# Introduction

- Amplitude Modulation System
- Angle Modulation System

**Frequency Modulation** 

Phase Modulation

- Digital Communication
- Elements of Information Theory
- Advanced Communication Techniques

# Tools for communication

- Fourier Series
- Fourier Transform

### **Fourier Series**

- Every composite periodic signal can be represented with a series of sine and cosine functions.
- The functions are integral harmonics of the fundamental frequency "f" of the composite signal.
- Using the series we can decompose any periodic signal into its harmonics.

# periodic signal

- Periodic signals repeat with some *period* T, while aperiodic, or non periodic, signals do not. We can define a periodic function through the following mathematical expression, where t can be any number and T is a positive constant:
- $f(t) = f(T + t) (1) \dots (1)$
- The *fundamental period* of our function, f(t), is the smallest value of T that the still allows equation-1 to be true.

# A periodic signal with period T<sub>0</sub>



#### An aperiodic signal



#### Fourier Transform

 Fourier Transform is a tool that changes a time domain signal to a frequency domain signal and vice versa.

# Why frequency domain analysis?

- Allows simple algebra rather than time-domain differential equations to be used
- Transfer functions can be applied to transmitter, communication channel and receiver
- Channel bandwidth, noise and power are easier to evaluate
- > Easy to pick out frequencies.

#### Basic analog communications system



# Introduction

- Definition of COMMUNICATION?
- Why do we need to communicate?
- Definition of COMMUNICATION SYSTEM (Electronic Communication System)?
- Why do we need to communicate electronically?
- How to communicate?
- Interference in communications? How to overcome?

#### Introduction to Communication Systems

- Communication Basic process of exchanging information from one location (source) to destination (receiving end).
- Refers process of sending, receiving and processing of information/signal/input from one point to another point.



Figure 1 : A simple communication system

- Electronic Communication System defined as the whole mechanism of sending and receiving as well as processing of information electronically from source to destination.
- Example Radiotelephony, broadcasting, point-to-point, mobile communications, computer communications, radar and satellite systems.

# Objectives

- Communication System to produce an accurate replica of the transmitted information that is to transfer information between two or more points (destinations) through a communication channel, with minimum error.
- Besides interactive purposes, business and social

# NEED FOR COMMUNICATION

- Interaction purposes enables people to interact in a timely fashion on a global level in social, political, economic and scientific areas, through telephones, electronic-mail and video conference.
- Transfer Information Tx in the form of audio, video, texts, computer data and picture through facsimile, telegraph or telex and internet.
- Broadcasting Broadcast information to masses, through radio, television or teletext.

#### **Terms Related To Communications**

- Message physical manifestation produced by the information source and then converted to electrical signal before transmission by the transducer in the transmitter.
- Transducer Device that converts one form of energy into another form.
- Input Transducer placed at the transmitter which convert an input message into an electrical signal.
- Example Microphone which converts sound energy to electrical energy.



- Output Transducer placed at the receiver which converts the electrical signal into the original message.
- Example Loudspeaker which converts electrical energy into sound energy.



 Signal – electrical voltage or current which varies with time and is used to carry message or information from one point to another.  Information – defined as what is being conveyed by the telecommunication environment or as knowledge or facts. It can be in analogue form (voice, video, music) or digital form ( binary-coded numbers, graphics symbols or database information).

# Basic Requirements of Communication Systems.

- Rate of Information Transfer defined as the amount of info that must be communicated from source to destination in a certain period of time. It determines the physical form and the techniques used to transmit and receive information.
- □ The rate of information transfer must be reasonable and acceptable rate for each communication system.

Purity of Received Signal – the received signal must be the same as the transmitted signal

Simplicity of the system – must be convenience in order to be effective and efficient/easy to use.

Reliability – User must be able to depend on a communication system. It must work when needed and transmit/receive information without errors or with an acceptable errors.

#### Elements of a Communication System

The basic elements are : Source, Transmitter, Channel, Receiver and Destination.



Figure : Basic Block Diagram of a Communication System

### Function of each Element.

- Information Source the communication system exists to send messages. Messages come from voice, data, video and other types of information.
- Transmitter Transmit the input message into electrical signals such as voltage or current into electromagnetic waves such as radio waves, microwaves that is suitable for transmission and compatible with the channel. Besides, the transmitter also do the modulation and encoding (for digital signal).

# Block Diagram of a Transmitter



Channel/Medium – is the link or path over which information flows from the source to destination. Many links combined will establish a communication networks.

There are 5 criteria of a transmission system; Capacity, Performance, Distance, Security and Cost which includes the installation, operation and maintenance.

2 main categories of channel that commonly used are; line (guided media) and free space (unguided media) Receiver – Receives the electrical signals or electromagnetic waves that are sent by the transmitter through the channel. It is also separate the information from the received signal and sent the information to the destination.

