Unit- 1

Theory of Relativity

- Frame of Reference
- The Michelson-Morley Experiment
- Einstein's Postulates
- The Lorentz Transformation
- Time Dilation and Length Contraction
- Addition of Velocities
- Experimental Verification
- Twin Paradox
- Space-time
- Doppler Effect
- Relativistic Momentum
- Relativistic Energy

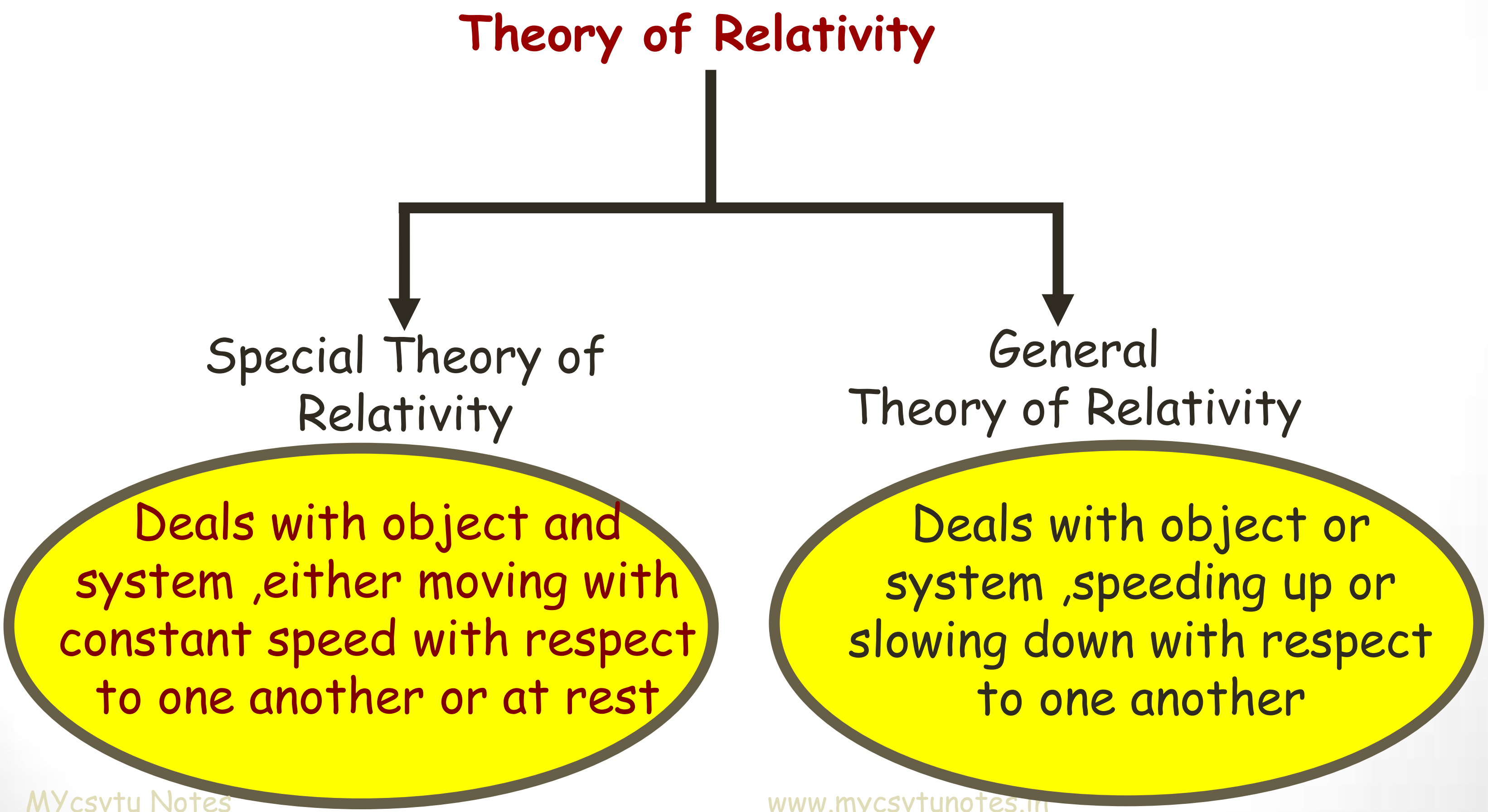
Albert Michelson
(1852-1931)

It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there is still a difficulty in the theory itself…

- Albert Michelson, 1907

Theory of Relativity

- The theory which deals with relativity of motion and rest is called theory of relativity.



Theory of Relativity

- Motion of body is described with respect to some well defined coordinate system, which is known as "Frame of Reference".

Frame of Reference

Inertial frame of reference

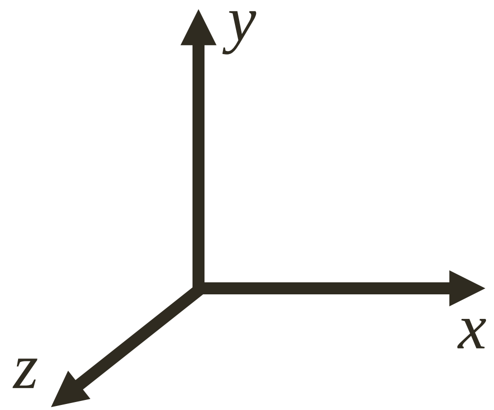
In this frame bodies not acted upon by external force, is at rest or moves with constant velocity and obey Newton's law of inertia

Non-inertial frame of reference

In this frame , bodies not acted upon by any external force, is accelerated not obey Newton's law of inertia.

- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**



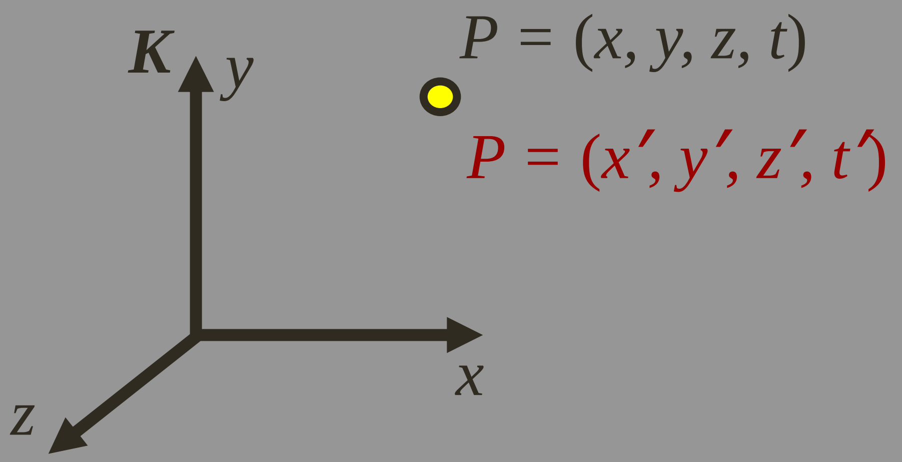
If the axes are also parallel, these frames are said to be **Inertial Coordinate Systems**

The Galilean Transformation

For a point P:

In one frame K: $P = (x, y, z, t)$

In another frame K': $P = (x', y', z', t')$



Conditions of the Galilean Transformation

- 1. Parallel axes
- 2. K' has a constant relative velocity (here in the *x*-direction) with respect to K.
- 3. Time (*t*) for all observers is a **Fundamental invariant**, i.e., it's the same for all inertial observers.

$$x' = x - vt$$

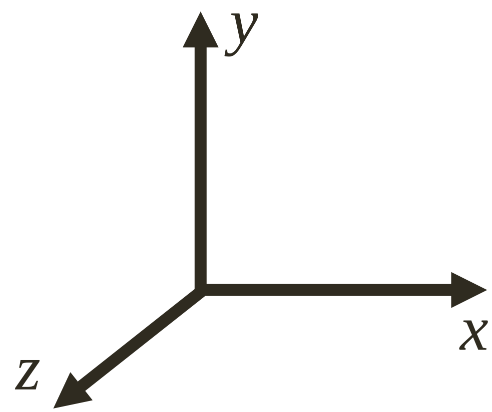
$$y' = y$$

$$z' = z$$

$$t' = t$$

Newtonian (Classical) Relativity

Newton's laws of motion must be implemented with respect to (relative to) some reference frame.



A reference frame is called an **inertial frame** if Newton's laws are valid in that frame.

Such a frame is established when a body, not subjected to net external forces, moves in rectilinear motion at **constant velocity**.

- Step 1. Replace -v with +v.
- Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed."

$$x = x' + vt'$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

- The wave nature of light seemed to require a propagation medium. It was called the **luminiferous ether** or just **ether** **(or aether)**.

- Ether was supposed to be invisible, mass less, perfectly transparent, perfectly non-resistive and at absolute rest.

