UNIT – II
BALANCING

2.1 Introduction

If all the rotating and reciprocating parts of high speed engines and other machines are not balanced, the dynamic forces are set up which increase the loads on bearings and stresses in various members. These forces also produce unpleasant and even dangerous vibrations.

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2.2 Balancing of Rotating Masses

Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force. Its effect is to bend the shaft and to produce vibrations in it. In order to prevent this effect another mass is attached to the opposite side of the shaft. Centrifugal forces of both the masses are made to be equal and opposite.

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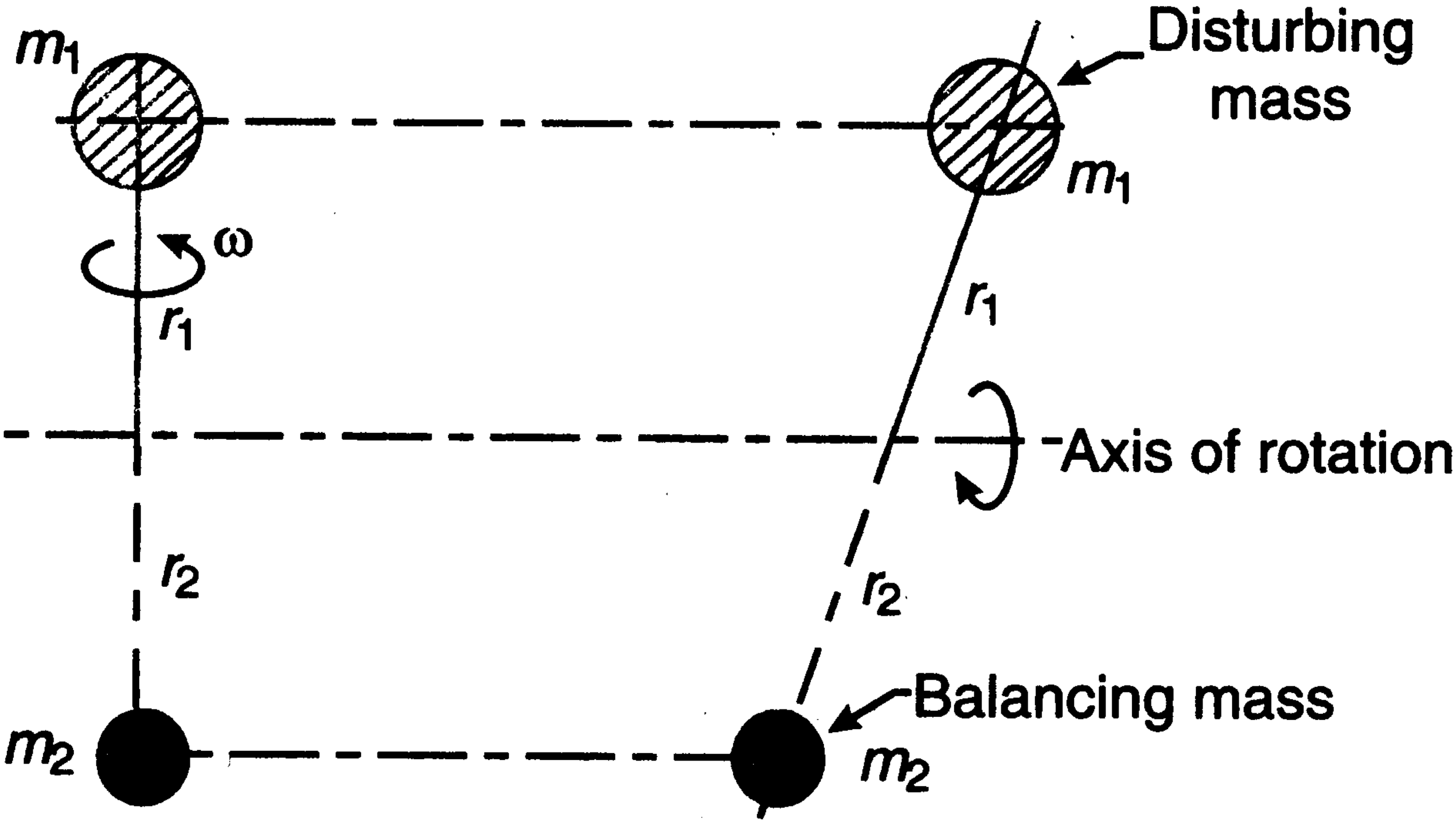
2.3 Balancing of a Single Rotating Mass by a Single Mass Rotating in the same plane.

Centrifugal force due to disturbing mass is equal to centrifugal force due to balancing mass.

$$F_{c1} = F_{c2} \quad \text{or} \quad m_1.\omega^2.r_1 = m_2.\omega^2.r_2$$



m_1
Disturbing mass
m_1
ω
r_1
r_1
Axis of rotation
r_2
r_2
Balancing mass
m_2
m_2

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2.4 Balancing of a Single Rotating Mass by Two Masses Rotating in Different Planes.

By introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. But this type of arrangement gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, the two balancing masses are placed in two different planes in such a way that they satisfy the following two conditions of equilibrium.

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1. The net dynamic force acting on the shaft is equal to zero. This requires that the centre of the masses of the system must lie on the axis of rotation. This is the condition for *static balancing*.

2. The net couple due to the dynamic forces acting on the shaft is equal to zero.

The condition (1) and (2) together give dynamic balancing. The following are two possible methods of attaching the two balancing masses.

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1. The plane of the disturbing mass may be in between the planes of two balancing masses.
2. The plane of the disturbing mass may be on the left or right of the two planes containing the balancing masses.

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The two conditions are discussed below.

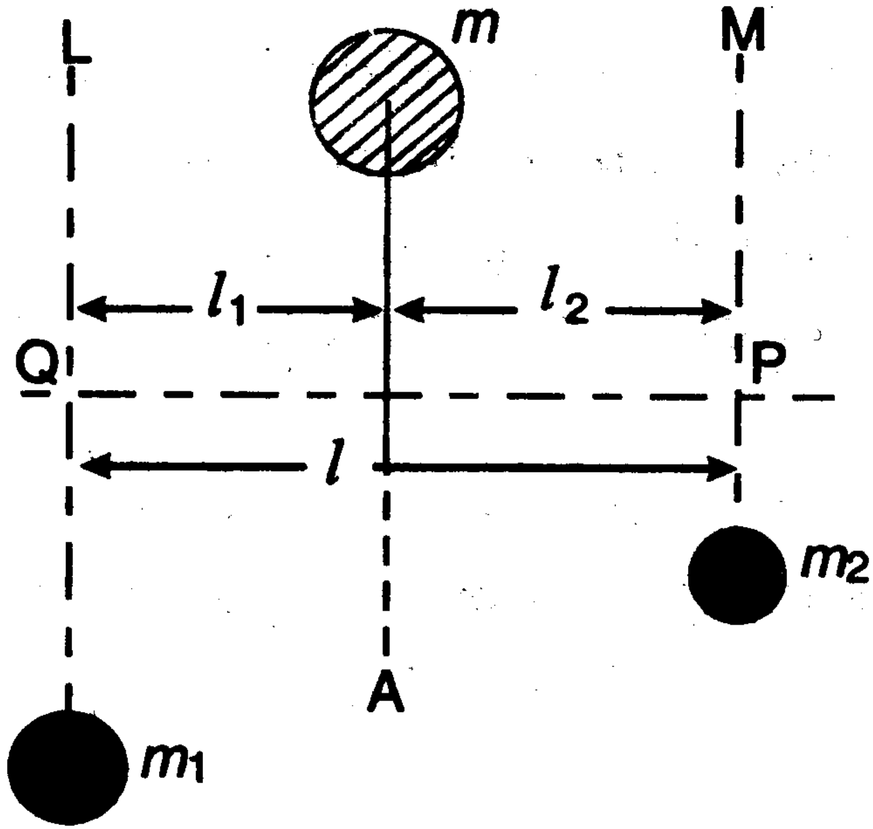
1. When the plane of the disturbing mass lies in between the planes of the two balancing masses.

   Centrifugal forces exerted by masses m, m_1 and m_2 are

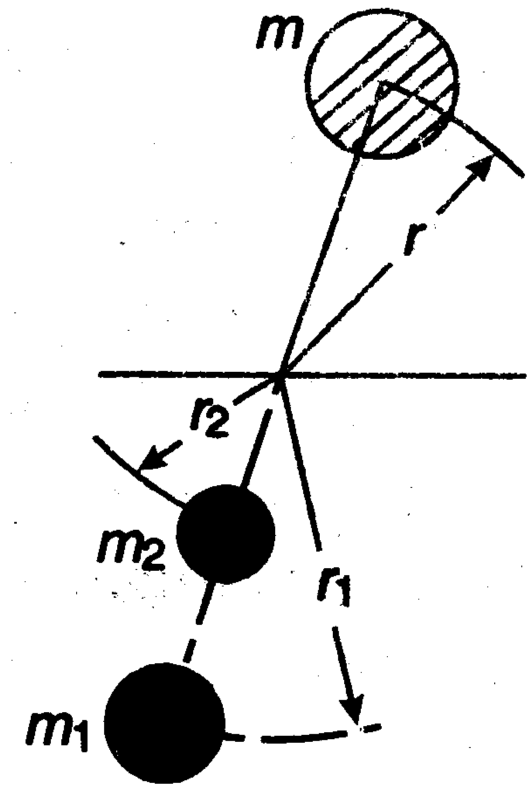
$$F_c = m.\omega^2.r$$

$$F_{c1} = m.\omega^2.r_1$$

$$F_{c2} = m.\omega^2.r_2$$



L
m
M
l_1
l_2
Q
P
l
m_2
A
m_1



m
r
r_2
m_2
r_1
m_1

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Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$m.\omega^2.r = m.\omega^2.r_1 + m.\omega^2.r_2 \qquad \text{(i)}$$

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Taking moment about P

$m_1.\omega^2.r_1 \times l = m.\omega^2.r \times l_2$ or $m_1.r_1 = m.r \times \frac{l_2}{l}$ (ii)

Taking moment about Q

$m_2.\omega^2.r_2 \times l = m.\omega^2.r \times l_1$ or $m_2.r_2 = m.r \times \frac{l_1}{l}$ (iii)

Equation (i) represents condition for static balance. For dynamic balance equations

(i), (ii) and (iii) must be satisfied.

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2. When the plane of the disturbing mass lies on one end of the planes of the two balancing masses.

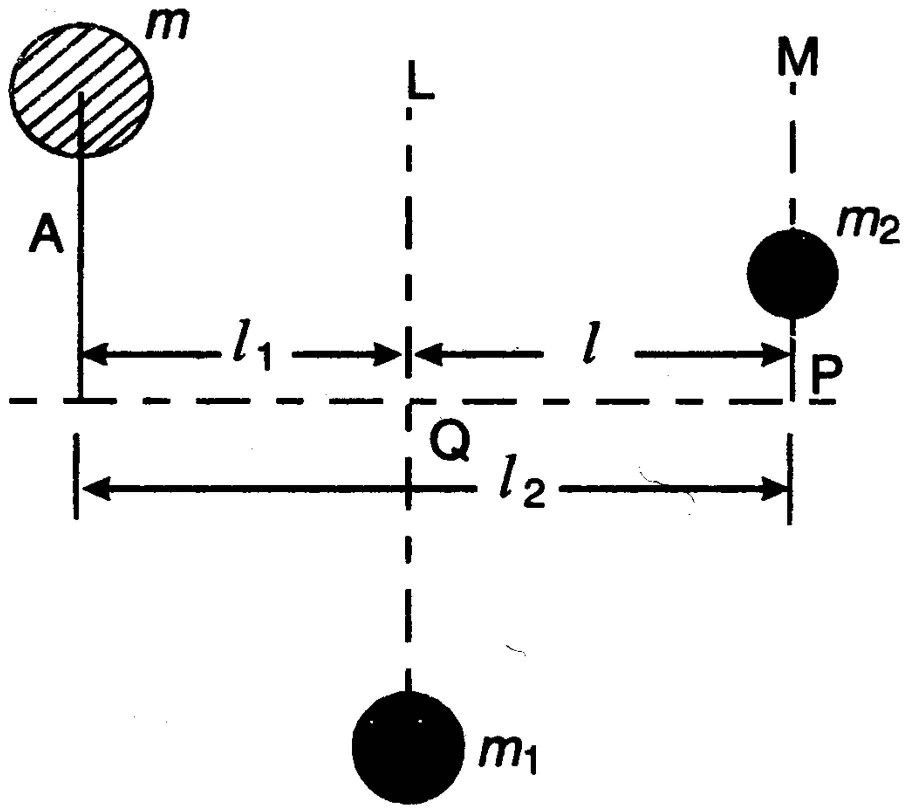
The following conditions must be satisfied in order to balance the system

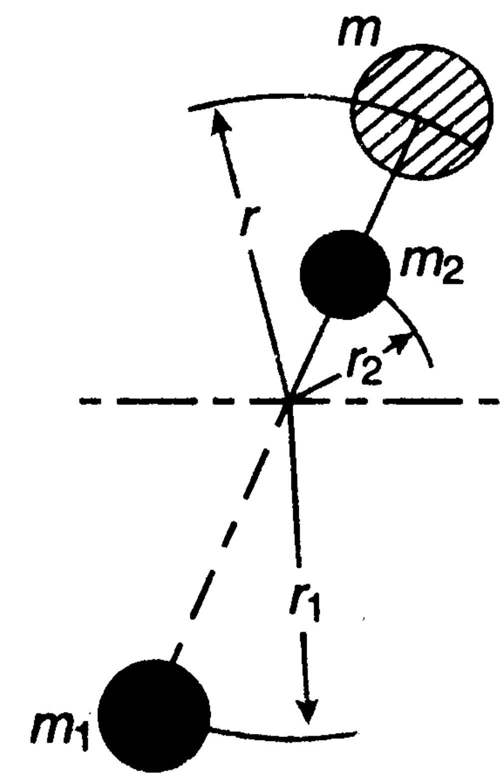
$$m.\omega^2.r + m_2.\omega^2.r_2 = m_1.\omega^2.r_1$$

$$m.r + m_2.r_2 = m_1.r_1 \qquad \text{(iv)}$$

Taking moment about P

$$m_1.\omega^2.r_1 \times l = m.\omega^2.r \times l_2$$


m
L
M
m2
A
l1
l
P
Q
l2
m1


m
r
m2
r2
r1
m1

.

or $m_1.r_1 = m.r \times \frac{l_2}{l}$

Taking moment about Q,

$m_2.\omega^2.r_2 \times l = m.\omega^2.r \times l_1$

or $m_2.r_2 = m.r \times \frac{l_1}{l}$

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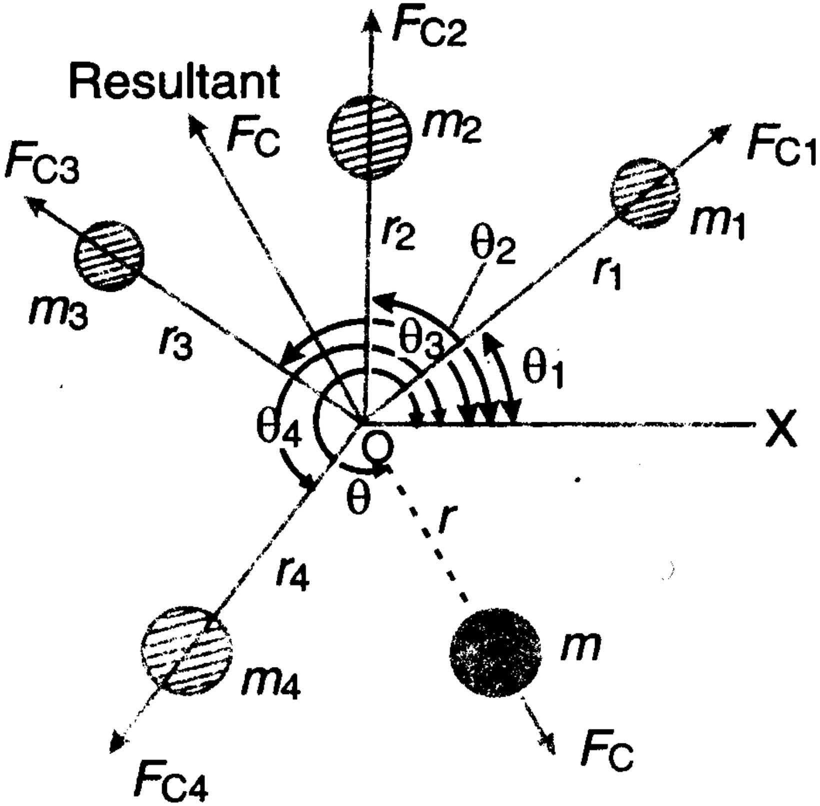
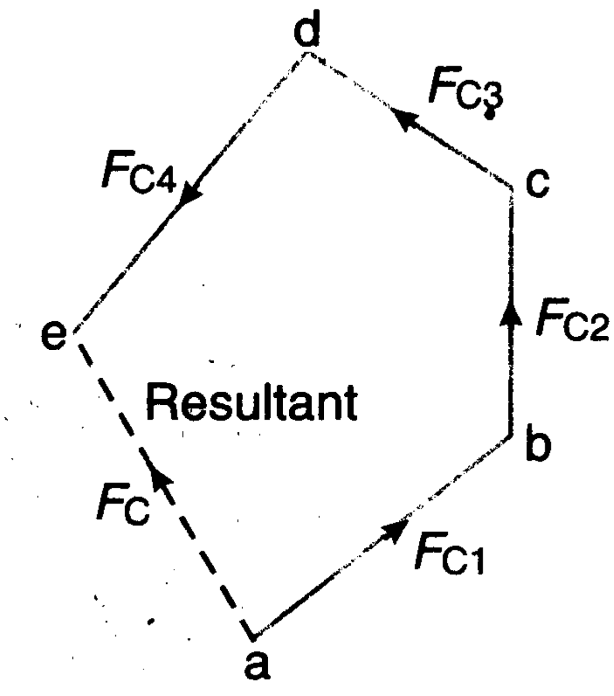
2.5 Balancing of several Masses Rotating in the Same Plane



(*a*) Space diagram.



(*b*) Vector diagram.

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1. Analytical Method

Sum of horizontal components of Centrifugal forces

$\Sigma H = m_1.r_1.\cos\theta_1 + m_2.r_2.\cos\theta_2 + \ldots\ldots..$

Sum of vertical components of Centrifugal forces

$\Sigma V = m_1.r_1.\sin\theta_1 + m_2.r_2.\sin\theta_2 + \ldots\ldots..$

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Magnitude of the resultant centrifugal force