Basically, a receiver consists of several stages of amplification, frequency conversion and filtering.

#### Block Diagram of a Receiver **Receiving Antenna** RF Amplifier Intermediate **Audio** Demodulator **Destination** Mixer Frequency Amplifier Amplifier Local **Oscillator**

Limitations in a Communication System

- There are 2 categories of limitations:-
  - Technological
  - Physical
- Technological Constraint includes equipment availability, economy and cost factor, national and international law (ITU-T) and interaction with existing systems.
- Physical Constraint includes bandwidth and noise.

# Cont.....physical constraint

- Bandwidth defined as the information carrying capacity of a system or the frequency content of a signal.
- BW is the difference between the upper frequency f<sub>H</sub> and the lower frequency f<sub>L</sub> of the signal.
- Example: The voice frequency ranges from 300 Hz to 3400 Hz. Therefore, the BW is 3.1 kHz.
- The limitation due to BW is applied to both signals and systems as a measure of speed. Means that the shorter transmission time will result in high-speed transmission of the signal, which require large BW and hence increasing the cost of the system

# Cont.....physical constraint

- Noise defined as unwanted electrical energy present in the usable passband of a communication circuit. It is unavoidable.
- Noise is measured in terms of Signal-to-Noise ratio (SNR).
- Its limitation, Higher SNR.....higher transmitted power and higher cost.
- Noise Factor, denoted by F and defined as the ratio of SNR at the input to the SNR at the output of a network.
  - F = (SNR)input / (SNR)output
  - Noise Figure = 10 log F dB
- Noise figure or noise factor is used to compare the performance of a communication system. Higher SNR, lower F is better in a communication systems.

# Unit-1

# Amplitude Modulation System

# NEED FOR MODULATION

- PRACTICABILITY OF ANTENNA
- "FOR EFFICIENT RADIATION AND RECEPTION THE TRANSMITTING AND RECEIVING ANTENNAS SHOULD HAVE LENGTHS COMPARABLE TO QUARTER WAVE LENGTHS OF FREQUENCY USED"
- NARROWBANDING
- BAND EDGE RATIO SHOULD BE APPROXIMATELY UNITY.
- MULTIPLEXING
- SIMULTANEOUS TRANSMISSION OF MULTIPLE MESSAGE
- COMMON PROCESSING
- REDUCED NOISE

#### **Amplitude Modulation**

## **Amplitude Modulation**

- Amplitude Modulation is the process of changing the amplitude of a relatively high frequency carrier signal in proportion with the instantaneous value of the modulating signal (information).
- Use in commercial broadcasting of both audio and video signals.
- Also used for two-way mobile radio communications, such as citizens band (CB) radio.

#### **AM Waveforms**



#### Equation of the AM Wave

 $e = (E_c + E_i sin \omega_i t) sin \omega_c t$ 

#### **AM Analysis**

- The instantaneous value of the AM waveform can be developed as follows.
- The equation for the amplitude of an AM waveform can be written as the carrier peak amplitude, Ec, plus the intelligence signal.

$$E = E_c + e_i$$

We know,  $e_i = E_i sin \omega_i t$ 

$$E = E_c + E_i \sin \omega_i t$$

#### **AM Analysis**

Ratio of  $E_i / E_c = m$  =Modulation index

 $E = E_c + mE_c \sin \omega_i t$ = E\_c(1 + m sin \omega\_i t)

The instantaneous value for the AM wave is the amplitude, E, times  $sin\omega_c t$ 

e = E sin 
$$ω_c t$$
  
= E<sub>c</sub>(1 + m sin  $ω_i t$ ) sin  $ω_c t$ 

#### **AM Analysis**

Remember the trigonometric identity :

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

We can write e as follows:

 $e = E_c \sin \omega_c t + [m E_c / 2] \cos(\omega_c - \omega_i) t - [m E_c / 2] \cos(\omega_c + \omega_i) t$
### **Frequency Spectrum**



# The AM Waveform

The three components that form the AM waveform are listed below:

- 1. The *lower-side frequency*  $(f_c f_i)$
- 2. The carrier frequency ( $f_c$ )
- 3. The upper-side frequency  $(f_c + f_i)$

### **AM Waveforms**



#### **Carrier and Side Frequency Components**



# **Upper and Lower Sidebands**

- In most systems the intelligence signal is a complex waveform containing components from roughly 200Hz to 3KHz.
- If this is used to modulate the carrier there would be a whole band of side frequencies.
- The band of frequencies above the carrier is term the upper sideband.
- The band of frequencies below the carrier is called the lower sideband.