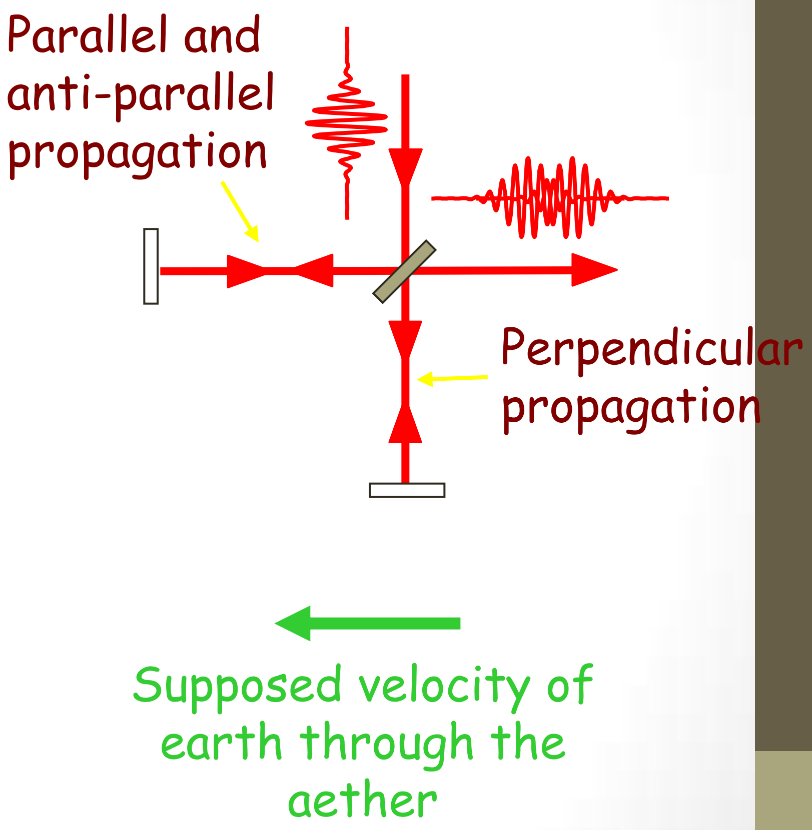
  - ✓ It had to have an elasticity to support the high velocity of light waves.

  - ✓ And somehow, it could not support longitudinal waves.

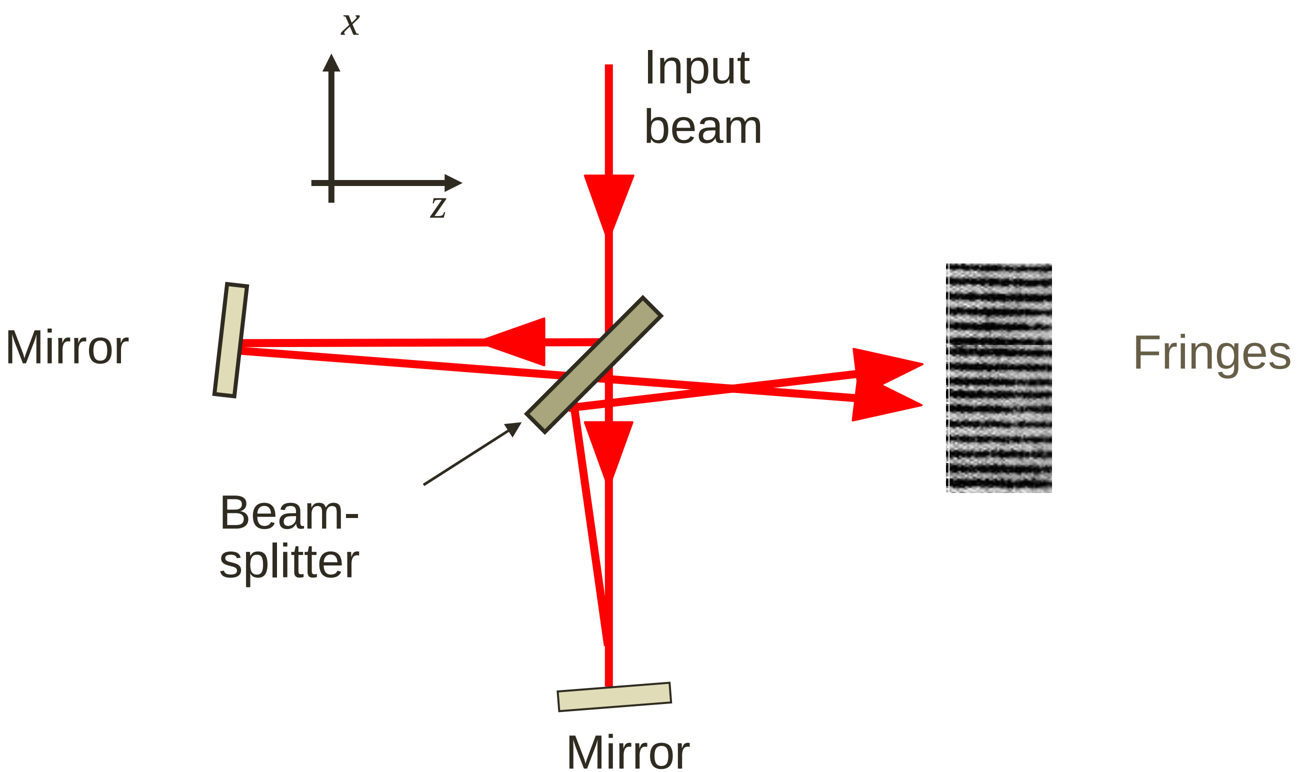
- And (it goes without saying…) light waves in the aether obeyed the Galilean transformation for moving frames.

Michelson-Morley experiment

Michelson and Morley realized that the earth could not always be stationary with respect to the ether. And light would have a different path length and phase shift depending on whether it propagated parallel and anti-parallel or perpendicular to the aether.



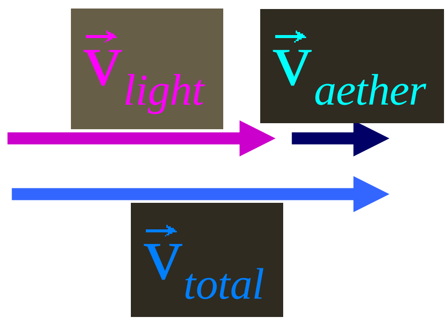
Michelson - Morley Experiment



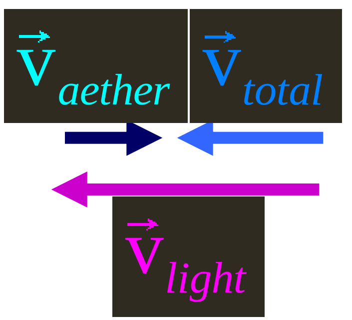
Michelson - Morley Experiment

If light requires a medium, then its velocity depends on the velocity of the medium. Velocity vectors add.

Parallel velocities



Anti-parallel velocities



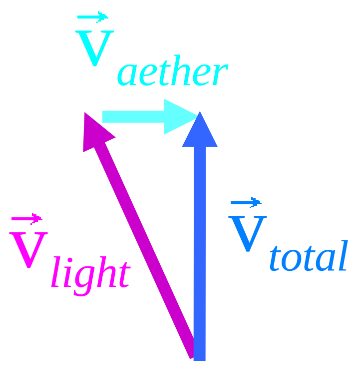
$$V_{total} = V_{light} + V_{aether}$$

$$V_{total} = V_{light} - V_{aether}$$

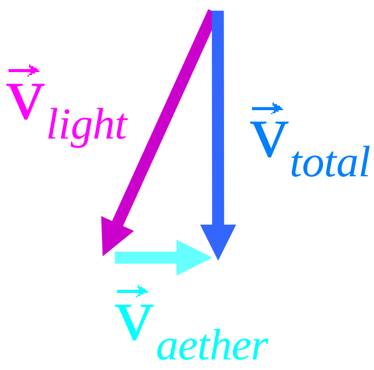
Michelson - Morley Experiment

In the other arm of the interferometer, the total velocity must be perpendicular, so light must propagate at an angle.

Perpendicular velocity to mirror



Perpendicular velocity after mirror



$$V_{total} = \sqrt{V_{light}^{2} - V_{aether}^{2}}$$

Michelson-Morley Experiment

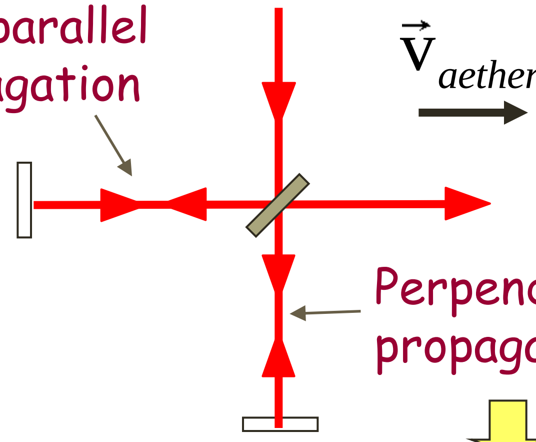
Let c be the speed of light, and v be the velocity of the aether.

$$\Delta t_{\parallel} = \frac{L}{c-\text{v}} + \frac{L}{c+\text{v}}$$

$$= \frac{L(c+\text{v})}{c^2-\text{v}^2} + \frac{L(c-\text{v})}{c^2-\text{v}^2}$$

$$= \frac{2Lc}{c^2-\text{v}^2}$$

$$= \frac{2L}{c}\frac{1}{[1-\text{v}^2/c^2]}$$



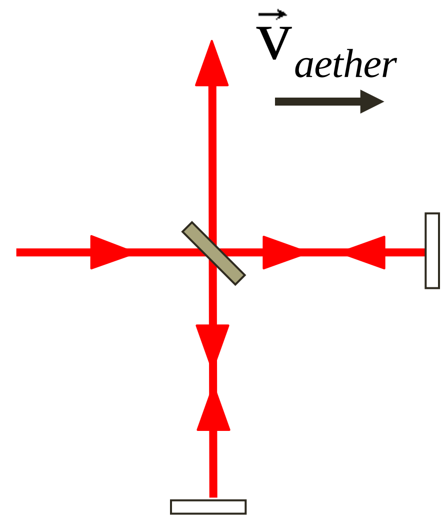
$$\Delta t_{\perp} = \frac{2L}{\sqrt{c^2-\text{v}^2}}$$

$$= \frac{2L}{c}\frac{1}{\sqrt{1-\text{v}^2/c^2}}$$

The delays for the two arms depend differently on the velocity of the aether!