$$F_c = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2}$$

If ɵ is the angle which the resultant makes with the horizontal, then

$\tan \theta = \Sigma V / \Sigma H$

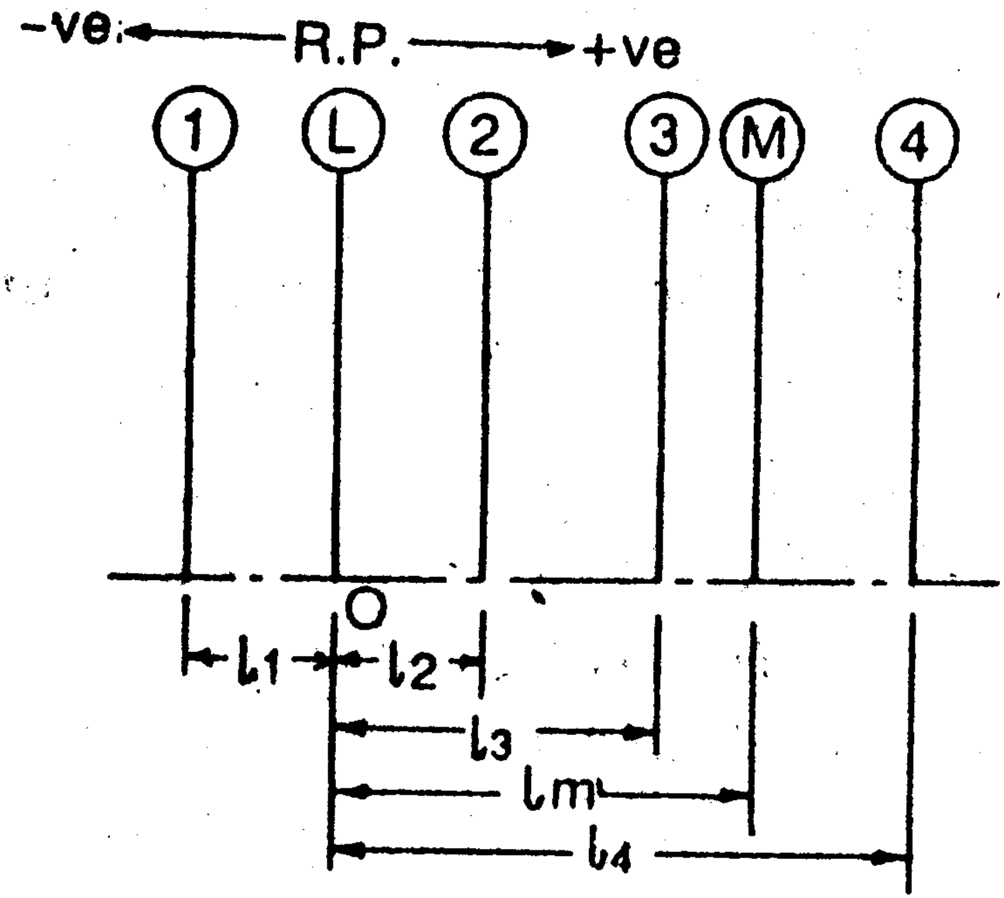
Magnitude of balancing mass

$F_c = m.r$

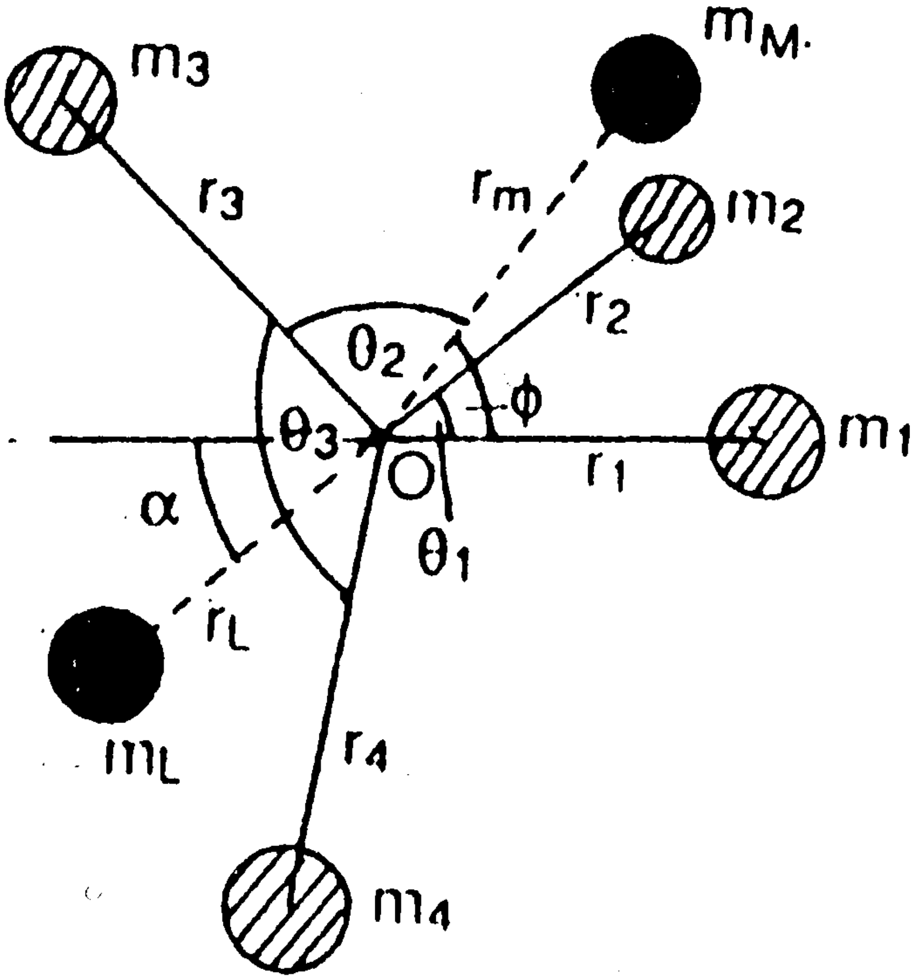
m = balancing mass

2.6 Balancing of several Masses Rotating in Different Planes

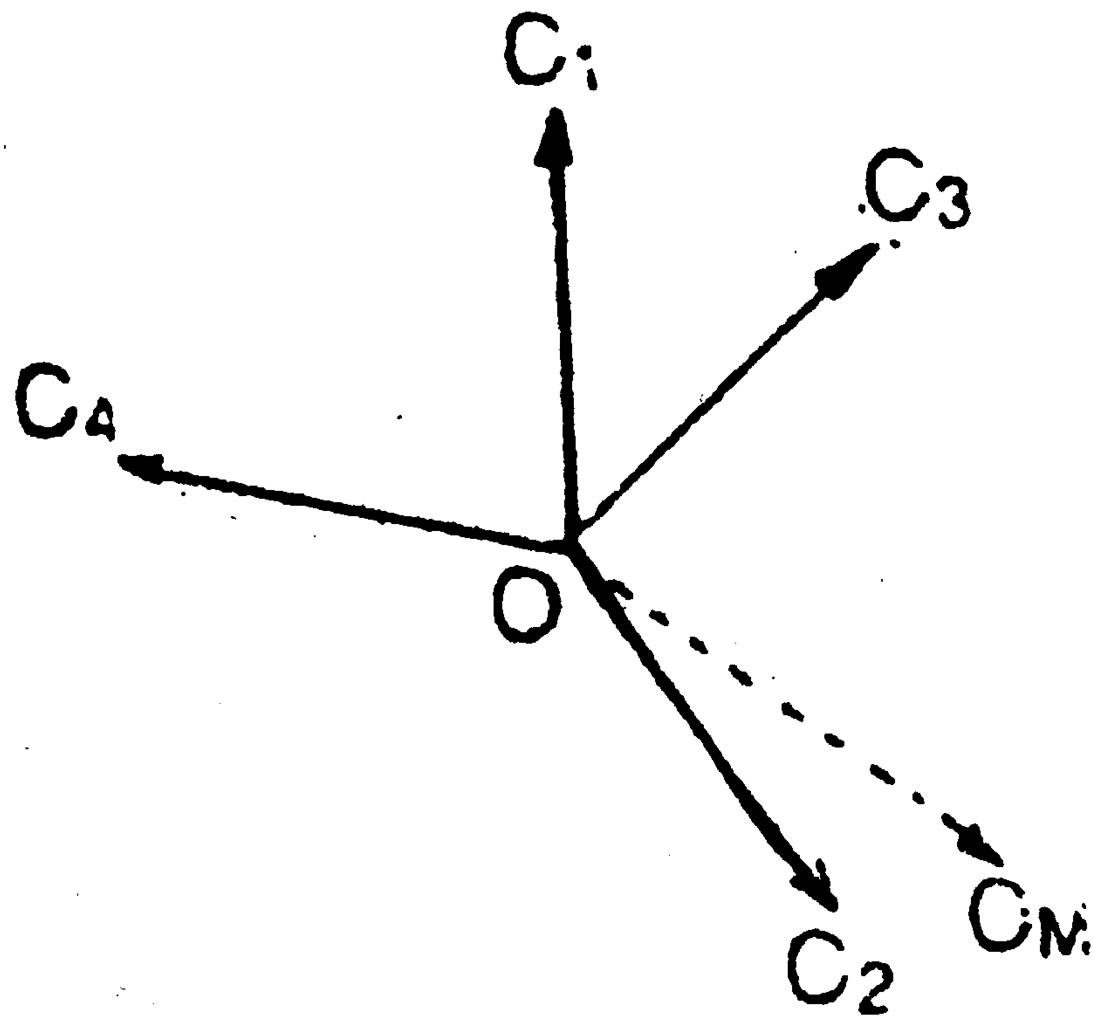
Plane (1)	*Mass (m)* (2)	*Radius (r)* (3)	*Cent. force* ÷ ω^2 *(m.r)* (4)	*Distance from plane L (l)* (5)	*Couple* ÷ ω^2 *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L (R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



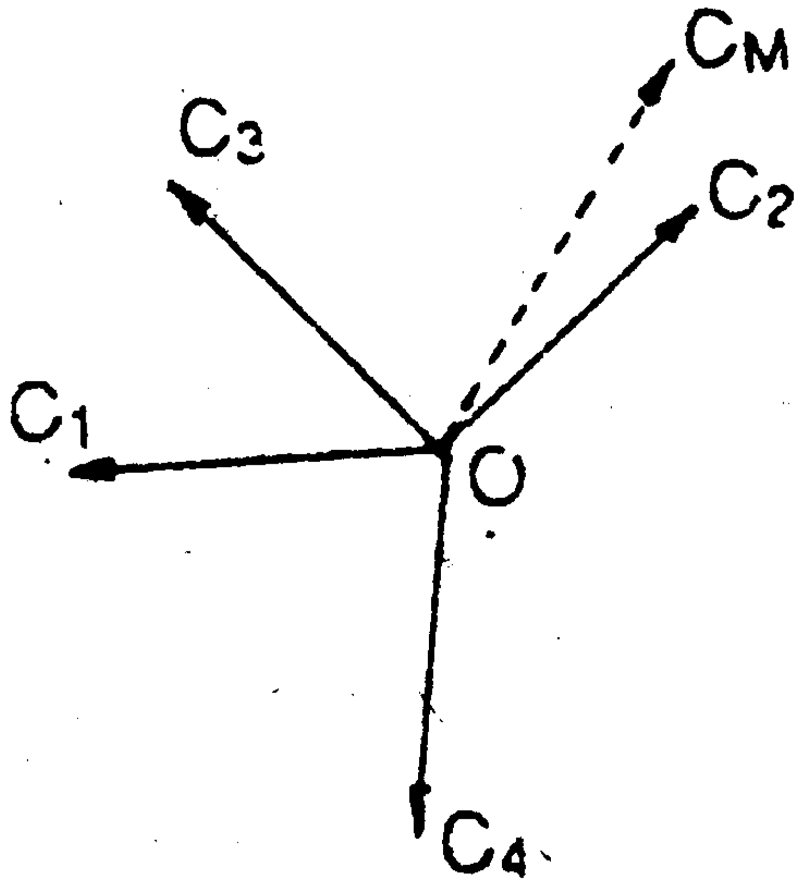
(*a*) Position of planes of the masses.



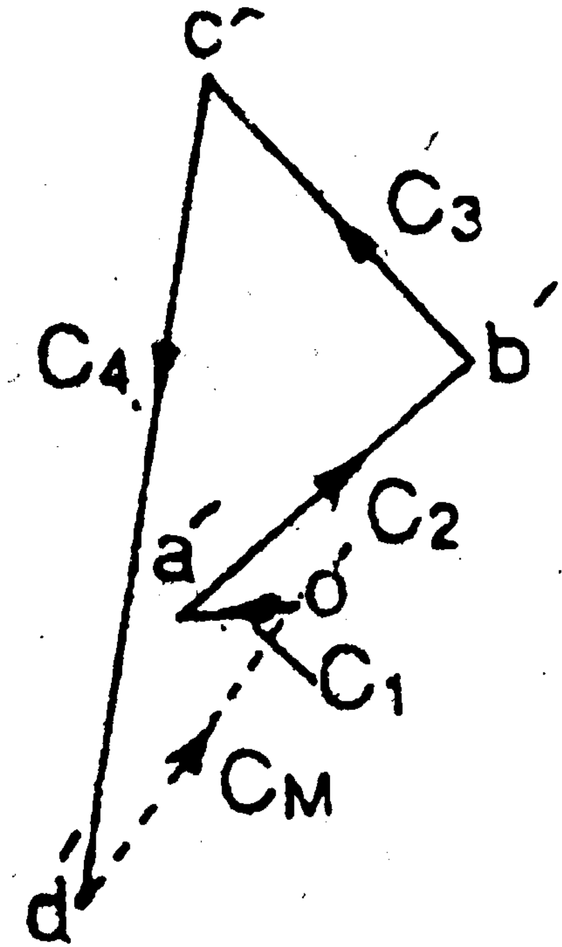
Angular position of masses



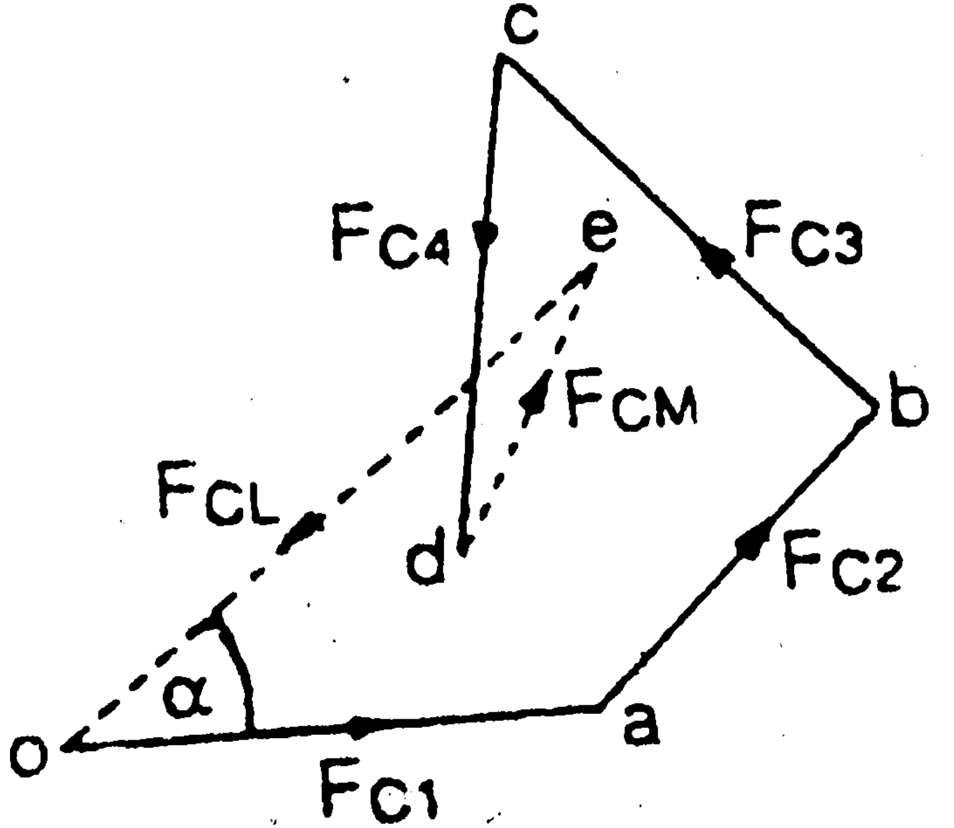
Couple vector diagram.



Couple vectors turned counter clockwise through a right angle



Couple polygon



Force polygon.

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2.7 Balancing of Reciprocating Masses

Consider a horizontal reciprocating engine mechanism.

Let F_R =Force required to accelerate the reciprocating parts

F_I = Inertia force due to reciprocating parts.

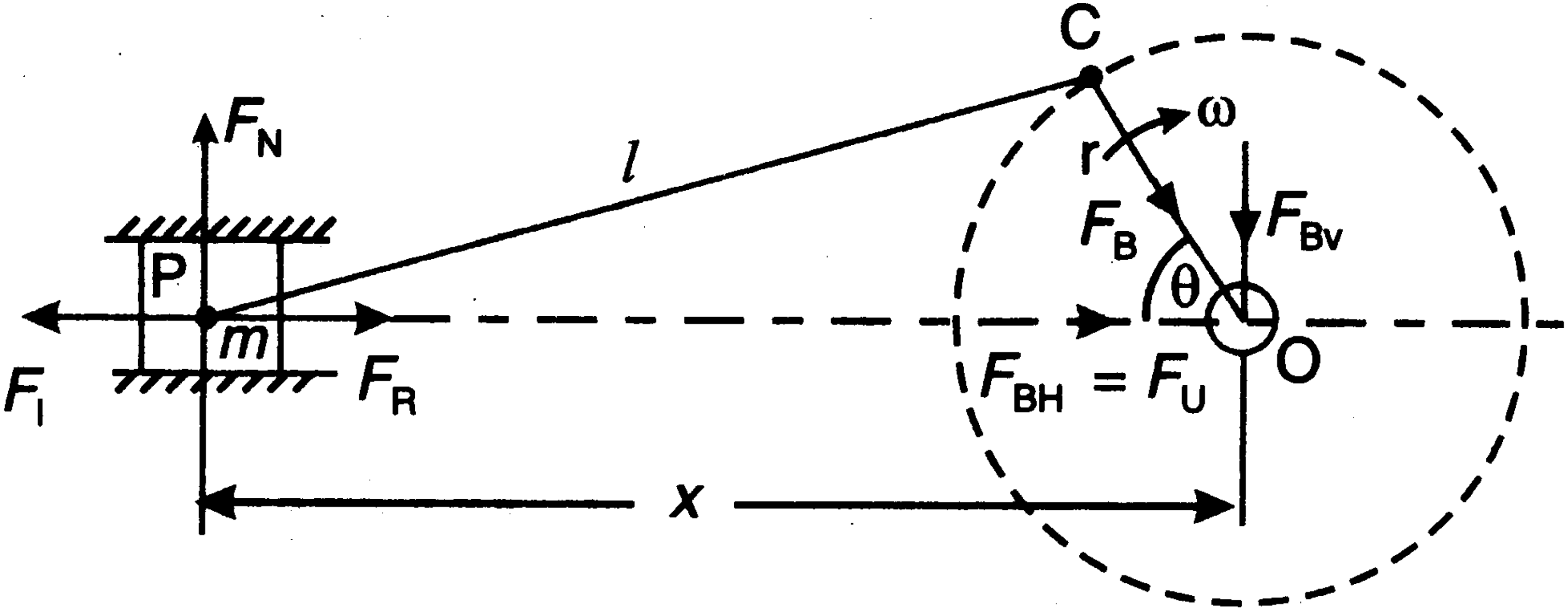

Fig. 22.1. Reciprocating engine mechanism.

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F_N = Force on the side of the cylinder walls or normal force acting on the cross-head guides.

F_B = Force acting on the crankshaft bearing or main bearing.

F_R and F_I balance each other.

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The force $F_{BH} = F_U$ is an unbalanced force or shaking force and is required to be properly balanced.

The force on the side of the cylinder walls (F_N) and the vertical component of F_B (F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$. Shaking force and shaking couple cause very objectionable vibrations.

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2.8 Primary and Secondary Unbalanced Forces of Reciprocating Masses.

Consider a reciprocating engine mechanism.

Let m = mass of the reciprocating parts

l = length of the connecting rod PC

r = radius of the crank OC

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ɵ = Angle of inclination of crank with the line of stroke PO,

ω = Angular speed of the crank,

n = ratio of length of connecting rod to the crank radius = l/r

Acceleration of the reciprocating parts is given by the relation

$$a_R = \omega^2 . r \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

Inertia force due to reciprocating parts

$$F_1 = F_R = m.\omega^2 . r \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

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Horizontal component of the force exerted on the crankshaft bearing (F_{BH}) is equal and opposite to the inertia force (F_I) and is denoted by F_U

$$F_U = m.\omega^2.r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= m.\omega^2.r\cos\theta + m.\omega^2.r \times \frac{\cos\theta}{n}$$

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The expression $m.\omega^2.r\cos\theta$ is known as

primary unbalanced force and $m.\omega^2.r \times \frac{\cos\theta}{n}$

is called secondary unbalanced force.

The primary unbalanced force is maximum

when ϴ = 0˚ or 180˚.

The secondary unbalanced force is maximum

when ϴ = 0˚, 90˚. 180˚ or 270˚.

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2.9 Partial Balancing of Unbalanced force in a Reciprocating Engine

The primary unbalanced force ($m.\omega 2.r \cos \theta$) may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius r. Balancing of primary force is considered as equivalent to balancing of a mass m rotating at the crank radius r.

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This is balanced by having a mass B at radius b, placed diametrically opposite to the crank pin C.

The primary force is balanced if

$B.\omega_2.b\cos\Theta = m.\omega^2.r\cos\Theta$ or $B.b = m.r.$

The centrifugal force produced due to revolving mass B has also a vertical component of magnitude $B.\omega^2.b.\sin\Theta$. This force remains unbalanced. As a compromise let a fraction ‘c’ of the reciprocating masses is balanced, such that

$c.m.r = B.b$

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Unbalanced force along the line of stroke

$= m.\omega^2.r \cos \theta – B.\omega^2.b.\cos \theta$

$= m.\omega^2.r \cos \theta – c.m.\omega^2.r \cos \theta$

$= (1 – c)\, m.\omega^2.r \cos \theta$

Unbalanced force along the perpendicular to the line of stroke

$= B.\omega^2.b.\sin \theta = c.m.\omega^2.b.\sin \theta$

Resultant unbalanced force at any moment

$$= \sqrt{\left[(1-c) m.\omega^2 .r \cos\theta \right]^2 + \left[c.m.\omega^2 .r \sin\theta \right]^2}$$

$$= m.\omega^2 .r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

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2.10 The two cylinder locomotive may be classified as:

Inside cylinder locomotives

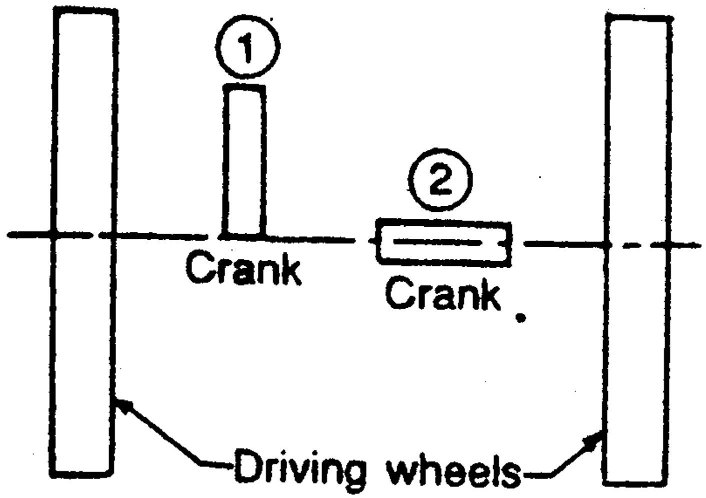
Outside cylinder locomotives.