# **Upper and Lower Sidebands**



# Example

A 1.4MHz carrier is modulated by a music signal that has frequency components from 20Hz to 10kHz. Determine the range of frequencies generated for the upper and lower sidebands.

## Solution



# Modulation index (m)

• The relationship between the modulating signal amplitude  $V_m$  and the carrier  $V_c$  is a ratio called the modulation index m defined as





 $m = \frac{V_m}{V_m}$ 

m = 90%

# Overmodulation

- *m* should range between 0 and 1.
- The condition in which m > 1 is called overmodulation and will result in distortion.



# Modulation index

• We can also determine the modulation index *m* from the maximum and minimum values of the envelope of  $v_{AM}$  $V = \frac{V_{max} - V_{min}}{V_{max} - V_{min}}$ 





### **Example Problem**

If a carrier signal with an amplitude of 9 V is mixed with a modulating signal with an amplitude of 7.5 V, what is the percentage modulation of the resulting signal?



# **Percentage Modulation**

- A measure of the extent to which a carrier voltage is varied by the intelligent signal.
- Also known as modulation index or modulation factor and is symbolized by m.

 $0 \le \mathbf{m} \le 1$ 

0 => no modulation 1 => 100% modulation

### **AM Waveforms**



# Example

 An unmodulated carrier is 300V p-p. Calculate %m when its maximum p-p value reaches 400, 500, 600.

### Overmodulation

- When an excessive intelligent signal overdrives an AM modulator producing percentage modulation exceeding 100 percent.
- Overmodulation produces sideband splatter. Distortion resulting in an overmodulated AM transmission creating excessive bandwidths.

### Overmodulation



# AM in the frequency domain



Carrier signal  $v_c$  (carrier frequency  $f_c = 5$ -kHz)

# AM in the frequency domain

• 
$$v_{AM}$$
 is given  $v_{AM} = (V_c + V_m \sin 2\pi f_m t) \sin 2\pi f_c t$   
=  $V_c \sin 2\pi f_c t + (V_m \sin 2\pi f_m t) (\sin 2\pi f_c t)$ 

• Applying the trigonometric identity for the product of two sine functions  $\sin A \sin B = \frac{\cos(A-B)}{2} - \frac{\cos(A+B)}{2}$ 

we can write  

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2} \sin 2\pi (f_c - f_m) t - \frac{V_m}{2} \sin 2\pi (f_c + f_m) t$$

# AM in the frequency domain

 This form shows that v<sub>AM</sub> consists of just three frequency components.

$$v_{AM} = \frac{V_m}{2} \sin 2\pi (f_c - f_m)t + V_c \sin 2\pi f_c t - \frac{V_m}{2} \sin 2\pi (f_c + f_m)t$$
Lower sideband Carrier frequency  $f_c$  Upper sideban

 $(f_{\rm LSB} = f_c - f_m)$ 

 $(f_{\text{USB}} = f_c + f_m)$ 





Amplitude modulated signal  $v_{AM}$ 

# Example

• Determine the maximum sideband power if the carrier output is 1 kW and calculate the total maximum transmitted power.

#### Solution

Max sideband power occurs when m = 1 or 100%. At this percentage modulation each side frequency is ½ the carrier amplitude.

$$E_{SF} = mE_c/2$$

Since power is proportional to the square of the voltage, each has ¼ of the carrier power.

¼ x 1kW = 250W

Total sideband power = 2 x 250 = 500W

Total transmitted power = 1kW + 500W = 1.5kW

#### Importance of High Percentage Modulation

- It in important to use a high a percentage of modulation as possible while ensuring that overmodulation does not occur.
- The sidebands contain the information and have maximum power at 100% modulation.
- Useful equation

$$P_t = P_c(1 + m^2/2)$$

 $P_t$  = Total transmitted power (sidebands and carrier)  $P_c$  = Carrier power Importance of High Percentage Modulation

$$I_t = I_c V(1 + m^2/2)$$

= Total transmitted current

m = Modulating index

 $\mathbf{I}_{t}$ 

# Example

- 1. A 500W carrier is to be modulated to a 90% level. Determine the total transmitted power.
- 2. An AM broadcast station operates at its maximum allowed total output of 50kW and at 95% modulation. How much of its transmitted power is intelligence (sidebands)?
- 3. The antenna current of an AM transmitter is 12A when unmodulated but increase to 13A when modulated. Calculate %m.

### **Effective Modulation Index**

If a carrier is modulated by more than a single sine wave, the effective modulation index is given by:

$$m_{eff} = V(m_1^2 + m_2^2 + m_3^2 + ....)$$

Note that the total modulation index must not exceed 1 or distortion will occur.

m<sub>eff</sub> can be used in all previously developed equations using m.