Michelson-Morley Experiment

Because we don't know the direction of the ether velocity, Michelson and Morley did the measurement twice, the second time after rotating the apparatus by 90°



$$\Delta t_{\parallel} = \frac{2L}{c}\frac{1}{[1 - v^2/c^2]}$$

The delay reverses, and any fringe shift seen in this second experiment will be opposite that of the first.

$$\Delta t_{\perp} = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

Actually, the rotated the apparatus continuously by 180° looking for a sinusoidal variation in the shift with this amplitude.

Michelson-Morley Experiment: Analysis

Copying: $\Delta t_{\parallel} = \frac{2L}{c}\frac{1}{[1 - v^2/c^2]}$ $\Delta t_{\perp} = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$

Upon rotating the apparatus by 90°, the optical path lengths are interchanged producing the opposite change in time. Thus the **time difference between path differences** is given by:

$$2\left(\Delta t_{\parallel} - \Delta t_{\perp}\right) = 2\frac{2L}{c}\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

Assuming v << c:

$$\approx 2\frac{2L}{c}\left[\left(1 + v^2/c^2\right) - \left(1 + v^2/2c^2\right)\right] = 2\frac{2L}{c}\frac{v^2}{2c^2}$$

$$\Rightarrow \quad 2\left(\Delta t_{\parallel} - \Delta t_{\perp}\right) \approx 2L\frac{v^2}{c^3}$$

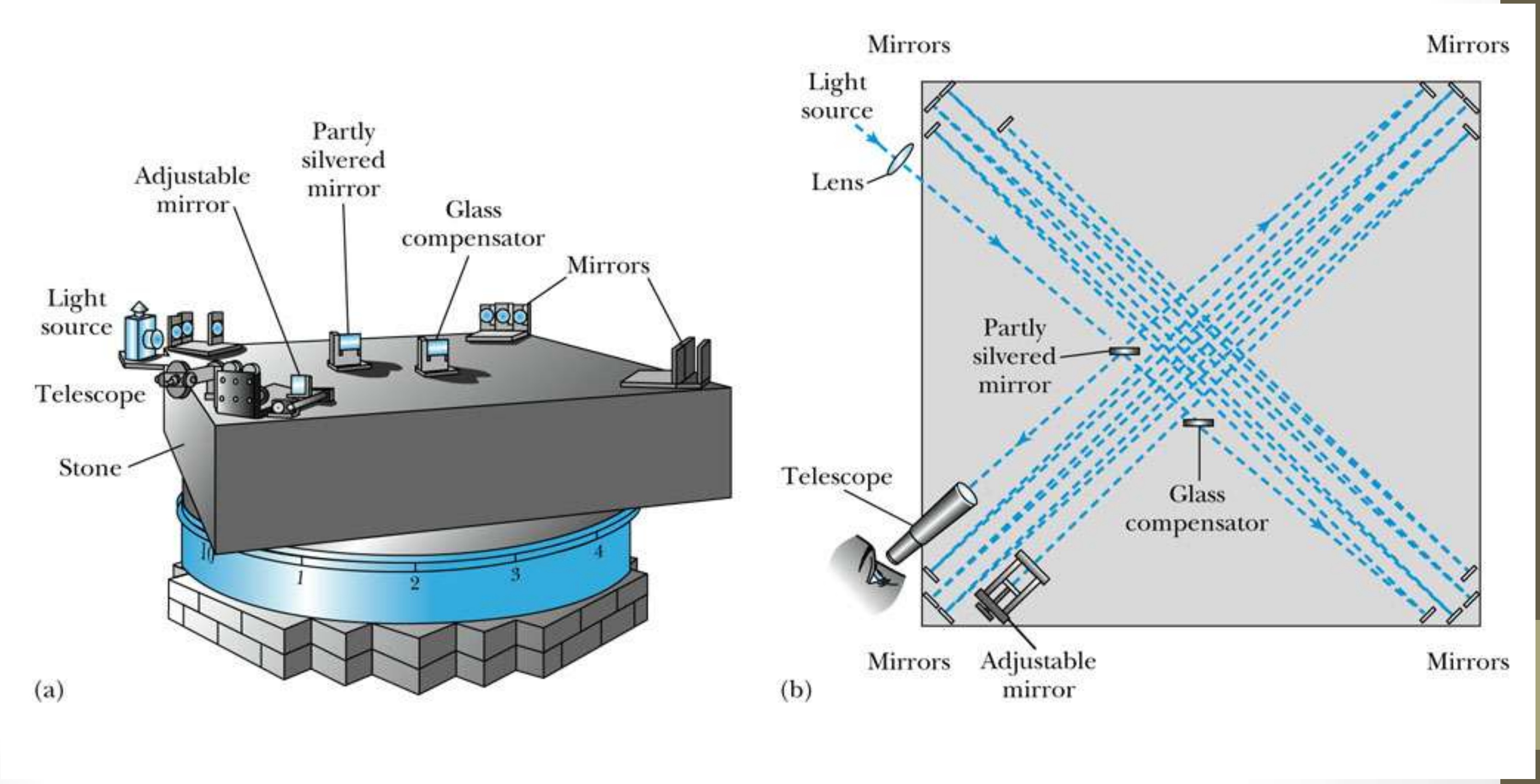
$$2\left(\Delta t_{\parallel} - \Delta t_{\perp}\right) \approx 2L\frac{v^2}{c^3}$$

Recall that the phase shift is ω times this relative delay:

$$2\omega L\frac{v^2}{c^3} \quad \text{or:} \quad 4\pi\frac{L}{\lambda}\frac{v^2}{c^2}$$

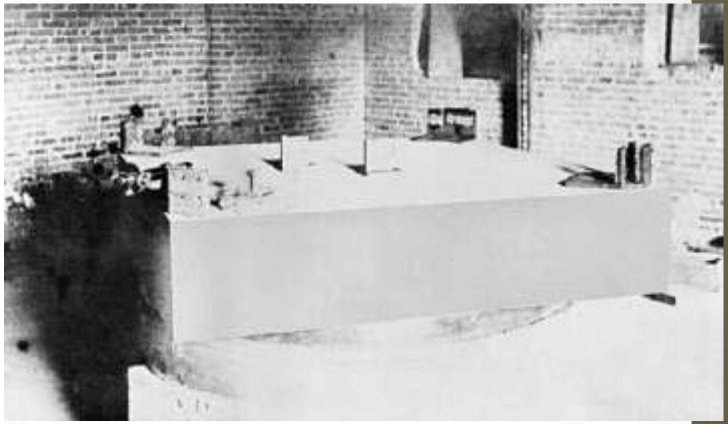
- The Earth's orbital speed is: $v = 3 \times 10^4$ m/s
-
- and the interferometer size is: $L = 1.2$ m
- So the time difference becomes: 8×10^{-17} s
- which, for visible light, is a phase shift of: 0.2 rad = 0.03 periods
- Although the time difference was a very small number, it was well within the experimental range of measurement for visible light in the Michelson interferometer, especially with a folded path.

They folded the path to increase the total path of each arm.

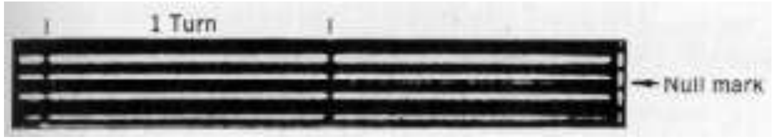


Michelson-Morley Experiment: Results

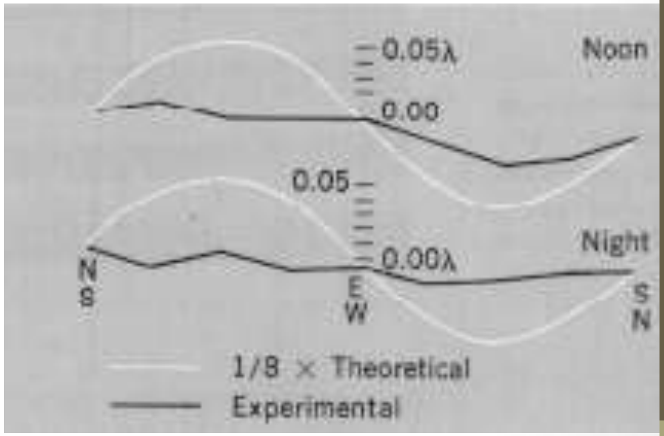
The Michelson interferometer should've revealed a fringe shift as it was rotated with respect to the aether velocity. MM expected 0.4 periods of shift and could resolve 0.005 periods. They saw none!



