Unit – V

Neural Fuzzy System

Fuzzy Set

- In the classical set, its characteristic function assigns a value of either 1 or 0 to each individual in the universal set,
- There by discriminating between members and nonmembers of the crisp set under consideration.
- The values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set.
- Larger values denote higher degree of set membership such a function is called a membership function and the set is defined by it is a fuzzy set.

Fuzzy Set…

- Introduced by **L. Zadeh** in 1965 as a way to manage complexity of systems.
- Using a term *principle of incompatibility*, Dr. Zadeh states

"*As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics*."

[Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. on Systems, Man, and Cybernetics,* Vol. 3, No. 1, 1973].

Fuzzy Set…

- Fuzzy sets are functions that map each member in a set to a real number in [0, 1] to indicate the *degree of membership* of that member.
- The ambiguity of real world definitions
  - *John is **OLD***
  - *David is **TALL***

  How "OLD" is old? 40 years, 50, or 60?

  How "TALL" is tall? 5 feet, 6 feet, or 7 feet?
- **Every thing is a matter of degree**
- The "degrees" of being old or tall can be quantitatively illustrated using *quantified meaning*

Fuzzy Set…

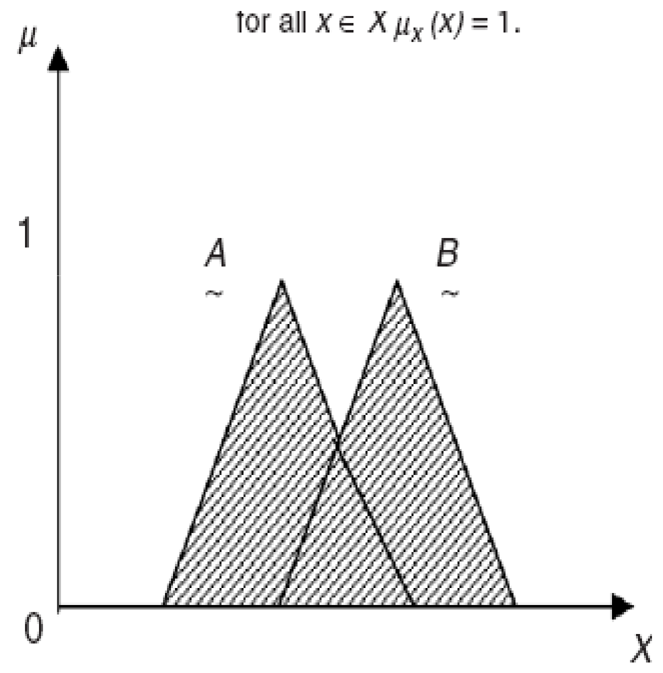
- Elements in a fuzzy set, because their membership need not be complete,
- can also be members of other fuzzy set on the same universe. Fuzzy set are denoted by a set symbol with a tilde understrike.
- Fuzzy set is mapped to a real numbered value in the interval 0 to 1. If an element of universe, say *x, is* a member of fuzzy set $\underset{\sim}{A}$
- , then the mapping is given by $\mu_{\underset{\sim}{A}}(x) \in [0,1]$

Fuzzy Set Operations

- Union
- Intersection
- Complement

Fuzzy Union

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$$



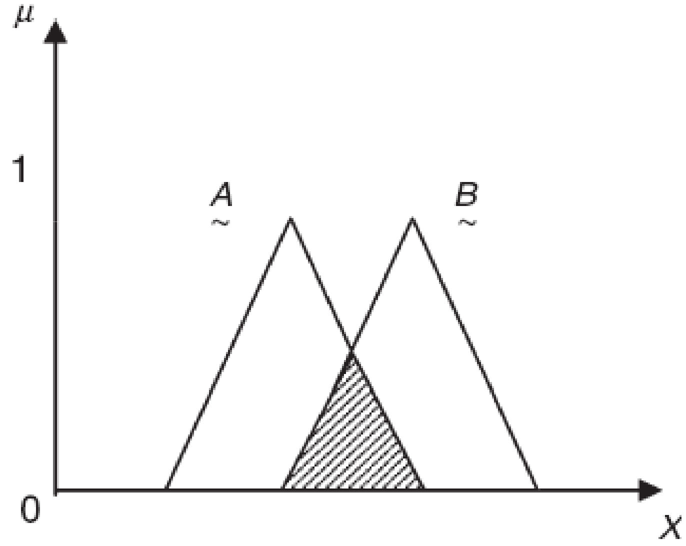
The most commonly used method for fuzzy union is to take the maximum. That is, given two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$

$$\mu_{AUB}(x) = \max(\mu_A(x), \mu_B(x))$$

Fuzzy Intersection

- It can represented as:

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \Lambda \mu_{\underset{\sim}{B}}(x)$$