# Example

 A transmitter with a 10kW carrier transmit 11.2kW when modulated with a single sine wave. Calculate the modulation index. If the carrier is simultaneously modulated with another sine wave at 50 percent modulation, calculate the total transmitted power.

### **Baseband vs Passband Transmission**

- Baseband signals:
  - Voice (0-4kHz)
  - TV (0-6 MHz)
- A signal may be sent in its baseband format when a <u>dedicated wired</u> channel is available.
- Otherwise, it must be converted to passband.



# Types of Amplitude Modulation (AM)

- <u>Double Sideband with carrier (we will call it AM</u>): This is the most widely used type of AM modulation. In fact, all radio channels in the AM band use this type of modulation.
- **Double Sideband Suppressed Carrier (DSBSC):** This is the same as the AM modulation above but without the carrier.
- <u>Single Sideband (SSB)</u>: In this modulation, only half of the signal of the DSBSC is used.
- <u>Vestigial Sideband (VSB)</u>: This is a modification of the SSB to ease the generation and reception of the signal.

#### Double Sideband Suppressed Carrier (DSBSC)

- Assume that we have a message signal m(t) with bandwidth  $2\pi B$  rad/s (or B Hz).  $m(t) \Leftrightarrow M(\omega)$ .
- Let c(t) be a carrier signal,  $c(t) = \cos(\omega_c t)$ ,  $\omega_c >> 2\pi B$
- $g_{DSBSC}(t) = m(t) \cdot \cos(\omega_c t)$

$$\Leftrightarrow (1/2) [M(\omega - \omega_c) + M(\omega + \omega_c)].$$



DSBSC Modulator (transmitter)

#### Time and Frequency Representation of DSBSC Modulation Process



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# **DSBSC** Demodulation



- $e(t)=g_{DSBSC}(t)\cdot\cos(\omega_c t)$   $= m(t)\cdot\cos^2(\omega_c t)$   $= (1/2) m(t) [1 + \cos(2\omega_c t)]$  c(t)DSBSC Demodulator (receiver)
  - $= (1/2) m(t) + (1/2) m(t) \cos(2 \omega_c t)$
- $E(\omega) \Leftrightarrow (1/2) M(\omega) + (1/4) [M(\omega 2 \omega_c) + M(\omega + 2 \omega_c)].$
- The output signal f(t) of the LPF will be  $f(t) = (1/2) m(t) \iff (1/2) M(\omega)$ .

#### Time and Frequency Representation of DSBSC Demodulation Process



# **Modulator Circuits**

- Basically we are after multiplying a signal with a carrier.
- There are three realizations of this operation:
  - Multiplier Circuits
  - Non-Linear Circuits
  - Switching Circuits

# Non-Linear Devices (NLD)

- A NLD is a device whose input-output relation is nonlinear. One such example is the diode (*i*<sub>D</sub>=e<sup>v<sub>D</sub>/v<sub>T</sub></sup>).
- The output of a NLD can be expressed as a power series of the input, that is
   y(t) = ax(t) + bx<sup>2</sup>(t) + cx<sup>3</sup>(t) + ...
- When x(t) << 1, the higher powers can be neglected, and the output can be approximated by the first two terms.
- When the input x(t) is the sum of two signal, m(t)+c(t), x<sup>2</sup>(t) will have the product term m(t)c(t)

### **Non-Linear Modulators**



DSBSC modulation using non-linear device

$$x_{1}(t) = c(t) + m(t) = \cos(\omega_{c}t) + m(t)$$

$$x_{1}(t) = c(t) - m(t) = \cos(\omega_{c}t) - m(t)$$

$$y_{1}(t) = a[\cos(\omega_{c}t) + m(t)] + b[\cos(\omega_{c}t) + m(t)]^{2}$$

$$= a\cos(\omega_{c}t) + am(t) + bm^{2}(t) + 2bm(t) \cdot \cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= am(t) + bm^{2}(t) + 2bm(t) \cdot \cos(\omega_{c}t) + a\cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= am(t) + bm^{2}(t) + 2bm(t) \cdot \cos(\omega_{c}t) + a\cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= am(t) + bm^{2}(t) + 2bm(t) \cdot \cos(\omega_{c}t) + a\cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$y_{2}(t) = a[\cos(\omega_{c}t) - m(t)] + b[\cos(\omega_{c}t) - m(t)]^{2}$$

$$= a\cos(\omega_{c}t) - am(t) + bm^{2}(t) - 2bm(t) \cdot \cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= -am(t) + bm^{2}(t) - 2bm(t) \cdot \cos(\omega_{c}t) + a\cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= -am(t) + bm^{2}(t) - 2bm(t) \cdot \cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= -am(t) + bm^{2}(t) - 2bm(t) \cdot \cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$

$$= -am(t) + bm^{2}(t) - 2bm(t) \cdot \cos(\omega_{c}t) + b\cos^{2}(\omega_{c}t)$$
# Switching Modulators

- Any periodic function can be expressed as a series of cosines (Fourier Series).
- The information signal, m(t), can therefore be, equivalently, multiplied by any periodic function, and followed by BPF.
- Let this periodic function be a train of pulses.
- Multiplication by a train of pulses can be realized by simple *switching*.