Their apparatus



Interference fringes showed no change as the interferometer was rotated.



Michelson and Morley's results from A. A. Michelson, *Studies in Optics*

Michelson's Conclusion

In several repeats and refinements with assistance from Edward Morley, he always saw *a null result.*

He concluded that the hypothesis of the stationary ether must be incorrect.

Thus, ether seems not to exist!

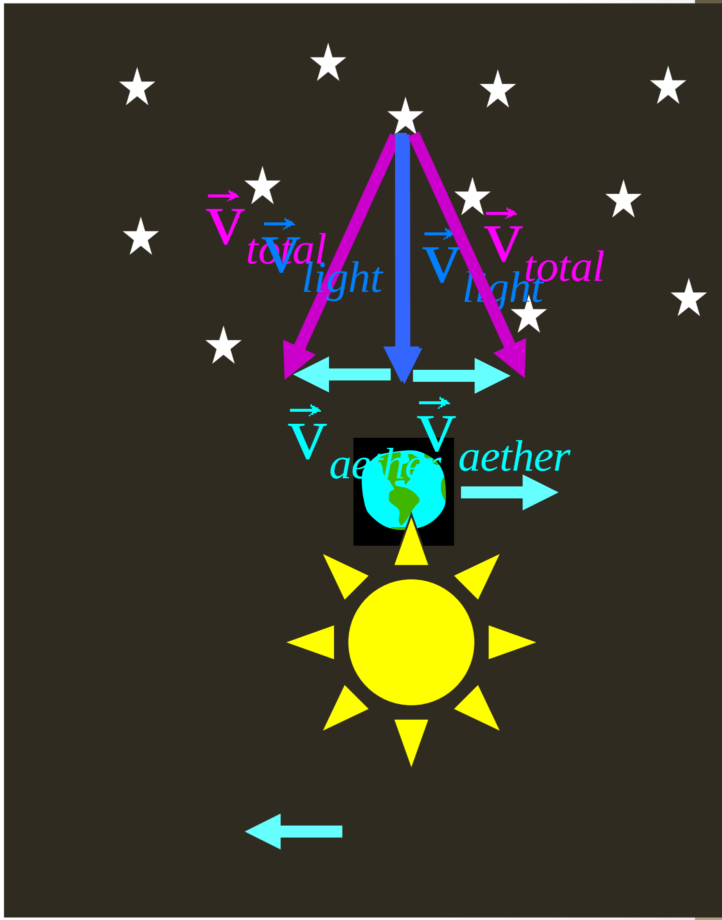
Albert Michelson
(1852-1931)

Edward Morley
(1838-1923)

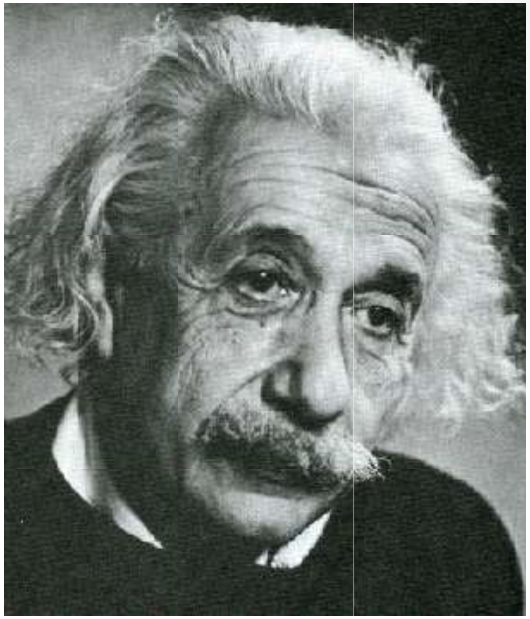
Possible Explanations for MM's Null Result

Many explanations were proposed, but the most popular was the **ether drag** hypothesis.

> This hypothesis suggested that the Earth somehow "dragged" the aether along as it rotates on its axis and revolves about the sun.
>
> This was contradicted by **stellar abberation** wherein telescopes had to be tilted to observe starlight due to the Earth's motion. If ether were dragged along, this tilting would not occur.



- Albert Einstein was only two years old when Michelson and Morley reported their results.
- At age 16 Einstein began thinking about Maxwell's equations in moving inertial systems.
- In 1905, at the age of 26, he published his startling proposal: the **Principle of Relativity**.
- It nicely resolved the Michelson and Morley experiment (although this wasn't his intention and he maintained that in 1905 he wasn't aware of MM's work…)



Albert Einstein (1879-1955)

It involved a fundamental new connection between space and time and that Newton's laws are only an approximation.

- With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposed the following postulates:

- **The principle of relativity**: All the laws of physics (not just the laws of motion) are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

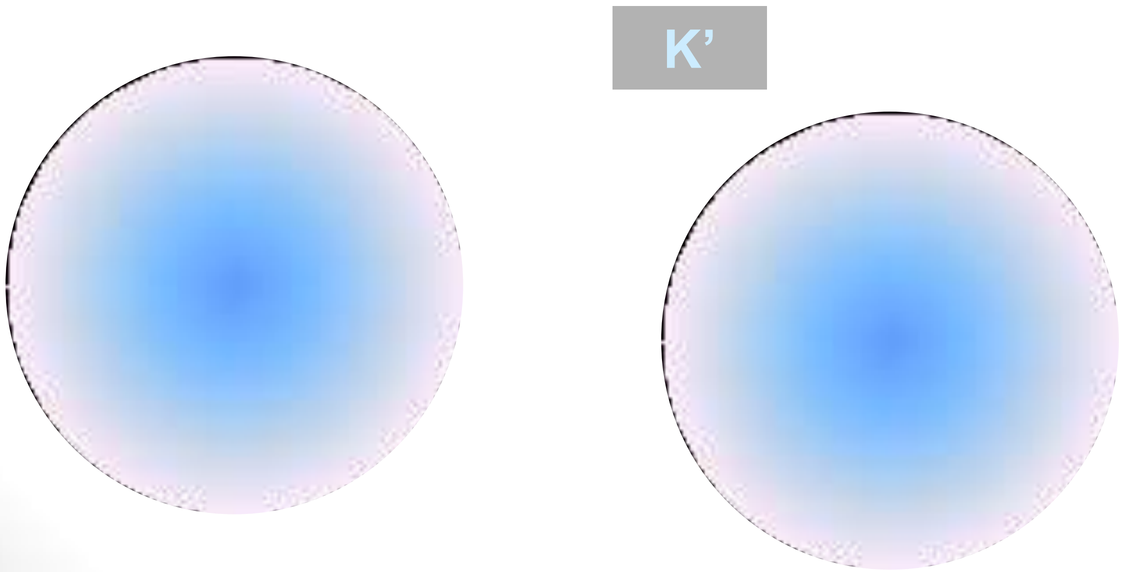
- **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The constancy of the speed of light

Consider the fixed system K and the moving system K’. *At t* = 0, the origins and axes of both systems are coincident with system K’ moving to the right along the *x* axis.

A flashbulb goes off at both origins when *t* = 0.

According to postulate 2, the speed of light will be *c* in both systems and the wavefronts observed in both systems must be spherical.



The constancy of the speed of light is not compatible with Galilean

- Spherical wavefronts in K: $$x^2 + y^2 + z^2 = c^2t^2$$
- Spherical wavefronts in K': $$x'^2 + y'^2 + z'^2 = c^2t'^2$$
- Note that this cannot occur in Galilean transformations:

$$x' = x - \mathrm{v}t$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

There are a couple of extra terms ($-2x\mathrm{v}t + \mathrm{v}^2t^2$) in the primed frame.

$$x'^2 + y'^2 + z'^2 = (x^2 - 2x\mathrm{v}t + \mathrm{v}^2t^2) + y^2 + z^2 \neq c^2t'^2$$

- What transformation will preserve spherical wave-fronts in both frames?
- Try $x' = \gamma\,(x - \mathrm{v}t)$ so that $x = \gamma'\,(x' + \mathrm{v}t')$, where γ could be anything.
- By Einstein's first postulate: $\gamma' = \gamma$
- The wave-front along the x'- and x-axes must satisfy: $x' = ct'$ and $x = ct$
- Thus: $ct' = \gamma\,(ct - \mathrm{v}t)$ or $t' = \gamma\, t\,(1 - \mathrm{v}/c)$
- and: $ct = \gamma'\,(ct' + \mathrm{v}t')$ or $t = \gamma' t'(1 + \mathrm{v}/c)$
- Substituting for t in $t' = \gamma\, t\,(1 - \mathrm{v}/c)$:

$$t' = \gamma^2 t' \left(1 - \frac{\mathrm{v}}{c}\right)\left(1 + \frac{\mathrm{v}}{c}\right)$$