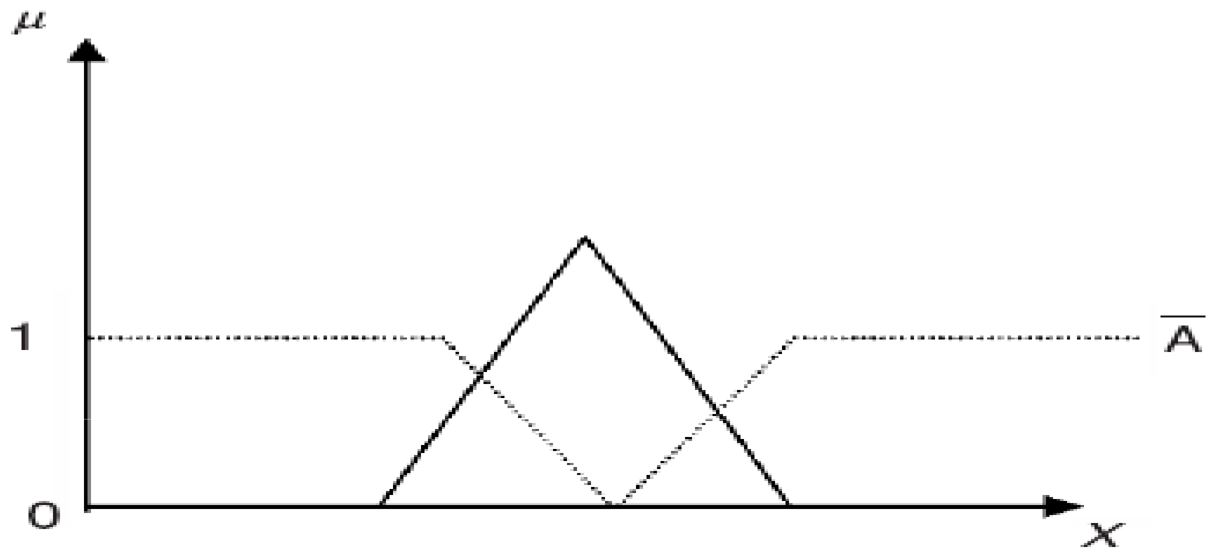
The most commonly adopted t-norm is the minimum. That is, given two fuzzy sets *A* and *B* with membership functions $\mu_A(x)$ and $\mu_B(x)$

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Fuzzy Complement

- It can represented as:

$$\mu_{\bar{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x).$$



Example 1

Suppose we have the following (discrete) fuzzy sets:

$$A = 0.4/1+0.6/2+0.7/3+0.8/4$$

$$B = 0.3/1+0.65/2+0.4/3+0.1/4$$

The union of the fuzzy sets *A* and *B*

$= 0.4/1+0.65/2+0.7/3+0.8/4$

The intersection of the fuzzy sets *A* and *B*

$= 0.3/1+0.6/2+0.4/3+0.1/4$

The complement of the fuzzy set A

$= 0.6/1+0.4/2+0.3/3+0.2/4$

Example 2

- Given two fuzzy sets A and B

a. Calculate the of union of the set A and set B

b. Calculate the intersection of the set A and set B

c. Calculate the complement of the union of A and B

$$A = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

$$B = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

Solution

a

$$\text{Union} = \max(A,B) = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

b

$$\text{Intersecti on} = \min(A, B) = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

c

$$\text{Complementof (b)} = 1 - \max(A,B) = 0.9/-2 + 0.6/-1 + 0.3/0 + 0.0/1 + 0.4/2 + 0.7/3 + 1.0/4$$

Example 3

- Consider two fuzzy sets *A* and *B* find Complement, Union, Intersection.

$$\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.6}{4} + \frac{0.2}{5} + \frac{0.6}{6} \right\},$$

$$\underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.8}{3} + \frac{0.4}{4} + \frac{0.7}{5} + \frac{0.3}{6} \right\}.$$

Solution

Complement

$$\bar{\underset{\sim}{A}} = \left\{ \frac{0}{2} + \frac{0.5}{3} + \frac{0.4}{4} + \frac{0.8}{5} + \frac{0.4}{6} \right\},$$

$$\bar{\underset{\sim}{B}} = \left\{ \frac{0.5}{2} + \frac{0.2}{3} + \frac{0.6}{4} + \frac{0.3}{5} + \frac{0.7}{6} \right\}.$$

Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \left\{ \frac{1}{2} + \frac{0.8}{3} + \frac{0.6}{4} + \frac{0.7}{5} + \frac{0.6}{6} \right\}$$

Solution…

Intersection

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.4}{4} + \frac{0.2}{5} + \frac{0.3}{6} \right\}$$

Properties of Fuzzy Sets

Commutativity

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \underset{\sim}{B} \cup \underset{\sim}{A},$$
$$\underset{\sim}{A} \cap \underset{\sim}{B} = \underset{\sim}{B} \cap \underset{\sim}{A}.$$

Associativity

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cup \underset{\sim}{C},$$
$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}.$$

Distributivity

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C}),$$
$$\underset{\sim}{A} \cap (\underset{\sim}{B} \cup \underset{\sim}{C}) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{A} \cap \underset{\sim}{C}).$$

Example

- For the given fuzzy set. Prove the associativity property for the above given sets.

$$\underset{\sim}{A} = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.4}{2.0} + \frac{0.35}{2.5} + \frac{0}{3.0} \right\},$$

$$\underset{\sim}{B} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0} \right\},$$

$$\underset{\sim}{C} = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0}{2.0} + \frac{0.25}{2.5} + \frac{0.5}{3.0} \right\}.$$

Solution

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C}$$

LHS

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right)$$

$$\left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left\{\frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0}\right\}$$

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left\{\frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0}\right\}$$

Solution…

RHS

$$\left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C}$$

$$\left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) = \left\{\frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0}\right\}$$

$$\left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C} = \left\{\frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0}\right\}$$

$$\text{LHS} = \text{RHS}$$

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C}$$

Fuzzy Relation

- Fuzzy relations are fuzzy subsets of X× Y , i.e., mapping from X → Y . It maps elements of one universe, X to those of another universe, say Y , through the Cartesian product of the two universes. A fuzzy relation R is mapping from the Cartesian space X × Y to the interval [0, 1] where the strength of the mapping is expressed by the membership function of the relation for ordered pairs from the two expressed as or μ_R (x, y). This can be expressed as

$$\underset{\sim}{R} = \{((x,y), \mu_{\underset{\sim}{R}}(x,y)) | (x,y) \in X \times Y\}$$

is called a fuzzy relation on *X × Y .*

Operations on Fuzzy Relations

- Let *R* and *T* be fuzzy relation on Cartesian space *X ×Y . Then the following* operations apply for the membership values for various set operations:

- *Union* $$\mu_{\underset{\sim}{R} \cup \underset{\sim}{T}}(x, y) = \max\left(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{T}}(x, y)\right)$$
- *Intersection* $$\mu_{\underset{\sim}{R} \cap \underset{\sim}{T}}(x, y) = \min\left(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{T}}(x, y)\right)$$
- *Complement* $$\mu_{\underset{\sim}{\bar{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y)$$

Properties of Fuzzy Relations

Commutativity

$$\mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \mu_{\underset{\sim}{S} \cup \underset{\sim}{R}}(x, y).$$

Associativity

$$\mu_{\left(\underset{\sim}{R} \cup \underset{\sim}{S}\right) \cup \underset{\sim}{T}}(x, y) = \mu_{\underset{\sim}{R} \cup \left(\underset{\sim}{S} \cup \underset{\sim}{T}\right)}(x, y)$$

Distributivity

$$\mu_{\left(\underset{\sim}{R} \cup \underset{\sim}{S}\right) \cap \underset{\sim}{T}}(x, y) = \mu_{\underset{\sim}{R} \cup \left(\underset{\sim}{S} \cap \underset{\sim}{T}\right)}(x, y)$$

Fuzzy Cartesian Product

- Let *A* be a fuzzy set on universe *X and B* be a fuzzy set on universe *Y* , then the Cartesian product between fuzzy sets *A* and *B* will result in a fuzzy relation *R* which is contained with the full Cartesian product space or

$$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R} \subset X \times Y,$$

- where the fuzzy relation *R* has membership function.

$$\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min\left(\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\right)$$

Example

- Consider two fuzzy sets *A and B. A represents universe of* three discrete temperatures *x = {x1, x2, x3} and B represents universe of two* discrete flow *y = {y1, y2}. Find the fuzzy Cartesian product between them:*

$$\underset{\sim}{A} = \frac{0.4}{x_1} + \frac{0.7}{x_2} + \frac{0.1}{x_3} \quad \text{and} \quad \underset{\sim}{B} = \frac{0.5}{\gamma_1} + \frac{0.8}{\gamma_2}$$

Solution

- *A* represents column vector of size 3 × *1 and B* represents column vector of size 1×*2. The fuzzy Cartesian product results in a fuzzy relation R* of size 3 × *2*

$$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.4 & 0.4 \\ 0.5 & 0.7 \\ 0.1 & 0.1 \end{bmatrix} \end{array}$$

Membership Functions

- Fuzziness in a fuzzy set is characterized by its membership functions.
- It classifies the element in the set, whether it is discrete or continuous. The membership functions can also be formed by graphical representations.
- The graphical representations may include different shapes.
- There are certain restrictions regarding the shapes used.
- The rules formed to represent the fuzziness in an application are also fuzzy.
- Membership value is between 0 and 1.

Features of Membership Function

- The feature of the membership function is defined by three properties. They are:

1. Core
2. Support
3. Boundary

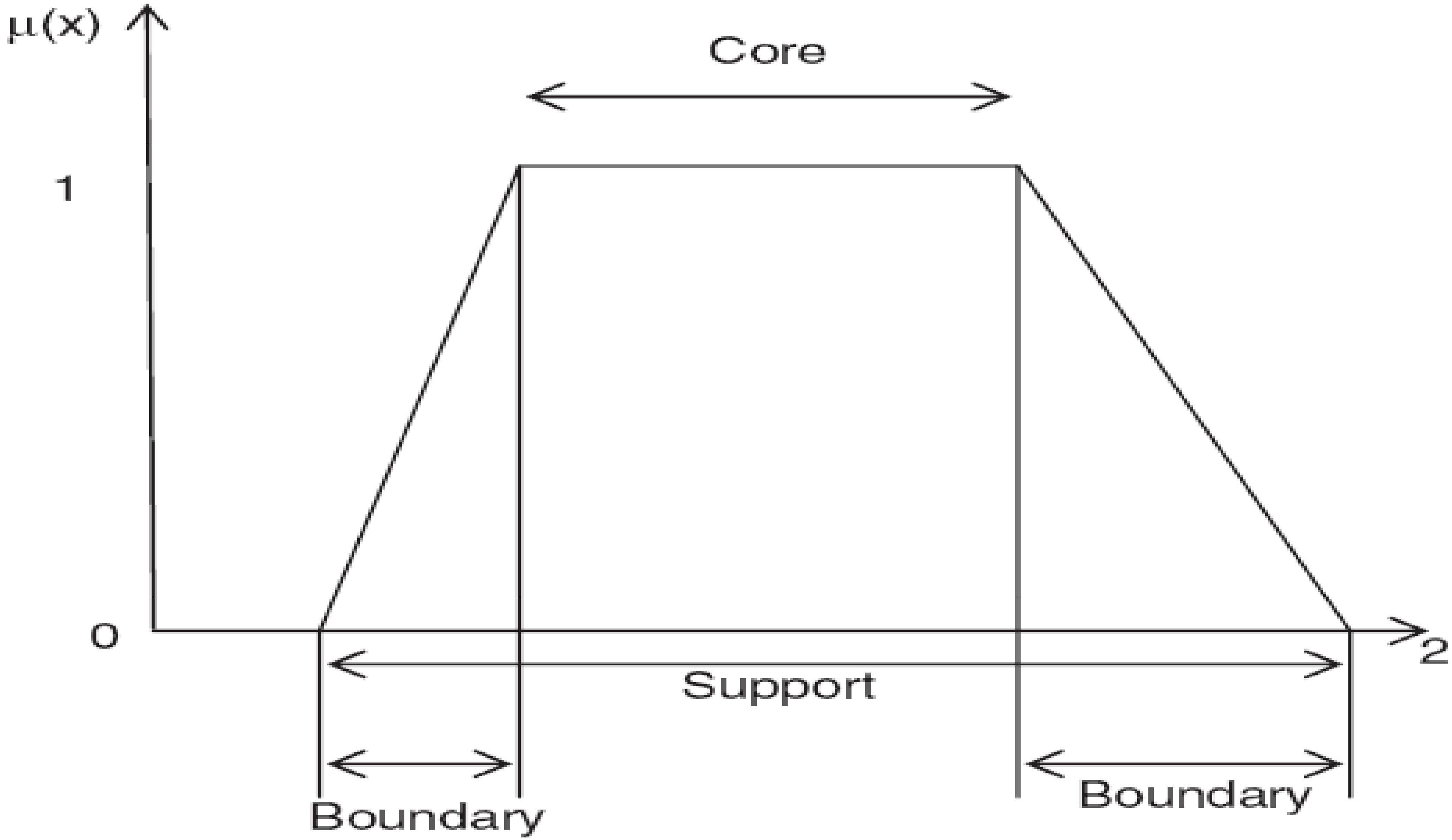
Features of Membership Function…

- **Core**
- If the region of universe is characterized by full membership (1) in the set *A* then this gives the core of the membership function of fuzzy at *A.*
- The elements, which have the membership function as 1, are the elements of the core, i.e., here $\mu_A(x) = 1$.
- **Support**
- If the region of universe is characterized by nonzero membership in the set *A*, this defines the support of a membership function for fuzzy set $A\sim$
- The support has the elements whose membership is greater than .$\mu_A(x) > 0$.

Features of Membership Function…

- **Boundary**
- If the region of universe has a nonzero membership but not full membership, this defines the boundary of a membership; this defines the boundary of a membership function for fuzzy set *A*:
- The boundary has the elements whose membership is between 0 and 1, $0 < \mu A(x) < 1$.

Features of Membership Function…



Features of Membership Function…

- **Crossover point**

  - The crossover point of a membership function is the elements in universe whose membership value is equal to 0.5, $\mu_A(x) = 0.5$.

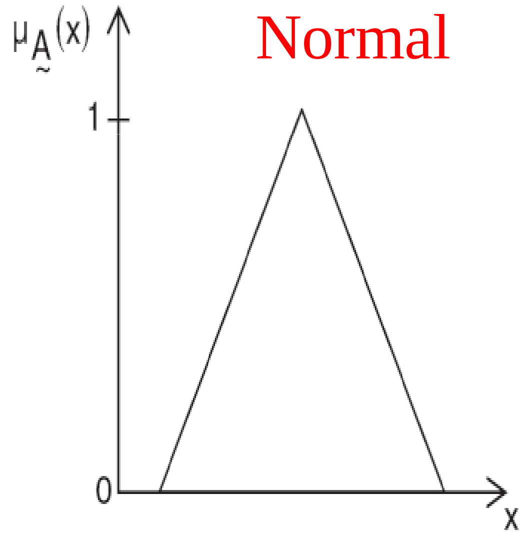
- **Height**

  - The height of the fuzzy set *A* is the maximum value of the membership function,
  - $\max(\mu_A(x))$

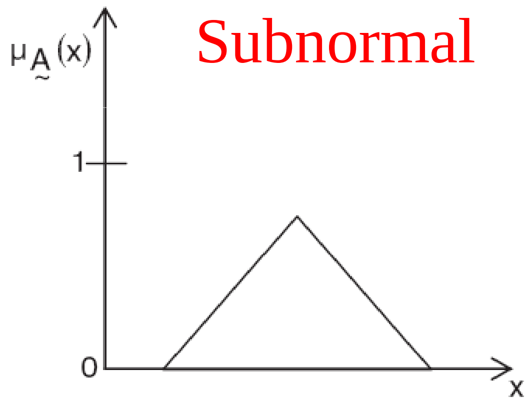
Classification of Fuzzy Sets

- The fuzzy sets can be classified based on the membership functions. They are:

- ***Normal fuzzy set****: If the membership function has at least one element in* the universe whose value is equal to 1, then that set is called as normal fuzzy set.



- ***Subnormal fuzzy set****: If the membership function has the membership values* less than 1, then that set is called as subnormal fuzzy set.



Fuzzification

- Fuzzification is the process where the crisp quantities are converted to fuzzy (crisp to fuzzy).
- By identifying some of the uncertainties present in the crisp values.
- The conversion of fuzzy values is represented by the membership functions.
- The assignment can be just done by intuition or by using some algorithms or logical procedures.

Defuzzification

- Defuzzification means the fuzzy to crisp conversions.
- The fuzzy results generated cannot be used as such to the applications, hence it is necessary to convert the fuzzy quantities into crisp quantities for further processing.
- The defuzzification has the capability to reduce a fuzzy to a crisp single-valued quantity or as a set, or converting to the form in which fuzzy quantity is present.
- Defuzzification can also be called as “rounding off” method. Defuzzification reduces the collection of membership function values in to a single sealer quantity.