# Switching Modulator Illustration



Figure 4.4 Switching modulator for DSB-SC.

# Switching Modulator: Diode Bridge



Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

# Switching Modulator: Ring



#### Figure 4.6 Ring modulator.

# Demodulation of DSBSC

- The modulator circuits can be used for demodulation, but replacing the BPF by a LPF of bandwidth *B* Hz.
- The receiver must generate a carrier frequency in phase and frequency synchronization with the incoming carrier.
- This type of demodulation is therefore called *coherent* demodulation (or detection).



DSBSC Demodulator (receiver)

# From DSBSC to DSBWC (AM)

- Carrier recovery circuits, which are required for the operation of coherent demodulation, are sophisticated and could be quite costly.
- If we can let m(t) be the envelope of the modulated signal, then a much simpler circuit, the *envelope detector*, can be used for demodulation (non-coherent demodulation).
- How can we make m(t) be the envelope of the modulated signal?

# Definition of ,

 Shift m(t) by some DC value "A" such that A+m(t) ≥ 0. Or A ≥ m<sub>peak</sub>

$$g_{AM}(t) = [A + m(t)]\cos(\omega_{C}t)$$
$$= A\cos(\omega_{C}t) + m(t)\cos(\omega_{C}t)$$

- Called DSBWC. Here will refer to it as Full AM, or simply AM
- Modulation index  $\mu = m_p / A$ .
- $0 \le \mu \le 1$



#### Spectrum of AM

$$g_{AM}(t) \Leftrightarrow \pi A \Big[ \delta(\omega - \omega_C) + \delta(\omega + \omega_C) \Big] + \frac{1}{2} \Big[ M(\omega - \omega_C) + M(\omega + \omega_C) \Big]$$



# The "Buy" and "Price" of AM

- Buy: Simplicity in demodulation.
- Price: Waste in Power

 $g_{AM}(t) = A\cos\omega_{c}t + m(t)\cos\omega_{c}t$ 

Carrier Power  $P_c = A^2/2$  (carries no information)

Sideband Power  $P_s = P_m/2$  (useful)

Power efficiency =  $\eta = P_s/(P_c + P_s) = P_m/(A^2 + P_m)$ 



# **Tone Modulation**

- $m(t) = B\cos(\omega_m t)$
- $g(t) = [A + B\cos(\omega_m t)] \cos \omega_c t = A[1 + \mu \cos(\omega_m t)] \cos \omega_c t$
- $\eta = (B^2/2)/(B^2/2 + A^2) = \mu^2/(2 + \mu^2)$
- Under best conditions,  $\mu$ =1  $\rightarrow \eta_{\text{max}}$ =1/3=33%
- For  $\mu$  = 0.5,  $\eta$  = 11.11%
- For practical signals,  $\eta < 25\%$

# **?** Would you use AM or DSBSC?

# Generation of AM

- AM signals can be generated by any DSBSC modulator, by using A+m(t) as input instead of m(t).
- In fact, the presence of the carrier term can make it even simpler. We can use it for switching instead of generating a local carrier.
- The switching action can be made by a single diode instead of a diode bridge.

#### **AM Generator**



- $v_R = [\cos \omega_c t + m(t)] [1/2 + 2/\pi (\cos \omega_c t 1/3 \cos 3\omega_c t + ...)]$ =  $(1/2) \cos \omega_c t + (2/\pi)m(t) \cos \omega_c t$  + other terms (suppressed by BPF)
- $v_o(t) = (1/2)\cos\omega_c t + (2/\pi)m(t)\cos\omega_c t$

#### AM Modulation Process (Frequency)



#### AM Demodulation: Rectifier Detector

 Because of the presence of a carrier term in the received signal, switching can be performed in the same way we did in the modulator.



# **Rectifier Detector: Time Domain**





#### **Rectifier Detector (Frequency Domain)**



### **Envelope Detector**



- When D is forward-biased, the capacitor charges and follows input.
- When D is reverse-biased, the capacitor discharges through *R*.



- The operations of the circuit requires careful selection of *τ=RC*
- If *RC* is too large, discharging will be slow and the circuit cannot follow a decreasing envelope.
- When *RC* is too small the ripples will be high.
- 1/(2πB) << τ << 1/ω<sub>c</sub>
- The ripples are finally removed by LPF.
- The DC value is blocked by a capacitor.