- which yields:

$$\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

- Some simple properties of γ:

$$1 - \frac{\mathrm{v}^2}{c^2} = \frac{1}{\gamma^2}$$

- which yields:

$$\frac{\mathrm{v}^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

- When the velocity is small:

$$\gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2 / c^2}} \approx \frac{1}{1 - \frac{1}{2}\mathrm{v}^2 / c^2} \approx 1 + \tfrac{1}{2}\mathrm{v}^2 / c^2$$

$$\gamma \approx 1 + \tfrac{1}{2}\mathrm{v}^2 / c^2$$

- Now substitute $x' = \gamma\,(x - \mathrm{v}\,t)$ into $x = \gamma\,(x' + \mathrm{v}\,t')$:
- $$x = \gamma\,[\gamma\,(x - \mathrm{v}\,t) + \mathrm{v}\,t'\,]$$
- Solving for t' we obtain: $x - \gamma^2\,(x - \mathrm{v}\,t) \;=\; \gamma\,\mathrm{v}\,t'$
- or: $$t' \;=\; x/\gamma\,\mathrm{v} - \gamma\,(\,x/\mathrm{v} - t\,)$$
- or: $$t' \;=\; \gamma t + x/\gamma\,\mathrm{v} - \gamma\,x/\mathrm{v}$$
- or: $$t' \;=\; \gamma t + (\gamma x/\mathrm{v})\,(\,1/\gamma^2 - 1\,)$$
- or: $$t' = \gamma(t - \mathrm{v}x/c^2)$$

v^2/c^2

$$x' = \frac{x - \mathrm{v}t}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \mathrm{v}x / c^2}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

A more symmetrical form:

$$\beta = \mathrm{v}/c$$

$$\gamma = \frac{1}{\sqrt{1-\mathrm{v}^2/c^2}}$$

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

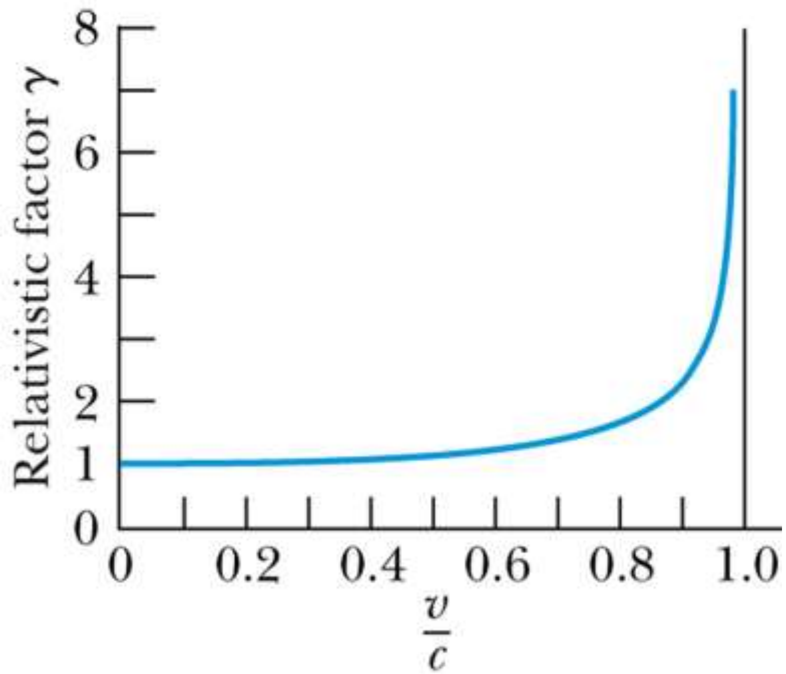
$$t' = \gamma(t - \beta x/c)$$

- Recall that $\beta = \mathrm{v} / c < 1$ for all observers.

γ equals 1 only when $\mathrm{v} = 0$.
In general:

$$\gamma \geq 1$$

Graph of γ vs. β:
(note $\mathrm{v} < c$)



$$x' = \frac{x - \mathrm{v}t}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \mathrm{v}x / c^2}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$x = \frac{x' + \mathrm{v}t'}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \mathrm{v}x' / c^2}{\sqrt{1 - \mathrm{v}^2 / c^2}}$$

- If $\mathrm{v} << c$, i.e., $\beta \approx 0$ and $\gamma \approx 1$, yielding the familiar Galilean transformation.
- Space and time are now linked, and the frame velocity cannot exceed c.

UNIT-IV (DIGITAL ELECTRONICS)

- **Note : Solve any two questions.**

- **Q.1.** State de Morgan's theorems. Verify them using truth table for two variables.
- **Q.2.** Why NAND and NOR gates are called universal gates?
- **Q.3.** Do the following operation:
- (i) (19.63)10 to binary
- (ii)(E91)16 + (2D3)16
- (iii)(2A4)16 * (A8)16
- (iv)(43)8 * (76)8
- (v)(11001)2 – (10110)2
- **Q.4.** Write electronic symbol, equivalent circuit and two input truth table for AND, EX-OR, NOR and NAND gates.
- **Q.5.** Differentiate between digital and analog circuits.

Lorentz Transformation

- Q.1: A light pulse is emitted at the origin of a frame of reference s' at time t'=0. Its distance x' from the origin after a time t' is given by $x'^2 = c^2t'^2$. Use the Lorentz Transformation to transform this equation to an equation in x and t show that this is $x^2 = c^2t^2$.

Lorentz Transformation

- Q.2: Prove that $x^2+y^2+z^2=c^2t^2$
- is invariant under Lorentz Transformation.

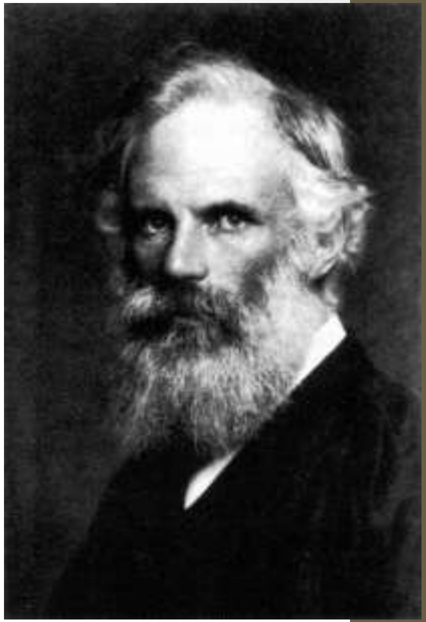
Lorentz-FitzGerald Contraction

- Another idea, proposed independently by Lorentz and FitzGerald, suggested that the length, *L*, in the direction of the motion **contracted** by a factor of:

$$\sqrt{1-\mathrm{v}^2/c^2}$$

velocity of frame (v) — **velocity of light** (c)

Hendrik A. Lorentz
(1853-1928)



George F. FitzGerald
(1851-1901)

So:

$$\Delta t_{\parallel} = \frac{2L}{c}\frac{1}{[1-\mathrm{v}^2/c^2]} \rightarrow \frac{2L\sqrt{1-\mathrm{v}^2/c^2}}{c}\frac{1}{[1-\mathrm{v}^2/c^2]} = \frac{2L}{c}\frac{1}{\sqrt{1-\mathrm{v}^2/c^2}} = \Delta t_{\perp}$$

thus making the path lengths equal and the phase shift always zero.

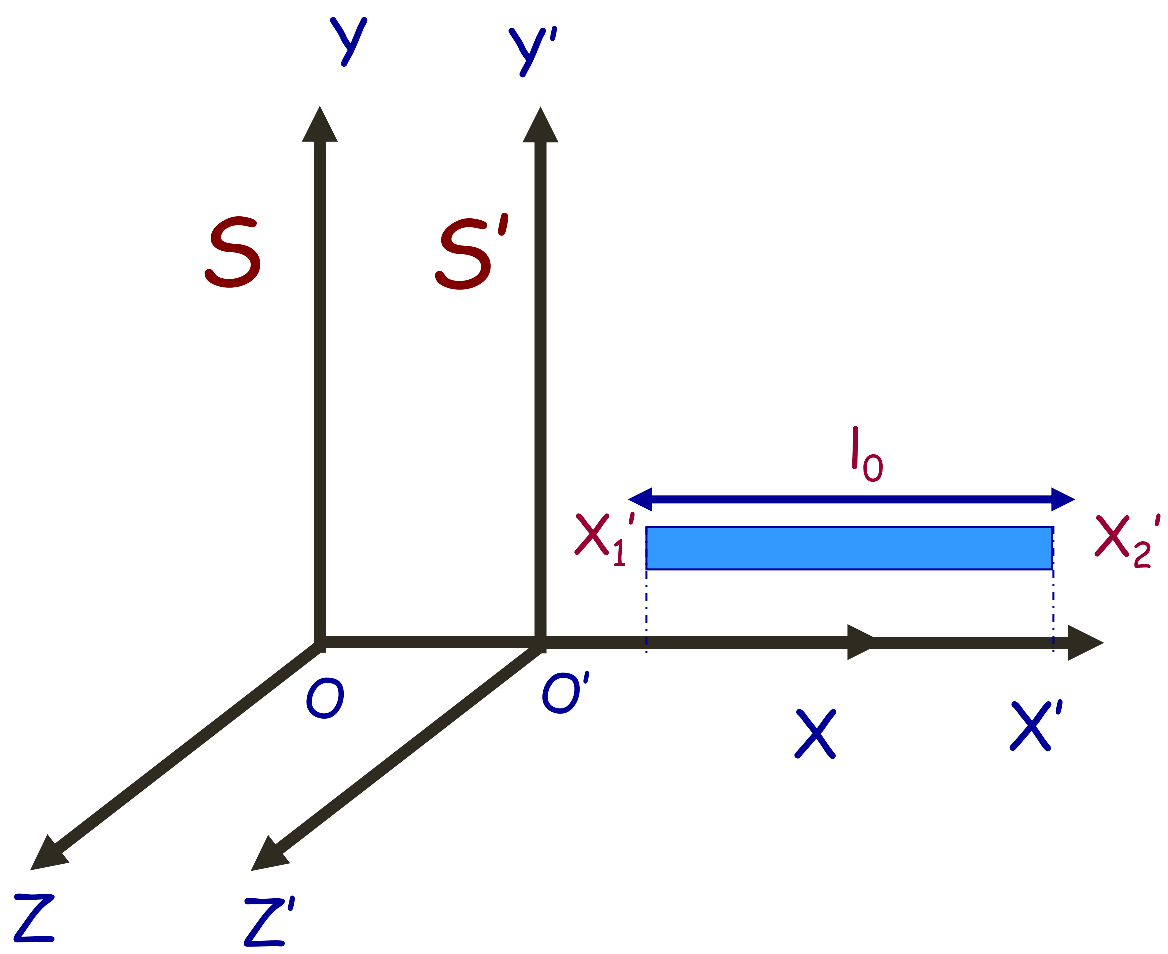
But there was no insight as to why such a contraction should occur.