Defuzzification

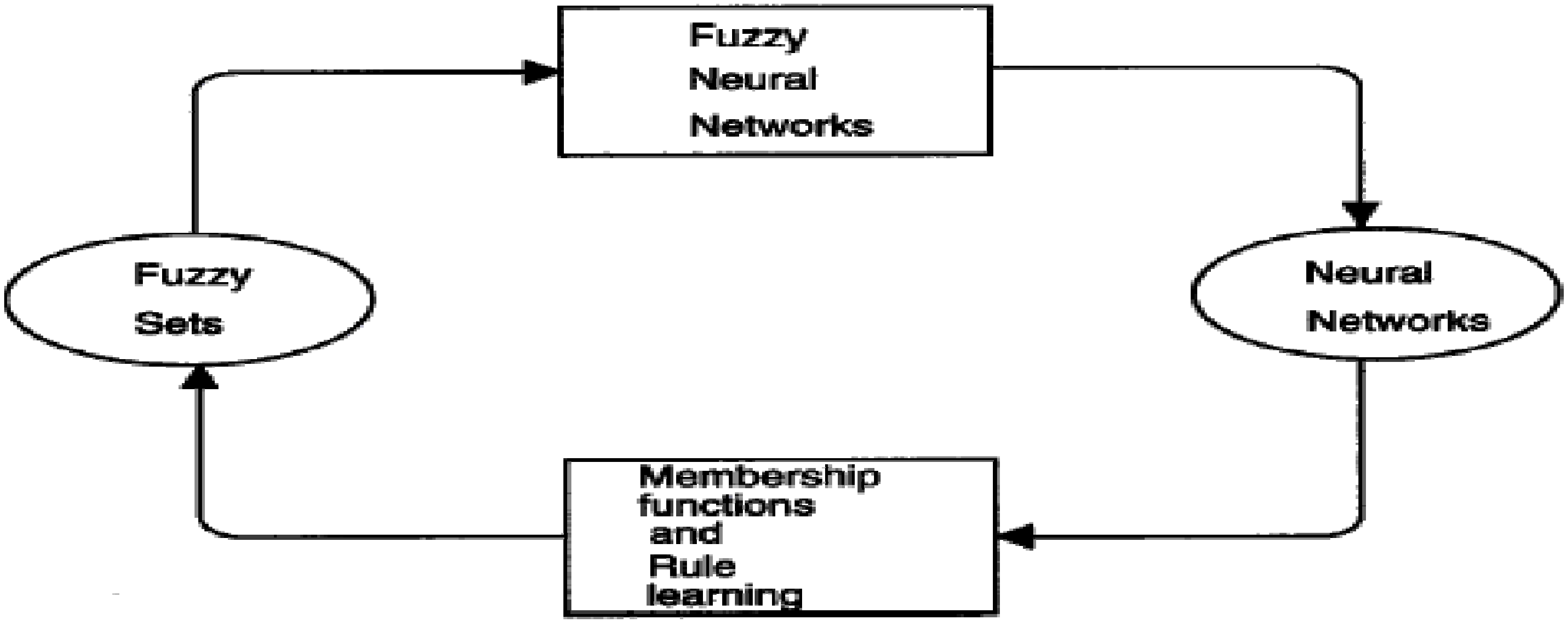
- There are seven methods used for defuzzifying the fuzzy output functions.

1. Max-membership principle,
2. Centroid method,
3. Weighted average method,
4. Mean–max membership,
5. Centre of sums,
6. Centre of largest area, and
7. First of maxima or last of maxima

Neuro-Fuzzy Systems

- Neuro-fuzzy systems
  - Soft computing methods that combine in various ways neural networks and fuzzy concepts
- ANN – nervous system – low level perceptive and signal integration
- Fuzzy part – represents the emergent "higher level" reasoning aspects

Neuro-Fuzzy Systems...



- “Fuzzification” of neural networks
- Endowing of fuzzy system with neural learning features

Neuro-Fuzzy Systems...

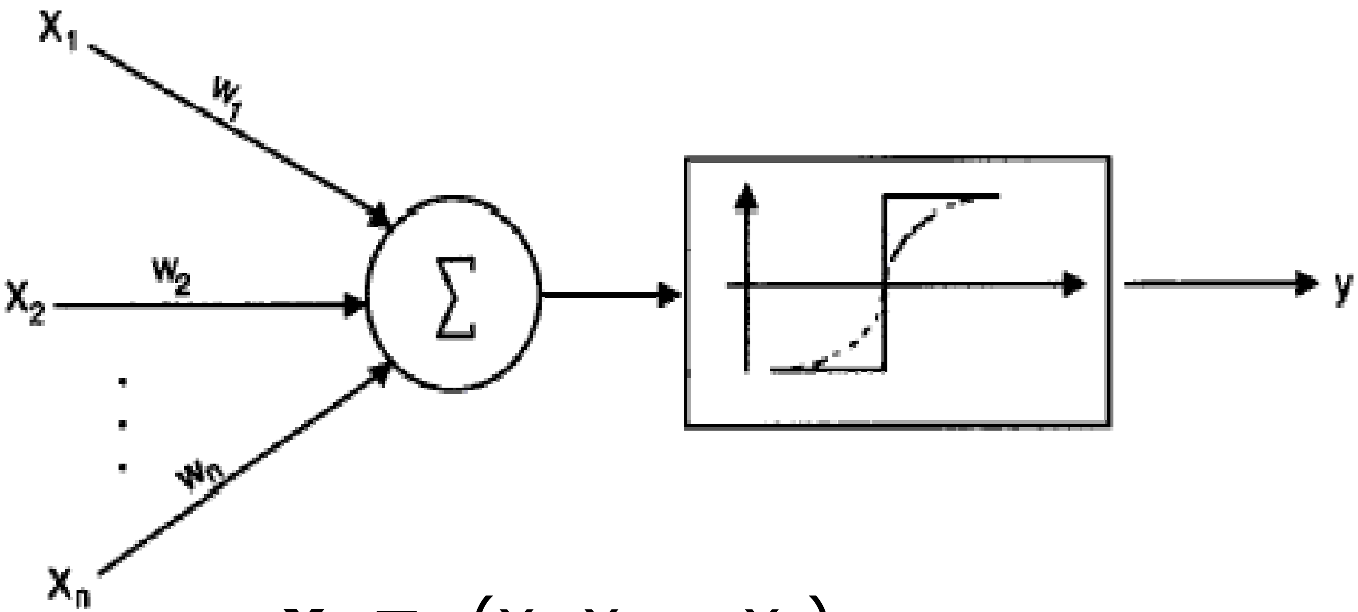
- Co-operative-neural algorithm adapt fuzzy systems
  - Off-line – adaptation
  - On-line – algorithms are used to adapt as the system operates
- Concurrent – where the two techniques are applied after one another as pre- or post-processing
- Hybrid – fuzzy system being represented as a network structure, making it possible to take advantage of learning algorithm inherited from ANNs