#### Quadrature Amplitude Modulation (QAM)

- In DSBSC or AM the modulated signal occupies double the bandwidth of the baseband signal.
- It is possible to send two signals over the same band, one modulated with a cosine and one with sine.
- Interesting enough, the two signals can be received separately after demodulation.



QAM Modulator/Demodulator



QAM Modulator/Demodulator with Demodulator Carrier Phase and/or Frequency Error

# Single-Side Band (SSB) Modulation

- DSBSC (as well as AM) occupies double the bandwidth of the baseband signal, although the two sides carry the same information.
- Why not send only one side, the upper or the lower?
- **Modulation:** similar to DSBSC. Only change the settings of the BPF (center frequency, bandwidth).
- **Demodulation:** similar to DSBSC (coherent)

#### **SSB** Representation



How would we represent the SSB signal in the time domain?

$$g_{USB}(t) = ?$$
  
 $g_{LSB}(t) = ?$ 

#### Time-Domain Representation of SSB (1/2)

 $M(\omega) = M_{+}(\omega) + M_{-}(\omega)$ Let  $m_{+}(t) \leftrightarrow M_{+}(\omega)$  and  $m_{-}(t) \leftrightarrow M_{-}(\omega)$ Then:  $m(t) = m_{+}(t) + m_{-}(t)$  [linearity] Because  $M_{+}(\omega)$ ,  $M_{-}(\omega)$  are not even  $\rightarrow m_{+}(t)$ ,  $m_{-}(t)$  are complex. Since their sum is real they must be conjugates.

 $m_{+}(t) = \frac{1}{2} [m(t) + j m_{h}(t)]$   $m_{-}(t) = \frac{1}{2} [m(t) - j m_{h}(t)]$ What is  $m_{h}(t)$ ?



#### Time-Domain Representation of SSB (2/2)

$$M(\omega) = M_{+}(\omega) + M_{-}(\omega)$$

$$M_{+}(\omega) = M(\omega)u(\omega); M_{-}(\omega) = M(\omega)u(-\omega)$$

$$\operatorname{sgn}(\omega) = 2u(\omega) -1 \twoheadrightarrow u(\omega) = \frac{1}{2} + \frac{1}{2} \operatorname{sgn}(\omega); u(-\omega) = \frac{1}{2} - \frac{1}{2} \operatorname{sgn}(\omega)$$

$$M_{+}(\omega) = \frac{1}{2} [M(\omega) + M(\omega)\operatorname{sgn}(\omega)]$$

$$M_{-}(\omega) = \frac{1}{2} [M(\omega) - M(\omega)\operatorname{sgn}(\omega)]$$
Comparing to:
$$m_{+}(t) = \frac{1}{2} [m(t) + j m_{h}(t)] \leftrightarrow \frac{1}{2} [M(\omega) + j M_{h}(\omega)]$$

$$m_{-}(t) = \frac{1}{2} [m(t) - j m_{h}(t)] \leftrightarrow \frac{1}{2} [M(\omega) - j M_{h}(\omega)]$$
We find

 $M_h(\omega) = -j M(\omega) \cdot \text{sgn}(\omega) \text{ where } m_h(t) \leftrightarrow M_h(\omega)$ 

# **Hilbert Transform**

- $m_h(t)$  is known as the Hilbert Transform (HT) of m(t).
- The transfer function of this transform is given by:
   H(ω) = -j sgn(ω)

•



# Hilbert Transform of $\cos(\omega_c t)$

$$cos(\omega_{c}t) \leftrightarrow \pi \left[\delta(\omega - \omega c) + \delta(\omega + \omega c)\right]$$
  

$$HT[cos(\omega_{c}t)] \leftrightarrow -j \operatorname{sgn}(\omega) \pi \left[\delta(\omega - \omega c) + \delta(\omega + \omega c)\right]$$
  

$$= j \operatorname{sgn}(\omega) \pi \left[-\delta(\omega - \omega c) - \delta(\omega + \omega c)\right]$$
  

$$= j \pi \left[-\delta(\omega - \omega c) + \delta(\omega + \omega c)\right]$$
  

$$= j \pi \left[\delta(\omega + \omega c) - \delta(\omega - \omega c)\right] \leftrightarrow \operatorname{sin}(\omega_{c}t)$$

Which is expected since:

 $\cos(\omega_c t - \pi/2) = \sin(\omega_c t)$ 

#### Time-Domain Operation for Hilbert Transformation

For Hilbert Transformation  $H(\omega) = -i \operatorname{sgn}(\omega)$ . What is h(t)?  $sgn(t) \leftrightarrow 2/(j\omega)$  [From FT table]  $2/(jt) \leftrightarrow 2\pi \operatorname{sgn}(-\omega)$  [symmetry]  $1/(\pi t) \leftrightarrow -i \operatorname{sgn}(\omega)$ Since  $M_h(\omega) = -j M(\omega) \cdot \text{sgn}(\omega) = H(\omega) \cdot M(\omega)$ Then  $m_h(t) = \frac{1}{\pi t} * m(t)$  $=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\alpha)}{t-\alpha}d\alpha$ 

Finally ...