Lorentz-FitzGerald Contraction

- Length of the rod moving with relativistic speed appears to be contracted in the direction of motion.

- The length in the direction perpendicular to the motion is unaffected.

Lorentz-FitzGerald Contraction



Lorentz-FitzGerald Contraction

- Frame S′ is moving with velocity v.
- Velocity of rod w.r.t. frame S is v.
- Length of the rod measured in frame S′ will be its " PROPER LENGTH ".
- Proper length $l_0 = x_2' - x_1'$.
- Length of the rod measured in frame S will be its " APPARENT LENGTH"
- Apparent length $l = x_2 - x_1$.

Lorentz-FitzGerald Contraction

From Lorentz Transformation

$$x' = \frac{(x - vt)}{\sqrt{(1-v^2/c^2)}}$$

$$x_2' - x_1' = \frac{(x_2 - x_1)}{\sqrt{(1-v^2/c^2)}}$$

$$l = \frac{l_0}{\sqrt{(1-v^2/c^2)}}$$

Lorentz-FitzGerald Contraction

Since $v \ll c$, $\sqrt{(1-v^2/c^2)} < 1$

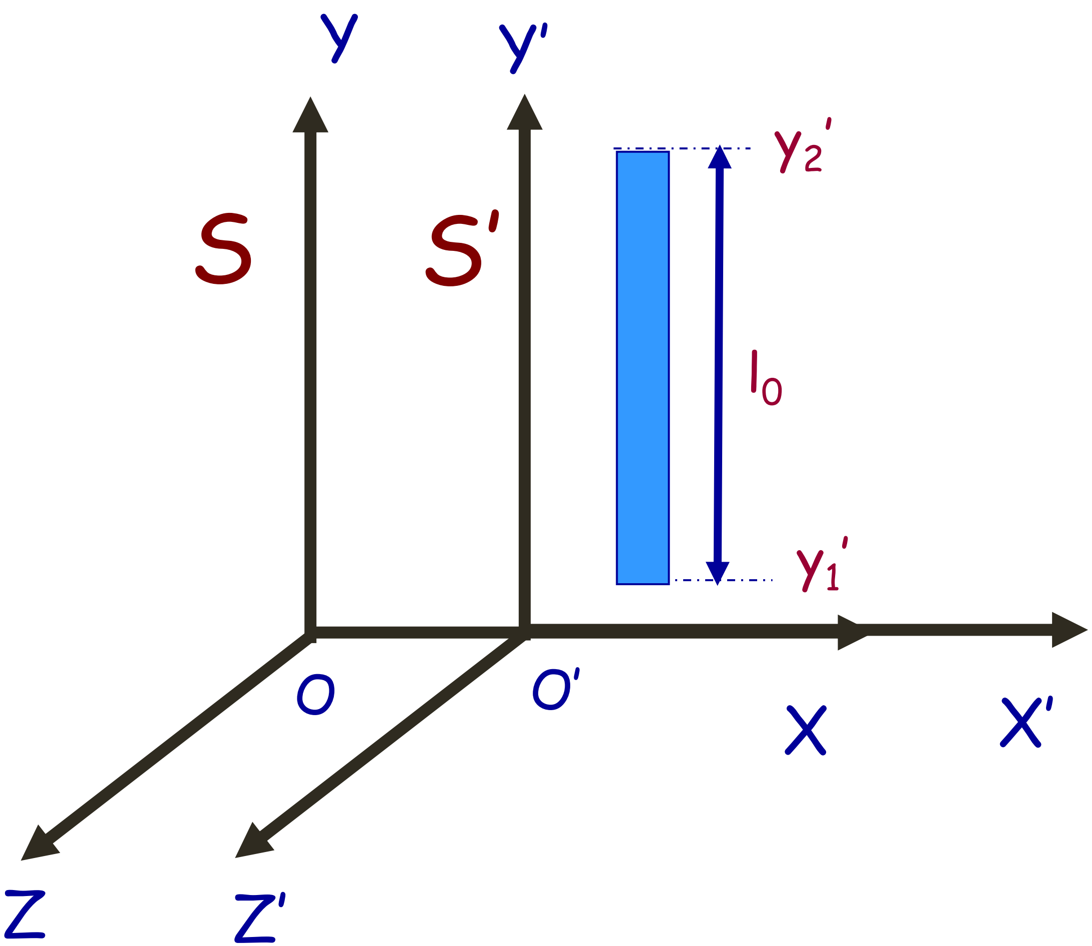
Therefore, $l < l_0$

The length is less than its proper length

"Length contraction"

Lorentz-FitzGerald Contraction

The length in the direction perpendicular to the motion is unaffected.



Lorentz-FitzGerald Contraction

$y_2' = y_2$ And $y_2 = y_1$

Therefore,

$$y_2' - y_1' = y_2 - y_1$$

$$l_0 = l$$

Lorentz-FitzGerald Contraction

Q. 1 A rod has 100cm. When the rod is in a satellite moving with a velocity that is one half of the velocity of light relative to the laboratory, what is the length of the rod as determine by an observer

(a) In the satellite.

(b) In the laboratory.

Lorentz-FitzGerald Contraction

Q. 2 A rocket ship is 100 m long on ground. When it is in flight, its length is 99m to an observer on the ground. What is its speed?

Lorentz-FitzGerald Contraction

Q.3 Calculate the percentage contraction in the length of a rod moving with a speed of .8c in the direction at an angle of 60^0 with its own length.

- In Newtonian physics, we previously assumed that $t' = t$.
  - With synchronized clocks, events in K and K' can be considered simultaneous.
- Einstein realized that each system must have its own observers with their own synchronized clocks and meter sticks.
  - Events considered simultaneous in K may not be in K'.
- Also, time may pass more slowly in some systems than in others.

Time Dilation

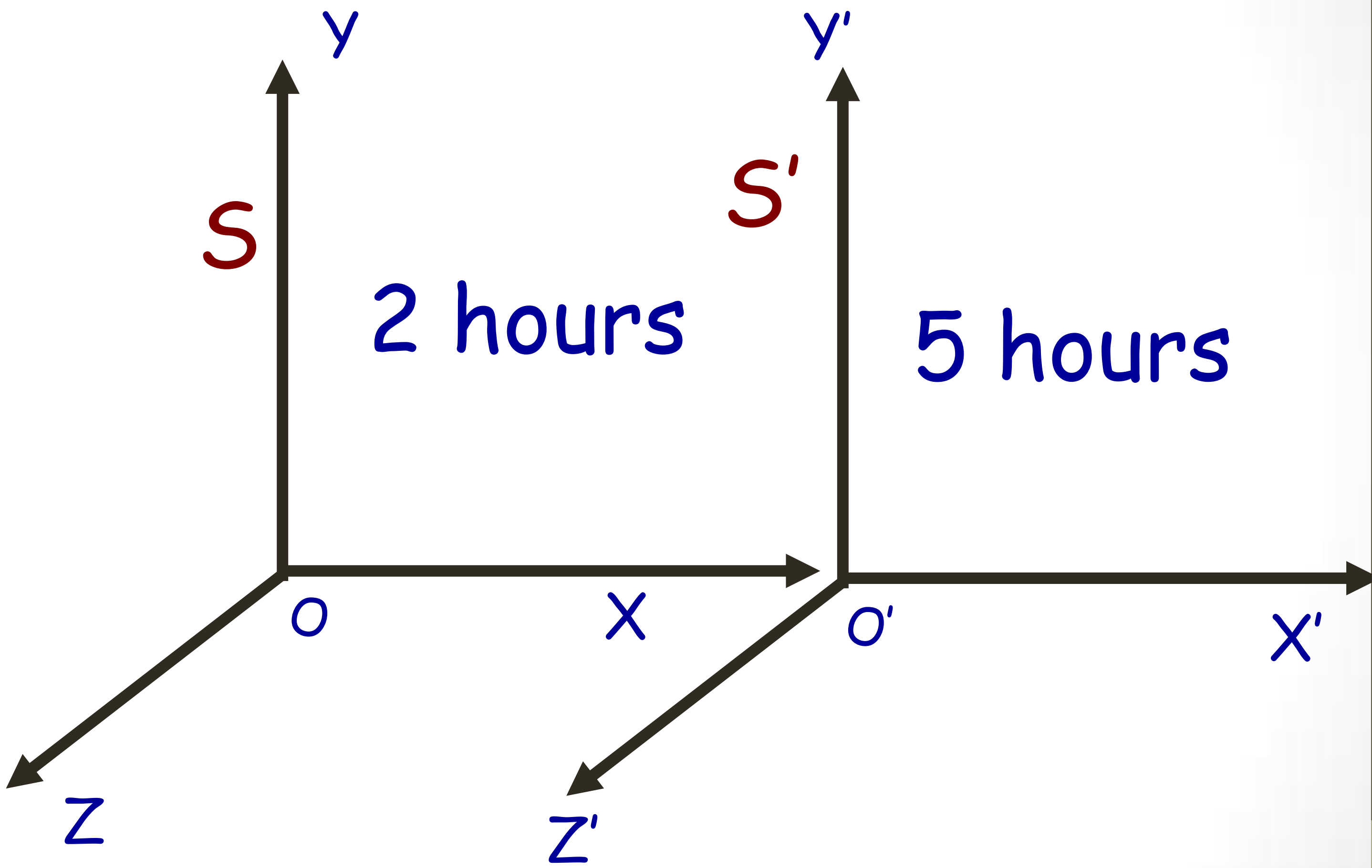
- The time interval of an event measured by a stationary observer is more than the interval of that event measured by a moving observer.
- Time Interval = ending time - starting time of the event .