The locomotives may be

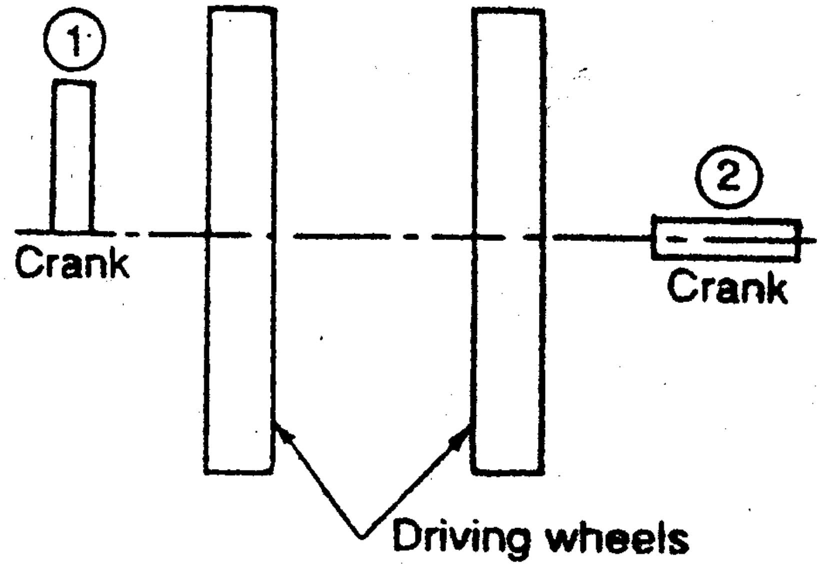
Single or uncoupled locomotives

Coupled locomotives

In coupled locomotives the driving wheels are connected to the trailing and leading wheels by an outside coupling rod.



(*a*) Inside cylinder locomotives.



(*b*) Outside cylinder locomotives.

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2.11 Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives.

Due to partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke; and 2. Swaying couple

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The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. The maximum magnitude of unbalanced force along the perpendicular to the line of stroke is known as hammer blow.

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2.12 Variation of Tractive Force

The resultant unbalanced force due to two cylinders along the line of stroke is known as tractive force.

Unbalanced force along the line of stroke for cylinder 1

$= (1 – c)\ m.\omega^2.r \cos \Theta$

Unbalanced force along the line of stroke for cylinder 2

$= (1 – c)\ m.\omega^2.r \cos (90^\circ + \Theta)$

Tractive force F_T

$= (1 – c)\ m.\omega^2.r (\cos \Theta – \sin \Theta)$

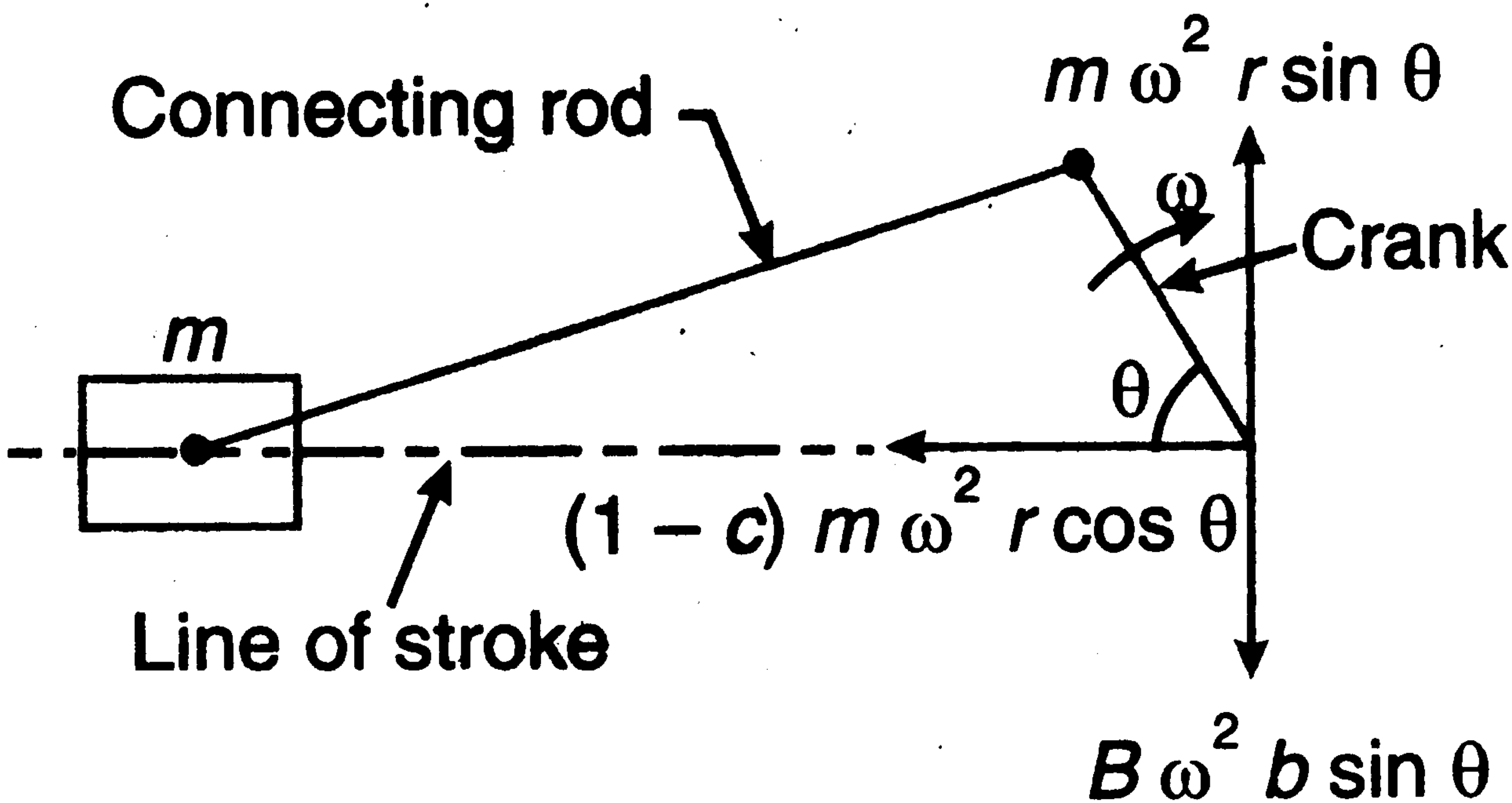

Fig. 22.4. Variation of tractive force.

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The tractive force is maximum or minimum when (cos ϴ – sin ϴ) is maximum or minimum. Therefore

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$

or ϴ = 135˚ or 315˚

Maximum or minimum value of tractive force

$= \pm (1- c)\, m.\omega^2.r\, (\cos 135^\circ - \sin 135^\circ)$

$= \pm \sqrt{2}\, (1 - c)\, m.\omega^2.r$

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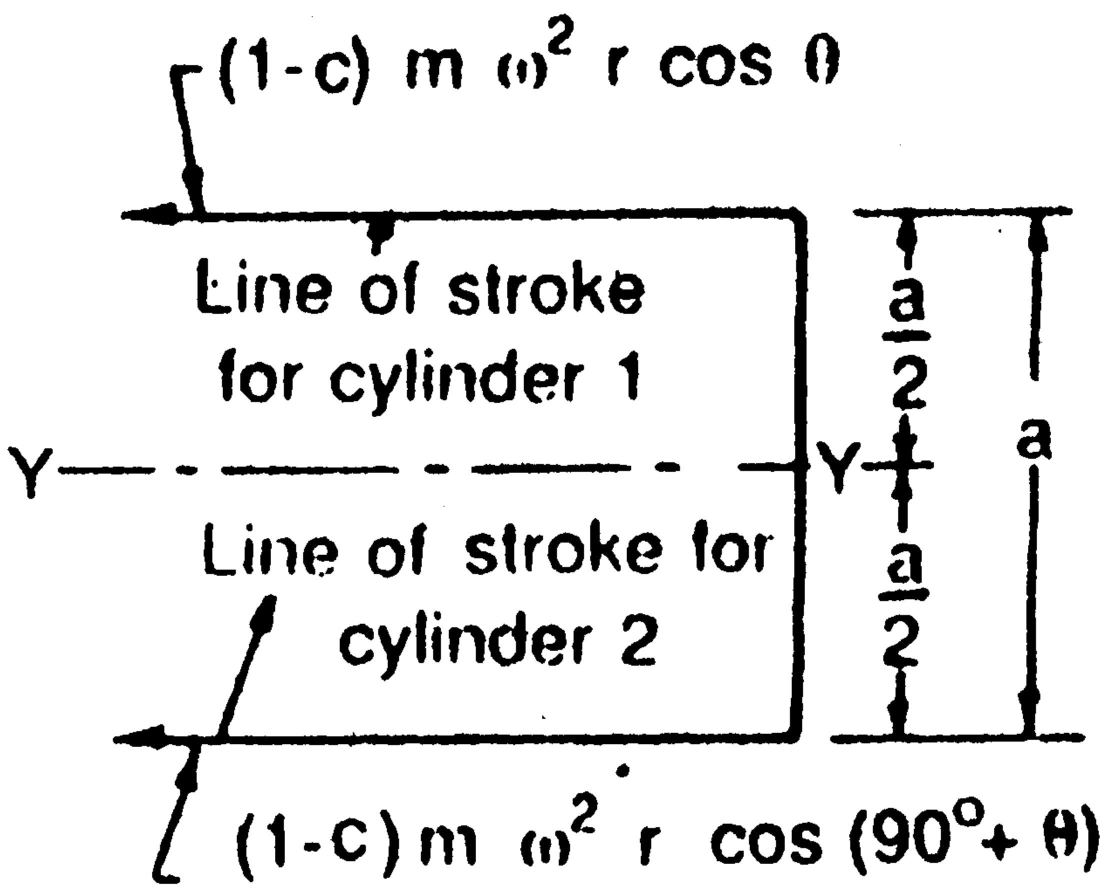
2.13 Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line YY between the cylinders.

This couple has swaying effect about a vertical axis.



Swaying Couple

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Swaying couple

$$= (1 - c)m.\omega^2.r\cos\theta \times \frac{a}{2}$$

$$-(1 - c)m.\omega^2.r\cos(90^\circ + \theta)\frac{a}{2}$$

$$= (1 - c)m.\omega^2.r\cos\theta \times \frac{a}{2}(\cos\theta + \sin\theta)$$

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The swaying couple is maximum or minimum when (cos Ө + sin Ө) is maximum or minimum or when Ө = 45˚ or 225˚.

Maximum swaying couple

$$= \pm (1-c) m.\omega^2.r \cos\theta \times \frac{a}{2} (\cos 45^\circ + \sin 45^\circ)$$

$$= \pm \frac{a}{\sqrt{2}} (1-c) m.\omega^2.r$$

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2.14 Hammer Blow

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as hammer blow.

Hammer blow = $B.\omega^2.b$

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail.

Let P be the downward pressure on the rails (or static wheel load).

Net pressure between the wheel and the rail = $P \pm B.\omega^2.b$.

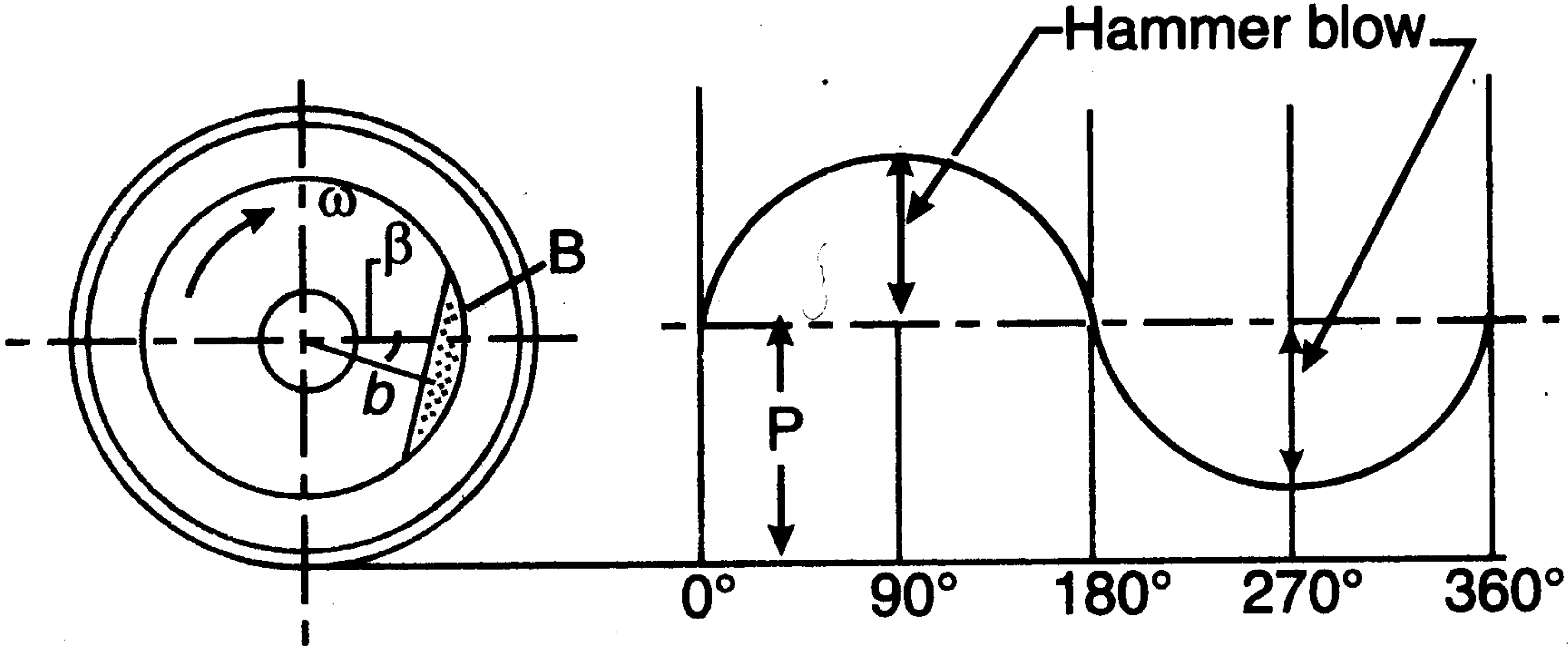

Fig. 22.6. Hammer blow.

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If $(P – B.\omega^2.b)$ is negative, then the wheel will be lifted from the rail.. Therefore the limiting condition that the wheel is not lifted from the rail is given by

$P = B.\omega^2.b$

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2.12 Balancing of Coupled Locomotives

In a coupled locomotive, the driving wheels are connected to the leading and trailing wheels by an outside coupling rod. The coupling rod cranks are placed diametrically opposite to the adjacent main cranks. The coupling rods together with cranks and pins may be treated as rotating masses and completely balanced by masses in the respective wheels. Hammer blow may be reduced by equal distribution of balanced mass (B) between the driving, leading and trailing wheels.

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2.13 Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same planes and on the same side of the centre line of the crankshaft, are known as In-line engines. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts.

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1. The algebraic sum of the primary forces must be equal to zero.
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero.

In order to give the primary balance of reciprocating parts of a multi-cylinder , it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.

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Notes: 1. For a two cylinder engine with cranks at 180°, condition (1) may be satisfied, but an unbalanced couple will remain.

2. For a three cylinder engine with cranks at 120° and same reciprocating masses per cylinder , condition (1) will be satisfied but unbalanced couples will remain.

3. For a four cylinder engine, primary balance is possible.

For a four-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided the number of cranks are not less than four.

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2.14 Balancing of Secondary Forces of Muti-cylinder In-line Engines

Secondary Force

$$F_s = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

The expression may be written as

$$F_s = m.(2\omega)^2.\frac{r}{4n} \times \cos 2\theta$$

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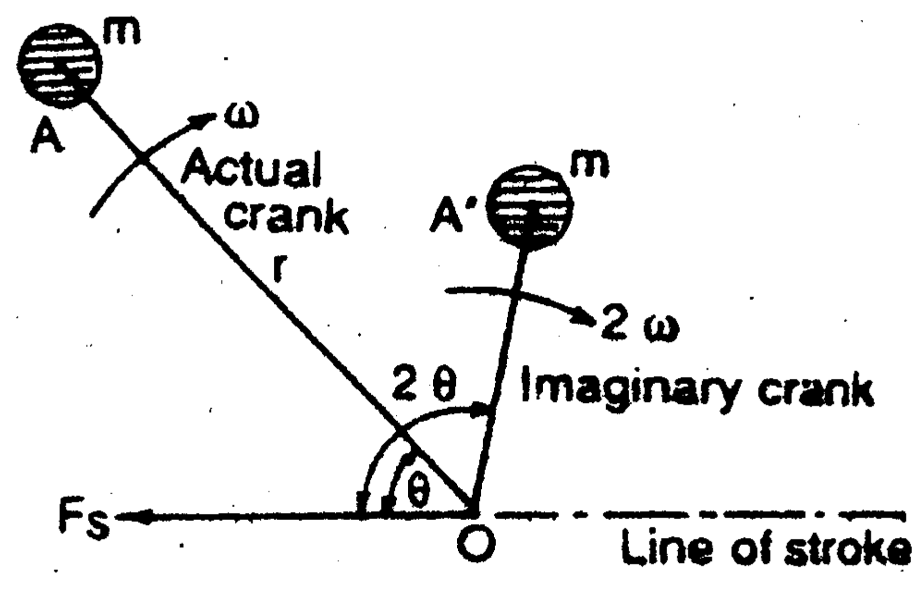
The secondary forces may be considered to be equivalent to the component , parallel to the line of stroke, of the centrifugal force produced by an equal mass at the imaginary crank of length r/4n and revolving at twice the speed of the actual crank.



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2.15 Balancing of Radial Engines (Direct and Reverse Cranks Method)

The Primary Forces

Let us suppose that mass (m) of the reciprocating parts is divided into two parts, each equal to m/2. It is assumed that m/2 is fixed at the primary direct crank pin C and m/2 at the secondary reverse crank pin C'.

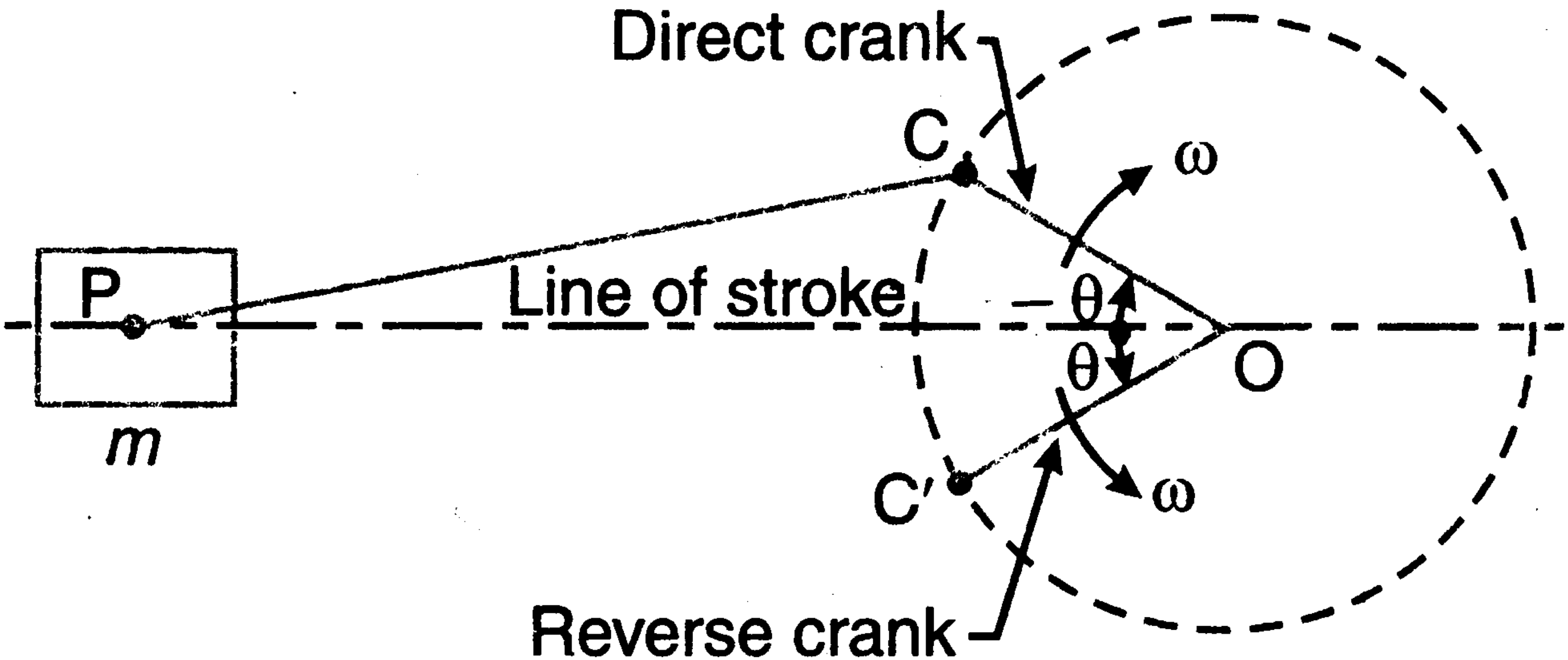

Fig. 22.27. Reciprocating engine mechanism.

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The centrifugal force acting on the primary direct and reverse crank = $\frac{m}{2} \times \omega^2 . r$

Component of the centrifugal force acting on the primary direct crank $= \frac{m}{2} \times \omega^2 . r \cos\theta$ (in the direction from O to P)

Component of the centrifugal force acting on the primary reverse crank $= \frac{m}{2} \times \omega^2 . r \cos\theta$ (in the direction from O to P)

Total component of the centrifugal force acting along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2 . r \cos\theta = m \times \omega^2 . r \cos\theta$$

$$= \Pr imary\, Force, F_P$$

Hence, for primary effects, the mass m of reciprocating parts at P may be replaced by two masses at C and C’ each of magnitude m/2.

The components in a direction perpendicular to the line of stroke are balanced.