 $g_{USB}(t) = m_{+}(t)e^{j\omega_{C}t} + m(t)e^{-j\omega_{C}t}$  $g_{LSB}(t) = m_{+}(t)e^{-j\omega_{C}t} + m_{-}(t)e^{j\omega_{C}t}$  $g_{USB}(t) = \frac{1}{2}m(t)e^{j\omega_{c}t} + \frac{1}{2}jm_{h}(t)e^{j\omega_{c}t}$  $+\frac{1}{2}m(t)e^{-j\omega_{c}t}-\frac{1}{2}jm_{h}(t)e^{-j\omega_{c}t}$  $= m(t)\cos(\omega_{c}t) - m_{h}(t)\sin(\omega_{c}t)$  $g_{LSB}(t) = \frac{1}{2}m(t)e^{j\omega_{c}t} - \frac{1}{2}jm_{h}(t)e^{j\omega_{c}t}$  $+\frac{1}{2}m(t)e^{-j\omega_{c}t}+\frac{1}{2}jm_{h}(t)e^{-j\omega_{c}t}$  $= m(t)\cos(\omega_{c}t) + m_{h}(t)\sin(\omega_{c}t)$ 



# Generation of SSB

- Selective Filtering Method
   Realization based on spectrum analysis
- Phase-Shift Method Realization based on time-domain expression of the modulated signal



# **Phase Shifting**



SSB Modulator

#### Phase-shifting Method: Frequency-Domain Illustration



# SSB Demodulation (Coherent)

$$g_{SSB}(t) = m(t)\cos(\omega_{C}t) \pm m_{h}(t)\sin(\omega_{C}t)$$

$$g_{SSB}(t)\cos(\omega_{C}t) = \frac{1}{2}m(t)[1 + \cos(2\omega_{C}t)] \pm \frac{1}{2}m_{h}(t)\sin(2\omega_{C}t)$$
LPF Output =  $\frac{1}{2}m(t)$ 

$$g_{SSB}(t) \longrightarrow M(t)$$

SSB Demodulator (receiver)

# FDM in Telephony

- FDM is done in stages
  - Reduce number of carrier frequencies
  - More practical realization of filters
- Group: 12 voice channels × 4 kHz = 48 kHz occupy the band 60-108 kHz
- Supergroup: 5 groups × 48 kHz = 240 kHz occupy the band 312-552
- Mastergroup: 10 S-G × 240 kHz = 2400 kHz occupy the band 564-3084 kHz

#### **FDM Hierarchy**


### Vestigial Side Band Modulation (VSB)

 What if we want to generate SSB using selective filtering but there is no guard band between the two sides?

→ We will filter-in a *vestige* of the other band.

• Can we still recover our message, without distortion, after demodulation? Yes. If we use a proper LPF.

### Filtering Condition of VSB

$$\begin{split} g_{DSBSC}(t) &= 2m(t)\cos(\omega_{C}t) \\ G_{DSBSC}(\omega) &= M(\omega - \omega_{C}) + M(\omega + \omega_{C}) \\ G_{VSB}(\omega) &= H_{VSB}(\omega) \Big[ M(\omega - \omega_{C}) + M(\omega + \omega_{C}) \Big] \\ X(\omega) &= H_{VSB}(\omega) \Big[ M(\omega - \omega_{C}) + M(\omega + \omega_{C}) \Big] \\ &+ H_{VSB}(\omega - \omega_{C}) \Big[ \underbrace{M(\omega - 2\omega_{C})}_{at + 2\omega_{C}} + \underbrace{M(\omega)}_{Baseband} \Big] \\ &+ H_{VSB}(\omega + \omega_{C}) \Big[ \underbrace{M(\omega)}_{baseband} + \underbrace{M(\omega + 2\omega_{C})}_{at - 2\omega_{C}} \Big] \\ g_{VSB}(t) & \longrightarrow \\ & (1 + M_{VSB}(\omega + \omega_{C})) \Big[ \underbrace{M(\omega)}_{baseband} + \underbrace{M(\omega + 2\omega_{C})}_{at - 2\omega_{C}} \Big] \\ &= H_{LPF}(\omega) \Big[ H_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C}) \Big] M(\omega) \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] M(\omega) \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C}) + H_{VSB}(\omega + \omega_{C})) \Big] \\ & (1 + M_{VSB}(\omega - \omega_{C$$



