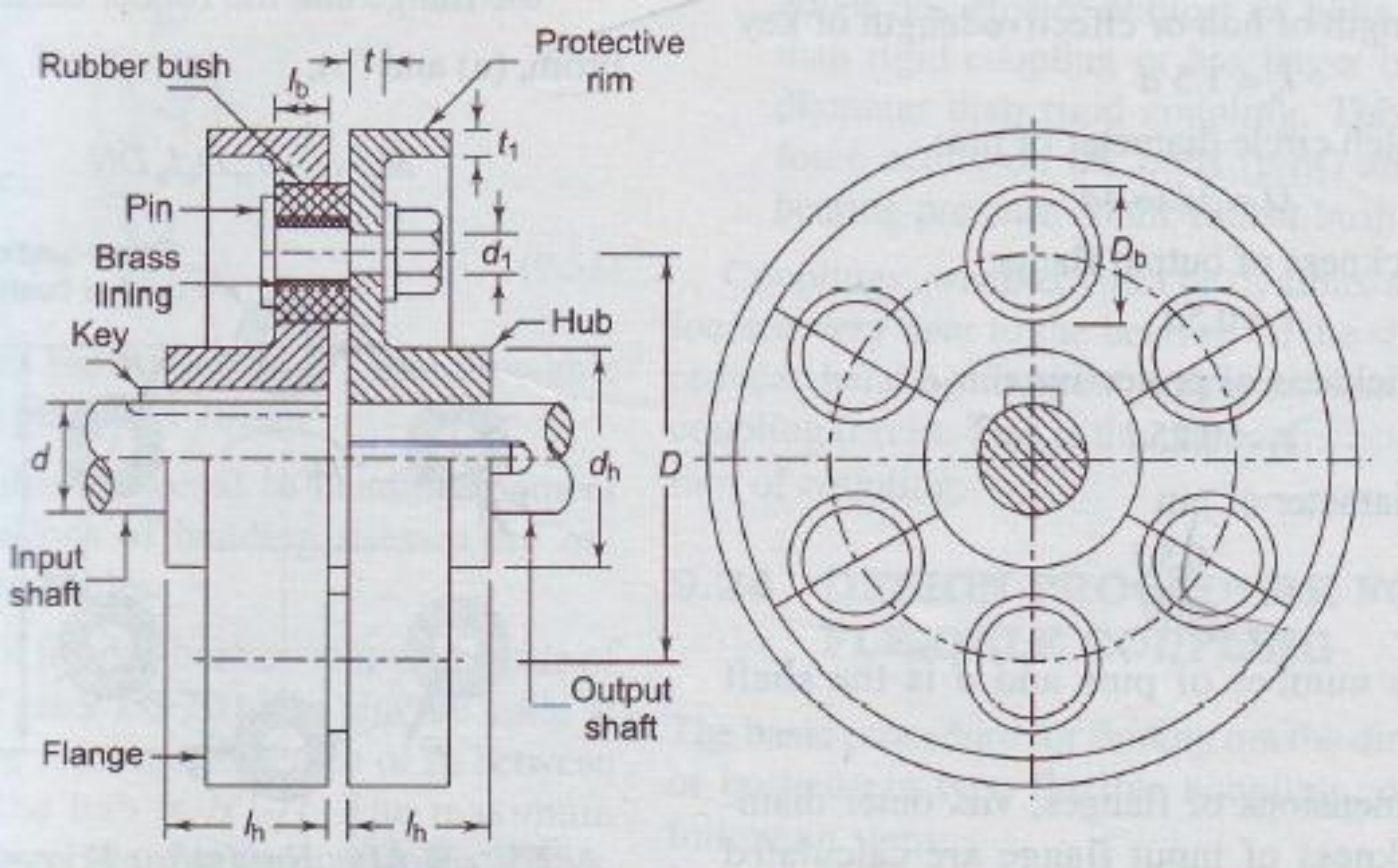


Fig. 9.39 *Flexible Coupling*

Design procedure for flexible coupling

Step 1. Shaft diameter:-

$$M_t = \frac{60\,P}{2\pi N} \quad and \quad \tau = \frac{16\,M_t}{\pi d^3}$$

Step 2 . Dimension of flange:-

- d_h= outside diameter of hub= 2d
- l_h= length of hub or effective length of key = 1.5d
- D= pitch circle diameter of pins= 3d to 4d
- t= thickness of output flanges = 0.5d
- t_1= thickness of protective rim= 0.25d
- The torsional shear stress in the hub is given by-

$$\tau = \frac{M_t r}{J} \quad where \quad J = \frac{\pi\left(d_h^{\,4} - d^4\right)}{32} \quad and \quad r = \frac{d_h}{2}$$

- The shear stress in the flange at the junction with the hub is given by:-

$$M_t = \frac{1}{2}\pi d_h^{\,2} t\tau$$

Step 3. Dimensions of bushes:-

- Outer diameter of rubber bush (D_b) is determined from the equation.

$$M_t = \frac{1}{2} D D_b^{\,2} n \times p_m$$

- Where p_m is pressure between bush and C.I. flange (usually 1N/mm^2)
- Effective length of the rubber bush ℓ_b= D_b

Step 4. Diameter of Pins:-

- Diameter of pins $d_1 = \dfrac{0.5d}{\sqrt{n}}$, Number of pins (n) is usually 4 or 6.
- Determine the shear stress in the pins by, $\tau = \dfrac{8M_t}{\pi d_1^2 D n}$

Cont…

Step 5. Check for bending stress in pins:

- Torque transmitted by the coupling:

$$M_t = P \times \frac{D}{2} \times n$$

From above equation force P on each rubber bush or pin is determined.