Fuzzy Neural Networks

- Introduction of fuzzy concepts into artificial neurons and neural networks
- For example, while neural networks are good at recognizing patterns, they are not good at explaining how they reach their decisions.
- Fuzzy logic systems, which can reason with imprecise information, are good at explaining their decisions but they cannot automatically acquire the rules they use to make those decisions.
- These limitations have been a central driving force behind the creation of intelligent hybrid systems where two or more techniques are combined in a manner that overcomes individual techniques

Fuzzy Neurons

- Fuzzy model of artificial neuron can be constructed by using fuzzy operations at single neuron level

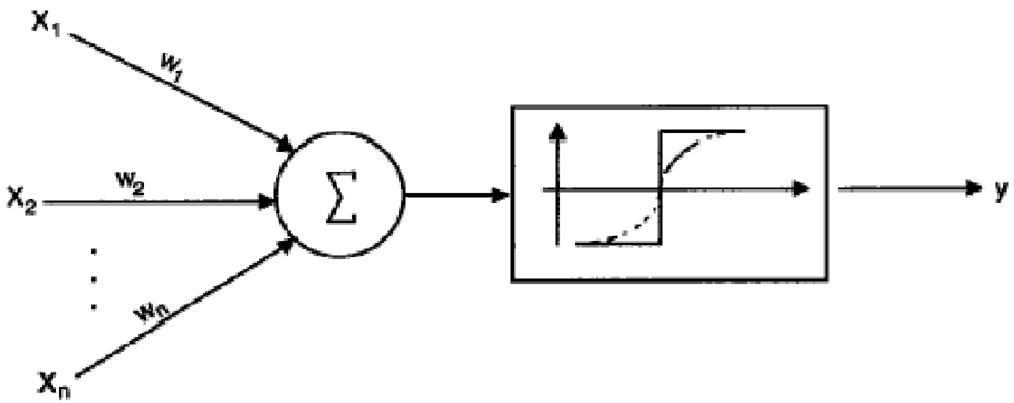


$y = g(\mathbf{w}.\mathbf{x})$

$\mathbf{x} = (x_1, x_2, \ldots\ x_n)$

$\mathbf{w} = (w_1, w_2, \ldots\ w_n)$

Fuzzy Neurons



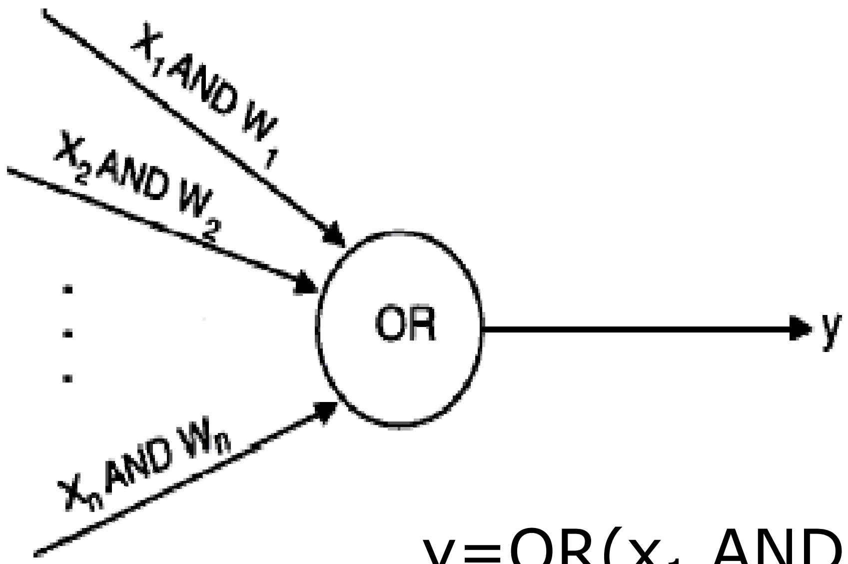
$$y = g(\mathbf{w.x})$$

$$\Downarrow$$

$$y = g(A(\mathbf{w,x}))$$

- Instead of weighted sum of inputs, more general aggregation function is used
- Fuzzy union, fuzzy intersection and, more generally, *s-norms* and *t-norms* can be used as an aggregation function for the weighted input to an artificial neuron