## VSB Filter: Special Case

• Condition For distortionless demodulation:

$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_C) + H_{VSB}(\omega + \omega_C)} ; |\omega| \le 2 \pi B$$

• If we impose the condition on the filter at the modulator:

$$H_{\scriptscriptstyle VSB}(\omega - \omega_c) + H_{\scriptscriptstyle VSB}(\omega + \omega_c) = 1 \ ; \ |\omega| \le 2 \ \pi B$$

Then  $H_{LPF} = 1$  for  $|\omega| \le 2 \pi B$  (Ideal LPF)

•  $H_{VSB}(\omega)$  will then have odd symmetry around  $\omega_c$  over the transition period.



## AM Broadcasting

- Allocated the band 530 kHz 1600 kHz (with minor variations)
- 10 kHz per channel. (9 kHz in some countries)
- More that 100 stations can be licensed in the same geographical area.
- Uses AM modulation (DSB + C)

### AM station Reception

• In theory, any station can be extracted from the stream of spectra by tuning the receiver BPF to its center frequency. Then demodulated.



- Impracticalities:
  - Requires a BPF with very high Q-factor (Q =  $f_c / B$ ).
  - Particularly difficult if the filter is to be tunable.

### Solution: Superheterodyne receiver

- Step 1: Frequency Translation from RF to IF
  Shift the desired station to another fixed pass band (called Intermediate Frequency IF = 455 kHz)
- Step 2: Bandpass Filtering at IF
  Build a good BPF around IF to extract the desired station.
  It is more practical now, because IF is relatively low
  (reasonable Q) and the filter is not tunable.
- Step 3: **Demodulation** Use Envelope Detector



## The Local Oscillator

• What should be the frequency of the local oscillator used for translation from RF to IF?  $f_{LO} = f_c + f_{IF}$  (up-conversion) or  $f_{LO} = f_c - f_{IF}$  (down-conversion)

• Tuning ratio =  $f_{LO, max} / f_{LO, min}$ 

- Up-Conversion: (1600 + 455) / (530+455) ≈ 2
- Down-Conversion: (1600–455) / (530–455) ≈ 12
- Easier to design oscillator with small tuning ratio.

# Image Station Problem

- While up-converting the desired station to IF, we are, at the same time, down-converting another station to IF as well.
- These two stations are called *image stations*, and they are spaced by 2x455=910kHz.
- Solution:

Before conversion, use a BPF (at RF) centered at  $f_c$  of the desired station.

The purpose of the filter is NOT to extract the desired station, but to suppress its image. Hence, it does not have to be very sharp.

### Superheterodyne Receiver Block Diagram



•The filter are designed with high gain to provide amplification as well.





-Multiplexing: Simultaneous transmission of multiple signals across a single data link. -In a multiplexed system, *n* lines share the bandwidth of one link.



- Link: The physical path.
- Channel: Portion of link that carries a transmission between a pair of lines.
- One link can have many (n) channels.



Categories of multiplexing



FDM

- Channels must be separated by guard band to prevent signals from overlapping.
- Carrier frequencies must not interfere with the original data frequencies.

#### (1) Multiplexing Process





- Each telephone generates a signal of similar frequency range.
- Signals are modulated onto different carrier frequencies( $f_1$ ,  $f_2$  and  $f_3$ ).

(2) Demultiplexing Process

- Demultiplexer uses a series of filters to decompose the multiplexed signal.



FDM demultiplexing example

### Example 1

Assume that a voice channel occupies a bandwidth of 4 KHz. We need to combine three voice channels into a link with a bandwidth of 12 KHz, from 20 to 32 KHz. Show the configuration using the frequency domain without the use of guard bands.

#### Solution

Shift (modulate) each of the three voice channels to a different bandwidth, as shown in Figure 6.6



Filter and shift

### Example 2

Five channels, each with a 100-KHz bandwidth, are to be multiplexed together. What is the minimum bandwidth of the link if there is a need for a guard band of 10 KHz between the channels to prevent interference?

#### Solution

For five channels, we need at least four guard bands. This means that the required bandwidth is at least

 $5 \times 100 + 4 \times 10 = 540$  KHz,

as shown in Figure 6.7



Example 2

### Example 3

Four data channels (digital), each transmitting at 1 Mbps, use a satellite channel of 1 MHz. Design an appropriate configuration using FDM

#### Solution

The satellite channel is analog. We divide it into four channels, each channel having a 250-KHz bandwidth. Each digital channel of 1 Mbps is modulated such that each 4 bits are modulated to 1 Hz. One solution is 16-QAM modulation. Figure 6.8 shows one possible configuration.



Example 3