- Bending moment on the pin is given by:

$$M_b = \left(5 + \frac{l_b}{2}\right) \times P$$

- Bending stress is checked by equation:

$$\sigma_b = \frac{32 M_b}{\pi {d_1}^3}$$

Cont…

Step 6. Dimensions of keys:

- Determine the standard cross-section of flat key from data book page no. 5.16.
- Length of the key = length of hub = l_h =1.5d
- with these dimensions of the key, check the shear stress and compressive stress by using,

$$\tau = \frac{2M_t}{dbl} \quad \text{and} \quad \sigma_c = \frac{4M_t}{dhl}$$

Problem 9. A bushed pin type flexible coupling is used to connect two shafts and transmit 5 kW power at 720 rpm. Shafts, keys and pins are made of commercial steel (S_{yt}=S_{yc}=240 N/mm^2) and the factor of safety is 3. The flanges are made of grey cast iron FG200 (S_{ut}= 200 N/mm^2) and the factor of safety is 6. Assume

$$S_{sy}=0.5\ S_{yt} \quad \text{and} \quad S_{su}=0.5\ S_{ut}$$

There are 4 pins. The pitch circle diameter of the pins is four times of shaft diameter. The permissible shear stress for pins is 35 N/mm^2. The bearing pressure for rubber bushing is 1 N/mm^2. The keys have square cross section. Calculate:

(i) diameter of the shafts

(ii) dimensions of the key

(iii) diameter of the pins

(iv) outer diameter and effective length of the bushes.

Problem 10. It is required to design a bushed pin type of flexible coupling for connecting the motor to a centrifugal pump shafts. The details of the duty required from the pump are:

Power to be transmitted = 18.5 kW

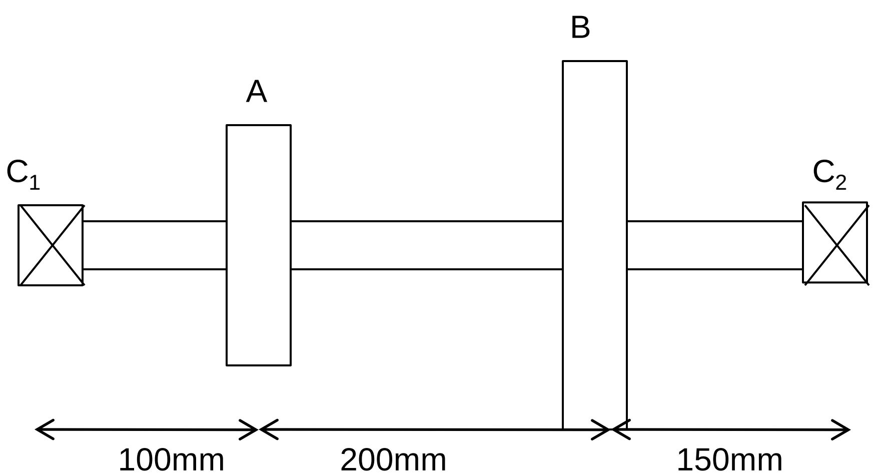
Speed in rpm = 1000

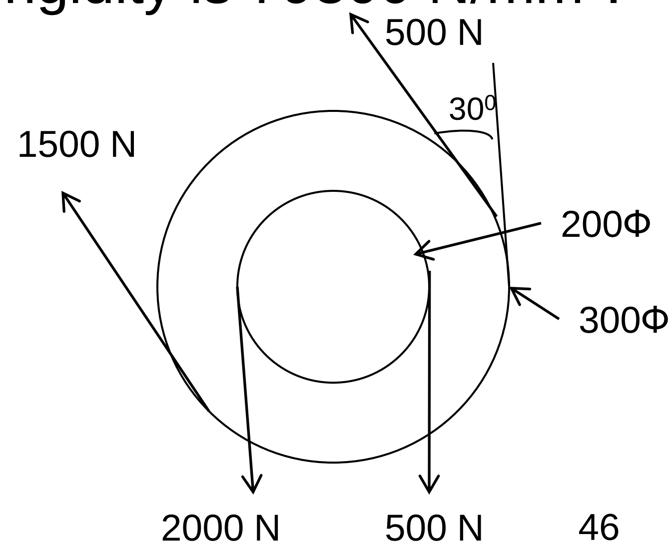
The diameters of the motor and pump shafts are 50 mm and 45 mm respectively. Take the bearing pressure on the rubber bush as 0.35 N/mm^2 and the working shear stress in the material of the pins as 20 N/mm^2. (**2009)**

Assignment 3

Que. 1 A transmission shaft supporting two pulleys A and B and mounted between two bearings C_1 and C_2 is shown in fig. Power is transmitted from the pulley A and B. The shaft is made of plain carbon steel 45C8 (S_{ut} = 600 and S_{yt} = 380 N/mm^2). The pulleys are keyed to the shaft. Determine the shaft diameter using the ASME code if k_b = 1.5 and k_t = 1.0.

Also determine the shaft diameter on the basis of torsional rigidity, if the permissible angle of twist between the two pulleys is 0.5^0 and the modulus of rigidity is 79300 N/mm^2.





Que. 2 A rigid coupling is used to transmit 45 KW power at 1440 rpm. The starting torque of the motor is 225 % of the rated torque. There are 8 bolts and their pitch circle diameter is 150 mm. The bolts are made of steel 45C8 (S_{yt}= 380 N/mm^2) and the factor of safety is 2.5. Determine the diameter of the bolts. Assume (S_{sy}= 0.577S_{yt})

Assume that the bolts are finger tight in reamed and ground holes.

Que.3 What is difference between rigid coupling and flexible coupling? Write advantages and disadvantages of both the couplings.