Secondary Force $= m \times (2\omega)^2 . \dfrac{r}{4n} \times \cos 2\theta$

$$= m.\omega^2 r \times \frac{\cos 2\theta}{n}$$

For secondary effects, the mass (m) of the reciprocating parts may be replaced by two masses (each m/2) placed at D and D' such that OD = OD' = r/4n

2.16 Balancing of V-engines

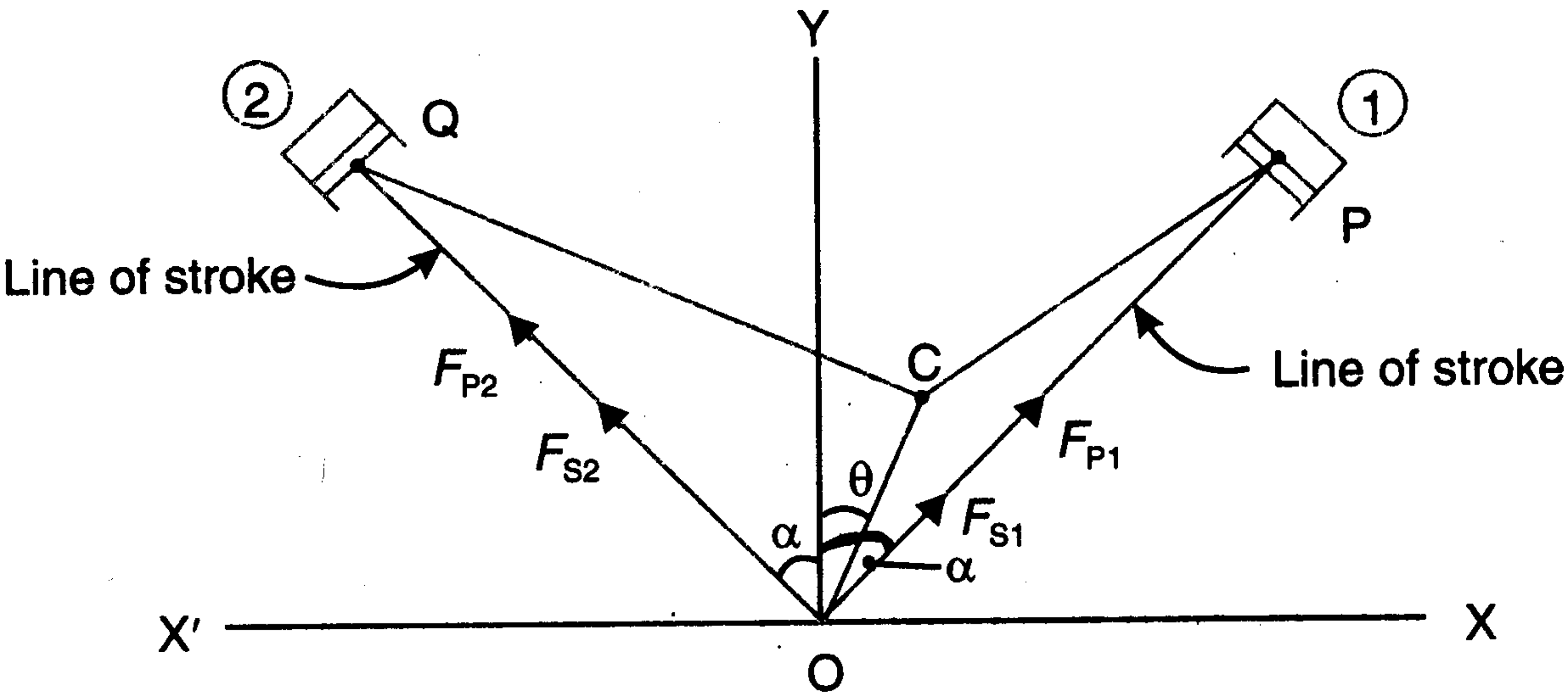

Fig.22.33. Balancing of V-engines.

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Inertia force due to reciprocating parts due to cylinder 1

$$= m.\omega^2.r\cos(\alpha-\theta) + \frac{\cos 2(\alpha-\theta)}{n}$$

Inertia force due to reciprocating parts due to cylinder 2

$$= m.\omega^2.r\cos(\alpha+\theta) + \frac{\cos 2(\alpha+\theta)}{n}$$

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Primary forces

Primary forces acting along the line of stroke of cylinder 1,

$$F_{P1} = m.\omega^2 \, r \cos(\alpha - \Theta)$$

Component of F_{P1} along the vertical line OY

$$F_{P1} \cos\alpha = m.\omega^2 \, r \cos(\alpha - \Theta) \cos\alpha$$

Component of F_{P1} along the horizontal line OX

$$F_{P1} \sin\alpha = m.\omega^2 \, r \cos(\alpha - \Theta) \sin\alpha$$

.

Primary forces acting along the line of stroke of cylinder 2,

$$F_{P2} = m.\omega^2 r \cos(\alpha + \Theta)$$

Component of F_{P2} along the vertical line OY

$$F_{P2} \cos\alpha = m.\omega^2 r \cos(\alpha + \Theta) \cos\alpha$$

Component of F_{P1} along the horizontal line OX'

$$F_{P2} \sin\alpha = m.\omega^2 r \cos(\alpha + \Theta) \sin\alpha$$

.

Total component of primary force along the vertical line OY

$F_{PV} = m.\omega^2 r \cos\alpha\,[\cos(\alpha - \Theta) + \cos(\alpha + \Theta)]$

$= 2\, m.\omega^2 r \cos^2\alpha.\cos\Theta$

$[\because \cos(\alpha - \Theta) + \cos(\alpha + \Theta)]$

$= 2 \cos\alpha \cos\Theta$

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Total component of primary force along the horizontal line OX

$= m.\omega^2\ r \sin\alpha\ [\cos(\alpha - \Theta) - \cos(\alpha + \Theta)]$

$= 2\ m.\omega^2\ r \sin^2\alpha.\sin\Theta$

Resultant primary force

$$F_P = \sqrt{\left(F_{PV}\right)^2 + \left(F_{PH}\right)^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2\alpha.\cos\theta)^2.+(\sin^2\alpha.\sin\theta)^2}$$

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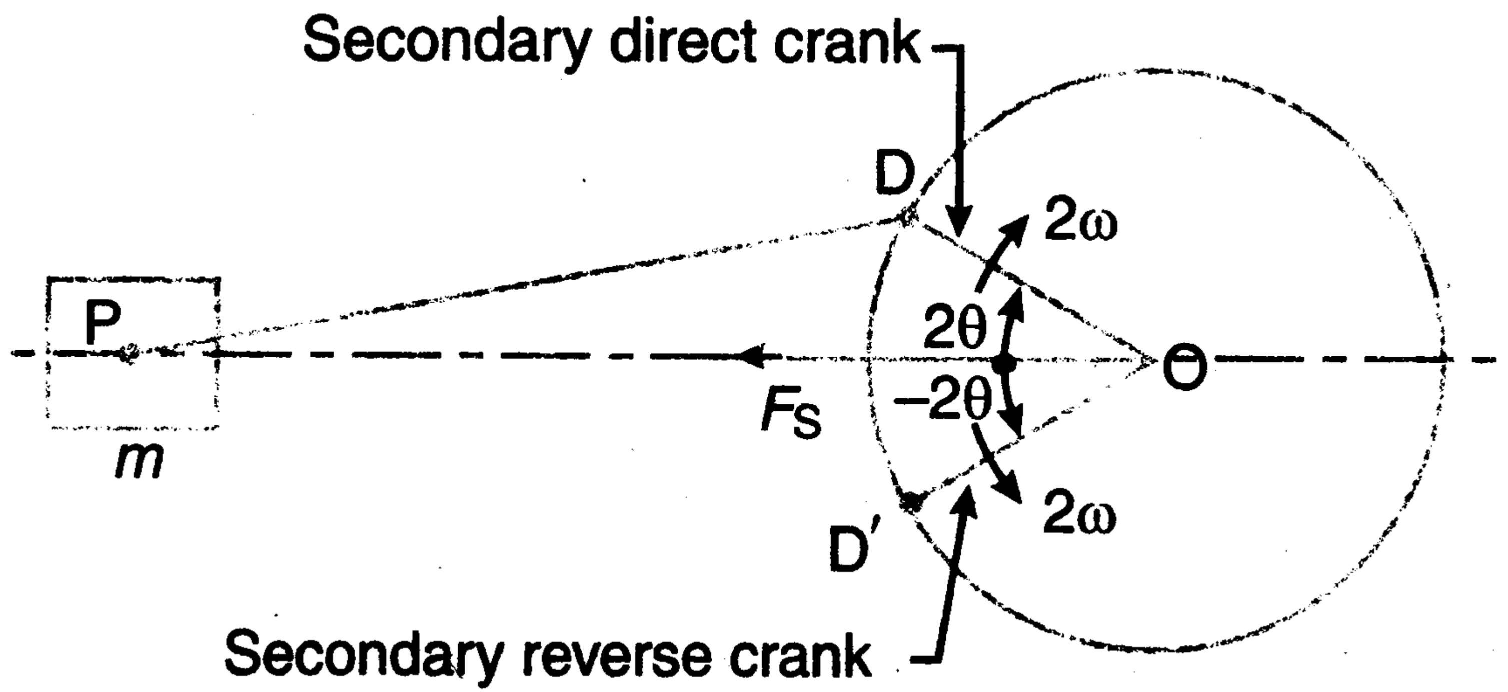
Secondary forces

Secondary force acting along the line of stroke of cylinder 1,

$$F_{S1} = m.\omega^2 r \times \frac{\cos 2(\alpha - \theta)}{n}$$

$\therefore$ component of F_{S1} along the vertical line OY

$$= F_{S1}\cos\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha - \theta)}{n} \times \cos\alpha$$



Secondary direct crank
D
2ω
P
2θ
O
-2θ
F_S
m
D′
2ω
Secondary reverse crank

.

component of F_{S1} along the horizontal line OX

$$F_{S1}\sin\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha-\theta)}{n} \times \sin\alpha$$

Secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n}$$

Component of F_{S2} along the vertical line OY

$$F_{S2}\cos\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n} \times \cos\alpha$$

.

component of F_{S2} along the Horizontal line OX'

$$F_{S2}\sin\alpha = m.\omega^2 r \times \frac{\cos 2(\alpha+\theta)}{n} \times \sin\alpha$$

Total component of secondary force along the vertical line OY,

$$F_{SV} = \frac{m}{n} \times \omega^2.r\cos\alpha\left[\cos 2\left(\alpha-\theta\right)+\cos 2\left(\alpha+\theta\right)\right]$$

$$= \frac{2m}{n} \times \omega^2.r\cos\alpha \times \cos 2\alpha\cos 2\theta$$

.

Total component of secondary force along the vertical line OX',

$$F_{SH} = \frac{m}{n} \times \omega^2 .r \sin\alpha \left[\cos 2\left(\alpha - \theta\right) - \cos 2\left(\alpha + \theta\right) \right]$$

$$= \frac{2m}{n} \times \omega^2 .r \sin\alpha \times \cos 2\alpha \sin 2\theta$$

Resultant secondary force

$$F_s = \frac{2m}{n} .\omega^2 .r \sqrt{(\cos\alpha . \cos 2\alpha . \cos 2\theta)^2 . + (\sin\alpha \sin 2\alpha . \sin 2\theta)^2}$$

.

Example 1: A shaft is rotating at a uniform angular speed. Two masses of 300 kg and 450 kg are attached rigidly to the shaft. The masses are rotating in the same plane. The corresponding radii of rotation are 200 mm and 150 mm. The angle between two masses is 90˚, find magnitude and direction of balancing mass in the same plane, if its radius of rotation is 200 mm.

(W08)

.

Solution:

$m_1.r_1 = 300 \times 0.2 = 60$ kg-m

$m_2.r_2 = 450 \times 0.15 = 67.5$ kg-m

Resultant R = $\sqrt{60^2 + 67.5^2}$ = 90.31 kg-m

m.r = R, m = $\frac{R}{r} = \frac{90.31}{0.2} = 451.56\,kg$

$$\tan\theta = \frac{450}{300} = 1.5$$

$\theta = 56.31^\circ$

.

Example 2: A disturbing mass of 500 kg attached to a shaft is rotating at a uniform angular velocity of ω rad/s. The distance of the centre of gravity of the disturbing mass from the axis of rotation is 250 mm. The disturbing mass is to be balanced by two masses in two different planes parallel to the plane of rotation of the disturbing mass. The distance of centre of gravity of the balancing masses from the axis of rotation is 400 mm. The distance between the two planes of balancing masses is 1500 mm.

.

The distance between the plane of disturbing mass and one plane of balancing mass is 300 mm. Determine the magnitude of balancing mass when:

(i) The planes of balancing masses are on the same side of plane of disturbing mass.

(ii) The planes of balancing masses are on either side of the plane of two disturbing masses. **(W07)**

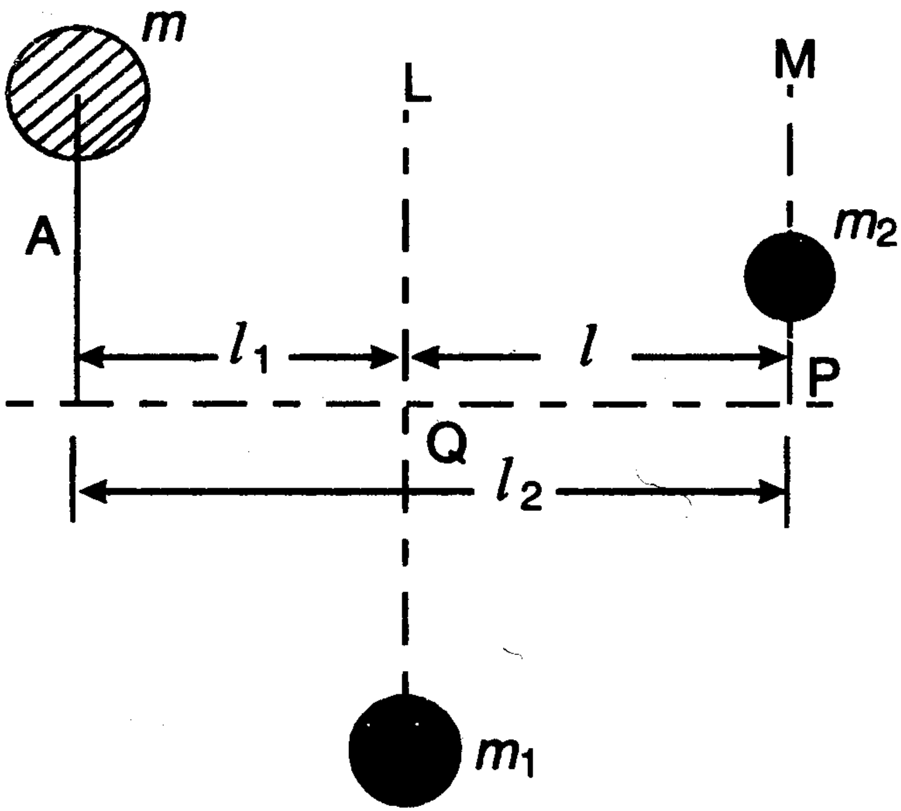
.

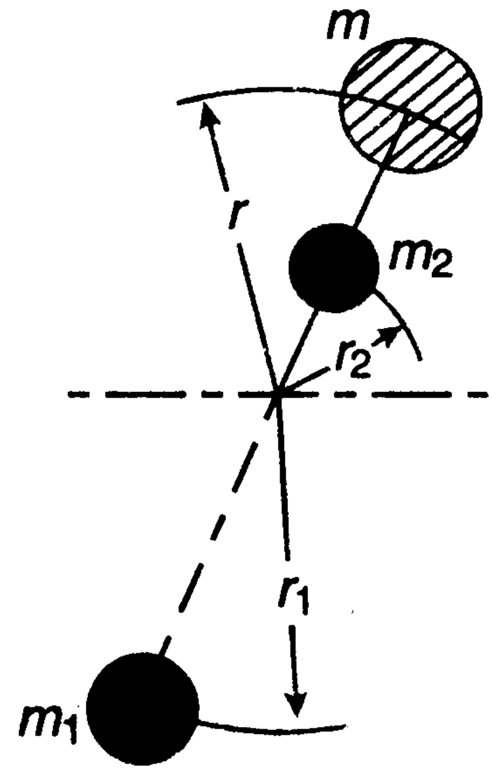
Solution:

Given: $m = 500$ kg; $l = 1.5$ m; $l_1 = 0.3$ m;

$l_2 = 1.8$ m.

(i) The planes of balancing masses are on the same side of plane of disturbing mass.


m
L
M
A
m_2
l_1
l
P
Q
l_2
m_1


m
r
m_2
r_2
r_1
m_1

.

Taking moment about P

$m_1 \times r_1 \times l = m \times r \times l_2$

$$m_1 = \frac{m \times r \times l_2}{r_1 \times l} = \frac{500 \times 0.25 \times 1.8}{0.4 \times 1.5} = 375\,kg$$

Taking moment about Q

$m_2 \times r_2 \times l = m \times r \times l_1$

$$m_2 = \frac{m \times r \times l_1}{r_2 \times l} = \frac{500 \times 0.25 \times 0.3}{0.4 \times 1.5} = 62.5\,kg$$

.

2. The planes of balancing masses are on either side of the plane of two disturbing masses.

Taking moment about P

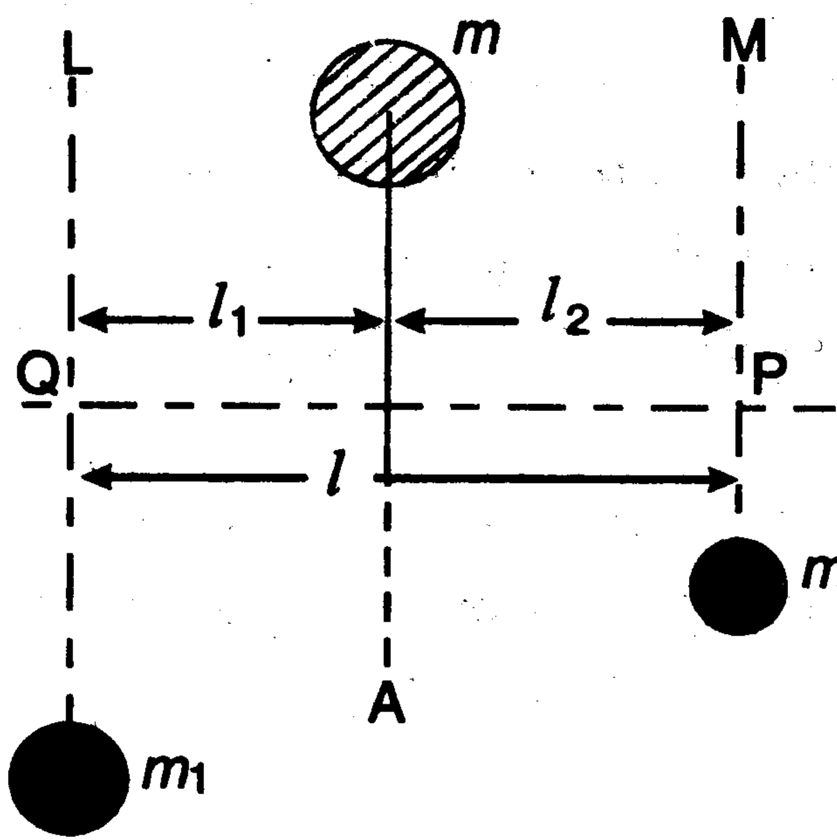
$m_1.r_1 .l = m.r.l_2$

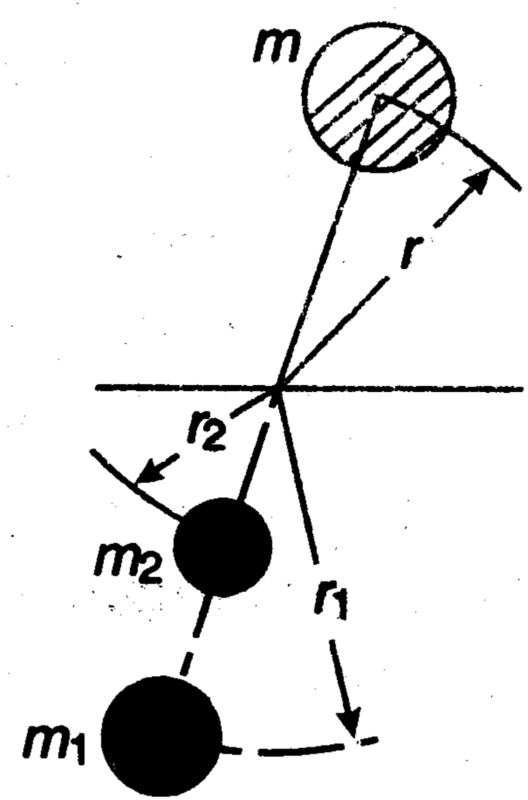
$$m_1 = \frac{m \times r \times l_2}{r_1 \times l} = \frac{500 \times 0.25 \times 1.2}{0.4 \times 1.5} = 250\,kg$$

Taking moment about Q

$m_2.r_2 .l = m.r.l_1$

$$m_2 = \frac{m \times r \times l_1}{r_2 \times l} = \frac{500 \times 0.25 \times 0.3}{0.4 \times 1.5} = 62.5\,kg$$


m
L
M
l_1
l_2
Q
P
l
m_2
A
m_1


m
r
r_2
m_2
r_1
m_1

.