OR Fuzzy Neuron

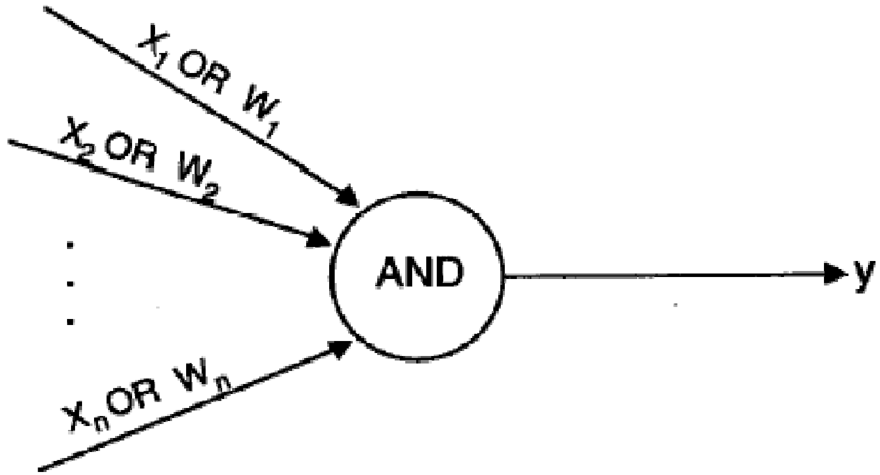


$OR:[0,1]x[0,1]^n->[0,1]$

$y=OR(x_1 \text{ AND } w_1, x_2 \text{ AND } w_2 \dots x_n \text{ AND } w_n)$

- Transfer function g is linear
- If $w_k=0$ then w_k AND $x_k=0$ while if $w_k=1$ then w_k AND $x_k= x_k$ independent of x_k

AND Fuzzy Neuron



AND:[0,1]x[0,1]^n->[0,1]

$$y=AND(x_1 \text{ OR } w_1, x_2 \text{ OR } w_2 \ldots x_n \text{ OR } w_n)$$

- In the generalized forms based on *t*-norms, operators other than *min* and *max* can be used such as algebraic and bounded products and sums

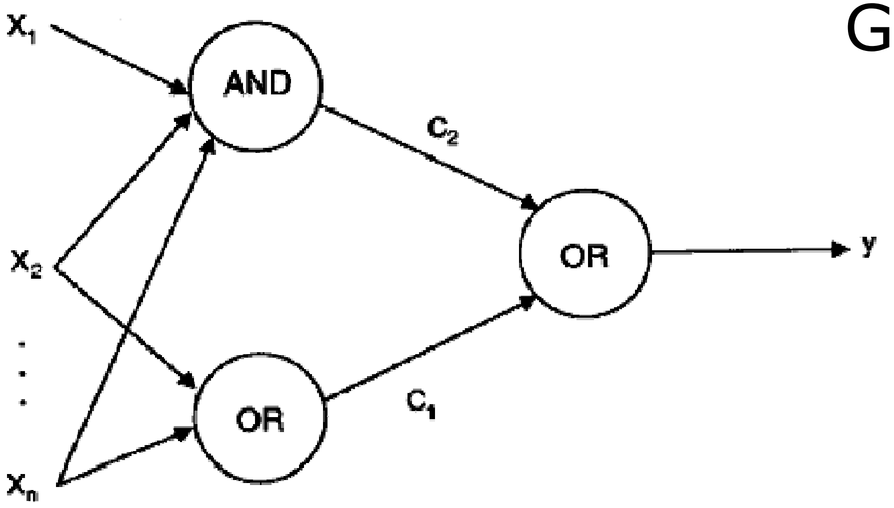
Fuzzy Neurons

- Both the OR and the AND logic neurons are excitatory in character, i.e. $\uparrow x_k \Rightarrow \uparrow y$
- Issue of inhibitory (negative) weights deserves a short digression
- In the realm of fuzzy sets operations are defined in [0,1]
- Proper solution to make a weighted input inhibitory is to take fuzzy complement of the excitatory membership value $\neg x = 1-x$
- Input $\mathbf{x}=(x_1,..x_n)$ is extended to $\mathbf{x}=(x_1,\ldots,x_n,\neg x_1,\ldots,\neg x_n)$

Fuzzy Neurons

- The weighted inputs x_i o w_i, where o is a t-norm and t-conorm, can be general fuzzy relations too, not just simple products as in standard neurons
- The transfer function *g* can be a non-linear such as a sigmoid

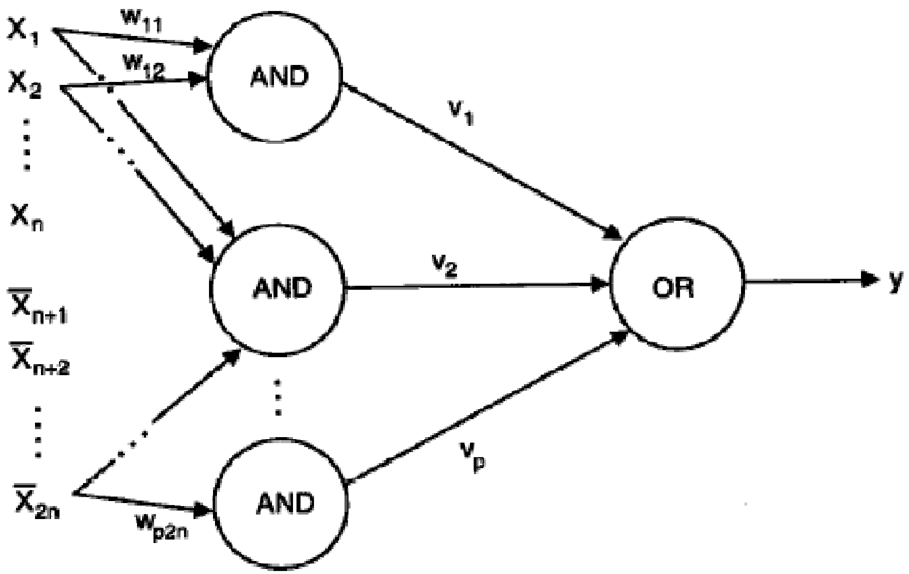
OR / AND Fuzzy Neuron



Generalization of the above simple fuzzy neurons

- This structure can produce a spectrum of intermediate behaviors that can be modified in order to suit a given problem
- If c_1 = 0 and c_2 = 1 the system reduces itself to pure AND neuron
- If c_1 = 1 and c_2 = 0 the behavior corresponds to that of a pure OR neuron