Time Dilation

A clock moving with relativistic speed relative to a stationary observer appears to go slow

"Time Dilation"

Time Dilation



Time Dilation

- Frame S' is moving relative to frame S with velocity v.
- The event starts at time t_1' and ends at time t_2' with respect to frame S'.
- Time interval of the event measured in frame S' is proper time,
- $T_0 = \Delta t' = t_2' - t_1'$.
- Time interval of the event measured in frame S is Apparent time.
- $T = \Delta t = t_2 - t_1$.

Time Dilation

From Lorentz Transformation

$$t = \frac{(t' + x'v/c^2)}{\sqrt{(1-v^2/c^2)}}$$

$$t_2 - t_1 = \frac{(t_2' - t_1')}{\sqrt{(1-v^2/c^2)}}$$

$$T = \frac{T_0}{\sqrt{(1-v^2/c^2)}}$$ OR $$T_0 = T\sqrt{(1-v^2/c^2)}$$

Time Dilation

Since $v \ll c$, $\sqrt{(1-v^2/c^2)} < 1$

Therefore, $T_0 < T$ OR $T > T_0$

The Apparent time interval is more than the proper time interval.

"Time Dilation"

Time Dilation

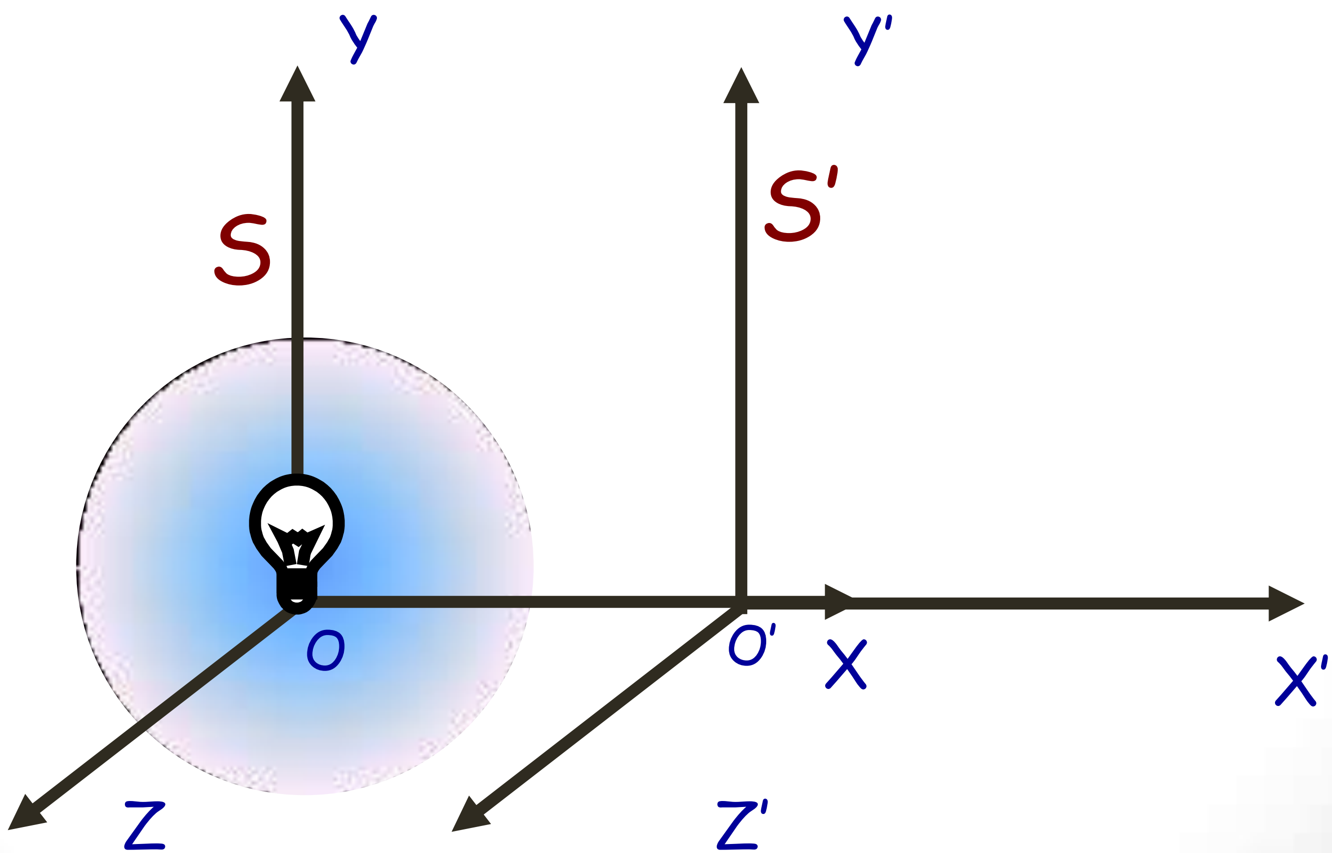
Q. 1 : A certain process requires 10^{-6} sec to occur in an atom at rest in laboratory. How much time will this process require to an observer in the laboratory when the atom is moving with a speed of $5 * 10^{7}$ m/sec.

Time Dilation

Q.2: What is the velocity of π- mesons whose observed mean life time is $2.5 * 10^{-7}$sec. The proper life of these π- mesons is $2.5 * 10^{-8}$sec.

Doppler's Effect

The apparent change in frequency of source(sound or light) due to relative motion between the source and observer



Doppler's Effect

- Frame S is at rest.
- Frame S′ is moving with velocity v along X-axis w.r.t. S.
- Source of proper frequency ν kept at origin O of frame S.
- Source emits two light signals at t=0 and t =T.
- First signal is received by observer in frame S′ at t′=0 at x′=0.
- Second signal received by him at time t′ at position x′.

Doppler's Effect

From Lorentz Transformation

$$x' = \frac{(x - vt)}{\sqrt{(1-v^2/c^2)}} = \frac{(-vT)}{\sqrt{(1-v^2/c^2)}}$$

$$t' = \frac{(t - xv/c^2)}{\sqrt{(1-v^2/c^2)}} = \frac{T}{\sqrt{(1-v^2/c^2)}}$$

Doppler's Effect

- Time taken by second signal to reach from x' to the origin

$$\Delta t' = x'/c = -vT/c\sqrt{(1-v^2/c^2)}$$

- Total time interval between the two signals received in frame S' at position x'= 0 is

$$T' = t' - \Delta t' = T \sqrt{1+(v/c)}/\sqrt{1-(v/c)}$$

Doppler's Effect

- In terms of frequency; v= 1/T

$$v' = v\sqrt{1+(v/c)}/\sqrt{1-(v/c)}$$

- In terms of wavelength; v=c/λ

$$c/\lambda' = c/\lambda\ \sqrt{1+(v/c)}/\sqrt{1-(v/c)}$$

$$\lambda'/\lambda = \sqrt{1+(v/c)}/\sqrt{1-(v/c)} = (1 + v/c)$$

Doppler's Effect

- Change in wavelength

$\Delta \lambda = \lambda' - \lambda = v \lambda / c$

Wavelength of light obtained from distant star appears to be increased to an observer on earth.