Example 3: A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45˚, B to C 70˚ and C to D 120˚

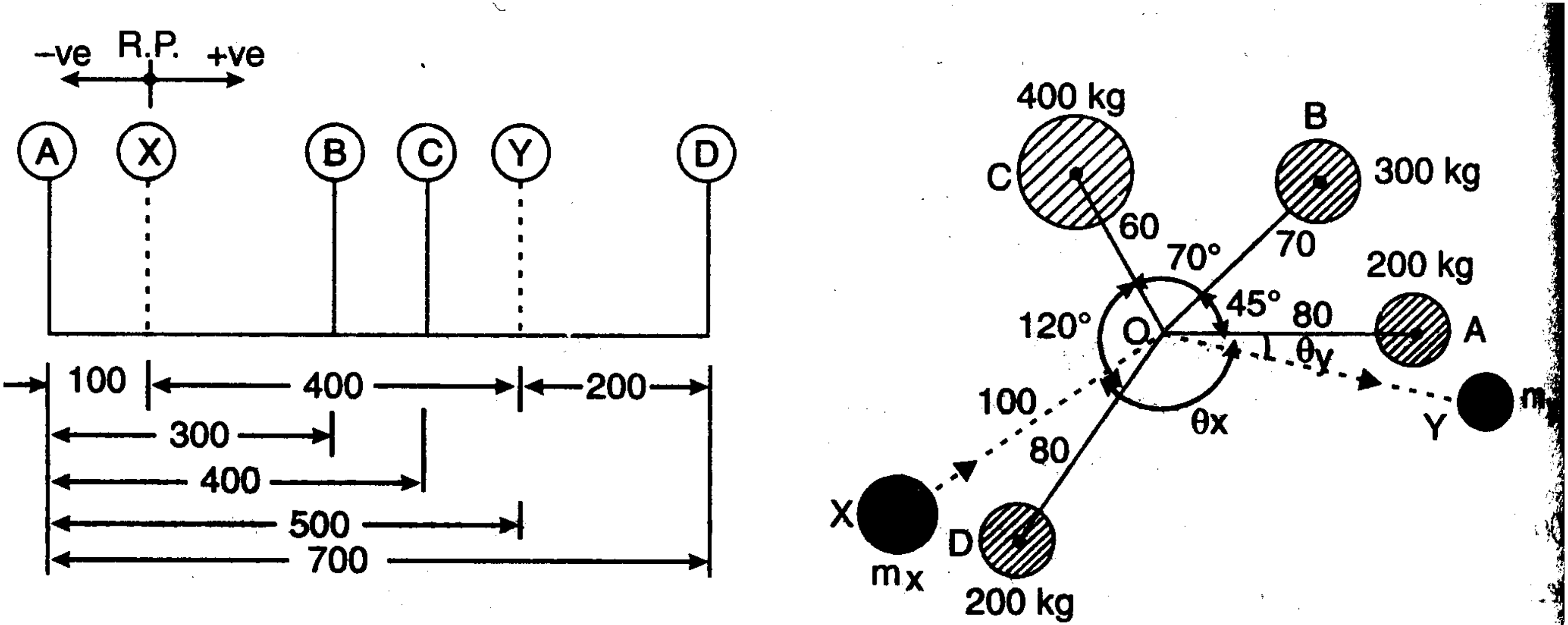
.

The balancing masses are to be placed in planes X and Y. the distance between the planes A and X are 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular position.

K/843/(S08)

Solution.

Plane (1)	Mass (m) kg (2)	Radius (r) m 3)	Cent. force÷ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple÷ω^2 (m.r.l) kg-m^2 (6)
A	200	0.08	16	0.1	1.6
X(R.P.)	m_X	0.1	$0.1m_X$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1m_y$	0.4	$0.04m_Y$
D	200	0.08	16	0.6	0.6



All dimensions in mm.

(*a*) Position of planes.

(*b*) Angular position of masses.

.

The vector d'o' represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y , therefore by measurements

0.04 m_Y = vector d'o' = 7.3 kg.m^2 or m_Y = 182.5 kg.

The angular position of mY is θ_Y= 12˚ in the clockwise direction from mass m_A.

.

Example 4: Four masses A, B, C and D are completely balanced. Masses C and D make angles of 90˚ and 195˚ respectively with that of mass B in the counter clockwise direction. The rotating masses have the following properties:

$m_b = 25$ kg	$r_a = 150$ mm
$m_c = 40$ kg	$r_b = 20$ mm
$m_d = 35$ kg	$r_c = 100$ mm, $r_d = 180$ mm

Planes B and C are 250 mm apart. Determine the

(i) Mass A and its angular position with that of mass B

(ii) position of all the planes relative to plane of mass A

. **R/486/(W07)**

.

Solution:

For complete balance

$$\sum m.r.\cos\theta = 0 \quad and \quad \sum m.r.\sin\theta = 0$$

$m_a \times 150 \times \cos \theta_a + 5000\times \cos 0^\circ$

$+ 4000\times \cos 90^\circ + 6300\times \cos 195^\circ = 0$

or $150\ m_a \cos \theta_a = 1085$ (i)

and $m_a \times 150 \times \sin \theta_a + 5000\times \sin 0^\circ$

$+ 4000\times \sin 90^\circ + 6300\times \sin 195^\circ = 0$

or $150\ m_a \sin \theta_a = -2369$ (ii)

Squaring and adding

.

m_a = 17.37 kg

Dividing (ii) by (i)

$$\tan\theta_a = \frac{-236.9}{108.5} = -2.184$$

Θ_a = 294.6˚

For complete balance , the couple equations are

$$\sum mrl\cos\theta = 0 \quad and \quad \sum mr\sin\theta = 0$$

.

Taking A as the reference plane

$5000\ l_b \cos 0^\circ + 4000\ l_c \cos 90^\circ$

$+ 6300\ l_d \cos 195^\circ = 0$

or $\quad l_b = 1.217\ l_d \qquad$ (iii)

$5000\ l_b \sin 0^\circ + 4000\ l_c \sin 90^\circ$

$+ 6300\ l_d \sin 195^\circ = 0$

or $l_c = 0.4078\ l_d \qquad$ (iv)

or $l_b + 250 = 0.4078$

or $1.217\ l_d + 250 = 0.4078$

.

l_d = -309 mm

l_b = 1.217 l_d = -376 mm

l_c = lb + 250 = - 126 mm

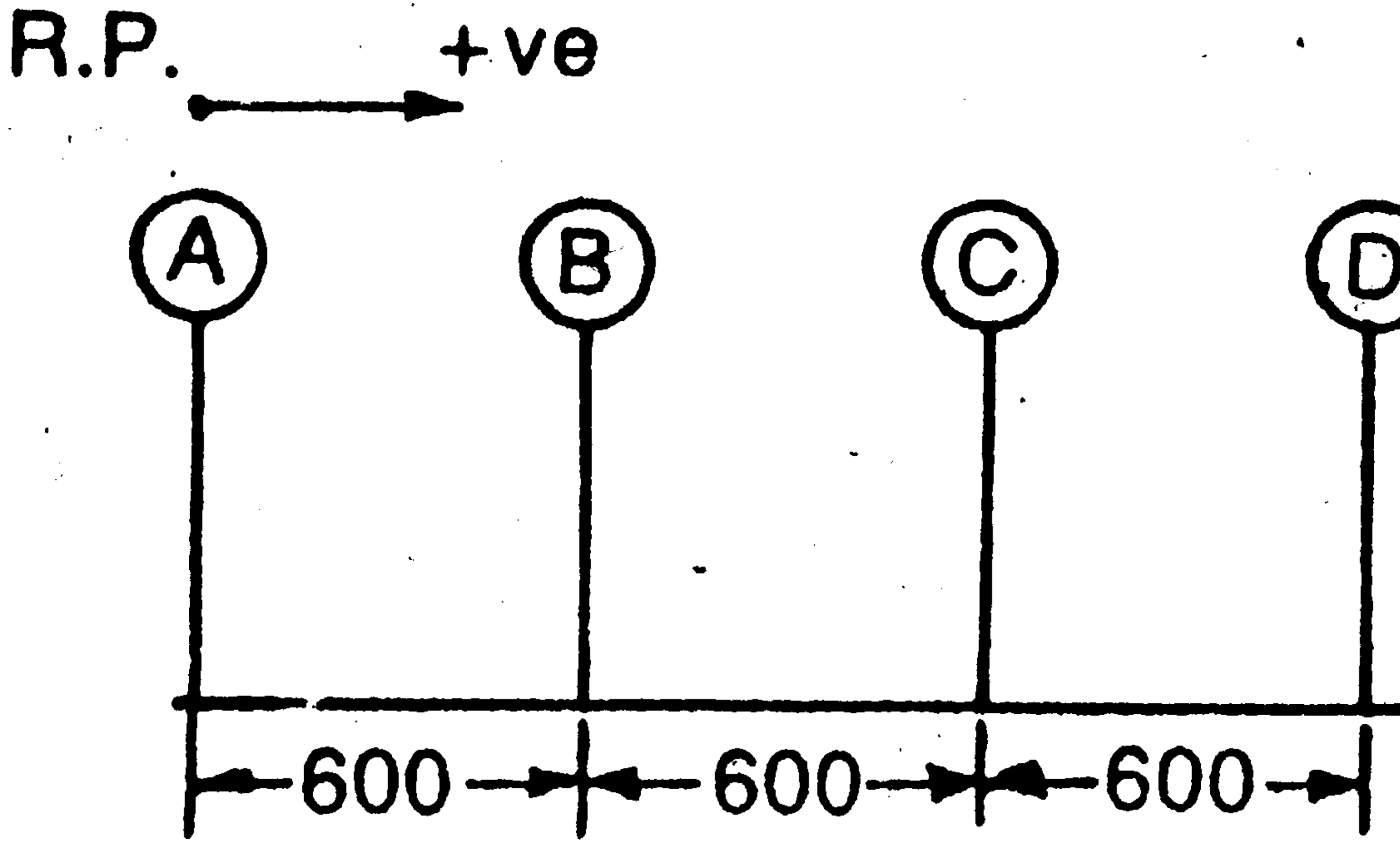
.

Example 5: A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C, and D are 10 kg, 5 kg and 4 kg respectively.

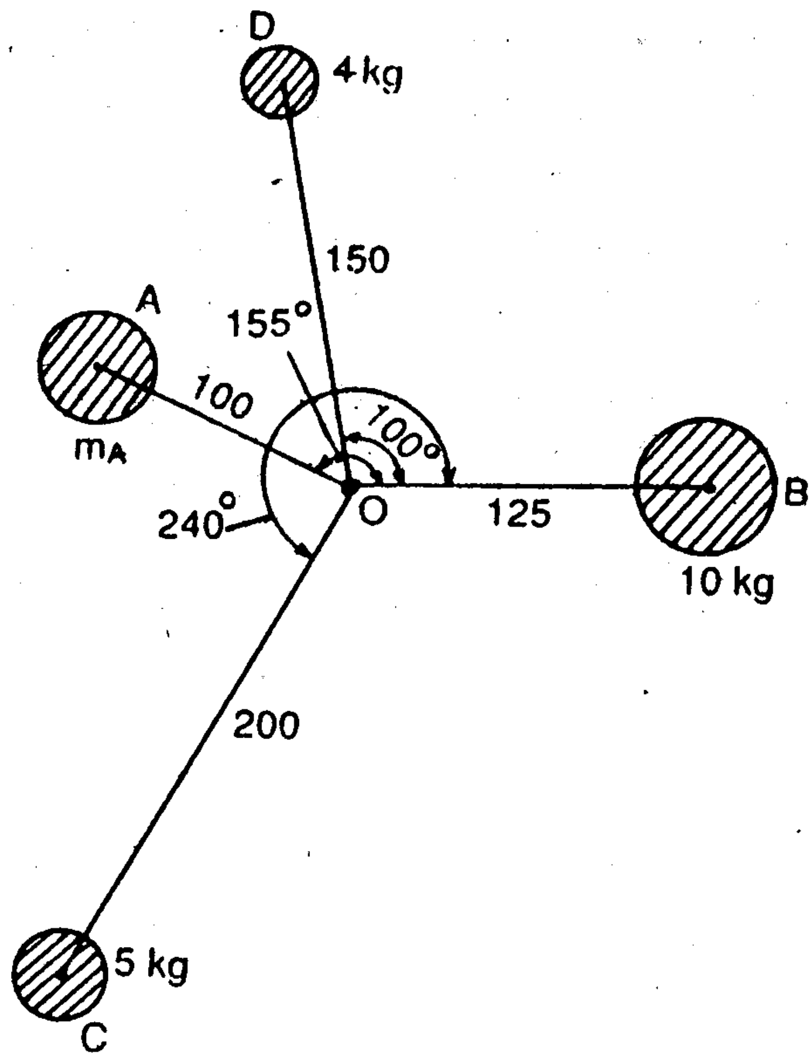
Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

K/847/(S09, W09)

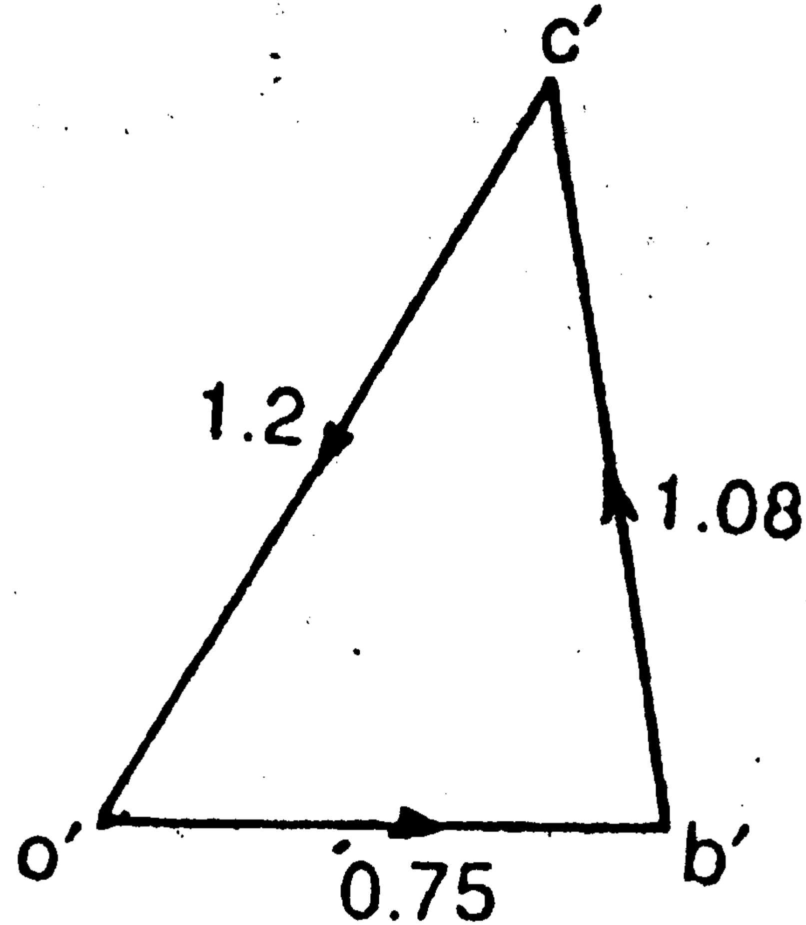
Solution:



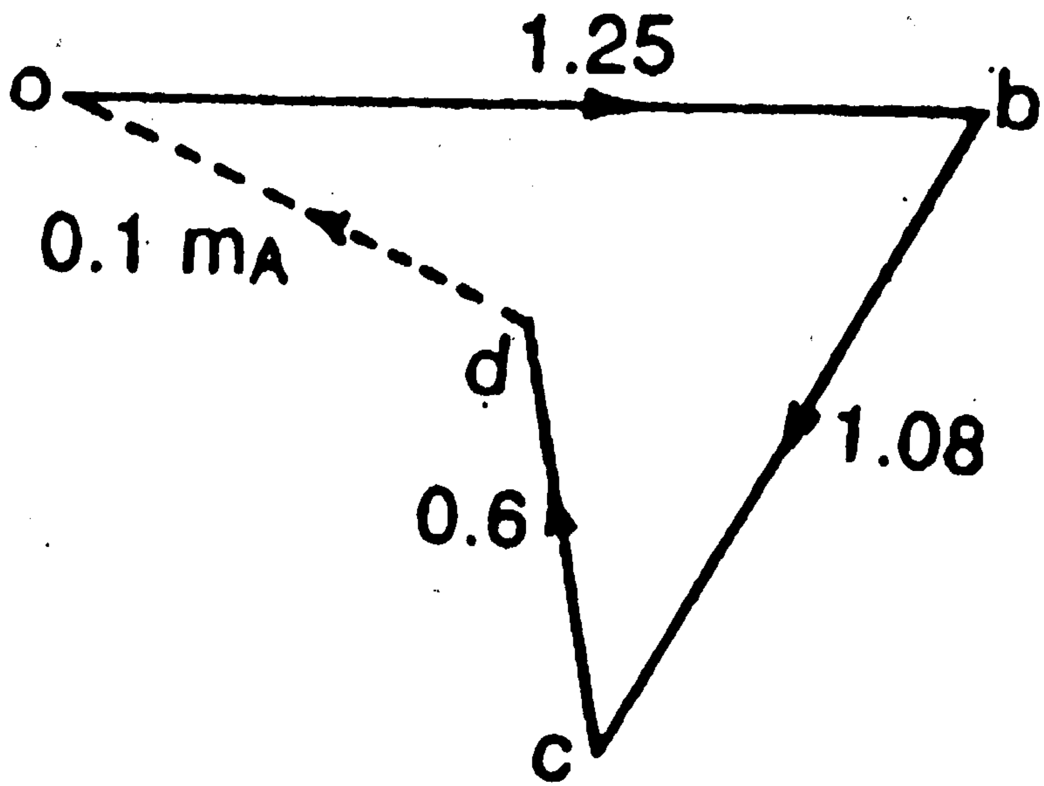
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08



(*b*) Angular position of masses.



(c) Couple polygon.



(*d*) Force polygon.

$\angle BOC = 240^\circ$

$\angle BOD = 100^\circ$

$m_a = 7kg$

$\angle BOA = 155^\circ$

.

Example 6: A single cylinder reciprocating engine has speed 240 rpm stroke 300 mm, mass of reciprocating parts 50 kg,, mass of revolving parts at 150 mm radius 37 kg. If two-third of reciprocating parts and all the revolving parts are to be balanced, find: 1. The balance mass required at a radius of 400 mm, and 2. The residual unbalanced force when the crank has rotated 60˚ from top dead centre.

.

Solution:

1, Balance mass required

$$B.b = (m_1. + c.m)\,r$$

B = 26.38 kg **Ans.**

2. Residual unbalanced force

$$= m.\omega^2.r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$

= **712.2 N**

.

Example 7: An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

.

The whole of the rotating masses and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 rpm. **K/865/(S08)**

.

Solution: Given: $a = 0.7$ m; $l_B = l_C = 0.6$ m; or

$r_B = r_C = 0.3$ m; m1 = 150 kg ; m2 = 180 kg;

$c = 2/3$; $r_A = r_D = 0.6$ m; N = 300 rpm

or $\omega = 31.42$ rad/s.

Equivalent mass of the rotating parts to be balanced perpendicular cylinder at the crank pin $= m_B = m_C = m_1 + c.m_2 = 270$ kg

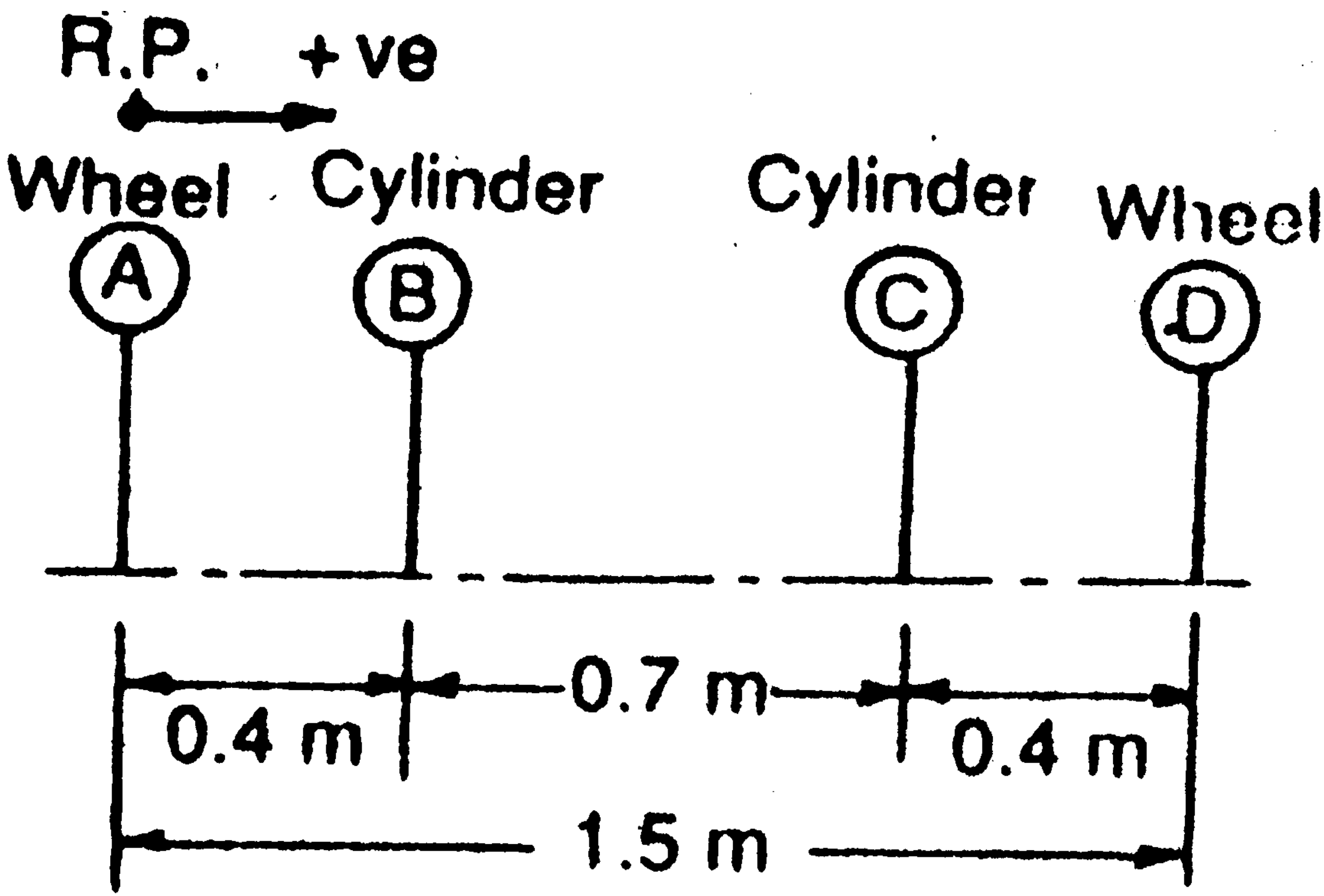
Magnitude and direction of the balancing masses

.