Multilayered Fuzzy Neural Networks



A second possibility is to have OR neurons in the hidden layer and a single AND neuron in the output layer

- If we restrict ourselves to the pure two-valued Boolean case, network represents an arbitrary Boolean function as a sum of minterms
- More generally, if the values are continuous members of a fuzzy set then these networks approximate certain unknown fuzzy function

Learning in Fuzzy Neural Networks

- Supervised learning in FNN consists in modifying their connection weights in a such a manner that an error measure is progressively reduced

- Its performance should remain acceptable when it is presented with new data

- Set of training data pairs ($\mathbf{x}_k$, d_k) for k=1,2..n

- $\mathbf{w}^{t+1}=\mathbf{w}^t + \Delta\mathbf{w}^t$, where weight change is a given function of difference between the target response *d* and calculated node output y $\Delta\mathbf{w}^t=F(|d^t-y^t|)$

Learning in Fuzzy Neural Networks

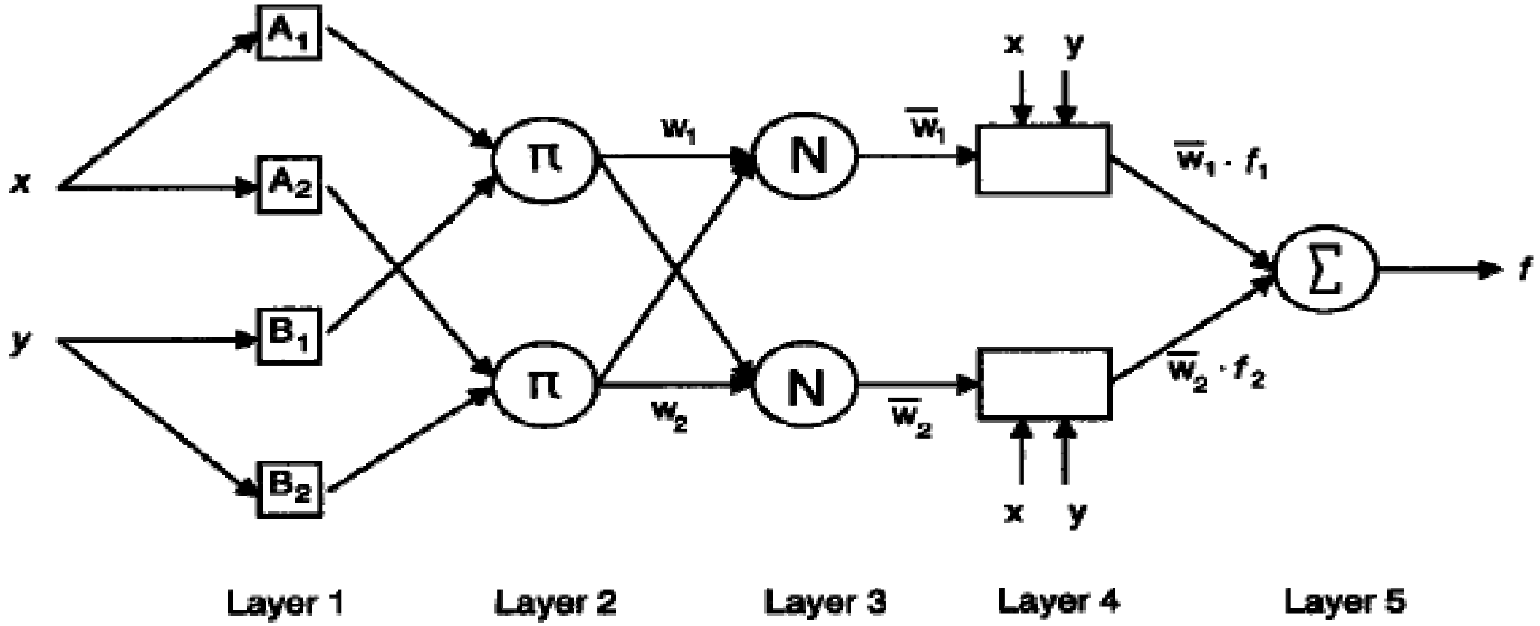
- Mean square error E – measure of how well the fuzzy network maps input data into the corresponding output
- $E(\mathbf{w}) = \frac{1}{2}\sum(d_k - y_k)^2$
- Gradient descent $\Delta w_{i,j} = -\eta \frac{\partial E}{\partial w_{ij}}$

ANFIS System

IF x is A_1 AND y is B_2 THEN $f_1 = p_1 x + q_1 y + r_1$
IF x is A_2 AND y is B_2 THEN $f_2 = p_2 x + q_2 y + r_2$

The reasoning mechanism for this model is:

$$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} = \bar{w}_1 + \bar{w}_2.$$



ANFIS System

- Learning algorithm is a hybrid supervised method based on gradient descent and Least-squares
- Forward phase: signals travel up to layer 4 and the relevant parameters are fitted by least squares
- Backward phase: the error signals travel backward and the premise parameters are updated as in backpropagation
- Fuzzy toolbox MATLAB
- Mackey-Glass prediction / excellent non-linear fitting and generalization / less parameters and training time is comparable with ANN methods

ANFIS System

- Since a wide class of fuzzy controllers can be transformed into equivalent adaptive networks, ANFIS can be used for building intelligent controllers that is, controllers that can reason with simple fuzzy inference and that are able to learn from experience in the ANN style.