"Galaxy is expanding"

Relativistic mass

- According to Newton's mechanics,

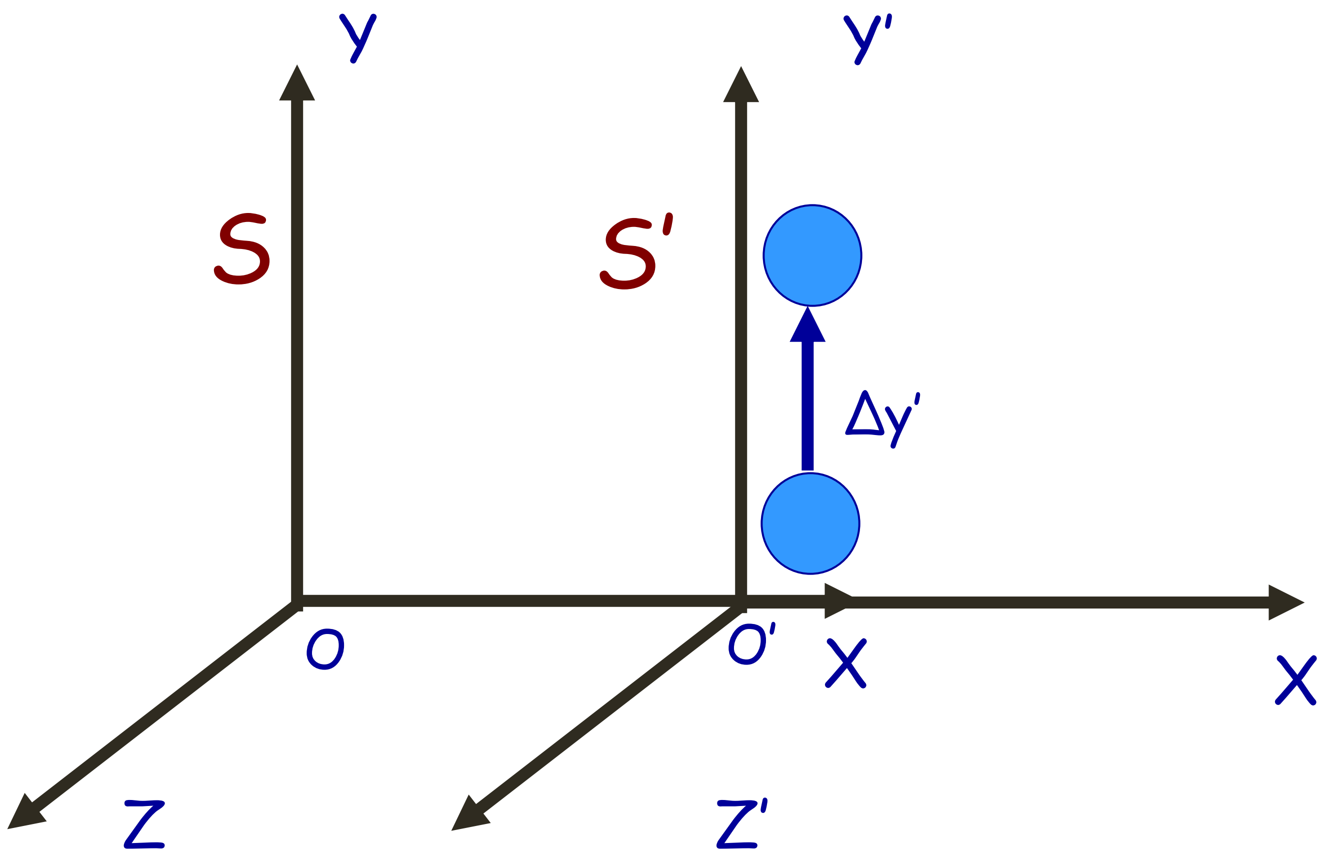
  "Mass is invariant quantity"

- For a stationary body of mass m, is acted upon by a finite and constant force F for time t and it acquires a velocity v, the gain i momentum, P = mv = Ft
- For m, F =constant , v α t
- By applying force infinite times, velocity of body can be increased to an infinite times.

Relativistic mass

- From theory of relativity $v_{max} = c$, Therefore , $P_{max} = mc$.
- Velocity remains α time as long as mass m remains constant.
- When $v \approx c$, then v is not α time but mass also increased with velocity.
- When $v = c$, then mass of the body becomes infinite.
- Momentum of the body is an invariant quantity.

Variation of mass with velocity



Variation of mass with velocity

- Frame of reference S is at rest.
- Frame S′ is moving with velocity v along +ve X-axis w.r.t. S.
- A particle of mass m_0 is also moving with frame S′.
- Moving mass of particle w.r.t. S is m.
- Consider the displacement of particle relative to frame S′ in time $\Delta t'$ is $\Delta y'$ along Y-axis.
- Velocity along Y-axis in frame S′ is $v_y' = \Delta y' / \Delta t'$.
- Momentum of particle w.r.t. S′ is $p_y' = m_0 v_y' = m_0 \Delta y' / \Delta t'$.

Variation of mass with velocity

- In frame S, if the same time interval measured to be Δt and the displacement of particle is Δy along Y-axis.
- Velocity of particle $v_y = \Delta y / \Delta t$.
- Momentum of particle $p_y = m \Delta y / \Delta t$.
- From Lorentz Transformation, $y = y'$ or $\Delta y = \Delta y'$

and

$$\Delta t = \frac{\Delta t'}{\sqrt{(1-v^2/c^2)}}$$

- Momentum $p_y = m \Delta y / \Delta t = \dfrac{m \Delta y' \sqrt{(1-v^2/c^2)}}{\Delta t'}$

Variation of mass with velocity

Momentum of the particle is invariant ;

$$P_y = p_y'$$

$$\frac{m\,\Delta y'\sqrt{(1-v^2/c^2)}}{\Delta t'} = m_0\,\Delta y' / \Delta t'$$

$$m = \frac{m_0}{\sqrt{(1-v^2/c^2)}}$$

Mass – Energy Equivalence

According to classical mechanics, mass and energy are separate entities which are conserved independently.

According to mass-energy equivalence, mass is one form of energy and mass and energy are inter-convertible.

It expressed through the relation;

$$E = mc^2$$

Relation between Energy & momentum

The total energy and momentum are related through the expression :

$$E = c\sqrt{p^2 + m_0c^2}$$

Numericals

Q. 1 :If the total energy of the particle is exactly thrice its rest energy, what is the velocity of the particle?

Mass – Energy Equivalence

Q.2 :Compute the mass m and velocity v of an electron having kinetic energy 1.5MeV. Given rest mass of electron $m_0 = 9.11 * 10^{-3}$ Kg, Velocity of light in vacuum $c = 3*10^8$ m/sec.

Mass – Energy Equivalence

Q.3 : An electron (rest mass 9.1 *10-31Kg is moving with speed 0.99c. What is its total energy ? Find the ratio of Newtonia kinetic energy to the relativistic energy.

Mass – Energy Equivalence

Q. 4 :Show that if the variation of mass with velocity is taken into account, the kinetic energy of a particle of rest mass m0 and moving with velocity v is given by

$$K = m_0c^2 \ [(1-v^2/c^2) - 1]$$

Where c is velocity of light.

- Solution:

Mass – Energy Equivalence

Q. 5 : (a) Show that for small velocities the relativistic kinetic energy of the body reduces to the classical kinetic energy which is less than the rest energy.

(b) If a particle could move with the velocity of light, how much kinetic energy it would possess?

Mass – Energy Equivalence

Q.6 : (a) Derive a relativistic expression of kinetic energy of a particle in terms of momentum and show that in the limit of small velocities the relativistic relation between kinetic energy and momentum tends to the classical relation.

(b) Obtain an expression for the velocity of a particle in terms of its relativistic momentum and energy.

UNIT TEST

UNIT-I

(THEORY OF RELATIVITY)

Date : 25/08/2006

Q.1. State fundamental postulates of special theory of relativity and deduce the Lorentz transformation.

OR

Q.2. Describe Michelson-Morley experiment and show how the negative results obtained from this experiment were interpreted.

Q.3. Show that the apparent length of rigid body in the direction of its motion with a uniform velocity v is reduced by factor $\sqrt{(1-v^2/c^2)}$. Discuss the result.

OR

Q.4. What is Time-Dilation in special theory? Deduce an expression for time dilation.

Q.5. Calculate the percentage contraction in the length of a rod moving with a speed of .8c in a direction at angle 600 with its own length.

OR

Q.6. If the total energy of a particle is exactly thrice its rest energy , what is the velocity of the particle?

- **Q.1:** A certain particle has a lifetime of 10^{-7} sec ,when measured at rest. How far does it go before decaying, if its speed is .99c when it is created.
- **Q.2:** Consider an electron which has been accelerated from rest through a potential difference from rest through a potential difference of 500kV. Find its
- (i) kinetic energy (ii) rest energy (iii) total energy (iv) speed.
- **Q.3:** A clock measures the proper time . With what velocity it should travel relative to an observer so that it appear to go slow by 30 sec. in a day.
- **Q.4:** Calculate speed and momentum of of an electron of kinetic energy 1.02MeV. Rest mass of electron is 9.1×10^{-31} kg.
- **Q.5:** An observer is moving with velocity 0.6c making an angle 300 with a rod of length 5m. Calculate the length and inclination of the rod with respect to the observer.