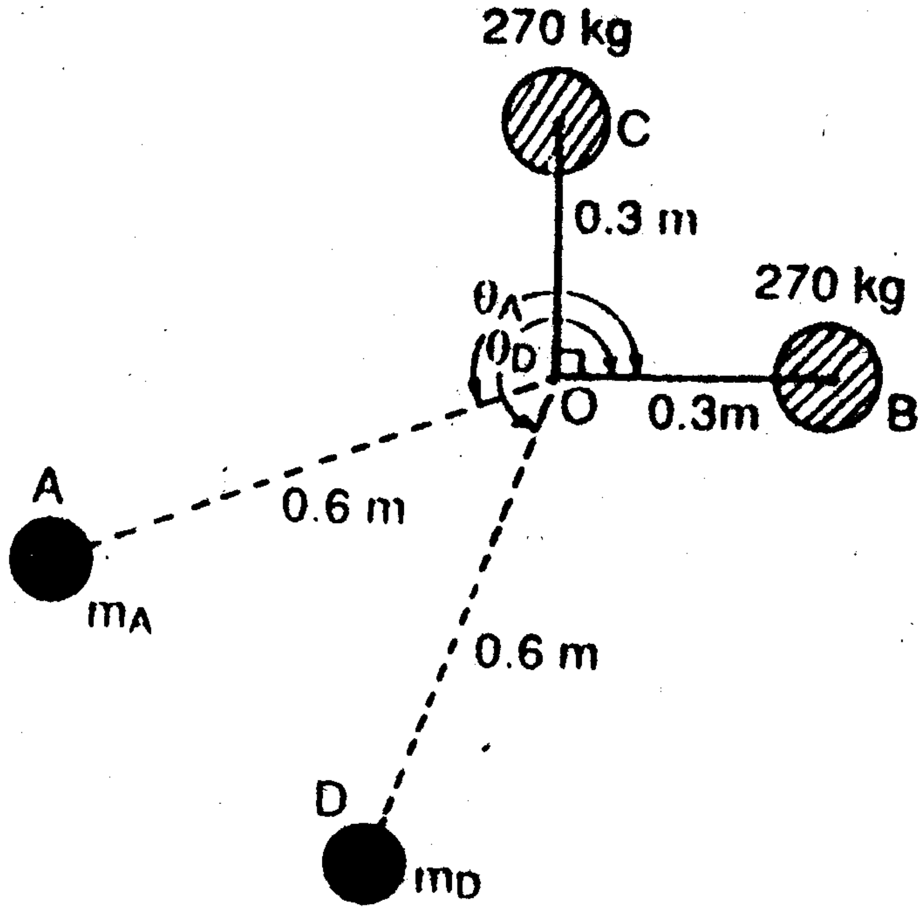
Let m_A and m_B = Magnitude of the balancing masses,

Θ_A and Θ_B = Angular positions of the balancing masses from the first crank B.

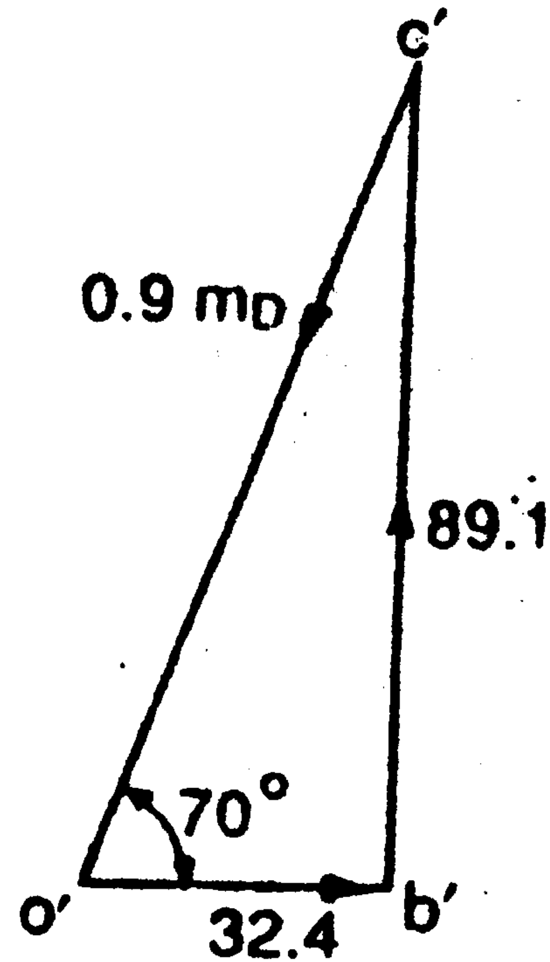
Plane (1)	*Mass.* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (4)	*Distance from plane A* (*l*)*m* (5)	*Couple* $\div \omega^2$ (*m.r.l.*) *kg-m*^{2} (6)
A (*R.P.*)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	0.6 m_D	1.5	0.9 m_D



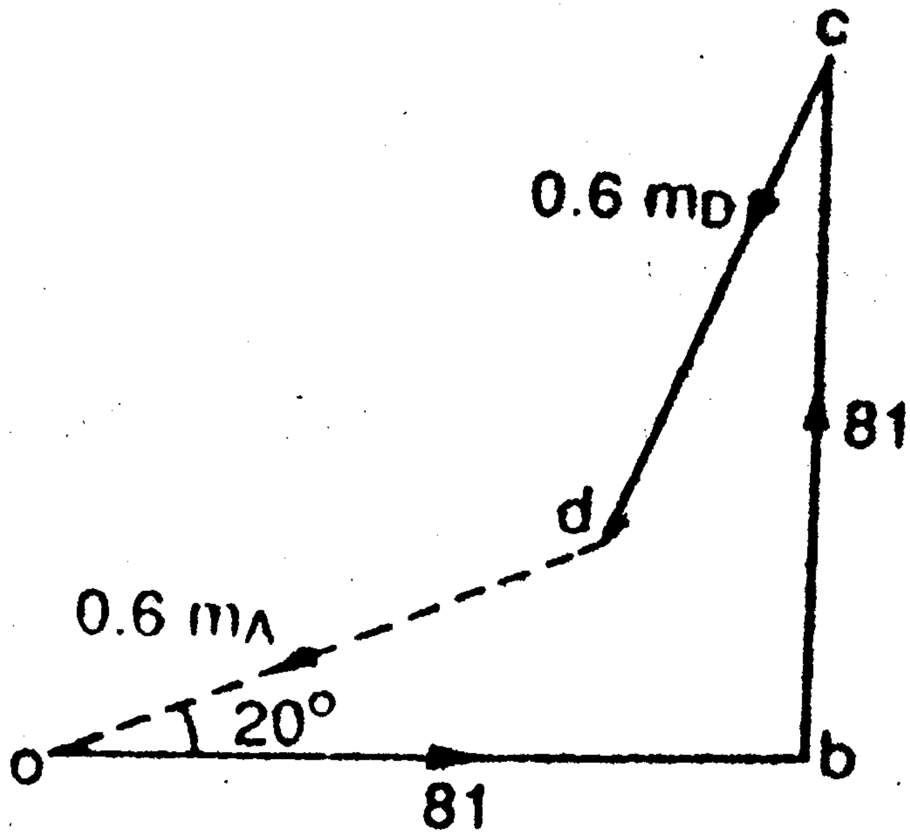
Position of planes



Angular position of masses



Couple polygon



Force diagram

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m_D = 105 kg

Θ_D = 250°, Θ_A = 200°

Fluctuation in rail pressure

Balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6$$

Fluctuation in rail pressure or hammer blow

$=B.\omega^2.b$ = 27602 N

.

Variation of tractive effort

$=\pm \sqrt{2}\,(1 - c)m_2.\omega^2.r = \pm\, 25\,127$ N

Swaying couple

$$= \frac{a\left(1-c\right)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\dfrac{2}{3}\right)}{\sqrt{2}} \times 180\left(31.42\right)^2.0.3\,N-m$$

$$= 8797\ \ N-m$$

.

Example 8: The following particulars relate to a two cylinder locomotive with two coupled wheels on each side:

Stroke	= 650 mm
Mass of reciprocating parts per cylinder	= 240 kg
Mass of revolving parts per cylinder	= 200 kg
Mass of each coupling rod	= 250 kg
Radius of centre of coupling rod pin	= 250 mm
Distance between cylinders	= 0.6 m
Distance between coupling rods	= 1.8 m

.

The main cranks are at right angles and the coupling rod pins are at 180˚ to their respective main cranks. The balance masses are to be placed at the wheels at a mean radius of 675 mm in order to balance whole of the revolving and 3/4th of the reciprocating masses. The balance mass for the reciprocating masses is to divided equally between the driving wheels and the coupled wheels.

.

Find: 1. The magnitude and angular position of the masses required for the driving and trailing wheels, and 2. The hammer blow at 120 km/h, if the wheels are 1.8 metre diameter.

K/874

.

Solution: Given: $L_C = L_D = 650$ mm or $r_C = r_D = 0.325$ m; $m_1 = 240$ kg; $m_2 = 200$kg; $m_3 = 250$ kg; $r_A = r_F = 0.25$ m; CD = 0.6 m; BE = 0.1.5 m; AF = 1.8 m
$r_B = r_E = 0.675$ m; c = ¾.

Mass of reciprocating parts per cylinder to be balanced = $c.m_1$ = ¾ × 240 = 180 kg.

90 kg is taken for driving wheels and 90 kg for trailing wheels.

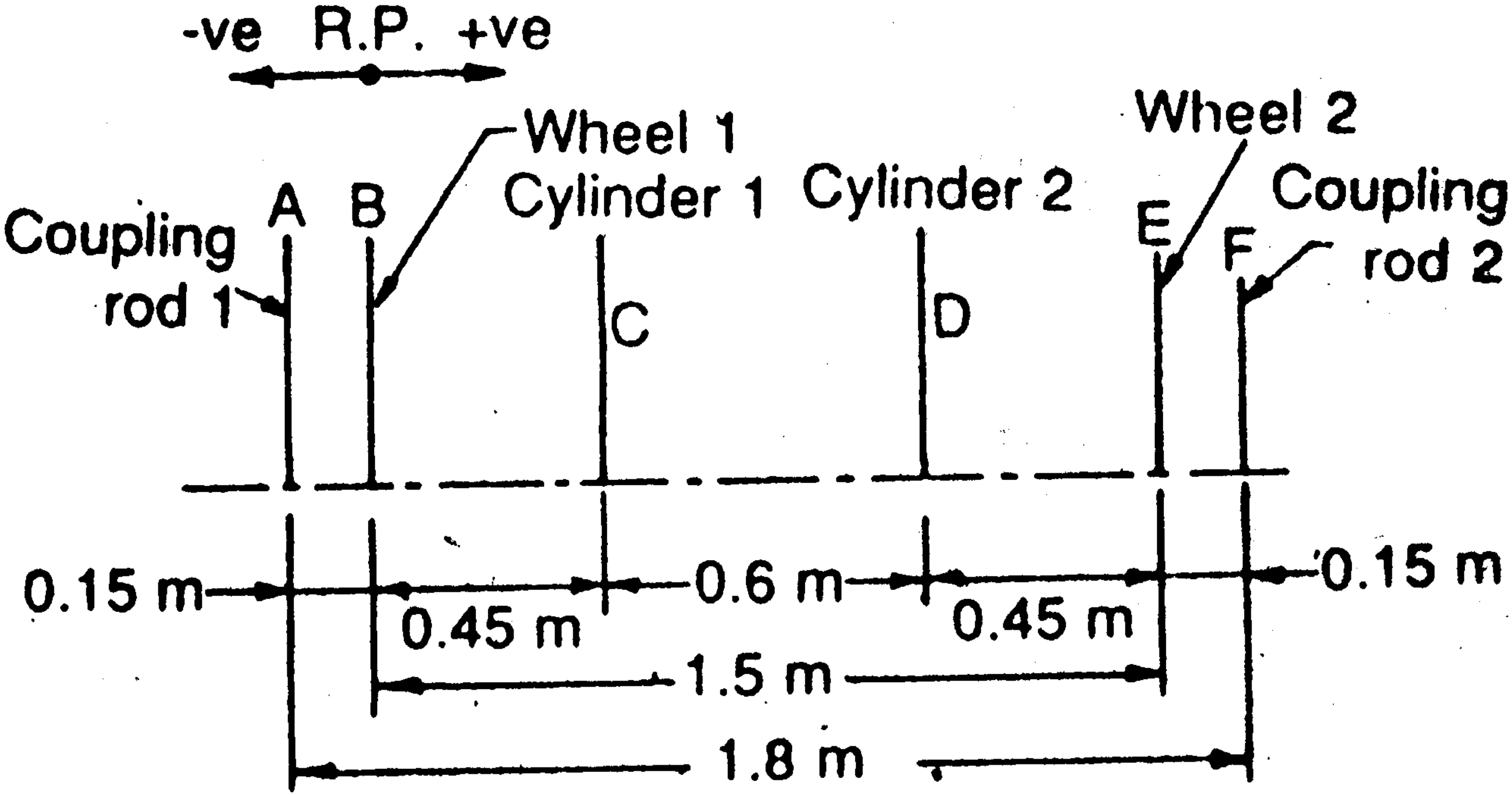
.

The masses at the coupling rods A and F to be balanced for each driving wheel are

$m_A = m_F = 125$ kg

Total mass at the cylinders C and D to be balanced at each driving wheel are

$m_C = m_D = 200 + 90 = 290$ kg and



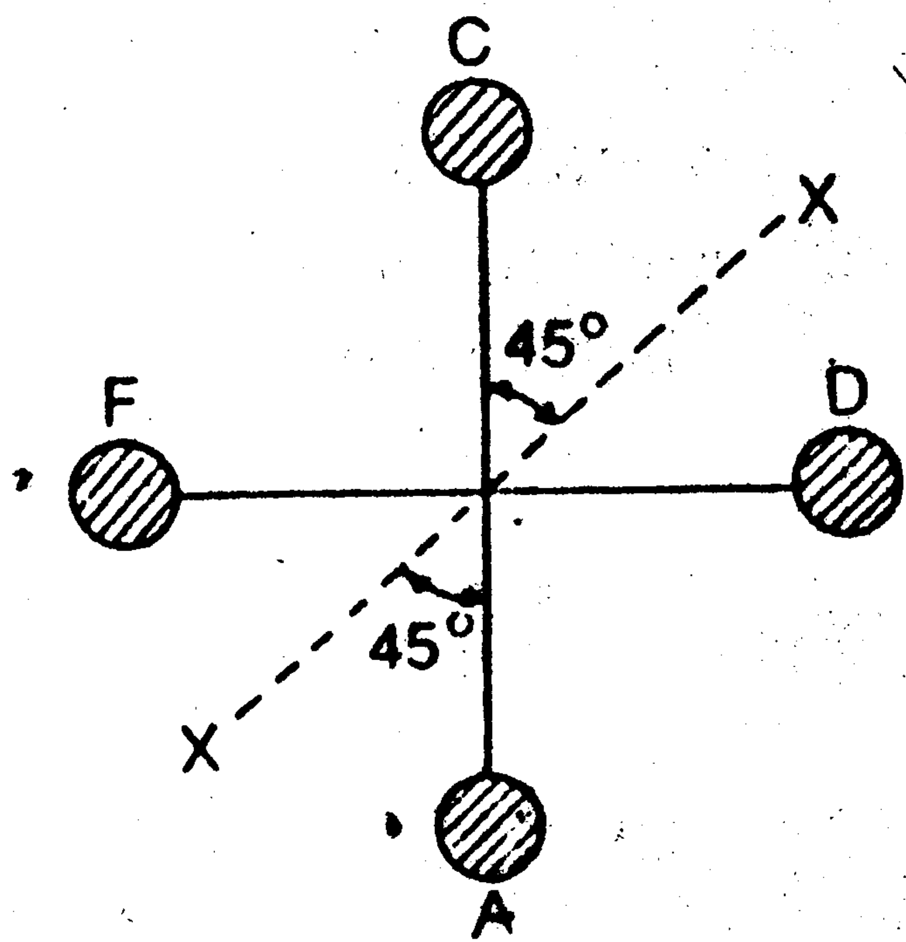
Position of planes

. Balance masses in the driving wheels

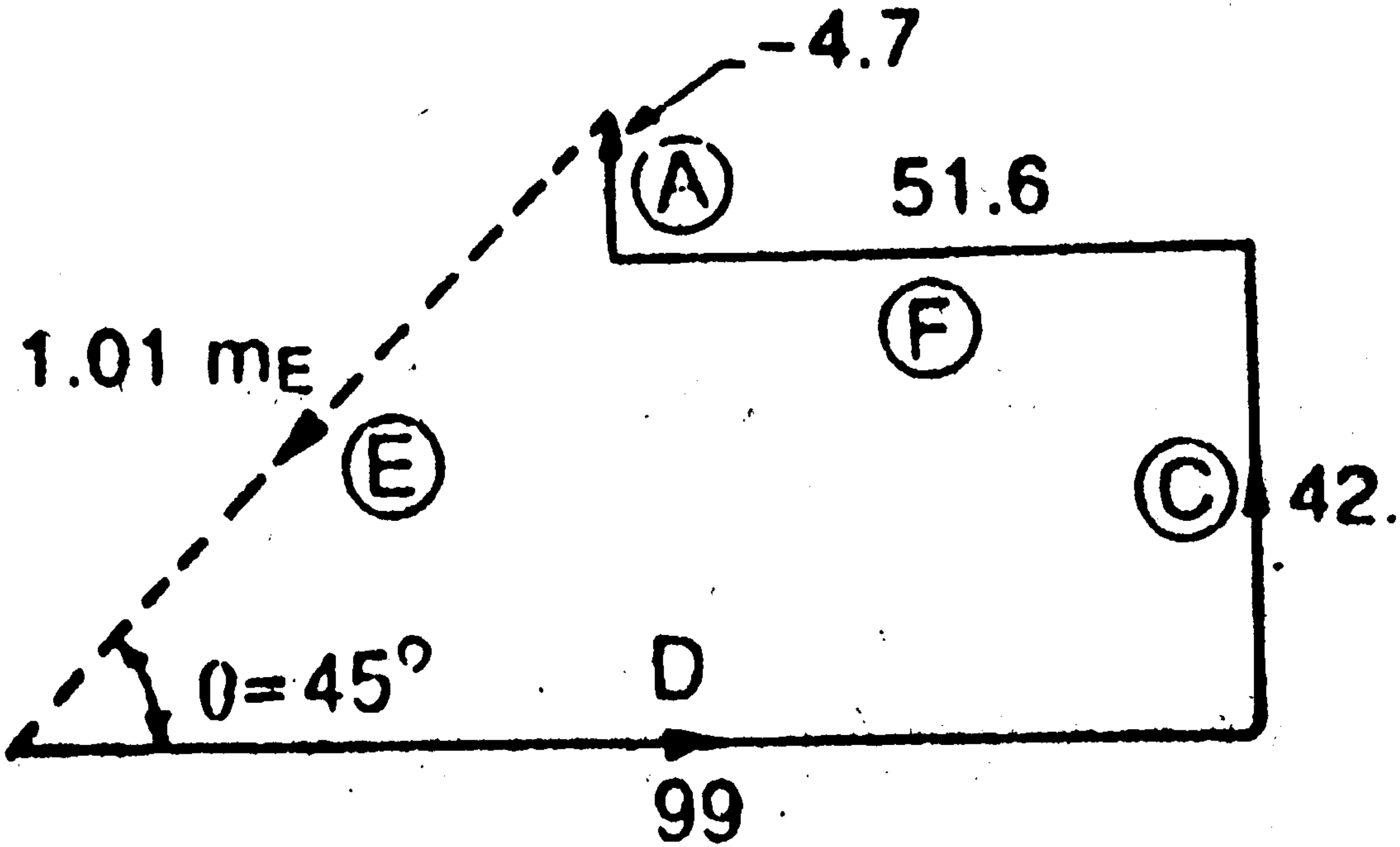
Let m_B and m_E be the balance masses placed in the driving wheels B and E respectively. Taking B as reference plane data is tabulated below.

Table 22.5. (For driving wheels)

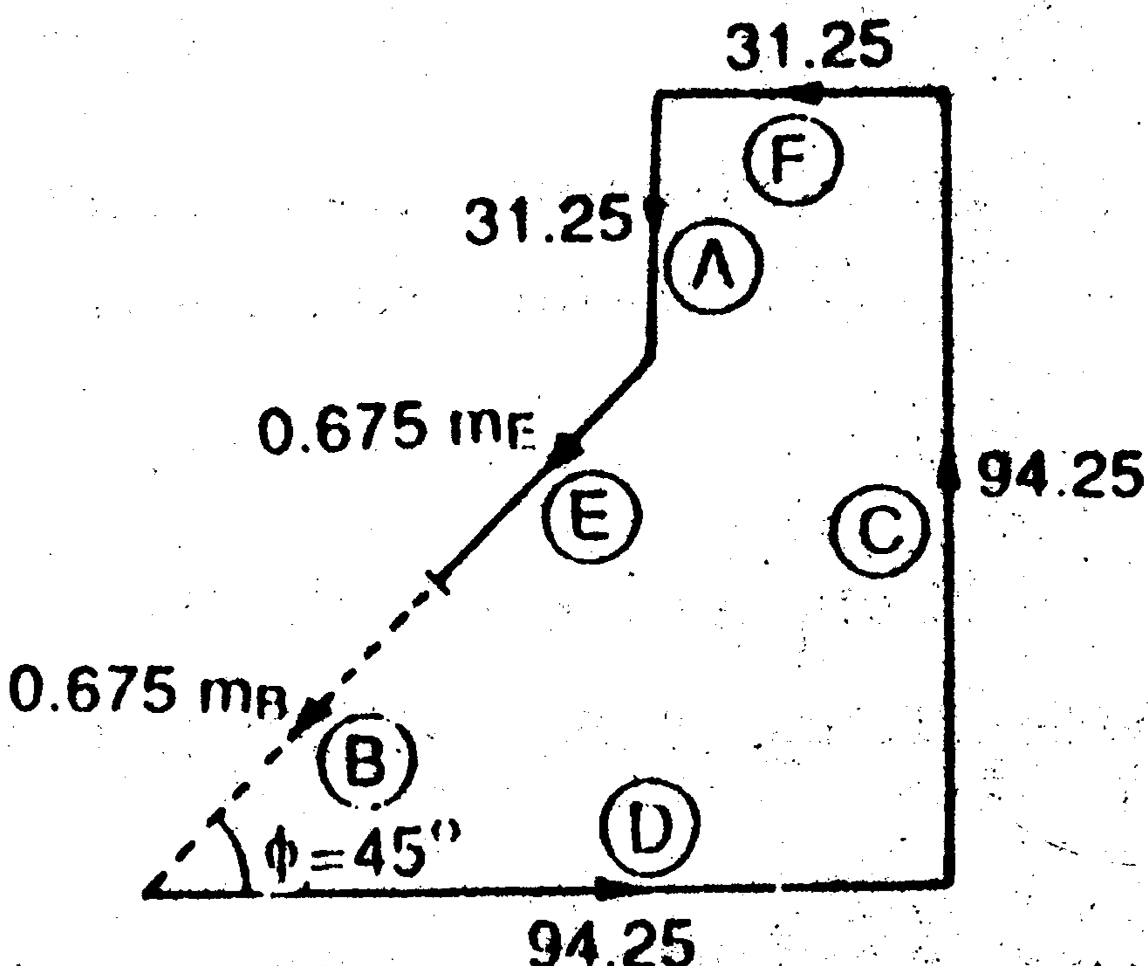
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	125	0.25	31.25	−0.15	−4.7
B (R.P.)	m_B	0.675	0.675 m_B	0	0
C	290	0.325	94.25	0.45	42.4
D	290	0.325	94.25	1.05	99
E	m_E	0.675	0.675 m_E	1.5	1.01 m_E
F	125	0.25	31.25	1.65	51.6



(*b*) Angular position of cranks and coupling pins.



(c) Couple polygon : Driving wheel *E*.



(*d*) Force polygon : Driving wheel *B*.

.

By measurement we find

$1.01\ m_E = 67.4$ kg-m^2 or $m_E = 66.7$ kg **Ans.**

$\Theta = 45^\circ$ **Ans.**

$0.67\ m_B = 45$ kg-m or $m_B = 66.7$ kg **Ans.**

$\Phi = 45^\circ$ **Ans.**

Balance masses in the trailing wheels

$m_A = m_F = 125$ kg

$m_C = m_D = 90$ kg

Table (For trailing wheels

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
A	125	0.25	31.25	−0.15	−4.7
B (R.P.)	m'_B	0.675	0.675 m'_B	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m'_E	0.675	0.675 m'_E	1.5	1.01 m'_E
F	125	0.25	31.25	1.65	51.6

In order to find the balance mass m'_E in the trailing wheel *E*, draw a couple polygon from the

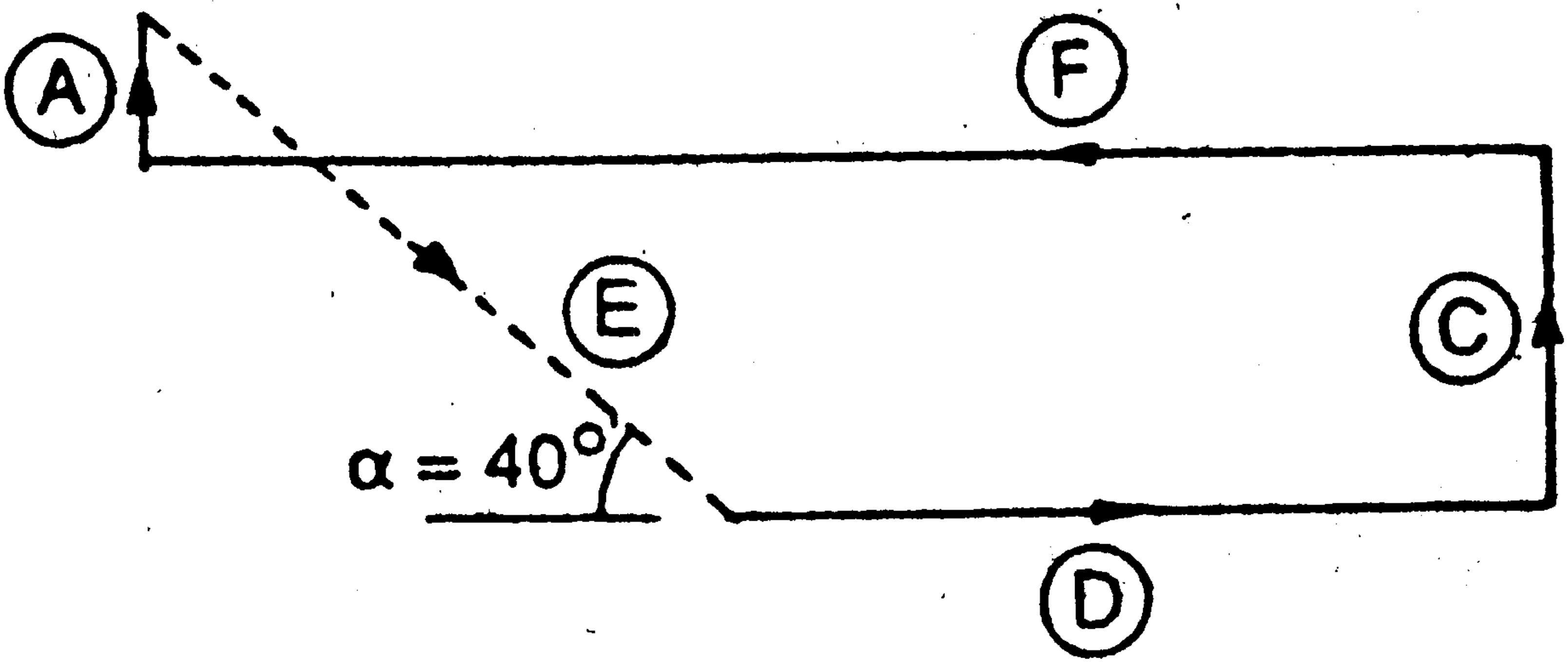
.

$1.01\, m'_E = 27.5$ kg-m^2 or $m'_E = 27.5$ kg **Ans.**

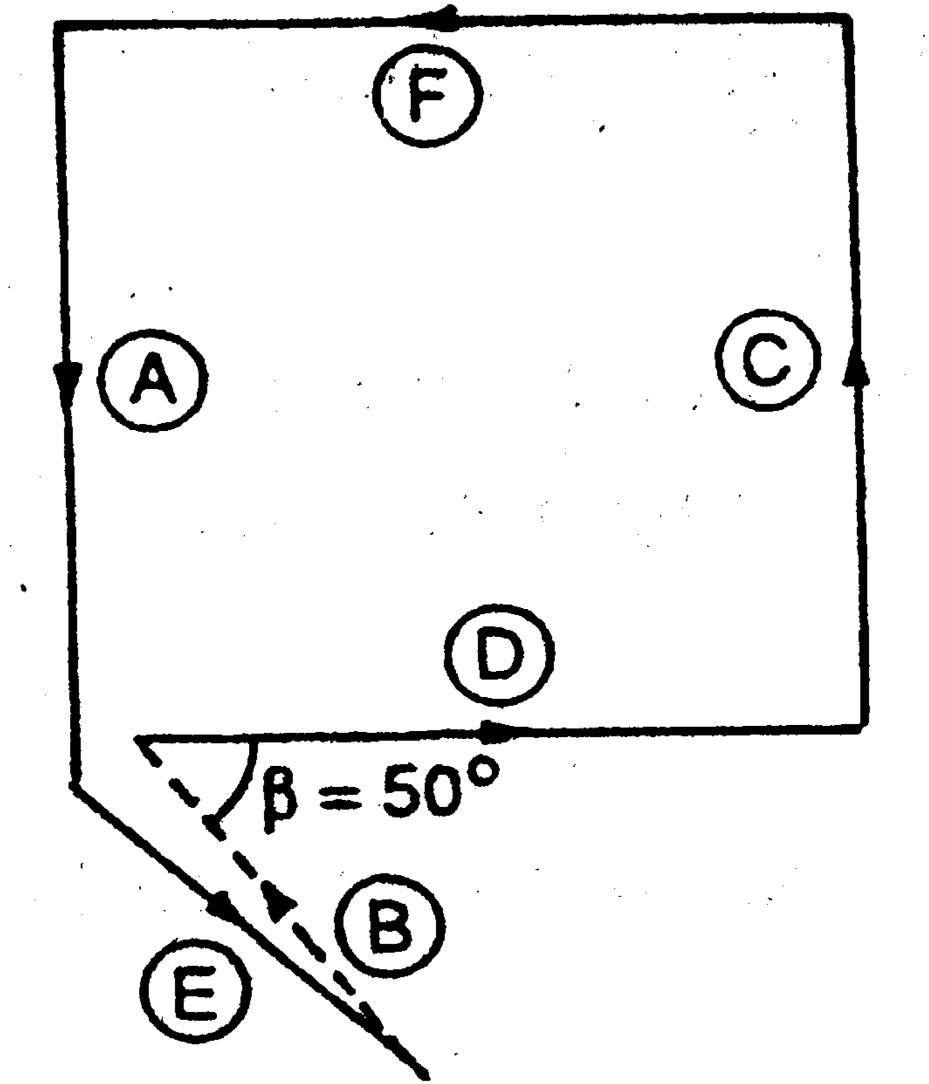
$\alpha = 40^\circ$ **Ans.**

$0.675\, m'_B = 18.35$ kg-m or $m'_B = 27.2$ kg **Ans.**

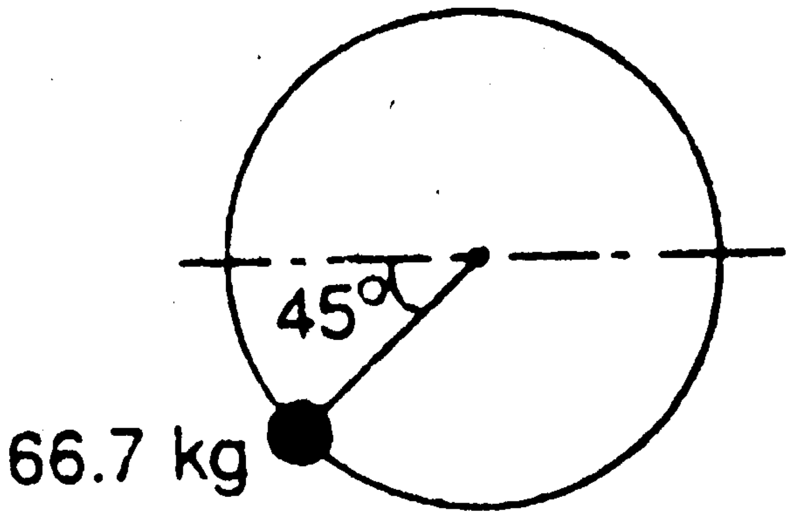
$\beta = 50^\circ$ **Ans.**



Couple polygon: Trailing wheel E

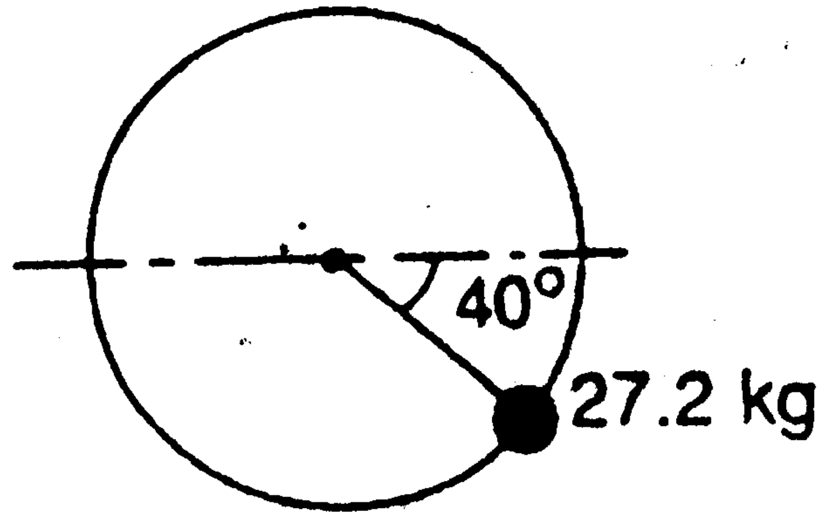


Force polygon: Trailing wheel B



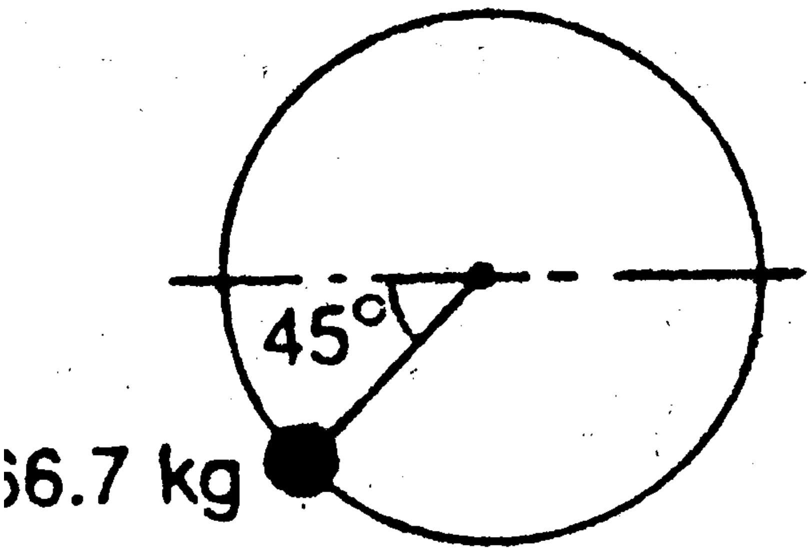
Driving wheel *E*.

(*a*)

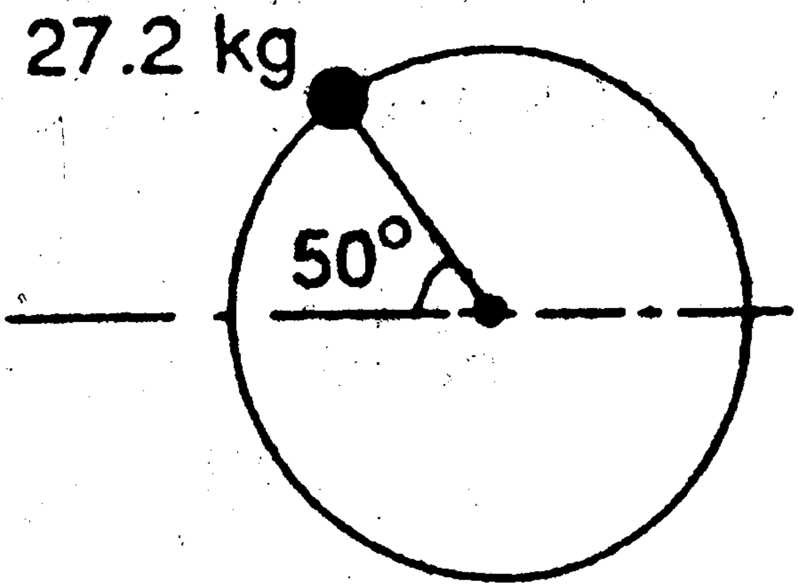


Trailing wheel *E*.

(*b*)



Driving wheel *B*.



Trailing wheel *B*.

•

Hammer blow

Table 22.7. (For hammer blow)

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent. force* ÷ ω^2 (*m.r*) *kg-m* (4)	*Distance from plane B* (*l*) *m* (5)	*Couple* ÷ ω^2 (*m.r.l.*) *kg-m*^{2} (6)
B (*R.P.*)	m''_B	0.675	0.675 m''_B	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m''_E	0.675	0.675 m''_E	1.5	1.01 m''_E

$1.01\ m''_E = 33$ kg = $\sqrt{(30.7)^2 + (13.2)^2} = 33.4$

$m''_E = 33$ kg

Linear speed of the wheel = 120 km/h

= 33.3 m/s

$\omega = \dfrac{v}{D/2}$ = 37 rad/s

Hammer blow = $\pm B.\omega^2.b$ = 33 (37)20.675

= ± 30 494 N **Ans**

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Example 9: A four cylinder vertical engine has cranks 150 mm long. The planes of the rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.

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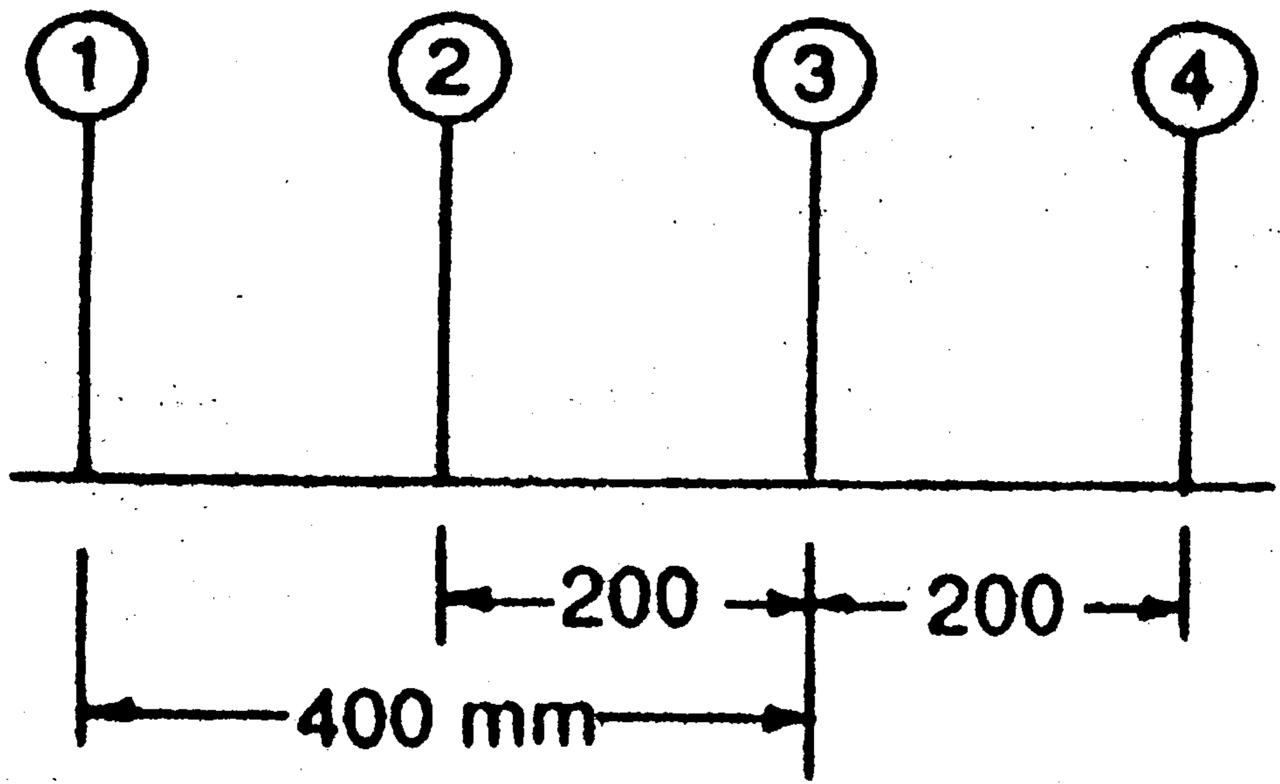
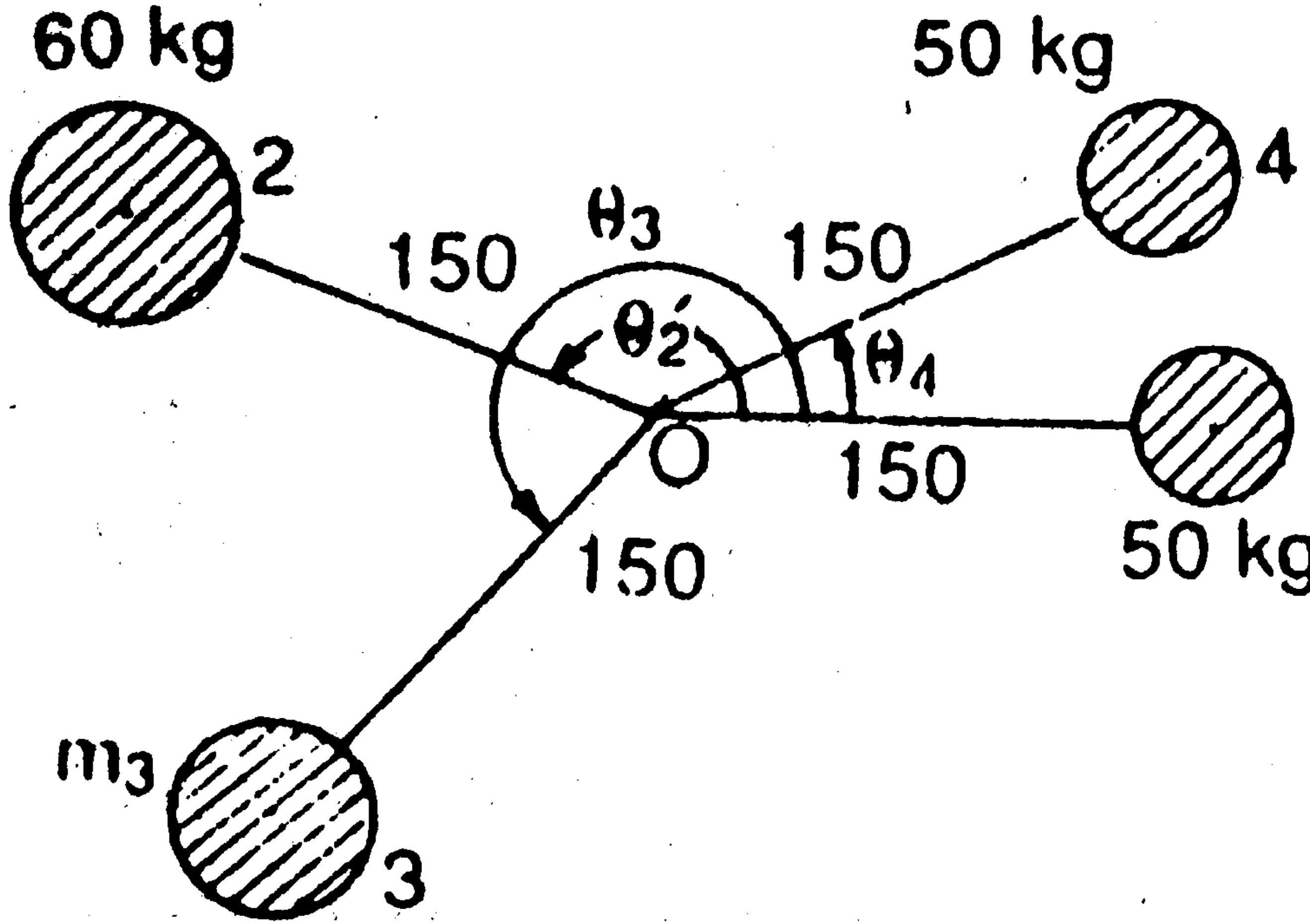
Solution:

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r) kg-m (4)	Distance from plane 3(l) m (5)	Couple÷ω^2 (m.r.l) kg-m^2 (6)
1	50	0.15	7.5	–0.4	-3
2	60	0.15	9	–0.2	-1.8
3(R.P.)	m_1	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

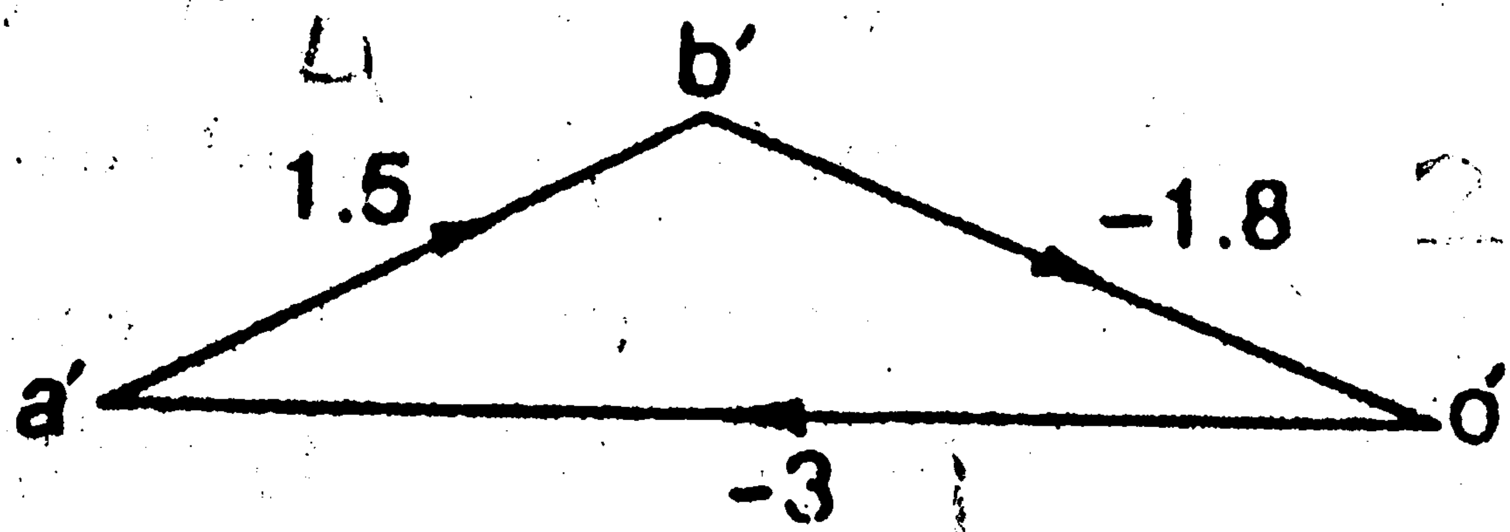
$\Theta_2 = 160^\circ$, $\Theta_4 = 26^\circ$, $m_3 = 60$ kg, $\Theta_3 = 227^\circ$



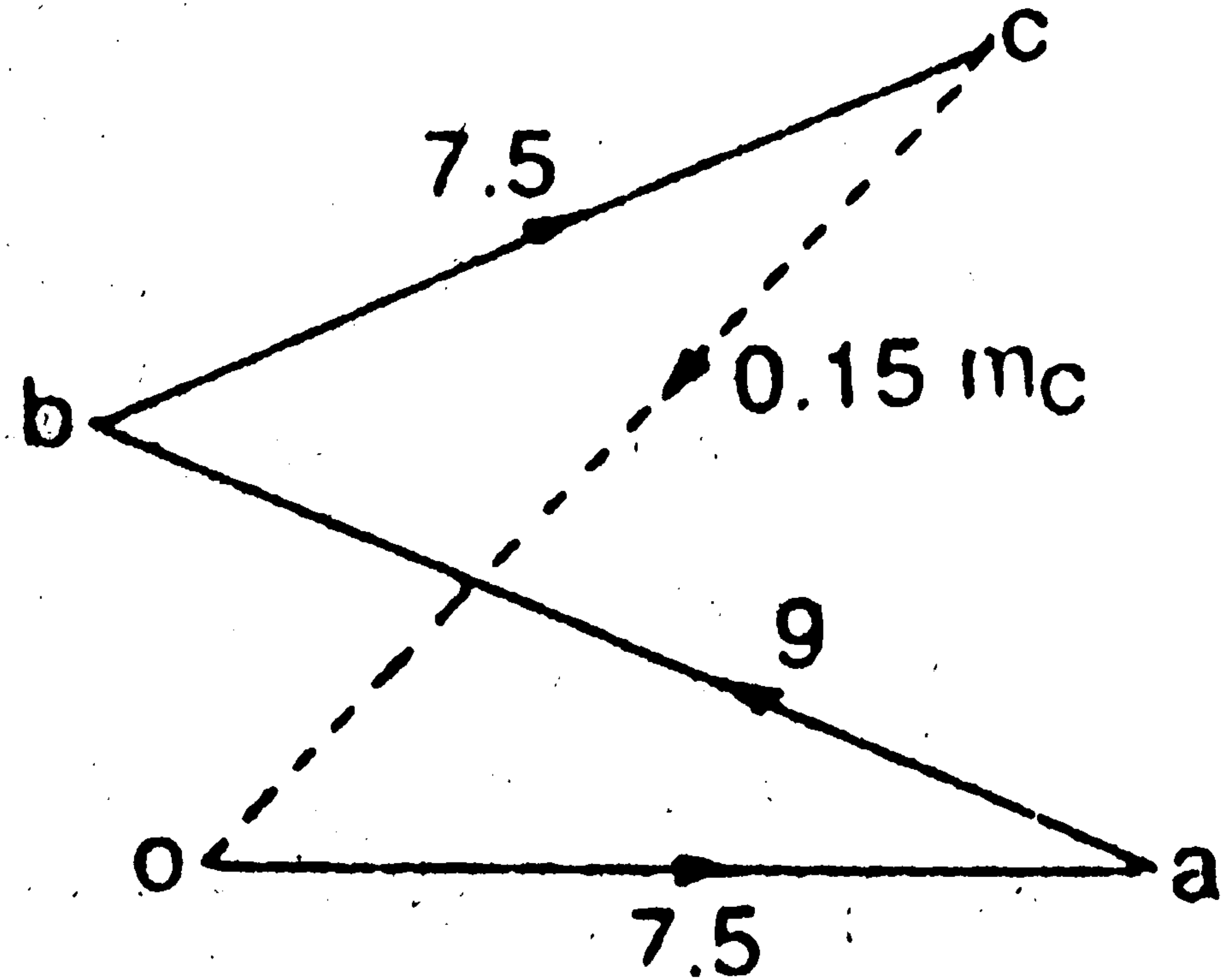
(a) Position of planes.



(*b*) Angular position of cranks.



(*c*) Couple polygon.



(*d*) Force polygon.

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Example 10: Three cylinders of an air compressor have their axes at 120˚ to one another, and their connecting rods are couples\d to a single crank. The stroke is 100 mm and length of each connecting rod is 150 mm. The mass of the reciprocating parts per cylinder is 1.5 kg. Find the maximum primary and secondary forces acting on the frame of the compressor when running at 3000 rpm. Describe clearly a method by which such forces may be balanced. **K/896/(W08)**

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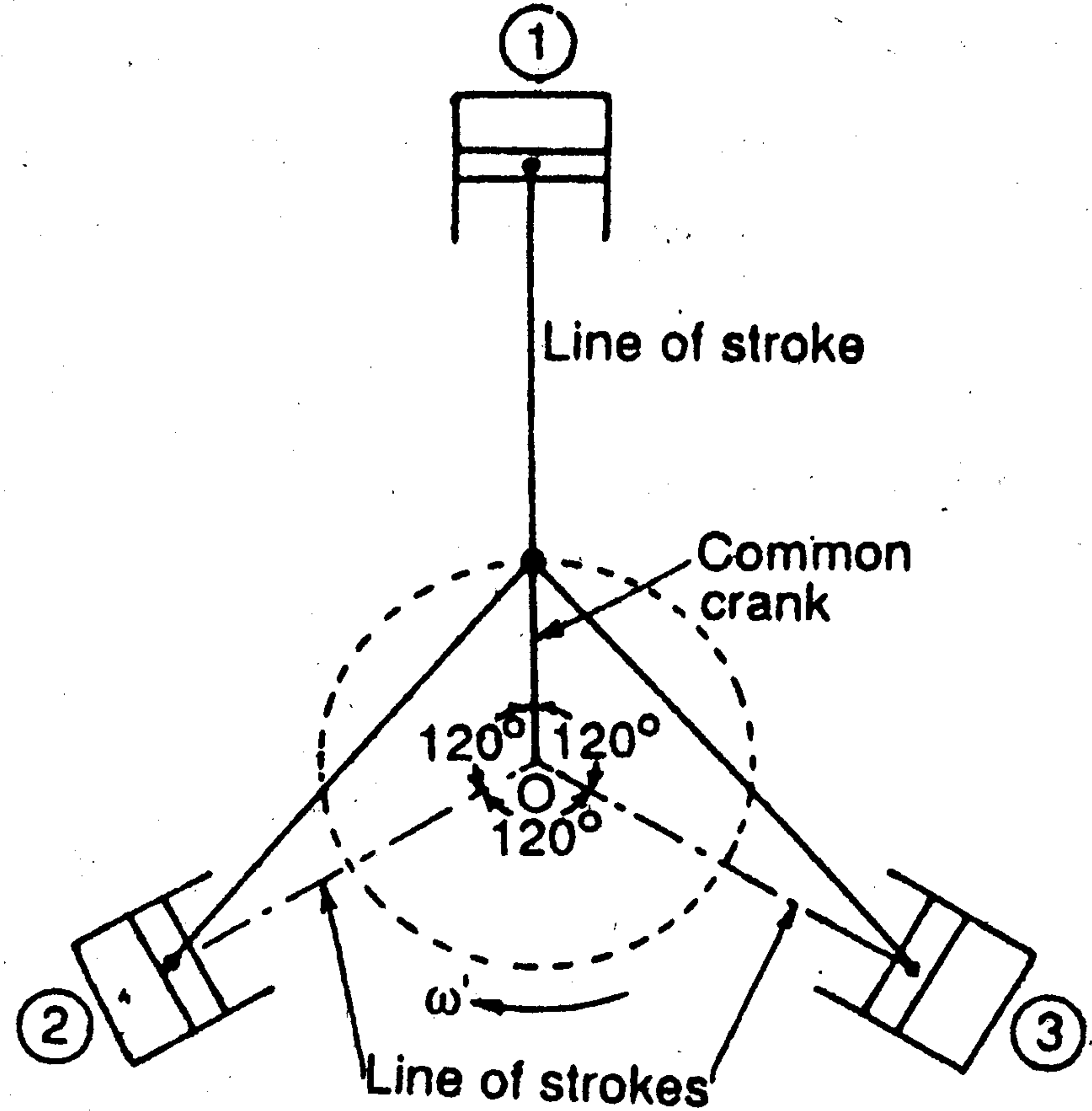
Solution:

Maximum primary force acting on the frame of the engine

$$= \frac{3m}{2} \times \omega^2 . r = 11/.106\,kN\ Ans.$$

The maximum primary force may be balanced by a mass attached diametrically opposite to the crank pin and rotating with the crank, of magnitude B_1 at radius b_1 such that

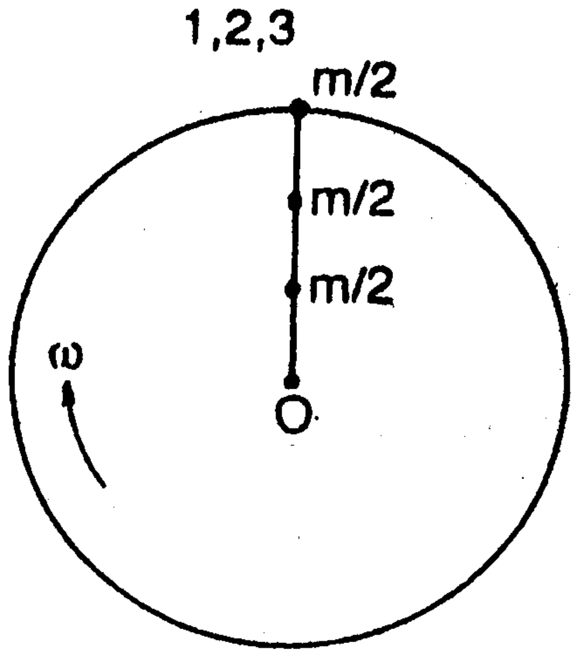

1
Line of stroke
Common
crank
120°
120°
O
120°
2
3
ω
Line of strokes

.

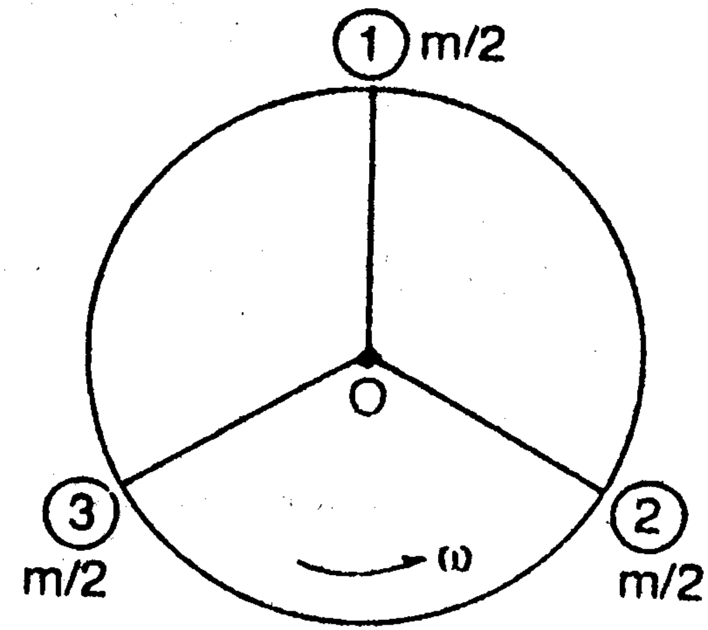
$$B_1b_1 = \frac{3m}{2} \times .r = 0.1125\, N - m\ Ans.$$

Maximum secondary force acting on the frame of the engine

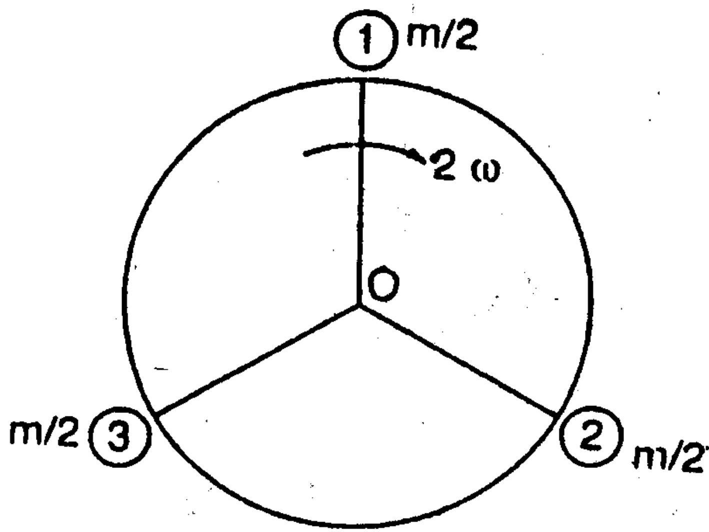
$$= \frac{3m}{2}(2\omega)^2 . \left[\frac{r}{4m} \right] = 3702 N\ Ans.$$



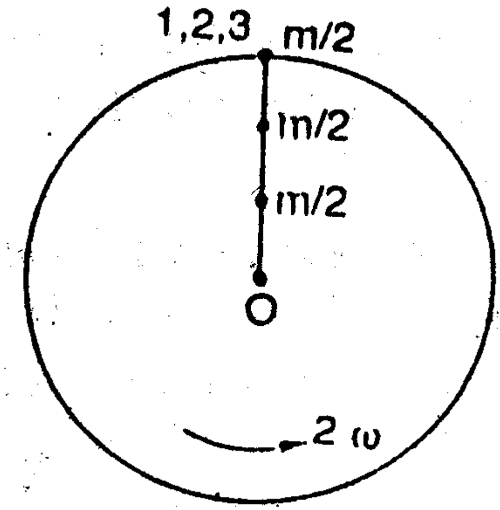
(*a*) Direct primary cranks.



(*b*) Reverse primary cranks.



(*a*) Direct secondary cranks.



(*b*) Reverse secondary cranks.

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The maximum secondary force may be balanced by a mass B_2 at radius b_2 attached diametrically opposite to the crankpin, and rotating anti-clockwise at twice the crank speed, such that

$$B_2 b_2 = \frac{3m}{2} \times . \frac{r}{4n} = 0.009375\, N - m\ Ans.$$

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Example 11: A vee-twin engine has the cylinder axis at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius id 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be **K/903**

Solution:

$$F_P = m.\omega^2.r$$

$$F_s = \sqrt{2} \times \frac{m}{n}. \times \omega^2.r \sin 2\theta$$

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This is maximum when sin 2Ɵ is maximum i.e. when 2Ɵ = ±1 or Ɵ = 45˚ or 135˚.

Maximum resultant secondary force = 8.36 N.

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Example: The reciprocating mass per cylinder in a 60˚ V-twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 rpm, determine the maximum and minimum values of the primary forces. Also find out the resultant secondary force.

K/903 (W07)

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Solution: Given 2α = 60° or α = 30°, m = 1.5 kg, Stroke = 100 mm or r = 0.05 m, l= 0.25 m, N = 250 rpm or ω = 261.8 rad/s

Maximum and Minimum values of primary forces

The resultant primary force

$$F_P = 2m.\omega^2.r\sqrt{\left(\cos^2\alpha.\cos\theta\right)^2+\left(\sin^2\alpha.\sin\theta\right)^2}$$

$$= 2m.\omega^2.r\sqrt{\left(\cos^2 30°.\cos\theta\right)^2+\left(\sin^2 30°.\sin\theta\right)^2}$$

$$= 2m.\omega^2.r\sqrt{\left(\frac{3}{4}\cos\theta\right)^2+\left(\frac{1}{4}\sin\theta\right)^2}$$

$$= \frac{m}{2}\times\omega^2 r\sqrt{9\cos^2\theta+\sin^2\theta}$$

The primary force is maximum when $\theta = 0^\circ$

$F_{P(max)}$ = 7710.7 N **Ans.**

The primary force is minimum when $\theta = 90^\circ$

$F_{P(min\,)}$ =2510.2 N **Ans.**

Resultant secondary force

$$F_S = \frac{2m}{n}.\omega^2.r\sqrt{\left(\cos\alpha\cos 2\alpha.\cos 2\theta\right)^2 + \left(\sin\alpha\sin 2\alpha.\sin 2\theta\right)^2}$$

$$= \frac{\sqrt{3}}{2} \times \frac{m}{n} \times \omega^2 r = 890.3N$$